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Implications of multiple numerical aspects for Carreau nanofluids with heat generation/absorption via nonuniform channels

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Abstract:

Nanomaterials are unique work fluids with preeminent thermal performance for improving heat dissipation. We present theoretical and mathematical insights into nanofluid heat transfer and flow dynamics in nonuniform channels utilizing a non-Newtonian fluid. Therefore, the impacts of heat absorption/generation and Joule heating in a magneto hydrodynamic flow of a Carreau nanofluid into a convergent channel with viscous dissipation are addressed in this mathematical approach. Brownian and thermophoresis diffusion are considered to investigate the behavior of temperature and concentration. The magnetic effects on the flow performance are measured. The leading nonlinear equations are solved numerically using the BVP4c solver and RK-4 (Runge–Kutta) along with the shooting algorithm using the computer software MATLAB. The obtained dual solutions are presented graphically. The consequences of the variable magnetic field, heat absorption/generation and numerous physical parameters on the temperature and concentration field are surveyed. The outcomes show that increasing the rates of the heat absorption/generation parameter and Eckert number enhances the thickness of the thermal profile of the convergent channels, while increasing the value of the Prandtl number expands the thickness of the momentum boundary layer of the convergent channels. The key findings related to the study models are presented and discussed. An assessment of solutions achieved in this article is made with existing data in the literature.

Keywords: Nanomaterials; Carreau fluid; Nonuniform channels; Heat transport; Viscous dissipations; Multiple solutions.

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1. Introduction

The idea of magneto hydrodynamics plays a vital role in the flow of Newtonian and non-Newtonian fluids. There are many promising applications of MHD, which appear in cooling systems with liquid metal, flow meters, sensors, blood flow measurement pumps, magnetic drugs, nuclear reactors, the formation of new stars, atmospheric heating, etc. Flow through nonuniform channels is a significant part of research due to many practical and industrial engineering and many physical applications, such as flow through rivers and canals, blood flow through arteries and capillaries, supersonic jets and nozzles. The two-dimensional flow of viscous fluid through a converging-diverging channel was introduced by Jeffrey [1] and Hamel [2]. They done pioneering work in this area. The Jeffrey-Hamel flow models are interesting and important to analyze the boundary layer separation in divergent channels. Jeffrey and Hamel provide a solution to Navier–Stokes equations by introducing the similarity transformation concept that depends on two nondimensional parameters, the Reynolds number and the angle of the channel width. This model was further analyzed by Axford [3] by considering the effects of an external magnetic field. He predicts that the magnetic field, the Reynold number, and angle of inclination act as control parameters. Imani *et al.* [4] extend the study by converting Maxwell’s equations and Navier–Stokes equations to nonlinear ordinary differential equations for the modeled problem of Jaffrey-Hamel flow in the presence of a high magnetic field and nanoparticles. The flow region in the divergent channel was analyzed for different values of the Hartmann number and angle of the channel. The obtained results were matched with the exact solution obtained by ADM. Makinde [5] studied the effects of MHD on classical Jaffrey-Hamel flow in a convergent-divergent channel using Pade approximations. They interpret various numbers of Reynolds numbers on the velocity field of the flow and constant shear rate in both convergent-divergent channels. Using an analytical (DRA) approach, Dogonchi et al. [6] explored a two-dimensional steady and incompressible viscous water-based MHD nanofluid from the origin between two stretchy/shrinkable walls. They found that the fluid velocity and temperature distribution increased with an increase in the stretching parameters. Reza et al. evaluated the copper-kerosine nanofluid in a channel across a stretched sheet under the action of a magnetic field. [7]. They used a three-stage Lobato IIIA formula to solve the nonlinear ODEs. Their outcomes show that solid volume friction decreases the velocity of nanoparticles close to the wall of the channels and escalates the thickness of the

thermal boundary layer of the channels. The study of magnetohydrodynamic flow of nanofluids over a rotating stretchable plate in the presence of a magnetic field was captured by Ram *et al.* [8]. The unsteady magnetic nanofluid over a stretchable rotating disk was analyzed.

A homogeneous concentration of nanosized magnetic particles (1-100 nm) in a base liquid with an exterior coating is known as a nanofluid. The most important feature of nanofluids is their high thermal conductivity compared to pure liquids. Choi *et al.* [9]. introduced a model and, for the first time, used the term nanofluid. After the groundbreaking idea, many researchers proposed different models to incorporate Brownian motion and thermophore effects. In the presence of stretched shrink walls, Mohyud-Din et al. [10] addressed the MHD flow of an incompressible fluid comprising nanosized particles between nonparallel walls. MHD nanofluid flow and heat transportation over two horizontal slabs in a rotating device were addressed by Sheikholeslami *et al.* [11]. The KKL (Koo–Kleinstreuer–Li) correlation was used to analyze the heat conduction and stiffness of the nanofluid. They noticed that with the increase in magnetic parameters, rotational parameters, and Reynolds number, the skin friction coefficient increased but decreased as the nanoparticle volume fraction increased. Mebarek-Oudina [12] examined the fluid flow structure of Titania nanofluids in a cylinder annulus containing a variety of base fluids. He used the finite volume strategy to address the Maxwell fluid model for convective energy transfer. In a few other articles, Mebarek-Oudina [12] analyzed the natural convection and heat transport stability of nanofluids in the presence of an applied magnetic field. The impact of molybdenum disulfide (MoS₂) nanoparticle shapes on the rotating flow of nanofluids along an elastic stretched sheet was examined by Usman *et al.*[13]. They discovered that as radiative heat parameters and Prandtl numbers improve, the local Nusselt number diminishes. Once the thermophysical parameters are improved, they improve. Alam et al. [14] explored the role of a magnetic field on the entropy rate of production of conductivity fluid flow in a converging-diverging channel. Khan *et al.* [15] provide a new track of such flows by taking velocity and temperature slip on flows in a convergent-divergent channel.

Viscous dissipation effects are often negligible in different flows; however, their involvement becomes significant when fluid viscosity is higher. It affects the heat field by playing an energy source role, which leads to a change in the heat transfer rate. Kayalvizhi and Ganga [16] analyzed the viscous and Joule dissipations on MHD flow over a stretchable porous region

immersed in porous medium for common fluids. Pal and Mandal [17] used the RKF algorithm to estimate a numerical solution for convective nanofluids over a stretchable/shrinkable surface underneath the impact of viscous dissipations. For large values of current shrinkable sheet parameters, they execute a dual solution for the temperature profile. In the presence of ohmic dissipations, Hayat et al. [18] address the boundary layer problem for MHD Williamson fluid past a porous stretchy sheet. They observed that the magnetic field expanded the surface drag force. The effect of viscous and ohmic dissipations of MHD nanofluids over an upright plate with energy generation/absorption was explored by Ganga *et al.* [19]. They observed that due to an increase in solid volume fraction, the temperature spreading declines. Singh *et al.* [20] explored the impact of MHD and velocity slip on a vertical plate in an alumina-water nanofluid. They observed that the energy transport ratio rises as the volume fraction of dense particles increases. Mishra *et al.* [21] conducted a numerical study for the current equations of MHD (Ag- H_2O) nanofluid flow through an upright cone subject to viscous dissipations. They conclude that the mass transference rate decreases as velocity slip limitations grow. Alamri *et al.* [22] explained the effect of the mass transport rate on the MHD flow of second-grade fluid. They establish that the fluid velocity declines as the magnetic parameter values. The effect of radiation, entropy generation and MHD nanofluid flow over a curved channel. They revealed that the lowest channel's flow rate was a result of the radiation parameter. Sarfraz *et al.* [23] analyzed the lake boiling heat transport features of nanosized iron oxide particles under the impact of a constant magnetic field.

The significance of nanoparticles and a high Lorentz force on Jeffery-Hamel flow was established by Sheikholeslami et al. [24]. The Adomian decomposition technique was used to explain the related nonlinear equations, and it was observed that boosting the Reynolds number caused a drop in velocity close to the walls. Pandey and Kumar [25] used an irregular stretched sheet to highlight the cumulative impact of radiation and heat absorption/generation on nanofluid movement. They also determined that as the heat absorption/generation values increased, the rate of heat transmission decreased. In addition to heat generation/absorption consequences on nanofluids, Ganga et al. [26] evaluated ohmic dissipation and viscous effects on MHD flow subjected to an upward plate. Hayat et al. [27] focused on the boundary layer stream and heat transmission consequences in a Jeffrey nanofluid flow with thermal conductivity, and they concluded that the radiation effect is temperature dependent. In addition, few recent research studies are offered in Refs. [28-39].

The current study is a mathematical attempt to analyze Carreau nanofluid flow in a convergent/divergent channel under the influence of a variable magnetic field subjected to the physical impact of viscous dissipations. The governing equations are highly nonlinear for this flow problem. The dimensionless ordinary differential equations are solved numerically using the shooting method along with the fourth-order RK-4 (Runge–Kutta Fehlberg) scheme. The obtained results are explained physically. The impact of different parameters on velocity, temperature and concentrations are plotted graphically.

2. Flow model and mathematical formulations

2.1 Flow analysis

The model addresses the flow of steady, incompressible, non-Newtonian Carreau fluid with the addition of nanoparticles in a nonuniform channel subject to an external magnetic field and constant pressure gradient. The fluid is electrically conducting and has nonlinear viscosity. The walls of the channels are located at an angle of 2α , as shown in Figure 1. To assess the flow under externally applied conditions, a variable magnetic field $B = (0, \frac{B_0}{r}, 0)$ is functional transversely across the channel. For this flow problem, the cylindrical polar coordinates (r, θ, z) are used for formulations. The fluid motion is purely in the radial direction, and there is no magnetic field in the z direction. Therefore, the fluid velocity can be taken as $V = u(r, \theta)$, while the v and w components are taken to be zero. Thus, the velocity field is subject to r and θ only. The nonuniform channels are convergent for $\alpha < 0$, and divergent if $\alpha > 0$. The effects of the induced magnetic field are negligible. The pressure gradient drives fluid motion, while viscous dissipation is a source of energy. The power-law model is used to incorporate the nondimensional governing equations.

Fig. 1. Flow structure of the proposed problem

Figure 1: The schematic of the governing model.

In cylindrical coordinates, the continuity and Navier–Stokes equation can be expressed as follows.

$$\frac{\rho_f}{r} \left(\frac{\partial(r u_r)}{\partial r} \right) = 0, \quad (1)$$

$$u \frac{\partial u_r}{\partial r} = -\frac{1}{\rho_f} \frac{\partial p}{\partial r} + v \left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{u_r}{r^2} \right] \left[1 + \Gamma^2 \left\{ 2 \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2 + \frac{2 u_r^2}{r^2} \right\} \right]^{\frac{n-1}{2}} - \frac{\sigma B_0^2 u_r}{\rho r^2}, \quad (2)$$

$$0 = -\frac{1}{\rho_f r} \frac{\partial p}{\partial \theta} + \frac{2v}{r^2} \left[1 + \Gamma^2 \left\{ 2 \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2 + \frac{2 u_r^2}{r^2} \right\} \right]^{\frac{n-1}{2}} \frac{\partial u_r}{\partial \theta}, \quad (3)$$

where u_r is the radial velocity, ρ is the fluid density, v describes the kinematic viscosity and p is the fluid pressure. Equations (2) and (3) are coupled into equation (4) by eliminating the pressure gradient term

$$\frac{\partial u_r}{\partial r} \frac{\partial u_r}{\partial \theta} + u \frac{\partial^2 u_r}{\partial r \partial \theta} = 2v \left[\frac{1}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{1}{r} \frac{\partial^2 u_r}{\partial r \partial \theta} \right] + v \left[\frac{\partial^3 u_r}{\partial r^2 \partial \theta} + \frac{1}{r} \frac{\partial^2 u_r}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial^3 u_r}{\partial \theta^3} - \frac{1}{r^2} \frac{\partial u_r}{\partial \theta} \right] \left[1 + \Gamma^2 \left\{ 2 \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2 + \frac{2 u_r^2}{r^2} \right\} \right]^{\frac{n-1}{2}} - \frac{\sigma B_0^2}{\rho r^2} \frac{\partial u_r}{\partial \theta}, \quad (4)$$

the associated conditions at the boundaries are:

$$u = U_{\max}, \quad \frac{\partial u}{\partial r} = 0, \quad \text{at } \theta = 0, \quad (5)$$

$$u = U_w, \quad \text{at } \theta = \pm \alpha,$$

From equation (1), the radial velocity is a function of r and θ , as provided in Eq. (5). Hence, Eq. (5) suggest the following form of radial velocity:

$$ru_r(r, \theta) = f(\theta), \quad (6)$$

Jaffrey-Hamel flow is the mechanism of the dual channels where at one end the fluid transports inside called convergent channel and at the other end the fluid removesward called divergent channel. Furthermore, it is well known that the peak fluid velocity is certainly achieved at $\theta = 0$. In fact, we find the following:

$$u_{\max} = \frac{f(0)}{r}, \quad (7)$$

Subsequently, $f(\theta) \leq f(0)$, in the range $-\alpha \leq \theta \leq \alpha$.

Incorporating Eq. (5) into Eq. (4), we subsequently obtain the following nondimensional ODE:

$$(f'''' + 4f') \left(1 + \frac{r^2}{r^4} (4f^2 + f'^2) \right)^{\frac{n-1}{2}} + \frac{2}{v} f' f - \frac{\sigma B_0^2}{\rho v} f' = 0, \quad (8)$$

With the aid of dimensionless variables:

$$F(\xi) = \frac{f(\theta)}{rU}, \quad \xi = \frac{\theta}{\alpha}, \quad (9)$$

Eq. (7), can be settled as

$$(F'''' + 4\alpha^2 F') \left(1 + We^2 (4\alpha^2 F^2 + F'^2) \right)^{\frac{n-1}{2}} + 2\alpha Re F F' - H_a \alpha^2 F' = 0, \quad (10)$$

With: $\xi \in [-1, 1]$

Where Reynold, can be expressed as:

$$Re = \frac{\alpha r U}{v} = \begin{cases} U > 0, \alpha > 0, \text{Divergent channel;} \\ U < 0, \alpha < 0, \text{Convergent channel;} \end{cases} \quad (11)$$

Moreover, $H_a = \sqrt{\frac{\sigma B_0^2}{\rho v}}$, denotes the Hartmann number, and $We = \left(\frac{r^2 U^2}{r^4 \alpha^2} \right)^{\frac{1}{2}}$, is the local

Weissenberg number. The associated boundary conditions of the hydrodynamic problem can be set as:

$$F(\xi) = 1, \quad F'(\xi) = 0, \quad \text{at } \xi = 0, \quad (12)$$

At the center of the channel

$$F(\pm\xi) = 0, \text{ at } \xi = 1, \quad (13)$$

2.2. Heat transfer analysis

This section presents the energy transport throughout the fluid drift through narrow and opening channels. In the presence of viscous dissipation due to nanoparticles, a heat generating/absorbing source is present within the system, and the heat Joule effects energy equation can be proposed as:

$$\begin{aligned} u_r \frac{\partial T}{\partial r} = & \frac{k}{(\rho c_p)_f} \left[\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] + \tau D_B \left[\frac{\partial T}{\partial r} \frac{\partial C}{\partial r} + \frac{1}{r^2} \frac{\partial T}{\partial \theta} \frac{\partial C}{\partial \theta} \right] + \frac{\tau D_T}{T_\infty} \left[\left(\frac{\partial T}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial T}{\partial \theta} \right)^2 \right] + \\ & \frac{\mu_0}{(\rho c_p)_f} \left[1 + \Gamma^2 \left\{ 2 \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2 + \frac{2u_r^2}{r^2} \right\} \right]^{\frac{n-1}{2}} \left[\left\{ 2 \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2 + \frac{2u_r^2}{r^2} \right\} \right] + \frac{\sigma B_0^2 u_r^2}{(\rho c)_f r^2} + \\ & \frac{Q_0 T}{(\rho c)_f r^2}, \end{aligned} \quad (14)$$

With the subsequent boundary conditions:

$$\frac{\partial T}{\partial \theta}, \text{ at } \theta = 0, \quad (15)$$

$$T = T_w, \text{ at } \theta = \pm\alpha,$$

Here, k and $(c_p)_f$ indicate the thermal conductivity and specific heat at constant pressure. Furthermore Q_0 indicate the heat generating/absorbing coefficient. $Q_0 < 0$, denotes heat absorption, while $Q_0 > 0$ represents heat generation. In addition, $\tau = \frac{(\rho c_p)_p}{(\rho c_p)_f}$, is the ratio of the thermal capacity ratio. D_B and D_T are Brownian and thermophoresis diffusion, respectively. ρ_p , signify the density of the nanoparticles. T_w and T_∞ demonstrate the wall and ambient temperature.

The following dimensionless transformations are considered:

$$F(\xi) = \frac{f(\theta)}{rU}, \quad g(\xi) = \frac{T}{T_w}, \quad \xi = \frac{\theta}{\alpha}, \quad (16)$$

In T_w indicate temperature at the wall. The partial differential equation in (14) is transmuted into ODE using (16), and we have

$$g'' + Pr N_B g' \phi' + Pr N_T g'^2 + Pr Ec \left[\left(1 + We^2 (4\alpha^2 F^2 + F'^2) \right)^{\frac{n-1}{2}} \right] (4\alpha^2 F^2 + F'^2) + \alpha^2 H_a Pr Ec F^2 + \alpha^2 H_g g = 0, \quad (17)$$

where $g(\xi)$ indicates the dimensionless thermal distribution. The associated boundary conditions in terms of $g(\xi)$ of the heat transport problem are

$$g(\xi) = 1, \quad \text{at } \xi = 1 \quad (18)$$

$$g'(\xi) = 0, \quad \text{at } \xi = 0,$$

where $Pr = \frac{v(\rho c_p)_f}{k}$, indicates the Prandtl number, $Ec = \frac{U^2}{T_w c_p}$, is the Eckert number, $H_g = \frac{Q_0}{k}$ is the heat generation/heat absorption parameter, and $N_B = \frac{\tau D_B C_w}{v}$, is the Brownian diffusion and $N_T = \frac{\tau D_T T_w}{v T_\infty}$, thermophoresis (diffusion).

2.3. Concentration analysis

The corresponding concentration equation for the flow model:

$$u \frac{\partial C}{\partial r} = D_B \left(\frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 C}{\partial \theta^2} \right) + \frac{D_T}{T_\infty} \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right), \quad (19)$$

Subject to the following restrictions at the boundaries:

$$\frac{\partial C}{\partial \theta}, \text{ at } \theta = 0, \quad (20)$$

$$C = C_w, \text{ at } \theta = \pm \alpha,$$

The following dimensionless transformations are offered:

$$F(\xi) = \frac{f(\theta)}{rU}, \quad \phi(\xi) = \frac{C}{C_w}, \quad \xi = \frac{\theta}{\alpha}, \quad (21)$$

Incorporating Eq. (16), into Eq. (19) to Eq. (20), we have

$$\phi'' + \frac{N_T}{N_B} g'' = 0. \quad (22)$$

$\phi(\xi)$ governs the concentration distribution that arises due to nanoparticles, and the concentrations within the central portion and at the wall are prescribed as:

$$\begin{aligned}\emptyset(\xi) &= 1, & \text{at } \xi &= 1, \\ \emptyset'(\xi) &= 0, & \text{at } \xi &= 0.\end{aligned}\tag{23}$$

3. Engineering parameters

The skin friction coefficient, Nusselt number, and Sherwood number are several physical quantities of interest.

$$C_f = \frac{\mu_f}{\rho_f U^2} \frac{1}{r} \left(\frac{\partial u}{\partial \theta} \right) \Big|_{\theta=\alpha} = F'(1), \quad Nu = -\frac{1}{T_w} \left(\frac{\partial u}{\partial \theta} \right) \Big|_{\theta=\alpha}, \quad \text{and } Sh = -\frac{1}{c_w} \left(\frac{\partial C}{\partial r} \right) \Big|_{\theta=\alpha}, \tag{24}$$

In the view of Eq. (10), (17) and (22),

$$ReC_f = F'(1), \quad \alpha Nu = -\theta'(1), \quad \text{and } \alpha Sh = -\emptyset'(1).\tag{25}$$

4. Results declaration and discussion

The nonlinear equations (10), (17) and (22) under the associated dimensionless conditions at the boundary in Eqs. (12), (18) and (23) are tickled. For numerical computations, the Runge–Kutta Fehlberg method with the Nachtsheim-Swigert Shooting approach is used, which is a rapid and adaptable numerical technique. The governing dimensionless boundary value problem (10), (17) and (22) must be reduced to an initial value problem using this method. Plotted graphs depict the numerical findings for flow velocity and temperature and concentration profiles for a variety of physical parameter values. The graphical phenomena illustrate the variation in velocity, temperature, and concentration under the influence of the aforementioned physical quantities portrayed in the range of $0 < \xi < 1$.

In this section, we have offered multiple solutions for convergent channels, while in a divergent channel, the outcomes are quite similar. The velocity distribution under the influence of **We, n, Ha** and opening angle α of channel are offered in Figs. 2-7. The fraction of elastic to viscous forces, named the Weissenberg number, enhances the fluid velocity distribution for growing negative values of the opening angle of the channel, as depicted in Fig. 2. Physically, the fluid viscosity climbs as the fluid approaches the splitting point, and the dominance of viscous force over elastic force promotes an amplification in fluid velocity. Figure 3 depicts the consequences of the Re number on the convergent flow fluid velocity. Rising the Reynold number

results in a flatter velocity profile in the channel's middle with significant gradients along the walls. As a response, the thickness of the boundary layer is diminished. For the convergent flow regime, it is self-evident that backflow is eliminated. Dropping of velocity is witnessed for shear thinning fluid $0 < n < 1$ and shear thickening fluid $n > 1$ in Figs. 4 and 5. Mounting n generates higher resistance in fluid flow. The power law index distinguishes between two types of fluids: $n < 1$ pseudoplastic and dilatant fluids $n > 1$. This is because increasing the power law index correlates to an increment in viscosity and consequently, a reduction in fluid velocity. The influence of a magnetic parameter Ha on the velocity distribution is visualized in Fig. 6. Physically, the Lorentz force attributes resistance to the magnetic field. The magnetic field suppress the fluid flow. As the magnetic number inside the convergent portion grows, the velocity boundary layer thickness improves; consequently, the fluid velocity declines. As the negative angles are increased, the velocity in the deviating channels climbs rapidly and reaches a maximum near the channel's middle, as seen in Fig. 7.

Fig. 2. Deviation in $F(\xi)$ under numerous values of We .

Fig. 3. Deviation in $F(\xi)$ under numerous values of Re .

Fig. 4. Deviation in $F(\xi)$ under numerous values of $n < 1$.

Fig. 5. Deviation in $F(\xi)$ under numerous values of $n > 1$.

Fig. 6. Deviation in $F(\xi)$ under numerous values of Ha .

Fig. 7. Deviation in $F(\xi)$ under numerous values of α .

Figs. 8-13 demonstrate that different parameters affect the thermal distribution. Fig. 8 expresses the thermal curves against a growing number of Pr . The ratio of momentum diffusivity thickness to thermal diffusivity is termed the Prandtl number. Because the thickness of the thermal boundary layer is wider than the thickness of the momentum boundary layer, magnified Prandtl numbers correlate to high thermal conductivity, i.e., heat spreads out more efficiently. The temperature evolution with expanding Brownian diffusion parameters Nb is shown in Fig. 9. Improved Brownian diffusion parameters raise the thermal field. The nanoparticle collides with a fluid molecule because of Brownian drift, and a part of the kinetic energy is turned to thermal energy, leading the fluid temperature to rise. In Fig. 10, the significance of the thermophoresis diffusion parameters Nt is prescribed. Thermophoresis transmits fluid molecules with a high thermal energy transfer from a hotter to a colder region, improving the fluid temperature. An increase in the Eckert number Ec provides an upsurge in temperature, as expected, as shown in Fig. 11. This enhancement occurs due to Ec , which directly affects the heat dissipation process and hence the thermal profile boosts. The viscous dissipation effect, which is always positive and indicates a source of heat due to frictional forces within the fluid molecules, is illustrated by the fourth term on the right-hand side of Eq. (14). In addition, as the Eckert number grows, the thickness of the

thermal boundary layer decreases. Figure 12 indicates the impact of the Hartmann number on temperature. In a convergent channel, the temperature is a decreasing function of the Hartmann number, as per this graphic. This is because a perpendicular magnetic field creates a resistive force in an electrically conducting fluid known as the Lorentz force. This force causes a resistance within the fluid particles by creating frictional force between its layers, dropping its temperature. Fig. 13 concludes that the temperature distribution grows for convergent channels against distinct values of the heat generation/absorption parameter Hg . Furthermore, with Hg , the corresponding boundary layer expands. Physically, large heat generation/absorption rates transfer more heat to the working fluid, driving the thermal profile to expand. Heat generation (heat source) is indicated by positive values of Hg , whereas heat absorption is indicated by a negative sign (heat sink). The term "heat source" describes the process of producing heat from the surroundings, which heats up in the flow field. As previously stated, the thermal phase of the fluid increases due to the presence of a heat source or a heat generating mechanism, causing the thermal boundary layer to increase.

Fig. 8. Deviation in $g(\xi)$ under numerous values of Pr .

Fig. 9. Deviation in $g(\xi)$ under numerous values of Nb .

Fig. 10. Deviation in $g(\xi)$ under numerous values of Nt .

Fig. 11. Deviation in $g(\xi)$ under numerous values of Ec .

Fig. 12. Deviation in $g(\xi)$ under numerous values of Ha .

Fig. 13. Deviation in $g(\xi)$ under numerous values of Hg .

The concentration profiles under diverse control flow parameters are plotted in Figs. 14-18. Strong magnetic resonance suppresses the boundary layer thickness; consequently, the concentration drops in the flow region, as prescribed in Figure. 14. The magnifying values of the Hartmann number Ha diminished the concentration profile in the flow regime. Strong magnetism suppresses the boundary layer thickness, and consequently, the concentration drops. Fig. 15 exhibits the impression of the Brownian diffusion parameter Nb on the concentration profile. The increase in Brownian motion promotes the random movement that disperses the nanoparticles, resulting in a rise in concentration. Dropping of concentration against growing thermophoresis parameters Nt is witnessed in Fig. 16. It is obvious that as the thermophoresis parameters are improved, the concentration of nanofluid drops, which may be interpreted by the increasing temperature of the flow field and the temperature of the boundaries. A decrease in concentration due to an enhancement in the Eckert number Ec is described in Fig. 17. Even at the boundary value, the concentration of nanoparticles diminishes for high Eckert number values. This is because as Ec rises, the rate of heat transmission at the surface drops. Dropping of the concentration in convergent channels due to a gradual increase in the heat generation absorption parameter Hg is depicted in Fig. 18. Since the concentration of the fluid does not vary with the fluctuation of heat

in the fluid, the heat generation/absorption coefficient, Hg , has little influence on the concentration distribution. The heat flux coefficient is responsible for increasing the fluid flow heat gradient, although it has no consequence on the fluid particle concentration levels.

Fig. 14. Deviation in $\phi(\xi)$ under numerous values of Ha .

Fig. 15. Deviation in $\phi(\xi)$ under numerous values of Nb .

Fig. 16. Deviation in $\phi(\xi)$ under numerous values of Nt .

Fig. 17. Deviation in $\phi(\xi)$ under numerous values of $\mathbf{Ec}$.

Fig. 18. Deviation in $\phi(\xi)$ under numerous values of Hg .

Figs. 19 and 20 are sketched to elaborate the effect of various parameters on the skin friction coefficient. The wall friction is diminished by the Hartmann number Ha , whereas the Weissenberg number We has the reverse effect. This is because when the Hartmann number grows, the fluid velocity decreases, resulting in a reduction in the volume flow rate. In the non-Newtonian fluid regime, the dimensionless skin friction escalates with increasing We , implying that the boundary layer flow is accelerated with diminishing viscosity effects. Figs. 21 and 22 depict the consequences of the Prandtl number Pr and Eckert number Ec on the local Nusselt number. The wall heat flux is significantly improved by Pr , as prescribed in Fig. 21. When a high Prandtl number is maintained, the convective mode of heat transport accelerates. Since the Nusselt number is related to convective heat transmission, it increases in the opposite direction. The Eckert number is inversely related to the enthalpy of the boundary layer. As a result, as the Eckert number jumps, the difference across the surface and ambient temperatures shrinks. In the end, the rate of heat transmission from the surface is reduced, as shown in Fig. 22. The greater values of the Brownian diffusion and thermophoresis diffusion parameter Nb and Nt show conflicting behavior for the Sherwood number, as deployed in Figs. 23 and 24. Brownian diffusion increases the boundary layer thickness and consequently the mass transport rate, while Nt reduces the mass transfer rate. Physically, the concentration boundary layer thickness increases with increasing thermophoresis,

while the rate of mass transfer boundary layer thickness diminishes. On the other hand, as the Brownian motion parameter is augmented, the concentration decreases while the rate of mass transfer increases.

Fig. 19. Fluctuation in skin friction C_f under numerous values of Ha .

Fig. 20. Fluctuation in skin friction C_f under numerous values of We .

Fig. 21. Deviation in Nu under numerous values of Pr .

Fig. 22. Deviation in Nu under numerous values of Ec .

Fig. 23. Deviation in Sh under numerous values of Nt .

Fig. 24. Deviation in Sh under numerous values of Nb .

Code validation

Table 1 shows the limiting case of our current work with the existing literature. If we set $\Gamma = 0$ or $n = 1$, our flow model meets the conventional Jaffrey Hamel flow model. Few results for the skin friction coefficient C_f and Nusselt number Nu for various values of α and an opening angle α of the channel are shown. Excellent agreement can be seen, which justifies the consistency of our numerical results.

Table 1. Assessment of numerical values $F'(1)$ and $-g'(1)$ for various values of angle α for fixed values of $Re = 50$, $Pr = 3.0$, $Nb = 0.4$, $Nt = 0.6$ and $\Gamma = 0$ or $n = 1$

α	Present Work		Dogonchi and Ganji [6]		Turkyilmazoglu [24]	
	$F'(1)$	$-g'(1)$	$F'(1)$	$-g'(1)$		
-5°	-5.1308	0.03159	-5.1309222926	0.0315761821	-5.130921689	0.03157845854
-5°	-4.6518	0.03158	-4.6521591354	0.0315761821	-4.652183982	0.03734696604
-5°	-2.8339	0.04216	-2.8339514330	0.0421517243	-2.833915413	0.04214811723
-5°	0	0.04641	0	0.0464015106	0	0.04640127099
-5°	3.6690	0.05023	3.6697111853	0.0502423154	3.654305033	0.05052578617
5°	-3.5083	0.03478	-3.5081031667	0.0347758169	-3.508090102	0.03475109986
5°	-1.1093	0.03998	-1.1093265266	0.0399820121	-1.109360533	0.03999321083
5°	0	0.04641	0	0.0464015106	0	0.04640127099

5. Conclusions

1. In both channels, the angle α and Reynolds Re results in the opposite trend for the velocity profile.
2. Similar effects are detected for Weissenberg number We and power index n in both channels.
3. A strong magnetic field Ha decreases the fluid flow rate and reduces the temperature of the fluid and can also be used to diminish the concentration of the fluid in both channels.
4. The rise in temperature is recorded due to growing values of Reynolds Re and Prandtl Pr in both channels.

5. Brownian and thermophoresis diffusion parameters Nb and Nt display similar effects in both channels. Decreases in temperature and concentration are noticed upon increasing these parameters.
6. Growing values of Eckert number Ec , temperature and concentration boosts.
7. The concentration and temperature are maximal in the middle of the channel and decrease rapidly near the wall of the channels.
8. The contribution of the heat generation parameter to the velocity and concentration profiles is quite minor, although the temperature of the nanofluid is rapidly growing.
9. The role of the Reynold number in the surface drag force and Nusselt number is contrary in narrow and deviating channels.

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