Study of the judder Characteristics of Friction Material for an Automobile Clutch and Test Verification

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Title page

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ORIGINAL ARTICLE

Study of the Judder Characteristics of Friction Material for an Automobile Clutch and Test Verification

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Abstract: The friction judder characteristics during clutch engagement have a significant influence on the NVH of a driveline. In this research, the judder characteristics of automobile clutch friction materials and experimental verification are studied. First, considering the stick-slip phenomenon in the clutch engagement process, a detailed six-degrees-of-freedom (DOF) model including the body, engine, clutch and friction lining, torsional damper, transmission and other driveline parts is established, and the calculation formula of friction torque in the clutch engagement process is determined. Second, the influence of the friction gradient characteristics on the amplification or attenuation of the automobile friction judder is analyzed, and the corresponding stability analysis and the numerical simulation of different friction gradient values are carried out with MATLAB/Simulink software. Finally, judder bench test equipment and a corresponding damping test program are developed, and the relationship between the friction coefficient gradient characteristics and the system damping is analyzed, to determine the evaluation basis of the test. The research results show that the friction lining with negative gradient characteristics of the friction coefficient will have a judder signal. When the friction gradient value is less than −0.005 s /m, the judder signal of the measured clutch cannot be completely attenuated, and the judder phenomenon occurs. When the friction gradient is greater than −0.005 s/m, the judder signal can be significantly suppressed and the system connection tends to be stable.

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Keywords: Clutch friction lining • judder • friction coefficient gradient • damping • bench test

1 Introduction

With the improvement of customers' requirements for automobile comfort, the vibration and noise problems of automobiles urgently need to be improved. As an important part of vehicle driveline, clutches are widely used in various types of automobiles. In the process of starting and shifting, the self-excited vibration [1] caused by the friction between the driving and driven discs of a clutch will cause the judder of the clutch, which destroys the stability of the clutch engagement, reduces the ride comfort, intensifies the wear on the clutch, and reduces the service life of the clutch. Therefore, it is necessary to study the phenomenon of friction judder of the clutch. Based on a 2-DOF dynamic model, Bostwick et al. [2] proposed a modeling and simulation method of clutch engagement judder process, and qualitatively analyzed the friction judder mechanism. Crowther et al. [3-4] developed a stick-slip algorithm to calculate the friction torque of the clutch and applied it to the powertrain with automatic transmission. The influence of applied pressure fluctuation of a clutch on the powertrain with or without automatic transmission was studied. Chen et al. [5] embedded the import damping model of a friction link into a 3-DOF dynamic model to study the judder mechanism from the perspective of friction import damping. References [6-8] established a 4-DOF model for a driveline,

analyzed the mechanism of clutch friction judder in detail, and gave the corresponding improvement measures. Yang [8] also added that the resistance torque excitation, which had the same effect on the friction judder as the pressure fluctuation. In addition, Yang [9] believed that the engine excitation had a significant effect on the clutch judder. When the crankshaft angular velocity corresponding to the engine harmonic order was equal to or close to the natural frequency of the driveline, the clutch judder increased significantly. Zhang Lijun [10] of Tongji University studied the relationship between modal coupling, the friction coefficient velocity slope and the stick-slip motion. According to the symbols of the friction coefficient and the velocity slope, the unstable region of modal coupling was formed into four parts, and the influence of the four parts on the stick-slip motion was analyzed. Shangguan Wenbin et al. [11-12] from the South China University of Technology studied the influence of the performance of the driven disc on vehicle judder and the car-shaking judder. It was found that increasing the static friction coefficient and reducing the moment of inertia of the driven disc and the second-stage torsional stiffness could reduce the joint judder and car-shaking judder of the vehicle during the starting process. Duan et al. [13] took a hydraulic torque converter as the research object, established a 3-DOF dynamic model, and analyzed the stick-slip characteristics during the connection of the hydraulic torque converter. Centea and Rahnejat et al. [14-16] established a torsional judder model with 7-DOF, calculated and analyzed the relationship between the friction coefficient and the relative velocity, and studied the influences of the relevant parameters of the driveline on the clutch judder. Based on the judder mechanism of a clutch, Gkinis and Paygude et al. [17-18] analyzed the influences of the temperature and the pressure on the friction lining from the point of view of the friction lining material, and they determined the judder performance of different friction linings. Gregor et al. [19-20] studied the performance of a friction lining with different resin contents in a judder test based on multivariate statistical analysis and the torque signals of an actual bench test, and they gave the evaluation criteria in a judder test. At the same time, a judder test bench was developed, that reduced the friction judder test time and accelerated the development of the friction lining.

Although previous scholars have conducted research on vehicle friction judder from different perspectives, most of them only focused on the theoretical analysis of friction judder, and there have been few studies involving theoretical analysis and related experimental verification. First, based on the research of previous scholars, this reasearch refines the mathematical model of clutch engagement judder, reduces the complexity of the modeling process, and solves the calculation formula of friction torque in a stick-slip condition. Second, the influence of the friction coefficient gradient on friction judder is analyzed and the corresponding stability and simulation analysis is carried out. Finally, a special judder test bench and the corresponding damping test program are developed. Through theoretical analysis, the relevant test standards are formulated, and the test evaluation basis is established. The influence of the friction coefficient gradient on the amplification or attenuation of the friction judder signal is verified.

2 Clutch Engagement Dynamics Analysis and Modeling

2.1 Clutch engagement dynamics model

Clutch engagement occurs in the starting and shifting process of a vehicle. It is a process in which the driving and driven parts begin to come into and produce relative sliding, and the speed of the driving and drivien parts reaches a consistent sliding stop. In this process, judder often occurs. In order to study this phenomenon in the process of clutch engagement, we establish a 6-DOF dynamic model including the body, engine, clutch and friction lining, torsional damper, transmission and other driveline parts based on the driveline structure shown in Figure 1. In the process of clutch engagement, there are two conditions of stick and slipping between the driving and driven parts of the clutch. When the driving part and the driven part of the clutch are in the slipping condition, the dynamic model is as shown in Fig. 2.

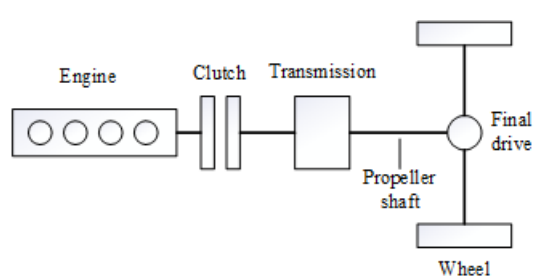


Figure 1 Structural diagram of driveline

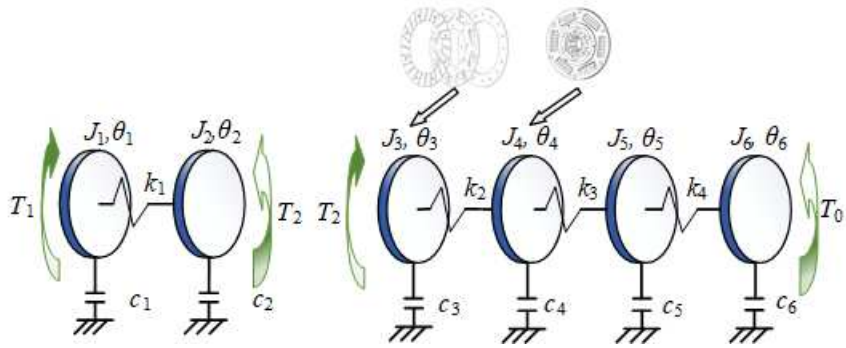


Figure 2 Dynamic model in slipping condition

In the model shown in Figure 2, J_1 is the equivalent moment of inertia of the engine part, J_2 is the equivalent moment of inertia of the flywheel and clutch driving part, J_3 is the equivalent moment of inertia of the friction lining and cushion spring, J_4 is the equivalent moment of inertia of the torsional damper, J_5 is the equivalent moment of inertia of the transmission, J_6 is the equivalent moment of inertia of the body and other driveline components, θ_1, θ_2, θ_3, θ_4, θ_5, θ_6 are the rotation angles of the corresponding components, T_1 is the engine excitation torque, and T_2 is the friction torque transmitted between the driving part and the friction lining. T_0 is the equivalent resistance moment from the road resistance moment to the torsional damper, c_1, c_2, c_3, c_4, c_5 and c_6 are the equivalent viscous damping coefficients of each part, k_1, k_2, k_3 and k_4 are the torsional stiffness between the engine and flywheel, the spring stiffness of the torsional damper, the equivalent torsional stiffness of the transmission input shaft and the equivalent torsional stiffness of the transmission output shaft, respectively.

Based on the dynamic model shown in Figure 2, Eq. (1) is derived from the Lagrange equation:

$$\begin{cases} J_1\ddot{\theta}_1 + c_1\dot{\theta}_1 + k_1(\theta_1 - \theta_2) = T_1 \\ J_2\ddot{\theta}_2 + c_2\dot{\theta}_2 - k_1(\theta_1 - \theta_2) = -T_2 \\ J_3\ddot{\theta}_3 + c_3\dot{\theta}_3 + k_2(\theta_3 - \theta_4) = T_2 \\ J_4\ddot{\theta}_4 + c_4\dot{\theta}_4 + k_3(\theta_4 - \theta_5) - k_2(\theta_3 - \theta_4) = 0 \\ J_5\ddot{\theta}_5 + c_5\dot{\theta}_5 + k_4(\theta_5 - \theta_6) - k_3(\theta_4 - \theta_5) = 0 \\ J_6\ddot{\theta}_6 + c_6\dot{\theta}_6 - k_4(\theta_5 - \theta_6) = -T_0 \end{cases} . \quad (1)$$

When the clutch driving and driven parts are in the sticking condition, the dynamic model is as shown in Figure 3.

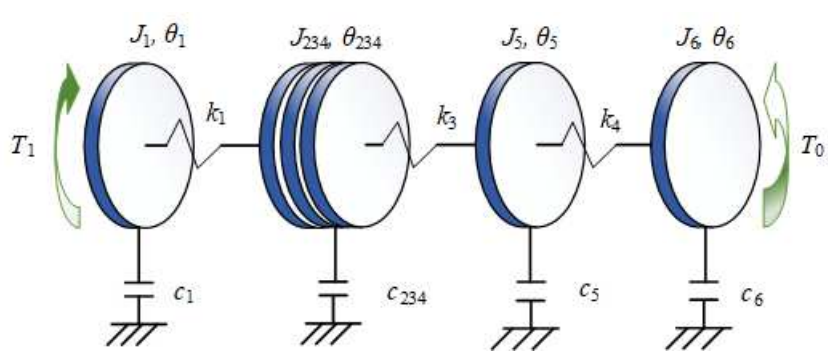


Figure 3 Dynamic model in sticking condition

In Figure 3, J_{234}, c_{123}, and θ_{234} are the equivalent rotational inertia, equivalent damping coefficient, and angular displacement of the clutch, respectively, and the remaining parameters are constant. The dynamics equations in the sticking condition are expressed as shown in Eq. (2):

$$\begin{cases} J_1\ddot{\theta}_1 + c_1\dot{\theta}_1 + k_1(\theta_1 - \theta_{234}) = T_1 \\ J_{234}\ddot{\theta}_{234} + c_{234}\dot{\theta}_{234} + k_3(\theta_{234} - \theta_5) - k_1(\theta_1 - \theta_{234}) = 0 \\ J_5\ddot{\theta}_5 + c_5\dot{\theta}_5 + k_4(\theta_5 - \theta_6) - k_3(\theta_{234} - \theta_5) = 0 \\ J_6\ddot{\theta}_6 + c_6\dot{\theta}_6 - k_4(\theta_5 - \theta_6) = -T_0 \end{cases} .(2)$$

Where $J_{234} = J_2 + J_3 + J_4$, $c_{234} = c_2 + c_3 + c_4$, $\theta_{234} = \theta_2 + \theta_3 + \theta_4$.

2.2 Friction torque of clutch under stick-slip condition

In the process of clutch engagement, there is friction torque between the driving and driven parts. There are sticking and sliding phenomena in the clutch engagement process, and the friction torque in the two conditions is not the same. When the clutch is slipping, the friction torque is expressed as shown in Eq. (3):

$$T_2 = n\mu R_m F \,, \tag{3}$$

where F is the pressing force acting on the surface of the friction lining, R_m is the equivalent friction radius of the clutch, μ is the slidng friction coefficient, and n is the number of friction surfaces. The calculation formula of the equivalent friction radius is expressed as shown in Eq. (4):

$$R_m = \frac{2(R_0^3 - R_i^3)}{3(R_0^2 - R_i^2)} \,, \tag{4}$$

where R_0 is the outer radius of the friction lining and R_i is the inner radius of the friction lining.

According to the classical Coulomb friction law [21-22], the magnitude of friction is independent of the velocity, but practice shows that the actual friction coefficient is not a constant value, but rather a function of the relative velocity [23]. Most studies define the two with nonlinear relationships. In order to facilitate analysis, the relationship between the friction coefficient and r the elative velocity of the clutch tested in this study is approximately linear. Ignoring the influence of pressure and temperature on the friction coefficient, there is a relationship between the friction coefficient and the relative sliding velocity of the clutch which is expressed as shown in Eq. (5):

$$\mu = \mu_0 + \mu^{'}(\dot{\theta}_2 - \dot{\theta}_3) \,, \tag{5}$$

where $\mu^{'}$ is the friction coefficient gradient, μ_0 is the static friction coefficient, and $\dot{\theta}_2 - \dot{\theta}_3$ is the velocity difference between the driving part and the friction lining. Substituting Eq. (5) into Eq. (3), the friction torque calculation Eq. (6) for the slipping condition is obtained:

$$T_2 = n\mu_0 R_m F + \mu^{'}(\dot{\theta}_2 - \dot{\theta}_3) n R_m F \tag{6}$$

When the clutch is in the sticking condition, the friction torque transmitted by the clutch is:

$$T_2 = \begin{cases} T_{full}, & \dot{\theta}_2 - \dot{\theta}_3 = 0, |T_{full}| \le T_{\max} \\ T_{\max} sign(T_{full}), & \dot{\theta}_2 - \dot{\theta}_3 = 0, |T_{full}| > T_{\max} \end{cases} . \tag{7}$$

In Eq. (7), T_{max} is the maximum static friction torque, T_{full} is the torque transmitted when the clutch is fully engaged, and the force analysis of the driving part and the friction lining can be carried out separately, as shown in Eqs. (8) and (9):

$$T_{full} = -J_2 \ddot{\theta}_2 - c_2 \dot{\theta}_2 + k_1(\theta_1 - \theta_2) \,. \tag{8}$$

(Analysis of the driving part)

$$T_{full} = J_3 \ddot{\theta}_3 + c_3 \dot{\theta}_3 + k_2(\theta_3 - \theta_4) \,. \tag{9}$$

(Analysis of the friction lining)

In summary, the friction torque of the clutch in the two conditions of sticking and slipping can be expressed as shown in Eq. (10):

$$T_2 = \begin{cases} n\mu_0 R_m F + \mu^{'}(\dot{\theta}_2 - \dot{\theta}_3) n R_m F & \dot{\theta}_2 - \dot{\theta}_3 \ne 0 \\ T_{full}, & \dot{\theta}_2 - \dot{\theta}_3 = 0, |T_{full}| \le T_{\max} \\ T_{\max} sign(T_{full}), & \dot{\theta}_2 - \dot{\theta}_3 = 0, |T_{full}| > T_{\max} \end{cases} . \tag{10}$$

3 Effect of Friction Coefficient Gradient on Judder Characteristics

Frictional judder is caused by the change of the friction coefficient relative to the sliding velocity, which may occur in any system that transmits force or torque through friction. The sensitivity of the system to friction judder is defined by the friction coefficient gradient, which is defined as the change of the friction coefficient μ relative to the sliding speed v or ω, the expression is shown in Eq. (11). According the existing research and analysis, the friction

coefficient gradient is considered to have three situations: positive gradient, zero gradient and negative gradient. The changes of the three friction coefficient gradients are shown in Fig. 4.

$$\mu' = \frac{d\mu}{d\Delta v}, \tag{11}$$

where Δv denotes the speed difference between the driving part and the driven part.

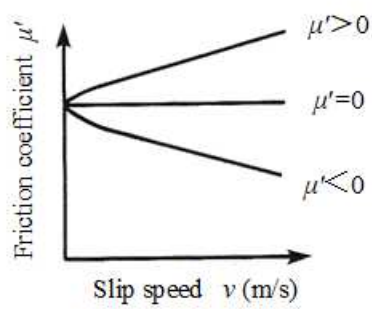


Figure 4 Defined as the gradient of the friction coefficient

According to the dynamic model of Figure 2, the dynamic equation can be written as:

$$T_2 = J_3\ddot{\theta}_3 + c_3\dot{\theta}_3 + k_2(\theta_3 - \theta_4). \tag{12}$$

Substituting Eq. (6) into Eq. (12):

$$n\mu_0 R_m F + \mu'(\dot{\theta}_2 - \dot{\theta}_3)nR_m F = J_3\ddot{\theta}_3 + c_3\dot{\theta}_3 + k_2(\theta_3 - \theta_4). \tag{13}$$

Finishing Eq. (13):

$$(\mu_0 + \mu'\dot{\theta}_2)nR_m F = J_3\ddot{\theta}_3 + (c_3 + \mu' nR_m F)\dot{\theta}_3 + k_2(\theta_3 - \theta_4). \tag{14}$$

The solution of the homogeneous equation corresponding to Eq. (14) is:

$$\theta_3 = e^{\alpha t}[\sqrt{C_1^2 + C_2^2}\cos(\beta t - \phi)]. \tag{15}$$

In Eq. (15):

$$\alpha = \frac{c_3 + \mu' nR_m F}{2J_3}, \tag{16}$$

$$\beta = \frac{\sqrt{(c_3 + \mu' nR_m F)^2 - 4J_3 k_2}}{2J_3}, \tag{17}$$

$$\phi = \arctan\frac{C_2}{C_1}. \tag{18}$$

In Eq. (18), C_1, C_2 are arbitrary constants, while C_1 is not 0.

The amplitude of the system is $e^{\alpha t}\sqrt{C_1^2 + C_2^2}$. Therefore, when $\mu' \geq 0$, $\alpha \leq 0$ and $e^{\alpha t} < 1$, the amplitude of the system attenuates, the judder disappears, and the system tends to be stable. When $\mu' < 0$, the friction coefficient decreases with the increasing sliding speed, at this time, $\alpha \geq 0$, $e^{\alpha t} > 1$, the amplitude of the system increases, the judder is strengthened, and the system is in an unstable state.

4 Gradient Stability Analysis of Clutch Friction Coefficient

For the dynamic model of the clutch engagement process established in Section 1.1, it can be seen from the analysis in Section 1.2 that only Eq. (1) needs to be analyzed in the calculation process. Eq. (1) is written in the following matrix form:

$$\boldsymbol{J}\ddot{\boldsymbol{\theta}} + \boldsymbol{C}\dot{\boldsymbol{\theta}} + \boldsymbol{K}\boldsymbol{\theta} = \boldsymbol{0}. \tag{19}$$

In Eq. (19),

$$\boldsymbol{J} = \begin{pmatrix} J_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_6 \end{pmatrix}, \tag{20}$$

$$\boldsymbol{C}=\begin{pmatrix} c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2+\mu' nR_m F & -\mu' nR_m F & 0 & 0 & 0 \\ 0 & -\mu' nR_m F & c_3+\mu' nR_m F & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6 \end{pmatrix}, \quad (21)$$

$$\boldsymbol{K}=\begin{pmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 \\ -k_1 & k1 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_2 & -k_2 & 0 & 0 \\ 0 & 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & 0 & 0 & -k_4 & k_4 \end{pmatrix}, \quad (22)$$

$$\boldsymbol{T}=\begin{pmatrix} T_1 \\ -\mu_0 nR_m F \\ \mu_0 nR_m F \\ 0 \\ 0 \\ -T_0 \end{pmatrix}. \quad (23)$$

In order to facilitate the numerical calculation, the dynamic Eq. (19) can be written as the initial value problem of differential equations :

$$\begin{cases} \boldsymbol{X}=\boldsymbol{A}\dot{\boldsymbol{X}}+\boldsymbol{B}\boldsymbol{U} \\ \boldsymbol{X}(0)=\boldsymbol{X}_0 \end{cases}. \quad (24)$$

In Eq. (24),

$\boldsymbol{X}=\begin{pmatrix}\theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_4 & \dot{\theta}_5 & \dot{\theta}_6\end{pmatrix}^{\mathrm{T}}$ is the state vector, and $\boldsymbol{A}=\begin{pmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{J}^{-1}\boldsymbol{K} & -\boldsymbol{J}^{-1}\boldsymbol{C} \end{pmatrix}$ is the Jacobi matrix of the system, $\boldsymbol{B}=\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{J}^{-1} \end{pmatrix}$, $\boldsymbol{U}=\boldsymbol{T}$.

Based on the Lyapunov stability theory, the eigenvalues of the Jacobian matrix A can be used as the basis for the stability determination for linear time-invariant systems. If the real parts of all of the eigenvalues of the Jacobian matrix A are less than 0, the system is stable, and its natural response will eventually approach equilibrium. If the maximum real part of all eigenvalues is equal to 0, the system is critically stable, but as long as there is an eigenvalue whose real part is greater than 0, the system will lose stability.

The moment of inertia, stiffness, damping and engine excitation parameters in the model are shown in Table 1. The actual vehicle is a damped system, but when analyzing the influence of the friction coefficient on the stability, the presupposed system is undamped; that is, c_1, c_2, c_3, c_4, c_5 and c_6 are all zero. The clutch static friction coefficient μ_0 is set to 0.3. In order to maintain the consistency of the theoretical analysis and the experiment, μ' is selected as the four parameter values of −0.01, −0.005, 0, 0.005, which are input into the state matrix A. The eigenvalue of each μ' is obtained. For the selection of the friction coefficient gradient value see Reference [1]. The results are shown in Table 2.

Table 1 Parameters of a vehicle

Variable	Implication	Value
$J_1(kg\cdot m^2)$	Moment of inertia of engine rotating part	0.2
$J_2(kg\cdot m^2)$	Inertia of flywheel and driving part of clutch	0.5
$J_3(kg\cdot m^2)$	Equivalent moment of inertia of friction lining and cushion spring	0.015
$J_4(kg\cdot m^2)$	Equivalent rotational inertia of clutch driven disc hub	0.04
$J_5(kg\cdot m^2)$	Equivalent rotational inertia of transmission	0.02
$J_6(kg\cdot m^2)$	Equivalent moment of inertia of body and other driveline components	4
$c_1(Nms\cdot rad^{-1})$	Part equivalent damping of engine	0
$c_2(Nms\cdot rad^{-1})$	Equivalent damping of flywheel and clutch driving part	0
$c_3(Nms\cdot rad^{-1})$	Friction lining damping	0
$c_4(Nms\cdot rad^{-1})$	Torsional damper damping	0
$c_5(Nms\cdot rad^{-1})$	Transmission equivalent damping	0
$c_6(Nms\cdot rad^{-1})$	Equivalent damping of body and other driveline components	0
$k_1(Nm\cdot rad^{-1})$	Equivalent torsional stiffness between engine and flywheel	20000
$k_2(Nm\cdot rad^{-1})$	Spring equivalent stiffness of torsional damper	2000
$k_3(Nm\cdot rad^{-1})$	Equivalent torsional stiffness of transmission input shaft	4000
$k_4(Nm\cdot rad^{-1})$	Equivalent torsional stiffness of transmission output shaft	6000
$T_1(N\cdot m)$	Engine excitation torque	300
$T_0(N\cdot m)$	Road resistance torque	100
$F(N)$	Clutch pressure	4000
$R_0(m)$	Outer radius of friction lining	0.2
$R_i(m)$	Inner radius of friction lining	0.134
μ_0	Static friction coefficient	0.3

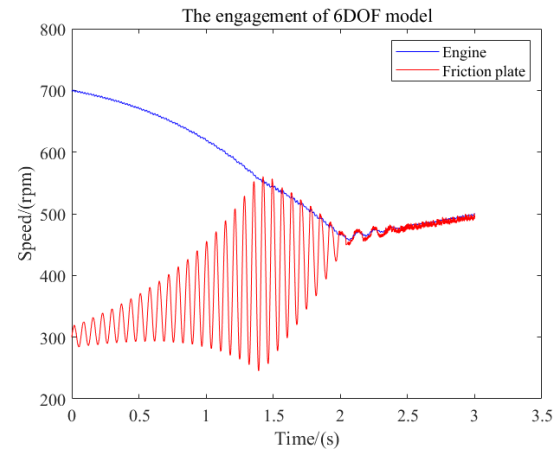
Table 2 Stability analysis results of different friction coefficient gradients

μ'	Real part	Imaginary part
-0.01	1.0755	0.0000i
	0.0002	±0.8476i
	0.0001	±0.3865i
	0.0210	±0.3187i
	0.0329	±0.0263i
	±0.0000	0.0000i
	-0.0000	0.0000i
-0.005	0.0021	±8.4772i
	0.0025	±3.8649i
	0.4665	±3.2391i
	2.4157	±0.5961i
	-0.3405	0.0000i
	0.1464	0.0000i
	-0.0000	±0.0000i
0	±0.0000	±3.8572i
	±0.0000	±8.4790i
	±0.0000	±4.2684i
	±0.0000	±2.0288i
	±0.0000	±0
	±0.0000	±0
0.01	-0.0012	±8.4772i
	-0.0025	±3.8649i
	-0.4665	±3.2391i
	-2.4157	±0.5961i
	-0.1464	0.0000i
	-0.1964	0.0000i
	0.0000	±0.0000i
	-0.0000	0.0000i

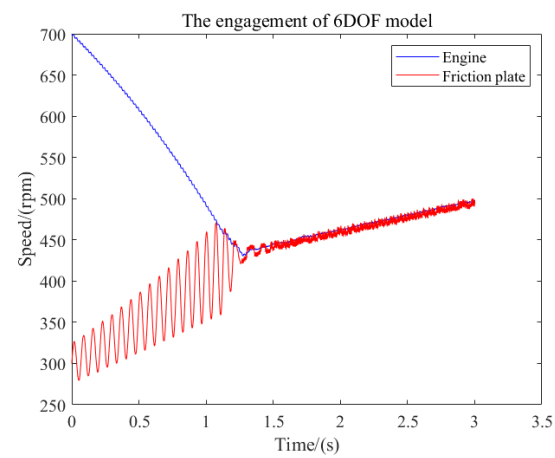
According to the calculation results, when the friction coefficient gradient is negative, the real part of the system eigenvalue appears to have a positive value, the system produces judder behavior, and the system loses stability. The smaller the friction coefficient gradient value is, the more unstable the system is. When the friction coefficient gradient is 0, the real part of the matrix eigenvalue is 0, and the system is in a critical stable state; when the friction coefficient is positive, the real part of the eigenvalues of the system matrix does not appear to have positive value, and the system is in a stable state.

5 Simulation Analysis of Clutch Friction Coefficient Gradient

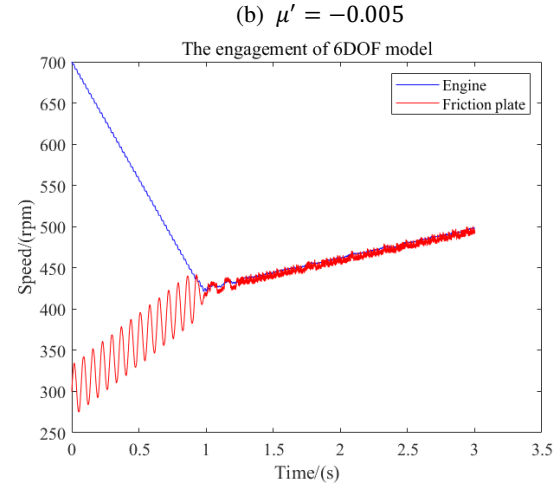
Based on MATLAB / Simulink software, the gradient values of the friction coefficient are selected as −0.01, −0.005, 0 and 0.005 for the simulation. The simulation results are shown in Fig. 5.



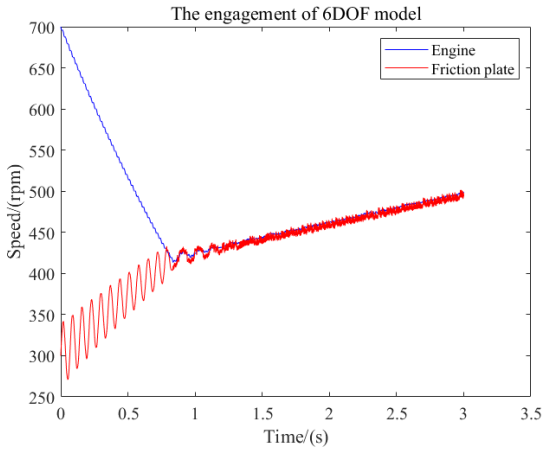
(a) $\mu' = -0.01$



(b) $\mu' = -0.005$



(c) $\mu' = 0$



(d) $\mu' = 0.005$

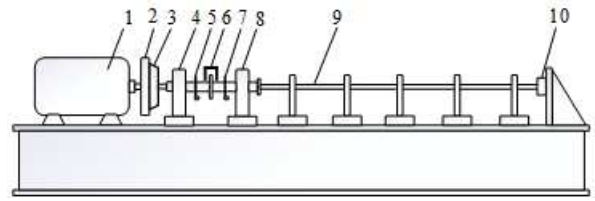
Figure 5 Simulation diagrams for different friction gradients

According to the simulation results, when the friction coefficient gradient is negative, the system has a strong judder phenomenon. Specifically, when the friction gradient is −0.01, the judder signal is significantly amplified. When the gradient value is nonnegative, the oscillation of the system is improved and the oscillation signal of the system is convergent. According to the comprehensive comparison, the theoretical analysis and simulation results are in good agreement.

6 Bench Test of Judder Characteristics of Friction Lining

6.1 Experimental equipment

The experimental equipment is shown in Figs. 6 and 7. The motor in the drive unit is used as the power source, and the torque is transmitted to the clutch at different speeds. The control cylinder achieves the separation and connection of the clutch by separating the fork. In the process of clutch connection, the torsional data are transmitted to the data collector by the speed sensor installed on the spline shaft holder. The driven disc is mounted on the spline shaft connected to the damping torsion bar, and the torque is transmitted to the torsion bar. One end of the torsion bar is fixed, so the torque transmitted makes the torsion bar torsional. Because the clutch is not fully connected, the clutch fails to completely transmit the torque to the torsion bar, so the torsion bar swings. With the full contact between the clutch disc and the pressure disc and the flywheel, the clutch friction lining does not judder, and the torsion bar stops at a specific angle and halts the torsion pendulum.



1 Driving unit, 2 Rotational speed, contact force, friction surface temperature sensor, 3 Clutch, 4 Bearing seat, 5 Torsional damper, 6 Additional damper, 7 Angle acceleration (receiver), 8 Bearing seat, 9 Torsion bar, 10 Torque measurement shaft

Figure 6 Judder test bench

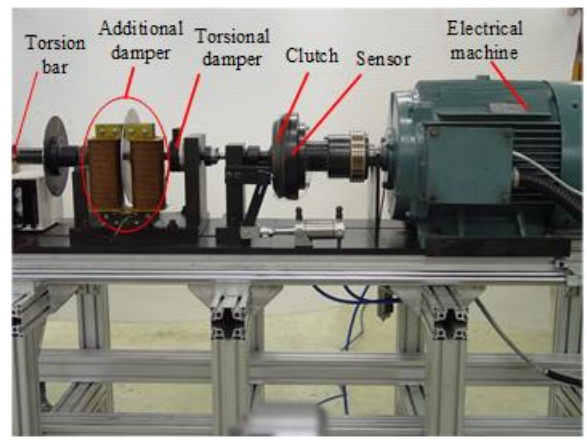


Figure 7 Actual diagram of judder test bench

6.2 Experimental method

The clutch judder test is performed using an ALLWAYS test bench. The motor power is 45 KW, test procedures refer to the internal standards of the 3102 test procedures. First, the weight of the whole cover assembly and the flywheel is determined, the clamping thickness of the driven disc is measured for the clamping load, and the weight of the driven disc is determined. In thesecond step, three cycles are tested (150 applications reach 15 kJ), followed by cooling at 800 rpm for two hours. The above steps are repeated and 37,40,80 cycles are performed (corresponding to 1850, 2000, 4000 applications) until 20,000 applications are performed. At the end of the test, additional judder tests are carried out at 200°C and 250°C. Finally, the clamping thickness of the driven disc is measured for the clamping load, and the weight of the driven disc is determined. The weight of the friction lining is also measured.

6.3 Test program

In order to obtain the damping value of the clutch friction lining at different times, the mechanism for obtaining the damping value of the friction lining is analyzed. Based on

the free vibration excitation of the single-DOF system [24], the friction lining is taken as the research object, and a theoretical analysis is applied to the longitudinal dynamics of the vehicle. The vibration time displacement curve is shown in Figure 8.

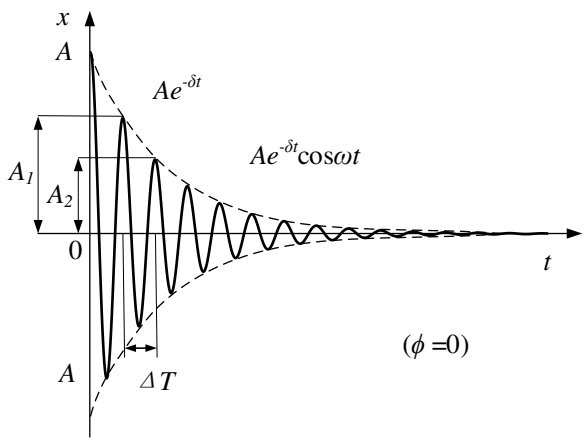


Figure 8 Damping vibration displacement time curve

The solution of Eq. (12) consists of the solution of the corresponding homogeneous equation and the solution of the non-homogeneous equation. Letting $2\sigma = \frac{c_3}{J_3}, \omega_0^2 = \frac{k}{J_3}$, and the corresponding homogeneous equation is:

$$\theta_3 + 2\sigma\dot{\theta}_3 + \omega_0^2\theta_3 = 0 . \tag{25}$$

Eq. (25) is solved to obtain Eq. (26):

$$\theta_3 = Ae^{-\sigma t}\cos\left(\sqrt{\omega_0^2 - \sigma^2}\,t + \phi\right), \tag{26}$$

where A denotes the maximum amplitude of the system, and the ratio of two adjacent amplitudes A_1 and A_2 is called the attenuation coefficient d:

$$d = \frac{A_1}{A_2} = \frac{Ae^{-\sigma t}}{Ae^{-\sigma(t+\Delta T)}} = e^{\sigma\Delta T} . \tag{27}$$

Taking the logarithms on both sides of Eq. (27) at the same time:

$$\ln\frac{A_1}{A_2} = \sigma\Delta T . \tag{28}$$

Converting Eq. (28) into a velocity expression:

$$\ln\frac{Speed_1}{Speed_2} = \sigma\Delta T . \tag{29}$$

From Eq. $2\sigma = \frac{c_3}{J_3}$ and Eq. (29):

$$c_3 = 2J_3\ln\frac{Speed_1}{Speed_2} \times \frac{1}{\Delta T} . \tag{30}$$

According to Eq. (30), based on the above analysis and principle derivation, a judder test program is developed to obtain the damping value of the friction lining at different times. The program calculates the damping value for the current condition according to the point we choose. The program interface is shown in Figure 9.

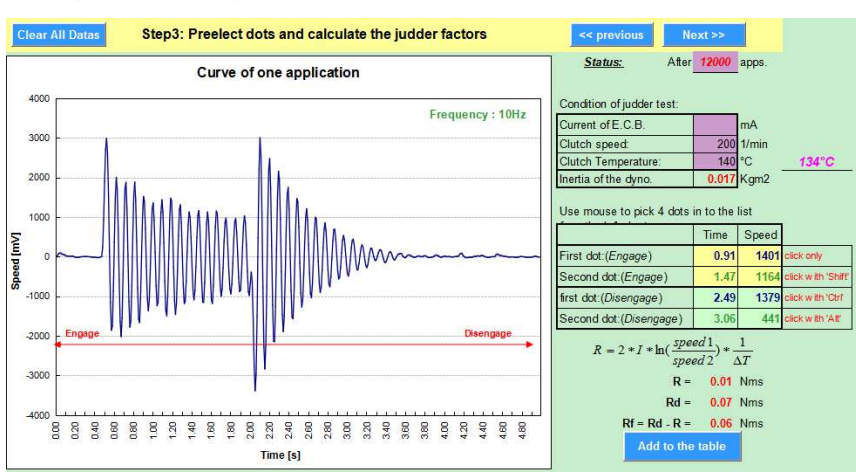


Figure 9 Test program interface

6.4 Evaluation basis

For vehicle friction judder, subjective evaluation and objective evaluation are generally adopted. Subjective evaluation is based on the feelings of drivers and passengers in the process of vehicle starting, while objective evaluation is based on the measured damping coefficient of the friction lining and the damping value set on the bench. Taking the friction lining as the research object, based on the viscous damping model, Eq. (31) is obtained:

$$T_2 = -c_3\dot{\theta}_3, \tag{31}$$

where simultaneous derivation of $\dot{\theta}_3$ is performed on both sides.

$$\frac{dT_2}{d\dot{\theta}_3} = -c_3 . \tag{32}$$

In the Coulomb friction zone, both sides of $\dot{\theta}_3$ are derived:

$$\frac{dT_2}{d\dot{\theta}_3} = \frac{d\left(n\mu R_m F\right)}{d\dot{\theta}_3} = nR_m F\frac{d\mu}{d\dot{\theta}_3} = nR_m F\frac{d\mu}{dv}\cdot\frac{dv}{d\dot{\theta}_3} = nR_m^2 F\frac{d\mu}{dv} = n\mu' R_m^2 F . \tag{33}$$

The simultaneous Eqs. (32) and (33) are solved:

$$c_3 + n\mu' R_m^2 F = 0 . \tag{34}$$

In this test example, the clutch friction lining of the Luk company with a diameter of 200 mm and a pressing force of 1500 N is used. Its number is 32411 R/3.5 (200 × 134 × 3.5 mm), and the number of contact surfaces of the friction lining is $n = 2$. The equivalent friction radius of the friction lining is 85.2 mm, and the critical friction coefficient gradient when the system judders is μ'=-0.005 [1]. The general automotive powertrain damping is considered to be 0.1 Nms [25], and the additional dampers are used to simulate the damping of the vehicle powertrain, anf attenuate the judder signal caused by the friction lining self-excited vibration. According to Eq. (34), the set value c_{add} of the additional damper in the bench device is as shown in Eq. (35):

$$c_{add} = -n\mu' R_m^2 F = 0.1\text{Nms} . \tag{35}$$

When the system damping is greater than 0.1 Nms due to the self-excited vibration of the friction lining, the damping value set by the additional damper cannot completely suppress the judder signal, and the system will produce judder behavior. When the system damping is less than 0.1 Nms due to self-excited vibration of the friction lining, the additional damper attenuates the judder signal and the system tends to be stable. The test results of the damping value caused by the self-excited vibration of the friction lining shown in Fig. 9 are 0.06 Nms, which is less than 0.1 Nms. The judder signal is attenuated by the additional damper, and the system tends to be stable, which has an inhibitory effect on the judder.

7 Conclusions

With the dynamic analysis of an automobile driveline, a 6 DOF dynamic model is established. Taking the friction lining as the research object, the influence of the friction coefficient gradient on the automobile friction judder is analyzed, and then the influence by the self-excited vibration of the friction lining on the automobile friction judder is analyzed. Based on the analysis of a test bench, the following conclusions can be drawn :

(1) There are two conditions of sticking and slipping in the clutch engagement process. By analyzing the dynamic models of the two conditions, the calculation model of the friction torque in the clutch engagement process is determined.

(2) When the friction characteristic of the friction lining in the clutch have a negative gradient, the system loses stability and judder occurs ; when the clutch friction characteristics have a zero gradient or a positive gradient, the system is in a stable state and the judder does not occur. Therefore, as far as possible, the friction gradient is chosen as a positive material to make the friction lining.

(3) Through theoretical and simulation analysis, it is determined that when the self-excited vibration of the clutch friction lining is too large and the system damping is not enough to attenuate the judder signal, the system loses stability. With the improvement of the friction gradient characteristics of the clutch friction material, the friction coefficient gradient value is greater than −0.005 s/m, which can effectively reduce the degree of system judder.

(4) Based on the existing test standards and the viscous damping model of the single-DOF system, a clutch friction lining judder test bench and the corresponding damping test program are designed and developed, and the experimental evaluation basis is established. Through the experimental analysis, it is concluded that the damping value set by the additional damper is 0.1 Nms. When the system damping caused by the self-excited vibration of the friction lining exceeds 0.1 Nms, the additional damper cannot completely suppress the judder signal, and the system produces judder behavior. When the system damping caused by the self-excited vibration of the friction lining does not exceed 0.1 Nms, the additional damper attenuates the judder signal and the system tends to be stable.

8 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author' contributions are as follows: Zheng-Feng Yan is responsible for the structure of the whole article; Hang-Sheng Li is responsible for writing manuscripts, dynamic modeling and simulation analysis, and stability analysis; Hai-Rui Lei is responsible for experimental equipment development and experimental data analysis; Mao-Qing Xie is the corresponding author and is responsible for the analysis of the effect of friction coefficient gradient on judder characteristics; Lei-Gang Wang is the co-corresponding author and responsible for the proofreading and review of the paper.

Competing interests

The authors declare no competing financial interests.

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Not applicable

Ethics approval and consent to participate

Not applicable

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Appendix

Appendix and supplement both mean material added at the end of a book. An appendix gives useful additional information, but even without it the rest of the book is complete: In the appendix are forty detailed charts. A supplement, bound in the book or published separately, is given for comparison, as an enhancement, to provide corrections, to present later information, and the like: A yearly supplement is issue.