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Calculations of the change of state of a gas by integrating the partial derivatives

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Abstract

It will not be denied that the calculations of the change of state for a gas is highly important in most engineering applications. For determining the gas's properties such as the pressure (P), the volume (V) and the temperature (T), engineers and scientists uses the Boyle's, Charles's and Gay-Lussac's (B-C-G) law of $P_1V_1/T_1=P_2V_2/T_2$. Although the B-C-G law provides the accurate property values of a gas, it give no detailed information embedded in the process when a gas changes its state. In this study, the author theoretically carried out the integrations of the partial differentials when differentiating the B-C-G law, which has not been tried by anyone up to now. The integration results of this study were thoroughly compared with the experimentally measured data and it was confirmed that the integration methods suggested in this study accurately provides the differential properties on ΔP , ΔV and ΔT . In addition to it, through the stepwise analysis of the integration of the partial differentials, it revealed that the efficiency in the change of state of a gas inherently exists higher than the Carnot cycle, which is operating between the same conditions. Therefore, the results of this study can be lead to the conclusion that all changes of state of all materials inevitably accompanies an energy loss and it is a natural phenomenon.

Keywords: Ideal gas law, Partial derivatives, Integration of partial derivatives, Efficiency of the change of state

Nomenclature

Alphabets

atm	: Atmospheric pressure (atm)
c	: Specific heat (kJ/kg·K)
m	: Mass (kg)
P	: Pressure (kPa)
Q	: Thermal Energy (kJ)
R	: Specific (individual) gas constant (kJ/kg·K)
T	: Temperature (K)
V	: Volume (m ³)
v	: Specific volume (m ³ /kg)
W	: Work (kJ)
Z	: Compressibility factor (--)

Greek letters

α	: Conversion factor (m ³)
β	: Conversion factor (kPa)
ρ	: Density (kg/m ³)

Subscripts

1	: State 1
2	: State 2
(A)	: An initial state to an intermediate state
(B)	: An intermediate state to an initial state
P	: At constant pressure
T	: At constant temperature
V	: At constant volume

Introduction

Before reading this article, it should be noticed that this study cited only a few research articles. The reason of it is resulted from the fact that the topic treated in this study is rarely

carried out up to now. In other words, it may be said that no one has tried to clarify the subject of this study.

This study is to suggest a new method for theoretically calculating the state variables of P , V and T , of a gas by integrating the partial differentials. As already well known, the change of P , V and T of a gas can only be expressed by using the partial differentials since those three variables are essentially interconnected [1-4]. As previously described, in spite of the author's long-standing trials, the author cannot find any other studies or thermodynamics textbooks on the integration of the partial differential for calculating the change of state for a gas such as ΔP , ΔV and ΔT . Although some recent studies tried to use the computational simulations for calculating the change of state for a gas [5-8], they were related with the flow of a gas. A few other studies [3, 4, 9-17] dealt with the partial derivatives induced from the multivariable functions for the thermodynamics properties, however those studies only described the partial derivatives and did not mentioned on the integration of them at all.

This study started from the authors' question why the pressure, volume and temperature of a gas, which are the most typical thermodynamic properties, have not been obtained through the integration of the partial derivatives derived from the ideal gas law. As mentioned in the above, since this study was purely originated from the authors' simple idea, the key references for this study are only two thermodynamics textbooks [1, 2] and one databook on air [18]. Nevertheless, only three key references cannot undermine the originality and novelty of this study.

Nearly, all processes of plants or engines even including electronics devices are related with a fluid flow. The fluids included in the processes can be liquids, gases or vapors. Unlike liquids, if the fluid involved in a process is a gas or a vapor, it is highly important to calculate the changes of their state in the stage of a process design.

So far, the most theoretical way to calculate the changes of state of a gas is to use the ideal gas law of Eq. (1).

$$PV = mRT \quad \text{Eq. (1)}$$

where P is the pressure of a gas system, R is the specific (individual) gas constant, m is the mass of a gas.

By contrast, the changes of state for a vapor are generally calculated by using a saturated vapor table, and the most familiar example of it would be the steam table. However, in the case of vapors, the phase change such as condensation and evaporation should be considered [19-21], so this study will address only the change of state of a gas.

When using the ideal gas law to calculate the change of state of a gas, it should be considered what process a gas changes through [1, 2, 22]. Due to the process paths, all thermodynamics textbooks mention on isochoric, isobaric, isothermal and reversible adiabatic processes.

As already well known, the state of a gas is fully specified by three variables of pressure, temperature and volume (P , V , T), in other words, the state functions or the thermodynamic properties. However, three variables of P , V and T are interrelated to one another, the change of any variable in the three variables causes the simultaneous changes of two other variables [1, 2, 22]. Therefore, if we want to know one variable's change in P , V and T , that variable is expressed as the function of two other variables. That is,

$$P = P(T, V), \quad \text{Eq. (2)}$$

$$T = T(P, V), \quad \text{Eq. (3)}$$

$$V = V(P, T). \quad \text{Eq. (4)}$$

Since P , V and T are mutually interconnected variables as seen in Eq. (2) to (4), the rate of change of these properties are expressed as the partial derivatives when a gas changes its state. According to the mathematical notation, the rate of changes of P , V and T , which are called total differential, are

$$dP = mR \left\{ \left. \frac{\partial P}{\partial T} \right|_V dT + \left. \frac{\partial P}{\partial V} \right|_T dV \right\}, \quad \text{Eq. (5)}$$

$$dV = mR \left\{ \left. \frac{\partial V}{\partial T} \right|_P dT + \left. \frac{\partial V}{\partial P} \right|_T dP \right\}, \quad \text{Eq. (6)}$$

$$dT = \frac{1}{mR} \left\{ \left. \frac{\partial T}{\partial P} \right|_V dP + \left. \frac{\partial T}{\partial V} \right|_P dV \right\}. \quad \text{Eq. (7)}$$

The partial differentials included in Eqs. (5) to (7) are

$$\left. \frac{\partial P}{\partial T} \right|_V = \frac{1}{V} \quad \text{and} \quad \left. \frac{\partial P}{\partial V} \right|_T = -\frac{T}{V^2}, \quad \text{Eq. (8)}$$

$$\left. \frac{\partial V}{\partial T} \right|_P = \frac{1}{P} \quad \text{and} \quad \left. \frac{\partial V}{\partial P} \right|_T = -\frac{T}{P^2}, \quad \text{Eq. (9)}$$

$$\left. \frac{\partial T}{\partial P} \right|_V = V \quad \text{and} \quad \left. \frac{\partial T}{\partial V} \right|_P = P. \quad \text{Eq. (10)}$$

Substituting Eqs. (8) to (10) into Eqs. (5) to (7),

$$dP = mR \left\{ \frac{1}{V} dT - \frac{T}{V^2} dV \right\}, \quad \text{Eq. (11)}$$

$$dV = mR \left\{ \frac{1}{P} dT - \frac{T}{P^2} dP \right\}, \quad \text{Eq. (12)}$$

$$dT = \frac{1}{mR} \{ V dP + P dV \}. \quad \text{Eq. (13)}$$

Since Eqs. (11) to (13) are the total differentials, the integration should be performed to obtain the amount of change of P , V and T . However, as previously described, the author has not been able to find any references and any studies on the integration of Eqs. (11) to (13) and their applications.

In this study, the author fully describes the results of the integration of Eqs. (11) to (13) and shows that the change of state of a gas cannot occur without the energy loss. That is, the energy loss is natural for the change of state. In addition to it, comparing the integration results with the experimentally measured air properties, the validity of this study will be demonstrated.

Integration of the partial differentials

Since each variable of P , V and T of a gas is the function depending on the other two variables, it should be taken into account along what path did the change of state take place. Consider the change of state of a gas from the state 1 to the state 2 as shown in Fig. 1 in which P is the dependent variable.

Fig. 1 Pressure change of a gas. The path of C is from the state 1 to the state 2 through C1 and C2, and the path of B is from the state of 1 to the state of 2 through B1 and B2.

As shown in Fig. 1, the gas initially in the state 1 of P_1 , V_1 and T_1 is changed to the state 2 of P_2 , V_2 and T_2 through the path B. At present, since the pressure of a gas is of interest, the pressure change will be determined by the variation of T and V .

In Fig. 1, the path A shows the transition from the state 1 to the state 2 by the simultaneous changes of T and V . On the other hand, the path B shows the process that V changes after T was changed while the path C shows the process that T changes after V was changed.

Since the variables of P , V and T are the state functions, the changes of P , V and T between the state 1 and the state 2 will be the same regardless of what path a gas follows. For calculating the pressure change of ΔP , we know that Eq. (11) should be integrated from the state 1 to the state 2.

$$\int_1^2 dP = mR \int_1^2 \left\{ \frac{1}{V} dT - \frac{T}{V^2} dV \right\},$$

$$\Delta P = P_2 - P_1 = mR \int_1^2 \frac{1}{V} dT - mR \int_1^2 \frac{T}{V^2} dV. \quad \text{Eq. (14)}$$

(A)
(B)

In the integration of the right-hand side of Eq. (14), V included in the term of (A) is a constant because the integral variable in the term of (A) is T , which means that the volume is not changed in the path B1. Likewise, T included in the term of (B) is also a constant when the integration is performed with respect to V . However, when integrating the term of (A) in Eq. (14), it should be determined whether V is V_1 or V_2 . Referring Fig. 1, since the term of (A) is the integration of a state function through the path B1 under the condition of a constant volume, $V=V_1$.

After the gas system changes its state along the path B1 as shown in Fig. 1, its state will be P' , V' and T' . However, since the gas system finally reaches P_2 , V_2 and T_2 with the constant temperature of T' , $T'=T_2$. T included in the integration of (B) in Eq. (14) is the constant temperature of T_2 . Therefore, Eq. (14) is

$$\Delta P = \frac{mR}{V_1} \int_1^2 dT - mRT_2 \int_1^2 \frac{1}{V^2} dV.$$

Since the integration of V^{-2} with respect to V is V^{-1} , the differential pressure of ΔP is

$$\Delta P = \frac{mR}{1V_4} (T_2 - T_1) + mRT_2 \left(\frac{1}{2V_4} - \frac{1}{4V_3} \right). \quad \text{Eq. (15)}$$

(A)
(B)

If substituting $PV=mRT$ into Eq. (15), it can be seen that both sides of Eq. (15) are equal to each other, that is $P_2 - P_1 = P_2 - P_1$. If comparing Boyle's, Charles's and Gay-Lussac's (B-C-G) law of $P_1V_1/T_1 = P_2V_2/T_2$ with Eq. (15), it remains unchanged that the values of two other variables should be known to calculate the value of one variable in the state 2 while the values of three variables in the state 1 are given. Certainly, B-C-G law simply allows to calculate the value of one variable if the other five variables in the six variables are known, but it provide no apparent information about the change of state.

However, it is necessary to consider the physical meanings of Eq. (15). Referring to Fig. 2, the path of B1 is the change of state under the condition of a constant volume. In order for the pressure of a gas system to be changed keeping its volume constant, a gas system should obtain a thermal energy from its surroundings, or a gas system should release a thermal energy to its surroundings. Therefore, the term of (A) in Eq. (15) is the pressure change by caused the thermal energy during the isochoric process.

When a gas system follows the path B2, the process is the change of state under the condition of a constant temperature. In order for the volume of a gas system to be changed keeping its temperature constant, a work should be supplied from the surroundings to a gas system, or a gas system supply a work to its surroundings. Therefore, the term of (B) in Eq. (15) is the pressure change during the isothermal process.

Fig. 2 Physical meaning of mRT/V in the path B.

The same concepts for calculating ΔP by integrating Eq. (11) can be applied to Eq. (12) and Eq. (13) for finding ΔV and ΔT . Table 1 shows ΔP , ΔV and ΔT calculated from the integrations of Eqs. (11) to (13).

Table 1 ΔP , ΔV and ΔT obtained through the integration of the partial differentials.

Integrations	Results	Eq. No.
$dP = mR \left\{ \left. \frac{\partial P}{\partial T} \right _V dT + \left. \frac{\partial P}{\partial V} \right _T dV \right\},$ $\int_1^2 dP = mR \int_1^2 \left\{ \frac{1}{V} dT - \frac{T}{V^2} dV \right\}$	$\Delta P = \frac{mR(T_2 - T_1)}{1} + mRT_2 \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$ <p>(A) Isochoric (B) Isothermal</p>	Eq. (15)
$dV = mR \left\{ \left. \frac{\partial V}{\partial T} \right _P dT + \left. \frac{\partial V}{\partial P} \right _T dP \right\},$ $\int_1^2 dV = mR \int_1^2 \left\{ \frac{1}{P} dT - \frac{T}{P^2} dP \right\}$	$\Delta V = \frac{mR(T_2 - T_1)}{1} + mRT_2 \left(\frac{1}{P_2} - \frac{1}{P_1} \right)$ <p>(A) Isobaric (B) Isothermal</p>	Eq. (16)
$dT = \frac{1}{mR} \left\{ \left. \frac{\partial T}{\partial P} \right _V dP + \left. \frac{\partial T}{\partial V} \right _P dV \right\},$ $\int_1^2 dT = \frac{1}{mR} \int_1^2 \{V dP + P dV\}$	$\Delta T = \frac{V_1(P_2 - P_1)}{1} + \frac{P_2(V_2 - V_1)}{1}$ <p>(A) Isochoric (B) Isobaric</p>	Eq. (17)

Verification of the integrations of the partial differentials

In this section, the results obtained by integrating the partial differentials, which are listed in Table 1, are verified by comparing them to the real measured data. Table 2 is the air properties cited from Arnold Engineering Development Center of USAF [18], in which the unit of pressure was converted to SI unit and the density and the specific volume were added into the original data.

Table 2 Air properties variations due to the changes of a temperature and a pressure^[1].

Temperature (K)	P ^[2] (kPa)	ρ/ρ_o ^[3] (--)	ρ (kg/m ³)	v (m ³ /kg)	Z ^[4] (--)
280	101.325	0.97546	1.26131	0.79283	1.00010
	202.650	1.95190	2.52388	0.39621	0.99955
	303.975	2.92910	3.78744	0.26403	0.99912
340	101.325	0.80292	1.03821	0.96320	1.00060
	202.650	1.60600	2.07662	0.48155	1.00050
	303.975	2.40900	3.11493	0.32103	1.00050
380	101.325	0.71829	0.92878	1.07668	1.00070
	202.650	1.43640	1.85732	0.53841	1.00080
	303.975	2.15440	2.78573	0.35897	1.00090

Note [1]: Table 2 is only a fraction of the ref. [18], and the gray colored conditions were calculated for the verification of Eqs. (15) to (17).

Note [2]: In the original data, the unit of pressure is atm.

Note [3]: In the original data, the air densities are specified as ρ/ρ_o , in which $\rho_o=1.29304$ kg/m³ is the air density at the standard state for a gas (0 °C and 1 atm).

Note [4]: Z is the compressibility factor. If Z=1.0, the behavior of a gas system perfectly follows the ideal gas law.

For the verification of Eqs. (15) to (17), the gray colored conditions in Table 1 were selected. The changes of states of an air system as shown in Fig. 3. First, the differential quantities of states were calculated after the air system changed from the state 1 to the state 2 (Path B of Fig. 3) and their values were compared to the values evaluated from Table 2. Second, it was checked that the calculated differential quantities of states had opposite sign when the direction of its process was reversed (Path B- of Fig. 3).

Fig. 3 Examples of the change of state of the air system with the mass of 1 kg.

For the simple calculation, the 1 kg of air was considered. Then, the ideal gas law of Eq. (1) will be

$$Pv = RT \quad \text{Eq. (18)}$$

where v is the specific volume of a gas. Therefore, when considering the gas system with the mass of 1 kg, the volume V should be changed to v in Eqs. (15) to (17).

Table 3 shows the comparisons the differential quantities of states after the air system changed its state, which were calculated by using Excel sheets. From Table 3, it can be confirmed that the differential values from Table 2 and the calculation results obtained by using Eqs. (15) to (17) have no difference. The detailed calculation examples using numbers is attached in S1 and S2 of the supplement in this study.

Table 3 Comparisons of the differential quantities of states between the values evaluated from the experimental properties and the calculated values of Eqs. (15) to (17).

	State 1	State 2	Path ^[1]	Differential properties from Table 2			Differential properties obtained from Eqs. (15), (16) and (17)		
				ΔP	$\Delta V^{[2]}$	ΔT	ΔP	$\Delta V^{[2]}$	ΔT
P	101.325	202.650	B	101.3	-0.311	60.0	101.3	-0.312	60.1
V	0.79283	0.48155	B-	-101.3	0.311	-60.0	-101.3	0.312	-60.1

T	280.0	340.0							
P	202.650	303.975	B	101.3	-0.123	40.0	101.2	-0.123	40.2
V	0.48155	0.35897							
T	340.0	380.0	B-	-101.3	0.123	-40.0	-101.2	0.123	-40.2
P	101.325	303.975	B	202.7	-0.434	100.0	202.5	-0.434	100.3
V	0.79283	0.35897							
T	280.0	380.0	B-	-202.7	0.434	-100.0	-202.5	0.434	-100.3

Note [1]: The path of B is the process from the state 1 to the state 2, and the path of B- is from the state 2 to the state 1, that is the reverse process of B.

Note [2]: Since all calculations were performed to the 1 kg of air, the volume of V is actually the specific volume of v .

New findings from the integration of the partial differentials.

In the section of 3, it was confirmed that the changes of states from the integration of Eqs. (15) to (17) is consistent with the differential quantities of states obtained from the experimentally measured data. In this section, the author mentions some new findings obtained by integrating the partial differentials of Eqs. (11) to (13).

Energies and efficiency in the pressure change of a gas, ΔP

In order for a gas system to change its state, it is necessary to exchange the energies with its surroundings and the involved energies are heat and work.

Since the pressure of a gas system is identical to mRT/V as seen from Eq. (1), Eq. (15) can be modified as

$$\Delta P = \left(\frac{mRT_2}{V_2} - P_1 \right) + \left(P_2 - \frac{mRT_2}{V_1} \right). \quad \text{Eq. (19)}$$

(A) (B)

As seen in Fig. 3 and Table 1, the term of (A) in Eq. (19) is the pressure change at the intermediate state (P' , V_1 , T_2) under the isochoric condition when a gas system goes from the state 1 (P_1 , V_1 , T_1) to the state 2 (P_2 , V_2 , T_2). Based on 1 kg of air,

$$\Delta P_{(A)} = P' - P_1 = \frac{mR(T_2 - T_1)}{v_1} - P_1,$$

$$P' = \frac{mR(T_2 - T_1)}{v_1}. \quad \text{Eq. (20)}$$

In order for the pressure of a gas to be changed under an isochoric condition, a heat energy should be supplied to a gas or released from it. It means that Eq. (20) must be related with a heat energy. For the isochoric process in the thermodynamics textbooks [1,2],

$$\begin{aligned} dQ &= mc_V dT, \\ \int_1^2 dQ &= mc_V \int_1^2 dT, \\ \Delta Q &= mc_V (T_2 - T_1). \end{aligned} \quad \text{Eq. (21)}$$

Certainly, Eq. (20) is caused by Eq. (21), however their units are different because the former has the unit of pressure and the latter has the unit of heat energy. Therefore, a new constant should be introduced in Eq. (21) to match the units of Eqs. (20) and (21).

$$\alpha_V = \frac{c_V v_1}{R}, \quad \text{Eq. (22)}$$

which is the coefficient to convert the pressure of a gas system into its energy and has the unit of m^3 . In Eq. (22), it should be noted that the specific heat of c_V is assumed a constant. The detailed derivation and its calculation of Eq. (22) were described in S3 of the supplement.

As seen in S3 of the supplement, if multiplying the conversion factor of Eq. (22), α_V , the heat energy for a gas to change its state under the constant volume condition is obtained, and the specific volume contained in α_V indicates the initial state of a gas.

The term of (B) in Eqs. (15) and (19) stands for the amount of the pressure change with the volume in the isothermal condition as shown in Fig. 3 and Eq. (5). Referring to some thermodynamics textbooks [1,2], the work required for the volume change of gas is

$$\begin{aligned} dW &= PdV = PV \frac{1}{V} dV, \\ \int_1^2 dW &= PV \int_1^2 \frac{1}{V} dV, \\ \Delta W &= mRT_1 \ln \left(\frac{V_2}{V_1} \right). \end{aligned} \quad \text{Eq. (23)}$$

Since Eq. (23) is the necessary work that a gas changes its state under the constant temperature, it should be noticed that $T_1 = T_2$.

As seen in Figs. 2 and 3, the term of (B) in Eq. (19) is the amount of the pressure change

when a gas system goes from the state of (P', V_1, T_2) to the state of (P_2, V_2, T_2) .

$$\Delta P_{(B)} = P_2 - P' = P_2 - \frac{mRT_2}{V_1},$$

$$P' = \frac{mRT_2}{V_1}. \quad \text{Eq. (24)}$$

Certainly, Eq. (24) corresponds to the term of (B) in Eq. (19) and is resulted from the work of Eq. (23), however their units are different each other. Therefore, to match the units of them, the new factor of α_T should be introduced same as the procedure to define α_V .

$$\alpha_T = v_1 \ln \left(\frac{v_2}{v_1} \right), \quad \text{Eq. (25)}$$

which has also the unit of m^3 and its verification was showed in S4 in the supplement.

As seen in S3 and S4 of the supplement, when the pressure of a gas changes from the state 1 to the state 2, the heat energy of 43.02 kJ/kg and the work energy of 47.92 kJ/kg are needed. However, since the gas should release 47.92 kJ/kg to the surroundings in total energy of 90.94 kJ/kg, the concept of the efficiency for the pressure change of a gas can be introduced.

$$\eta_{\Delta P} = \frac{\Delta Q}{\Delta Q + |\Delta W|} \quad \text{Eq. (26)}$$

in which the denominator $(\Delta Q + |\Delta W|)$ is total energy necessary for the change of state of a gas and the numerator (ΔQ) is the pure energy held by a gas out of total energy. If applying Eq. (26) to the change of state described in S1 of supplement,

$$\eta_{\Delta P} = \frac{43.02}{90.94} \approx 0.473.$$

Energies and efficiency in the volume change of a gas, ΔV

Since the volume of a gas is identical to mRT/P as seen from Eq. (1), Eq. (16) can be the modified as

$$\Delta V = \left(\frac{mRT_2}{P_1} - V_1 \right) + \left(V_2 - \frac{mRT_2}{P_2} \right). \quad \text{Eq. (27)}$$

(A)
(B)

As seen in Fig. 3 and Table 1, the term of (A) in Eq. (27) is the volume change at the intermediate state (P_1, V', T_2) under the isobaric condition when a gas system goes from the

state 1 (P_1, V_1, T_1) to the state 2 (P_2, V_2, T_2). Based on 1 kg of air,

$$\Delta V_{(A)} = V' - V_1 = \frac{mRT_2}{P_1} - V_1,$$

$$V' = \frac{mRT_2}{P_1}. \quad \text{Eq. (28)}$$

In order for the volume of a gas to be changed under an isobaric condition, a heat energy should be supplied to a gas or released from it. It means that Eq. (28) must be related with a heat energy. For the isobaric process in the thermodynamics textbooks [1,2],

$$dQ = mc_p dT,$$

$$\int_1^2 dQ = mc_p \int_1^2 dT,$$

$$\Delta Q = mc_p (T_2 - T_1). \quad \text{Eq. (29)}$$

As described in the section 4.1, Eq. (28) is clearly caused by Eq. (29), however their units are also different because the former has the unit of volume and the latter has the unit of heat energy. Therefore, a new constant should be introduced in Eq. (28) to match the units of Eqs. (28) and (29).

$$\beta_p = \frac{c_p P_1}{R}, \quad \text{Eq. (30)}$$

which is the factor to convert the volume of a gas into its energy and has the unit of a pressure. In Eq. (30), it should be noted that the specific heat of c_p is assumed a constant. The detailed derivation and its calculation of Eq. (30) were described in S5 of the supplement.

The term of (B) in Eqs. (16) and (27) stands for the amount of the volume change with the pressure in the isothermal condition as shown in Fig. 3 and Eq. (6). Referring to some thermodynamics textbooks [1,2], the work required for the volume change of gas is

$$dW = VdP = PV \frac{1}{P} dP,$$

$$\int_1^2 dW = PV \int_1^2 \frac{1}{P} dP,$$

$$\Delta W = mRT_1 \ln \left(\frac{P_2}{P_1} \right). \quad \text{Eq. (31)}$$

Similar to the descriptions on Eq. (23), Eq. (31) is the necessary work that a gas changes

its state under the constant temperature. Therefore, it should be also noticed that $T_1=T_2$. Since the term of (B) in Eq. (27) is the amount of the volume change when a gas system goes from the state of (P_1, V', T_2) to the state of (P_2, V_2, T_2) ,

$$\Delta V_{(B)} = V_2 - V' = V_2 - \frac{mRT_2}{P_1},$$

$$V' = \frac{mRT_2}{P_1}, \quad \text{Eq. (32)}$$

which clearly corresponds to the term of (B) in Eq. (27) and is caused by the work of Eq. (31). However, if comparing the units of Eqs. (31) and (32), their units are different each other. Therefore, to match the units of them, the new factor of β_T should be introduced same as the procedure to define α_T .

$$\beta_T = P_1 \ln \left(\frac{P_2}{P_1} \right), \quad \text{Eq. (33)}$$

which has also the unit of a pressure and its verification was showed in S6 in the supplement.

As seen in S5 and S6 of the supplement, when the volume of a gas changes from the state 1 to the state 2, the heat energy of 60.36 kJ/kg and the work energy of 60.69 kJ/kg are necessary. However, since the gas should release 60.69 kJ/kg to the surroundings in total energy of 121.05 kJ/kg in the isothermal process, the efficiency of the volume change is

$$\eta_{\Delta V} = \frac{60.36}{120.05} \approx 0.503.$$

Energies and efficiency in the temperature change of a gas, ΔT

Since the temperature of a gas is identical to PV/mR as seen from Eq. (1), Eq. (17) can be the modified as

$$\Delta T = \underbrace{\left(\frac{P_2 V_1}{mR} - T_1 \right)}_{(A)} + \underbrace{\left(T_2 - \frac{P_2 V_1}{mR} \right)}_{(B)} = T_2 - T_1, \quad \text{Eq. (34)}$$

which simply comes back to the temperature difference unlike Eqs. (19) and (27).

In the subsection of 4.1 and 4.2, four factors were introduced to convert the pressure or the volume of a gas into the energy because the pressure and the volume are not the energy but the properties of a gas. The conversion factors of α and β have the units of a volume (m^3) and a pressure (kPa), respectively. Then, by multiplying α to P and β to V , the mechanical energy

$(P \cdot V)$ for the change of state of a gas were evaluated. Considering that all kinds of mechanical energies eventually dissipates into heat energy, the result of Eq. (34) is not surprising.

If adding the additional descriptions on Eq. (34), the change of state of a gas is indicated by the changes of three variables (P , V , T). As already mentioned it, since these three variables have an interconnected relationship called the B-C-G law ($P \cdot V/T = C$). Therefore, if two variables are given, one rest variable is determined. As seen in Eqs. (15) and (16) of Table 1, since the effect of the temperature difference of a gas, ΔT , has already been reflected in the changes of a pressure and a volume, Eq. (34) does not provides any additional information.

Conclusion

The properties such as P , V and T of a gas, even if it is a real gas, can be satisfactory evaluated by using the B-C-G law of an ideal gas for the purpose of engineering applications. When a gas changes its state, the B-C-G law gives the properties of a gas after any process of a gas is completed. However, the B-C-G law provides only the results after the change of state of a gas and nothing about the information contained in a process. The motivation of this study was started from the author's simple curiosity as follows;

- The partial differentials of the ideal gas law, which are shown in Eqs. (5) to (10), are well known in most thermodynamics textbooks.

- Nevertheless, why don't those textbooks mention on the integration method of the partial differentials to determine the properties of a gas after completing a process?

- Is it unfeasible to integrate the partial differentials to get the property changes of a gas?

Frankly speaking, if we limit our focus to the engineering applications, this study will be unnecessary because the B-C-G law provides the sufficient information for a design. However, to get the detailed information embedded in the change of state of a gas, this study will be highly useful. In other words, this study is not for engineers but for scientists.

In this study, the integrations of the differentials originated from the ideal gas law were successfully carried out (See Table 1). To verify the integration results, they were compared to the differential properties of ΔP , ΔV and ΔT evaluated from the experimental data. As seen in S1 of the supplement, the integration results showed that they were consistent with the experimental data (See Table 3). That is, the integration methods of the partial differentials, which is for determining the properties of a gas after finishing a process, suggested in this study were correct.

In addition to it, this study thoroughly and stepwisely analyzed the processes included in the change of state of a gas. From this analysis, the conversion factors, which enable the pressure and the volume of a gas to convert the energy, were newly introduced. By multiplying those factors to P or V of a gas, the energies (ΔQ or ΔW) related with the change of state were evaluated, which intuitively leads to efficiency of the change of state of a gas.

$$\eta = \eta_{\Delta P} \cdot \eta_{\Delta V} \quad \text{Eq. (34)}$$

If substituting all parameters derived in this study into Eq. (34),

$$\begin{aligned} \eta &= \frac{\alpha_V}{(\alpha_V + |\alpha_T|)} \cdot \frac{\beta_P}{(\beta_P + |\beta_T|)} \\ &= \frac{1}{\left[1 + \frac{R}{c_V} \left| \ln \left(\frac{v_2}{v_1} \right) \right| \right]} \cdot \frac{1}{\left[1 + \frac{R}{c_P} \left| \ln \left(\frac{P_2}{P_1} \right) \right| \right]}, \end{aligned} \quad \text{Eq. (35)}$$

which is for 1 kg of a gas and the absolute symbol is for avoiding the minus results when a gas is compressed.

If calculating Eq. (35) from the state 1 to the state 2 (P_2, V_2, T_2) shown in the path B of Fig. 3, the efficiency of the change of state is $\eta = \eta_{\Delta P} \cdot \eta_{\Delta V} = 0.473 \times 0.503 \approx 0.238$. This calculated efficiency indicates that the energy loss is the unavoidable and inherent phenomenon when a gas changes its state. On the surface, the calculated efficiency of 0.238 may seem very low. However, its value of 0.238 is higher than the efficiency of the Carnot cycle operating between the hot temperature of 340 K and the cold temperature of 240 K.

From the comparison of two efficiencies, which are the efficiency of the change of state of a gas and the efficiency of the Carnot cycle, it can be concluded that the efficiency of the change of state of a gas $>$ the Carnot cycle $>$ a real cycle. Although the author considers some other studies related with the topic of this study, he is pleased to imagine that Table 1 of this study will be included in most thermodynamics textbooks in future.

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Author Contributions

S.Kim and S.H Choi wrote the manuscript and prepared the figures and tables. S.H.Choi conceived of the work.

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