Prescribed Performance Tracking Control for High-Order Uncertain Nonlinear Systems

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Abstract This paper considers the prescribed performance tracking control for high-order uncertain nonlinear systems. For any initial system condition, a state feedback control is designed, which guarantees the prescribed tracking performance and the boundedness of closed-loop signals. The proposed controller can be implemented without using any approximation techniques for estimating unknown nonlinearities. In this respect, a significant advantage of this article is that the explosion of complexity is avoided, which is raised by backstepping-like approaches that are typically employed to the control of uncertain nonlinear systems, and a low-complexity controller is achieved. Moreover, contrary to the existing results in existing literature, the restrictions on powers of high-order nonlinear systems are relaxed to make the considered problem having stronger theoretical and practical values. The effectiveness of the proposed scheme is verified by some simulation results.

Keywords Prescribed performance · High-order nonlinear systems · Tracking control · Low-complexity control

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1 Introduction

Tracking control problem of uncertain nonlinear systems has attracted many scholars' attention. The objective of tracking control is to drive the system output to track a prescribed reference signal asymptotically or with some prescribed accuracy, and much effort has been devoted to different types of systems via adaptive backstepping control scheme, such as the strict-feedback nonlinear systems in [1,2], nonlinear pure-feedback systems in [3,4], stochastic nonlinear systems in [5,6]. Furthermore, the nonlinear systems have different unknown issues or/and requirements of performances, such as the systems with unknown control directions [7], with actuator faults [8], full state-constrained [9,10], and so on.

During the past several years, great progresses of tracking control have been achieved for high-order nonlinear systems, which have the general form as

$$\begin{cases} \dot{x}_j = \psi_1(\overline{x}_j)x_{j+1}^{q_j} + f_j(\overline{x}_j), \quad j = 1, \cdots, n-1 \\ \dot{x}_n = \psi_n(\overline{x}_n)u^{q_n} + f_n(\overline{x}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $\overline{x}_j = [x_1, \cdots, x_j]^T \in \mathbb{R}^j$, $j = 1, \cdots, n$ are measurable state variables; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are input and output, respectively; $f_j(\overline{x}_j)$ and $\psi_j(\overline{x}_j)$ are nonlinear functions; q_j is the power of system (1). If there exists at least one $q_j > 1$, system (1) is called the high-order nonlinear system. Most of works about tracking control for system (1) concentrate on $q_j \in \mathbb{R}_{odd}^{\geq 1} = \{p/q : p$ and q are odd integers, $p/q \geq 1\}$ (see [11–20] and the references therein). Using the technique of changing supply rate, an adaptive tracking controller was designed for cascade systems with nonlinear parameterization [12]. By adding a power integrator, a systematic approach is developed to construct an adaptive practical tracking controller [13]. Using a high gain function to compensate the serious system uncertainties, an adaptive controller with a dynamic high gain is proposed [17]. Without imposing feasibility conditions, an adaptive practical tracking control scheme was designed for full-state constrained systems [18]. Different from aforementioned works, in [21], Man and Liu relaxed the restrictions on powers such that the powers are not only odd positive constants, but also positive continuous functions or positive constants. In this situation, terms $x_{i+1}^{q_i}$ and u^{q_n} in system (1) are written as $\lceil x_{i+1} \rceil^{q_i}$ and $\lceil u \rceil^{q_n}$, where the notation $\lceil x \rceil^q = \text{sign}(x)|x|^q$, with $\text{sign}(x)$ denoting its sign function, for any $x \in \mathbb{R}$.

Despite the recent progress of the tracking control for system (1), there are still shortcomings as in the following aspects: (1) A common characteristic is that only the convergence of the tracking error can be guaranteed to a residual set, whose size depends on explicit design parameters and some unknown bounded terms, but transient performance and the convergence rate cannot be prescribed. (2) The aforementioned works resorted to adaptive technique and approximation structure to deal with the uncertain terms of the systems, which may increase extra calculations and lead to complexity of controller designs.

(3) Strict assumptions need to be imposed on the nonlinearities or/and the powers of the high-order systems. To the authors' knowledge, for high-order nonlinear systems, only a few works have achieved the prescribed transient performance [22,23]. In [22], the tracking performance of the proposed control is ensured within preassigned bounds, regardless of high-power virtual control variables. In [23], a scheme of practical tracking control was developed within the funnel control framework, which rendered the tracking error to forever evolve within a prescribed performance funnel. However, the powers of the system in [22,23] are limited to positive odd integers. Therefore, relaxing restrictions on powers is still an open issue.

In order to achieve predefined transient and steady-state bounds on the output tracking errors, prescribed performance control (PPC) technology was proposed by Bechlioulis and Rovithakis in [24], and has been developed to different types of nonlinear systems [25–31]. As further development, Zhang et al. proposed a fault-tolerant control scheme which can guarantee prescribed tracking performance of systems with unknown control directions [32]. Yang et al. solved the problem of PPC for pure-feedback nonlinear systems with uncertainties [33]. Fotiadis and Rovithakis gave the PPC for multi-input multi-output nonlinear systems with discontinuous reference signals [34]. However, none of these works considered and dealt with high-order control variables of nonlinear systems.

Motivated by the above discussions, we consider the prescribed performance tracking control for a class of high-order uncertain nonlinear systems in this paper. Given any initial system condition and any output performance requirements, a tracking controller only composing of transformed error surfaces and designed constants is proposed. Compared with most of related works, the main contributions of this paper are threefold:

- In contrast with most existing tracking control results, the proposed state feedback control can achieve the steady state values of tracking error, the convergence rate and the bound of overshoot.
- In contrast with the relevant literatures, the controller design does not utilize derivatives of virtual control signals, and approximation structures (i.e., neural networks or fuzzy logic systems), as well as some analytical techniques (i.e., adaptive dynamic gain or/and the power integrator). Thus, the explosion of complexity and the burden of additional computational cost are avoided, and the controller can be implemented easily.
- Compared with the existing works of PPC [22,23], this work not only relax the restriction on powers but also consider the external disturbances in systems.

The remainder of this paper is organized as follows. Section 2 provides the preliminaries of the prescribed performance control problem. Section 3 is to establish the tracking control scheme and summarize the main results of this paper. Section 4 provides simulation results. Finally, some conclusions are given in Section 5.

Notations We use the following notations for convenience throughout this paper. $\mathbb{R}^i$ denotes the set of all real i-dimensional vectors; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\lceil x \rceil^q = \mathrm{sign}(x)|x|^q$, where $\mathrm{sign}(x)$ denotes its sign function.

2 Preliminaries

2.1 Prescribed performance

The prescribed performance control means that the output tracking error converges to a predefined arbitrary small residual set with convergence rate no less than a certain prespecified value, and displays maximum overshoot less than a predefined value [26]. In this respect, consider a measurable tracking error $\varepsilon(t):\ \mathbb{R}_+ \to \mathbb{R}$ and a prescribed function $\gamma(t) = (\ell_0 - \ell_\infty)e^{-\mu t} + \ell_\infty$ with $\ell_0,\ \ell_\infty$ and μ being positive constants, and $\ell_0 > \ell_\infty$. Given any initial condition $\varepsilon(0)$, by choosing ℓ_0 such that $\ell_0 > |\varepsilon(0)|$, the prescribed performance is achieved if the following inequality

$$-\gamma(t) < \varepsilon(t) < \gamma(t), \quad \forall t \geq 0$$

holds. Clearly, $-\gamma(0)$ and $\gamma(0)$ represent the lower bound of an undershoot and the upper bound of an overshoot of $\varepsilon(t)$, respectively. The scalar μ represents the convergence rate. $\ell_\infty = \lim_{t\to+\infty}\gamma(t)$ represents the maximum size of $\varepsilon(t)$ at the steady state, which can be set arbitrarily small, and $\varepsilon(t)$ can converge to zero. Thus, the appropriate choice of prescribed function $\gamma(t)$ reflects performance characteristics of the tracking error $\varepsilon(t)$.

2.2 Problem statement

Consider the following high-order nonlinear systems

$$\begin{cases} \dot{x}_i = \psi_i(\overline{x}_i)\lceil x_{i+1}\rceil^{q_i} + f_i(\overline{x}_i) + \varrho_i(t), \quad i = 1, \cdots, n-1 \\ \dot{x}_n = \psi_n(x)\lceil u \rceil^{q_n} + f_n(x) + \varrho_n(t) \\ y = x_1 \end{cases} \tag{2}$$

where $\overline{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$ and $x = \overline{x}_n$ are measurable system states; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are control input and system output, respectively; $f_i(\overline{x}_i)$ and $\psi_i(\overline{x}_i)$ are nonlinear functions with unknown analytical expressions; $\varrho_i(t) \in \mathbb{R}$ is a bounded unknown piecewise continuous function; $q_i \geq 1$ is the power of system (2). In this work, there exists at least one power $q_i > 1$. The functions $f_i(\overline{x}_i)$ and $\psi_i(\overline{x}_i)$ are locally Lipschitz in $\overline{x}_i$.

The objective of this work is that: consider the high-order uncertain nonlinear system (2) and the desired output tracking trajectory $y_r(t)$. Given any initial condition $x(0) \in \mathbb{R}^n$, a low-complexity state feedback controller is designed such that: (1) the system output $y(t)$ tracks $y_r(t)$ with prescribed transient and steady-state performance; (2) all the signals of the closed-loop systems remain bounded.

In order to design the required control scheme, the following assumptions and lemmas are needed.

Assumption 1 For control coefficients $\psi_i(\overline{x}_i)$, $i = 1, \cdots, n$, there exists unknown constant $\underline{\psi}_i$ such that $0 < \underline{\psi}_i \leq \psi_i(\overline{x}_i)$.

Assumption 2 The trajectory signal $y_r(t)$ and its first-order derivative $\dot{y}_r(t)$ are bounded.

Lemma 1 [21] For any constant $q \geq 1$, and any real-valued functions ω_1 and ω_2, the following inequality

$$|\lceil\omega_1\rceil^q - \lceil\omega_2\rceil^q| \leq |\omega_1 - \omega_2|^q + q \cdot |\omega_1 - \omega_2| \cdot (|\omega_1|^{q-1} + |\omega_2|^{q-1})$$

holds.

Lemma 2 [35] For any constant $q > 0$, and any real-valued functions ω_1 and ω_2, the following inequality

$$|\omega_1 + \omega_2|^q \leq m(|\omega_1|^q + |\omega_2|^q)$$

holds, where $m = 1$ when $q < 1$, and $m = 2^{q-1}$ when $q \geq 1$.

Lemma 3 [36] For any constants $q_1 > 0$, $q_2 > 0$ and $c > 0$, and any real-valued functions ω_1 and ω_2, the following inequality

$$|\omega_1|^{q_1}|\omega_2|^{q_2} \leq c\frac{q_1}{q_1 + q_2}|\omega_1|^{q_1+q_2} + c^{-\frac{q_1}{q_2}}\frac{q_2}{q_1 + q_2}|\omega_2|^{q_1+q_2}$$

holds.

Remark 1 In most related results of high-order nonlinear systems, some restrictive assumptions on powers and nonlinearities of systems are needed. These assumptions can be described as: (1) Powers are allowed only to be odd integers or $q_i \in \{p/q : \ p \text{ and } q \text{ are odd integers}, \ p/q \geq 1\}$ [11–19, 37–39]. Some results were achieved even under the assumption that odd integer powers q_i meet $q_1 \geq \cdots q_n \geq 1$; (2) The function $f_i(\cdot)$ satisfies extra growth condition, such as $f_i(\overline{x}_i) \leq \overline{f}_i(\overline{x}_i)\sum_{j=1}^{i}|x_j|^{q_j}$, where $\overline{f}_i(\overline{x}_i)$ is known positive smooth function. In this work, we only assume that q_i is any positive constant satisfying $q_i \geq 1$, $i = 1, \cdots, n$, and the nonlinear function $f_i(\overline{x}_i)$ is not imposed any extra growth condition.

Remark 2 When $q_i = 1$, system (2) can be regarded as a general representation of strict-feedback nonlinear systems [1, 2, 9]. If there exists at least one power satisfying $q_i > 1$, system (2) is called the high-order nonlinear system. Compared with some related works of tracking control, this work only assumes the tracking signal $y_r(t)$, its first-order derivative $\dot{y}_r(t)$, and the unmatched disturbances $\varrho_i(t),\ i=1,\cdots,n$ are bounded, but no preliminary knowledge for $y_r(t)$, $\dot{y}_r(t)$ and $\varrho_i(t)$ is available.

Next, we shall design a computationally efficient controller and prove that the controller can fulfill the control objective.

2.3 Control design

Define the function $F:(-1,1)\to\mathbb{R}$ with the form $F(*)=\ln(\frac{1+*}{1-*})$. Given a desired trajectory signal $y_r(t)$ and any initial system condition $x(0)\in\mathbb{R}^n$. In order to ensure a satisfactory tracking behavior, a group of performance functions are introduced

$$\gamma_i(t)=(\ell_i-\ell_{i,\infty})e^{-\mu_i t}+\ell_{i,\infty},\quad i=1,\ \cdots,\ n, \tag{3}$$

where the positive parameter $\ell_{i,\infty}$ denotes the maximum steady state error, and $\mu_i>0$ being the required minimum exponential convergence rate, freely chosen by the designer. The positive parameter ℓ_i needs to meet some initial conditions, which is described in the following design process, and $\ell_i>\ell_{i,\infty}$.

The detailed design process is given as follows.

Step I: Select performance function $\gamma_1(t)$ to satisfy $\gamma_1(0)>|x_1(0)-y_r(0)|$, and design the first virtual control signal as

$$\alpha_1(x_1,t)=-\beta_1 F\Big(\frac{x_1-y_r(t)}{\gamma_1(t)}\Big) \tag{4}$$

with β_1 being a positive constant.
Step II: Select performance function $\gamma_2(t)$ only satisfying $\gamma_2(0)>|x_2(0)-\alpha_1(x_1(0),0)|$, and design the virtual control signal as

$$\alpha_2(x_1,x_2,t)=-\beta_2 F\Big(\frac{x_2-\alpha_1(x_1,t)}{\gamma_1(t)}\Big) \tag{5}$$

with β_2 being a positive constant.
Step III: Select performance functions $\gamma_i(t)$ satisfying $\gamma_i(0)>|x_i(0)-\alpha_{i-1}(x_1(0),\cdots,x_{i-1}(0),0)|$, $i=3,\cdots,n-1$, repeat step II for all remaining virtual control signals

$$\alpha_i(x_1,\cdots,x_i,t)=-\beta_i F\Big(\frac{x_i-\alpha_{i-1}(x_1,\cdots,x_{i-1},t)}{\gamma_i(t)}\Big) \tag{6}$$

with β_i being a positive constant.
Step IV: Finally, design the control input as

$$u(x_1,\cdots,x_n,t)=-\beta_n F\Big(\frac{x_n-\alpha_{n-1}(x_1,\cdots,x_{n-1},t)}{\gamma_n(t)}\Big) \tag{7}$$

by selecting a performance function $\gamma_n(t)$ satisfying $\gamma_n(0) > |x_n(0)-\alpha_{n-1}(x_1(0),\cdots,x_{n-1}(0),0)|$, where β_n is a positive constant.

Remark 3 The controller proposed in (4)–(7) has simple structure that can be easily computed. First, no approximation structures (i.e., neural networks or fuzzy logic systems) have been employed to deal with unknown nonlinearities of the system. Second, the virtual control signals and the actual control in (4)–(7) do not comprise derivatives of $\alpha_i(t)$, and not using additional filters of $\alpha_i(t)$. Thus, no hard calculations exist in the design process, and the control input can be implemented easily.

Remark 4 The selection of the control gains β_i only depend on the low bound of the control coefficients $\psi_i(\overline{x}_i),\ i=1,\cdots,n$, and has no connection with any other prior knowledge of system nonlinearities and desired trajectory signal $y_r(t)$. The details of selection shall be given in Theorem 1.

3 Performance Analysis

We first define $e_1 = x_1 - y_r$, $e_i = x_i - \alpha_{i-1}$, $i = 2,\ \cdots,\ n$, and the normalized state errors as follows:

$$\varepsilon_i = \frac{e_i}{\gamma_i(t)}, \quad i = 1,\ \cdots,\ n. \tag{8}$$

From (4)–(7), the virtual control signals and the control law are written as

$$\alpha_i(x_1,\cdots,x_i,t) = \alpha_i(\varepsilon_i) = -\beta_i \ln\frac{1+\varepsilon_i(t)}{1-\varepsilon_i(t)}, \quad i = 1,\ \cdots,\ n-1, \tag{9}$$

$$u(x_1,\cdots,x_n,t) = u(\varepsilon_n) = -\beta_n \ln\frac{1+\varepsilon_n(t)}{1-\varepsilon_n(t)}. \tag{10}$$

By (8), the state variables can be represented as

$$\begin{cases} x_1 = \varepsilon_1\gamma_1(t) + y_r(t) \\ x_i = \varepsilon_i\gamma_i(t) + \alpha_{i-1}(\varepsilon_{i-1}), \quad i = 2,\cdots,n. \end{cases} \tag{11}$$

Taking derivative of ε_i with respect to time and invoking (2) as well as (11), it yileds

$$\begin{aligned} \dot{\varepsilon}_1 &= \Phi_1(t,\varepsilon_1,\varepsilon_2) \\ &= \frac{1}{\gamma_1(t)}\Big[\psi_1\big(\varepsilon_1\gamma_1(t)+y_r(t)\big)\lceil\varepsilon_2\gamma_2(t)+\alpha_1(\varepsilon_1)\rfloor^{q_1} \\ &\qquad + f_1\big(\varepsilon_1\gamma_1(t)+y_r(t)\big) + \varrho_1(t) - \dot{y}_r(t) - \dot{\gamma}_1(t)\varepsilon_1\Big], \end{aligned} \tag{12}$$

$$
\begin{aligned}
\dot{\varepsilon}_i &= \Phi_i(t, \varepsilon_1, \cdots, \varepsilon_{i+1}) \\
&= \frac{1}{\gamma_i(t)} \Big[\psi_i\big(\varepsilon_1\gamma_1(t) + y_r(t), \cdots, \varepsilon_{i+1}\gamma_{i+1}(t) + \alpha_i(\varepsilon_i)\big) \big\lceil \varepsilon_{i+1}\gamma_{i+1}(t) + \alpha_i(\varepsilon_i) \big\rceil^{q_i} \\
&\quad + f_i\big(\varepsilon_1\gamma_1(t) + y_r(t), \cdots, \varepsilon_{i+1}\gamma_{i+1}(t) + \alpha_i(\varepsilon_i)\big) + \varrho_i(t) \\
&\quad - \frac{d\alpha_{i-1}}{d\varepsilon_{i-1}} \Phi_{i-1}(t, \varepsilon_1, \cdots, \varepsilon_i) - \dot{\gamma}_i(t)\varepsilon_i \Big], \quad i = 2, \cdots, n-1, \qquad (13) \\
\dot{\varepsilon}_n &= \Phi_n(t, \varepsilon_1, \cdots, \varepsilon_n) \\
&= \frac{1}{\gamma_n(t)} \Big[\psi_n\big(\varepsilon_1\gamma_1(t) + y_r(t), \cdots, \varepsilon_n\gamma_n(t) + \alpha_{n-1}(\varepsilon_{n-1})\big) \big\lceil u(\varepsilon_n) \big\rceil^{q_n} \\
&\quad + f_n\big(\varepsilon_1\gamma_1(t) + y_r(t), \cdots, \varepsilon_n\gamma_n(t) + \alpha_{n-1}(\varepsilon_{n-1})\big) + \varrho_n(t) \\
&\quad - \frac{d\alpha_{n-1}}{d\varepsilon_{n-1}} \Phi_{n-1}(t, \varepsilon_1, \cdots, \varepsilon_n) - \dot{\gamma}_n(t)\varepsilon_n \Big]. \qquad (14)
\end{aligned}
$$

The normalized error vector $\varepsilon = [\varepsilon_1,\ \varepsilon_2,\ \cdots,\ \varepsilon_n]^T$ can be written as

$$
\dot{\varepsilon} = \Phi(t, \varepsilon) = \begin{bmatrix} \Phi_1(t, \varepsilon_1, \varepsilon_2) \\ \vdots \\ \Phi_n(t, \varepsilon_1, \cdots, \varepsilon_n) \end{bmatrix}. \qquad (15)
$$

Define the open set $\Omega = \Omega_1 \times \Omega_2 \times \cdots \Omega_n$ with $\Omega_i = (-1, 1),\ i = 1, \cdots, n$, and the signal $t \to \zeta_i(t)$ with

$$
\zeta_i(t) = \ln \frac{1 + \varepsilon_i(t)}{1 - \varepsilon_i(t)}, \quad i = 1, \cdots, n. \qquad (16)
$$

Then, (9) and (10) are rewritten as follows:

$$
\begin{aligned}
\alpha_i(\varepsilon_i) &= -\beta_i\zeta_i(t), \quad i = 1, \cdots, n-1, \qquad (17) \\
u(\varepsilon_n) &= -\beta_n\zeta_n(t). \qquad (18)
\end{aligned}
$$

Performance Analysis Philosophy: If $\varepsilon \in \Omega$, then $\zeta_i(t)$ is well defined. On the other hand, if $\zeta_i \in \mathcal{L}^\infty,\ i = 1, \cdots, n$, one can deduce that ε evolves strictly within a compact subset of Ω. Furthermore, the boundedness of all signals in the closed-loop systems can be guaranteed. Therefore, we shall first prove the existence and uniqueness of a maximal solution $\varepsilon :\ [0,\ T_m) \to \Omega$ of (15). Sencondly, we prove that $\zeta_i \in \mathcal{L}^\infty,\ i = 1, \cdots, n$. Consequently, the control objective can be achieved by the proposed control scheme.

Next, we summarize the main results of this paper as follows.

Theorem 1: Consider the high-order nonlinear system (2) under the Assumption 1, and the desired trajectory $y_r(t)$ obeys Assumption 2. Given any initial conditions meeting $\varepsilon_i(0) < 1$, $i = 1, \cdots, n$, the proposed control schemes (4)–(7) guarantee the following:

1) The system output tracks the trajectory $y_r(t)$ with prescribed performance, i.e.

$$|e_1(t)| < \gamma_1(t), \quad t \geq 0.$$

2) The remainder closed-loop signals are bounded.

Proof: We proceed in two parts to prove Theorem 1. In Part I, the existence and uniqueness of a maximal solution $\varepsilon(t) : [0, T_m) \to \Omega$ of (15) is guaranteed (i.e., $\varepsilon(t) \in \Omega$, $\forall t \in [0, T_m)$). Then, in Part II, we prove that the control schemes (4)–(7) guarantee the boundedness of all closed-loop signals of (9)–(14) and $\varepsilon(t)$ evolves in a compact subset of Ω for all $t \in [0, T_m)$. Further, we obtain $T_m = +\infty$.

Part I: We have chosen the performance functions satisfying $\gamma_1(0) > |x_1(0) - y_r(0)|$ and $\gamma_i(0) > |x_i(0) - \alpha_{i-1}(\overline{x}_{i-1}(0), 0)|$, $i = 2, \cdots, n$, so $|\varepsilon_i(0)| < 1$ can be guaranteed, which results in $\varepsilon(0) \in \Omega$. Therefore, the set Ω is nonempty. Additionally, $\gamma_i(t)$ and $y_r(t)$ are bounded and continuous differentiable, the functions f_i, ψ_i and $\varrho_i(t)$ are locally Lipschitz and piecewise continuous in their arguments, and the signals α_i and u are smooth over Ω. Therefore, $\Phi(t, \varepsilon)$ is bounded and continuous in t and locally Lipschitz in ε over the set Ω. By Theorem 54 in [40], there exists a unique maximal solution $\varepsilon : [0, T_m) \to \Omega$ of (15) such that $\varepsilon(t) \in \Omega$ for all $t \in [0, T_m)$.

Part II: In Part I, $\varepsilon(t) \in \Omega$ has been proved, which leads to $\zeta_i(t)$ in (12) being well defined for all $t \in [0, T_m)$. Note that the following discussions are based on the time interval $[0, T_m)$, and $|\varepsilon_i(t)| < 1$ for $t \in [0, T_m)$.

Step 1: Define the Lyapunov function $V_1 = \frac{1}{2}\zeta_1^2$. Taking derivative of V_1 with respect to time, we get

$$\begin{aligned}\dot{V}_1 = & \frac{2\zeta_1}{(1-\varepsilon_1^2)\gamma_1(t)}\big(\psi_1(\lceil x_2 \rceil^{q_1} - \lceil \alpha_1 \rceil^{q_1}) + \psi_1 \lceil \alpha_1 \rceil^{q_1} \\ & + f_1 + \varrho_1(t) - \dot{y}_r(t) - \dot{\gamma}_1(t)\varepsilon_1\big). \end{aligned} \tag{19}$$

Using Lemma 1, it follows that

$$\begin{aligned}|\lceil x_2 \rceil^{q_1} - \lceil \alpha_1 \rceil^{q_1}| &\leq |x_2 - \alpha_1|^{q_1} + q_1|x_2 - \alpha_1| \cdot (|x_2|^{q_1-1} + |\alpha_1|^{q_1-1}) \\ &= \gamma_2^{q_1}(t)|\varepsilon_2|^{q_1} + q_1\gamma_2(t)|\varepsilon_2|\big(|x_2|^{q_1-1} + |\alpha_1|^{q_1-1}\big). \end{aligned} \tag{20}$$

Furthermore, by Lemma 2, it follows that

$$|x_2|^{q_1-1} \leq m\big(\gamma_2^{q_1-1}(t)|\varepsilon_2|^{q_1-1} + |\alpha_1|^{q_1-1}\big). \tag{21}$$

Substituting (21) into (20) leads to

$$|\lceil x_2 \rceil^{q_1} - \lceil \alpha_1 \rceil^{q_1}| \leq (1 + mq_1)\gamma_2^{q_1}(t)|\varepsilon_2|^{q_1} + (1+m)q_1\gamma_2(t)|\varepsilon_2||\alpha_1|^{q_1-1}, \tag{22}$$

then substituting (22) into (19) leads to

$$\dot{V}_1 \leq \frac{2}{(1-\varepsilon_1^2)\gamma_1(t)}\big(|\zeta_1|\Delta_{1,1} + \Delta_{1,2}|\zeta_1||\alpha_1|^{q_1-1}\big) + \frac{2\zeta_1\psi_1}{(1-\varepsilon_1^2)\gamma_1(t)}\lceil \alpha_1 \rceil^{q_1}, \tag{23}$$

where

$$\begin{aligned}\Delta_{1,1} &= \psi_1(1+mq_1)\gamma_2^{q_1}(t)|\varepsilon_2|^{q_1} + |f_1 + \varrho_1(t) - \dot{y}_r(t) - \dot{\gamma}_1(t)\varepsilon_1|,\\ \Delta_{1,2} &= \psi_1(1+m)q_1\gamma_2(t)|\varepsilon_2|.\end{aligned}$$

We establish the boundedness of $\Delta_{1,1}$ and $\Delta_{1,2}$ as follows:
1) Clearly, $\gamma_i(t) \in \mathcal{L}^\infty$, $\dot{\gamma}_i(t) \in \mathcal{L}^\infty$, $\varrho_1(t) \in \mathcal{L}^\infty$, and $\varepsilon_i < 1$, $i = 1,\ 2$.
2) By Assumption 2, $y_r(t) \in \mathcal{L}^\infty$, $\dot{y}_r(t) \in \mathcal{L}^\infty$.
3) Notice that $x_1 = \varepsilon_1\gamma_1(t) + y_r(t)$, one has $x_1 \in \mathcal{L}^\infty$.
4) In view that $f_1(x_1)$ and $\psi_1(x_1)$ are locally Lipschitz in x_1 with $x_1 \in \mathcal{L}^\infty$, one can get $f_1(x_1) \in \mathcal{L}^\infty$ and $\psi_1(x_1) \in \mathcal{L}^\infty$.

From the above analysis, the variables $\Delta_{1,1}$ and $\Delta_{1,2}$ are bounded over $[0,\ T_m)$, that is, there exist positive constants $\eta_{1,1}$ and $\eta_{1,2}$ satisfying $\Delta_{1,1} \leq \eta_{1,1}$ and $\Delta_{1,2} \leq \eta_{1,2}$. According to (17) for $i = 1$, we obtain

$$|\zeta_1||\alpha_1|^{q_1-1} = \beta_1^{q_1-1}|\zeta_1|^{q_1}, \quad \zeta_1\lceil\alpha_1\rfloor^{q_1} = -\beta_1^{q_1}|\zeta_1|^{q_1+1}. \tag{24}$$

With the help of Lemma 3, the following inequalities

$$\Delta_{1,1}|\zeta_1| \leq \eta_{1,1}|\zeta_1| \leq |\zeta_1|^{q_1+1} + q_1(\frac{\eta_{1,1}}{1+q_1})^{1+\frac{1}{q_1}}, \tag{25}$$

$$\Delta_{1,2}|\zeta_1||\alpha_1|^{q_1-1} \leq \eta_{1,2}\beta_1^{q_1-1}|\zeta_1|^{q_1} \leq |\zeta_1|^{q_1+1} + q_1^{q_1}(\frac{\eta_{1,2}\beta_1^{q_1-1}}{1+q_1})^{1+q_1} \tag{26}$$

hold, which together with (23)–(24), lead to

$$\dot{V}_1 \leq \frac{2}{(1-\varepsilon_1^2)\gamma_1(t)}\big(\eta_{1,3} - (\underline{\psi}_1\beta_1^{q_1} - 2)|\zeta_1|^{q_1+1}\big), \tag{27}$$

where

$$\eta_{1,3} = q_1(\frac{\eta_{1,1}}{1+q_1})^{1+\frac{1}{q_1}} + q_1^{q_1}(\frac{\eta_{1,2}\beta_1^{q_1-1}}{1+q_1})^{1+q_1}.$$

Design the control gain β_1 such that $\beta_1 > (2/\underline{\psi}_1)^{1/q_1}$. Therefore, $\dot{V}_1 < 0$ as $|\zeta_1|^{1+q_1} > \eta_{1,3}/(\underline{\psi}_1\beta_1^{q_1} - 2)$. By the definition of V_1, it holds that

$$|\zeta_1| \leq \overline{\zeta}_1 = \max\Big\{|\zeta_1(0)|,\ (\frac{\eta_{1,3}}{\underline{\psi}_1\beta_1^{q_1} - 2})^{\frac{1}{1+q_1}}\Big\}, \quad t \in [0,\ T_m). \tag{28}$$

By the solution of inverse logarithmic function in (16) for $i = 1$, we get

$$-1 < \frac{e^{-\overline{\zeta}_1} - 1}{e^{-\overline{\zeta}_1} + 1} = \underline{\varepsilon}_1 \leq \varepsilon_1(t) \leq \overline{\varepsilon}_1 = \frac{e^{\overline{\zeta}_1} - 1}{e^{\overline{\zeta}_1} + 1} < 1, \quad t \in [0,\ T_m). \tag{29}$$

Consequently, by (17), $|\alpha_1| \leq \beta_1\overline{\zeta}_1$, i.e. $\alpha_1 \in \mathcal{L}^\infty$. From (11), we can get $x_2 \in \mathcal{L}^\infty$ by the fact that $\alpha_1 \in \mathcal{L}^\infty$, $\gamma_1(t) \in \mathcal{L}^\infty$ and $\varepsilon_2 \in \Omega_1$. Taking the derivative of α_1, gives

$$\dot{\alpha}_1 = \frac{d\alpha_1}{d\varepsilon_1}\Phi_1(t,\varepsilon_1,\varepsilon_2) = -\frac{2\beta_1}{(1-\varepsilon_1^2)\gamma_1(t)}\Phi_1(t,\varepsilon_1,\varepsilon_2). \tag{30}$$

Taking (12) into account, $\varPhi_1(t,\varepsilon_1,\varepsilon_2)$ can be rewritten as

$$\varPhi_1(t,\varepsilon_1,\varepsilon_2)=\frac{1}{\gamma_1(t)}\big(\psi_1(x_1)\lceil x_2\rceil^{q_1}+f_1(x_1)+\varrho_1(t)-\dot{y}_r(t)-\dot{\gamma}_1(t)\varepsilon_1\big). \quad (31)$$

Clearly, the right handside of (31) is bounded, which leads to $\varPhi_1(t,\varepsilon_1,\varepsilon_2)\in\mathcal{L}^\infty$. From (30), it is straightforward to deduce $\dot{\alpha}_1\in\mathcal{L}^\infty$.

Step i ($i=2,\cdots,n-1$): Define the Lyapunov function $V_i=\frac{1}{2}\zeta_i^2$. Taking derivative of V_i with respect to time gives

$$\dot{V}_i=\frac{2\zeta_i}{(1-\varepsilon_i^2)\gamma_i(t)}\big(\psi_i\lceil x_{i+1}\rceil^{q_i}+f_i+\varrho_i(t)-\dot{\alpha}_{i-1}-\varepsilon_i\dot{\gamma}_i(t)\big).$$

Applying recursively in line with the aforementioned proof of Step 1, it yields

$$\dot{V}_i\le\frac{2}{(1-\varepsilon_i^2)\gamma_i(t)}\big(\varDelta_{i,1}|\zeta_i|+\varDelta_{i,2}|\zeta_i||\alpha_i|^{q_i-1}\big)+\frac{2\zeta_i\psi_i}{(1-\varepsilon_i^2)\gamma_i(t)}\lceil\alpha_i\rceil^{q_i}, \quad (32)$$

where

$$\begin{aligned}\varDelta_{i,1}&=\psi_i(1+mq_i)\gamma_{i+1}^{q_i}(t)|\varepsilon_{i+1}|^{q_i}+|f_i+\varrho_i(t)-\dot{\alpha}_{i-1}-\varepsilon_i\dot{\gamma}_i(t)|,\\ \varDelta_{i,2}&=\psi_i(1+m)q_i\gamma_{i+1}(t)|\varepsilon_{i+1}|.\end{aligned}$$

Similar to Step 1, $\varDelta_{i,1}\in\mathcal{L}^\infty$ and $\varDelta_{i,2}\in\mathcal{L}^\infty$ are ensured recursively, i.e., $\varDelta_{i,1}\le\eta_{i,1}$ and $\varDelta_{i,2}\le\eta_{i,2}$ hold, where $\eta_{i,1}$ and $\eta_{i,2}$ are positive constants. According to (17), one can get

$$|\zeta_i||\alpha_i|^{q_i-1}=\beta_i^{q_i-1}|\zeta_i|^{q_i},\quad \zeta_i\lceil\alpha_i\rceil^{q_i}=-\beta_i^{q_i}|\zeta_i|^{q_i+1}. \quad (33)$$

Furthermore, we can derive

$$\dot{V}_i\le\frac{2}{(1-\varepsilon_i^2)\gamma_i(t)}\big(\eta_{i,3}-(\underline{\psi}_i\beta_i^{q_i}-2)|\zeta_i|^{q_i+1}\big), \quad (34)$$

where

$$\eta_{i,3}=q_i(\frac{\eta_{i,1}}{1+q_i})^{1+\frac{1}{q_i}}+q_i^{q_i}(\frac{\eta_{i,2}\beta_i^{q_i-1}}{1+q_i})^{1+q_i}. \quad (35)$$

Designing the control gains $\beta_i,\ i=2,\cdots,n-1$ such that $\beta_i>(2/\underline{\psi}_i)^{1/q_i}$, a similar conclusion with (28) is

$$|\zeta_i|\le\bar{\zeta}_i=\max\Big\{|\zeta_i(0)|,\ (\frac{\eta_{i,3}}{\underline{\psi}_i\beta_i^{q_i}-2})^{\frac{1}{1+q_i}}\Big\},\quad t\in[0,\ T_m). \quad (36)$$

Correspondingly, from (16), we obtain

$$-1<\frac{e^{-\bar{\zeta}_i}-1}{e^{-\bar{\zeta}_i}+1}=\underline{\varepsilon}_i\le\varepsilon_i(t)\le\bar{\varepsilon}_i=\frac{e^{\bar{\zeta}_i}-1}{e^{\bar{\zeta}_i}+1}<1,\quad i=2,\cdots,n-1. \quad (37)$$

As a result, $|\alpha_i| \le \beta_i \bar{\zeta}_i,\ i = 2, \cdots, n-1$. From (11), the state variables $x_i,\ i = 3, \cdots, n$ are bounded due to the fact that $\alpha_{i-1} \in \mathcal{L}^\infty$, $\gamma_i(t) \in \mathcal{L}^\infty$ and $\varepsilon_i \in \Omega_i$. Furthermore, the derivative of α_i is

$$\dot{\alpha}_i = \frac{d\alpha_i}{d\varepsilon_i} \Phi_i(t, \varepsilon_1, \cdots, \varepsilon_i) = -\frac{2\beta_i}{(1-\varepsilon_i^2)\gamma_i(t)} \Phi_i(t, \varepsilon_1, \cdots, \varepsilon_i) \tag{38}$$

with

$$\Phi_i(t, \varepsilon_1, \cdots, \varepsilon_i) = \frac{1}{\gamma_i(t)} \big(\psi_i(\overline{x}_i) \lceil x_{i+1} \rfloor^{q_i} + f_i(\overline{x}_i) + \varrho_i(t) - \dot{\alpha}_{i-1} - \dot{\gamma}_i(t)\varepsilon_i\big).$$

Similar to (31), we can get $\Phi_i(t, \varepsilon_1, \cdots, \varepsilon_i) \in \mathcal{L}^\infty$, then $\dot{\alpha}_i \in \mathcal{L}^\infty$ is confirmed for $i = 2, \cdots, n-1$.

Step n: Define the Lyapunov function $V_n = \frac{1}{2}\zeta_n^2$. Taking derivative of V_n with respect to time gives

$$\begin{aligned} \dot{V}_n &= \frac{2\zeta_n}{(1-\varepsilon_n^2)\gamma_n(t)} \big(\psi_n(x) \lceil u \rfloor^{q_n} + f_n(x) + \varrho_n(t) - \dot{\alpha}_{n-1} - \dot{\gamma}_n(t)\varepsilon_n\big) \\ &\le \frac{2}{(1-\varepsilon_n^2)\gamma_n(t)} \big(|\zeta_n| \Delta_{n,1} - \psi_n(x) \beta_n^{q_n} |\zeta_n|^{q_n+1}\big) \end{aligned} \tag{39}$$

with $\Delta_{n,1} = |f_n(x) + \varrho_n(t) - \dot{\alpha}_{n-1} - \dot{\gamma}_n(t)\varepsilon_n|$. Since $x_i \in \mathcal{L}^\infty, i = 1, \cdots, n$, then $x = [x_1, \cdots, x_n]^T \in \mathcal{L}^\infty$. Furthermore, $f_n(x)$ is locally Lipschitz in x, so $f_n(x) \in \mathcal{L}^\infty$. By the boundedness of $f_n(x)$, $\varrho_n(t)$, $\dot{\alpha}_{n-1}$, $\dot{\gamma}_n(t)$ and ε_n, $\Delta_{n,1} \in \mathcal{L}^\infty$ holds, i.e., there exists positive constant $\eta_{n,1}$ satisfying $\Delta_{n,1} \le \eta_{n,1}$, which together with Lemma 3, lead to

$$|\zeta_n| \Delta_{n,1} \le |\zeta_n| \eta_{n,1} \le |\zeta_n|^{q_n+1} + q_n \big(\frac{\eta_{n,1}}{1+q_n}\big)^{1+\frac{1}{q_n}}. \tag{40}$$

Based on (40) and Assumption 1, it follows that

$$\dot{V}_n \le \frac{2}{(1-\varepsilon_n^2)\gamma_n(t)} \big(\eta_{n,3} - (\underline{\psi}_n \beta_n^{q_n} - 1)|\zeta_n|^{q_n+1}\big) \tag{41}$$

where $\eta_{n,3} = q_n(\frac{\eta_{n,1}}{1+q_n})^{1+\frac{1}{q_n}}$. Designing the control gain $\beta_n > (1/\underline{\psi})^{1/q_n}$, when $|\zeta_n|^{1+q_n} > \eta_{n,3}/(\underline{\psi}_n \beta_n^{q_n} - 1)$, we arrive at $\dot{V}_n < 0$. Hence,

$$|\zeta_n| \le \bar{\zeta}_n = \max\Big\{|\zeta_n(0)|,\ \big(\frac{\eta_{n,3}}{\underline{\psi}\beta_n^{q_n} - 1}\big)^{\frac{1}{1+q_n}}\Big\}, \quad t \in [0,\ T_m). \tag{42}$$

From (16) for $i = n$, we obtain

$$-1 < \frac{e^{-\bar{\zeta}_n} - 1}{e^{-\bar{\zeta}_n} + 1} = \underline{\varepsilon}_n \le \varepsilon_n(t) \le \overline{\varepsilon}_n = \frac{e^{\bar{\zeta}_n} - 1}{e^{\bar{\zeta}_n} + 1} < 1, \quad t \in [0,\ T_m). \tag{43}$$

Therefore, the control law u defined in (18) is bounded.

From (29), (37) and (43), it follows that $\varepsilon(t) \in \Omega_\varepsilon = [\underline{\varepsilon}_1,\ \overline{\varepsilon}_1] \times \cdots \times [\underline{\varepsilon}_n,\ \overline{\varepsilon}_n] \subset \Omega$, for $t \in [0,\ T_m)$, which means that the solutions of (12)–(14)

remain bounded within a compact subset of Ω. According to Theorem 3.3 in [41], we can extend the solution to $T_m = +\infty$.

Owing to the analysis given in Step 1–Step n, all closed-loop signals remain bounded and $\varepsilon(t) \in \Omega_\varepsilon$ for all $t \geq 0$. Moreover, invoking (29), one can get that

$$-\gamma_1(t) < \frac{e^{-\bar{\zeta}_1} - 1}{e^{-\bar{\zeta}_1} + 1}\gamma_1(t) \leq e_1(t) \leq \frac{e^{\bar{\zeta}_1} - 1}{e^{\bar{\zeta}_1} + 1}\gamma_1(t) < \gamma_1(t), \quad \forall t \geq 0. \tag{44}$$

Thus, the output tracking with prescribed performance is achieved. This completes the proof.

4 Simulation results

The effectiveness of the proposed controller is verified by a underactuated, unstable mechanical system described in the form as [42]

$$\begin{cases} \ddot{\vartheta} = \dfrac{g}{l} + \dfrac{k_s}{m_2 l}\lceil \varpi - l\sin(\vartheta)\rceil^{q_1}\cos(\vartheta) \\ \ddot{\varpi} = -\dfrac{k}{m_1}\varpi - \dfrac{k_s}{m_1}\lceil \varpi - l\sin(\vartheta)\rceil^{q_1} + \dfrac{u}{m_1} \end{cases} \tag{45}$$

where $\vartheta \in (-\pi/2,\ \pi/2)$ is the angle of the pendulum from the vertical, ϖ is the displacement of mass m_1, and u is a control torque applied to the mass m_1, $g = 9.8m/s^2$ is the acceleration of gravity, l is the length of rod, k and k_s are spring coefficients.

If the parameters m_1, m_2, l, k, k_s are known, the smooth state feedback control law given in [11]. In practice, the system parameters are time varying as the changes of working environment. In this work, we consider a more realistic situation that m_1, m_2, l, k_s are unknown constant parameters but belong to a known interval $[\underline{d},\ \overline{d}]$ with $\underline{d} > 0$.

Using the changes of coordinates

$$x_1 = \vartheta, \quad x_2 = \dot{x}_1, \quad x_3 = \varpi - l\sin\vartheta, \quad x_4 = \dot{x}_3 \tag{46}$$

defined on $(\vartheta, \dot{\vartheta}, \varpi, \dot{\varpi}) \in (-\pi/2, \pi/2) \times \mathbb{R}^3$, and introducing uncertain time-varying disturbance $\varrho(t)$, system (45) can be transformed into

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{k_s}{m_2 l}\lceil x_3\rceil^{q_2} + \dfrac{g}{l}\sin x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{u}{m_1} - \dfrac{k}{m_1}(x_3 + l\sin x_1) - \dfrac{k_s}{m_1}\lceil x_3\rceil^{q_2} + l x_2^2 \sin x_1 - g\sin x_1\cos x_1 \\ \qquad - \dfrac{k_s}{m_2}\lceil x_3\rceil^{q_2}\cos^2 x_1 + \varrho(t) \end{cases} \tag{47}$$

with $\varrho(t) = \sin(0.2t)$. Obviously, system (47) is in the form of (2), and Assumption 1 holds. For the simulation, we assume that $m_1 = 1, m_2 = 0.5, l = 10, k = 5, k_s = 10$. The initial conditions of the state variables are chosen as $x_1(0) = 2$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$.

The control objective is to design a state-feedback controller u to force the output y tracking the trajectory $y_{1r} = \sin(1.5t) - 0.3\cos t$. We require the steady state of the output tracking error $y - y_r(t)$ no more than 0.1, the minimum speed of convergence being obtained by the exponential e^{-3t}, and select $\gamma_{1,0} = 9.9 > 2|x_1(0) - y_r(0)|$. Therefore, the output performance function is $\gamma_1(t) = (9.9 - 0.1)e^{-3t} + 0.1$. Following the design procedure presented in Section 3.1, the intermediate performance functions are selected as $\gamma_2(t) = (10-0.3)e^{-3t}+0.3$, $\gamma_3(t) = (10-0.2)e^{-t}+0.2$ and $\gamma_4(t) = (9.8-0.2)e^{-\mu_i t}+0.2$, satisfying $\gamma_{2,0} > |x_2(0) - \alpha_1(0)|$, $\gamma_{3,0} > |x_3(0) - \alpha_3(0)|$, and $\gamma_{4,0} > |x_3(0) - \alpha_2(0)|$. Since $k_s/(m_2 l) = 2$ and $1/m_1 = 1$, the control gains are designed as $\beta_1 = 5$, $\beta_2 = 2$, $\beta_3 = 4$ and $\beta_4 = 5$ to produce reasonable control effort.

The simulation results are described in Fig. 1–Fig. 6. The tracking trajectories are plotted in Fig. 1. The output tracking error $x_1 - y_r(t)$ and the prescribed performance function $\gamma_1(t)$ are illustrated in Fig. 2, while the required control input u is pictured in Fig. 6. The trajectories of state error e_i and the prescribed performance function $\gamma_i(t)$, $i = 2, 3, 4$, are illustrated in Fig. 3–Fig. 5, respectively. The simulation results reveal that the proposed control can guarantee the tracking performance and the closed-loop stability despite unknown nonlinearities and unknown time-varying external disturbances.

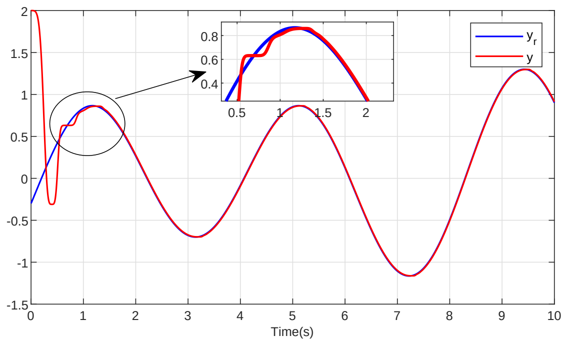

Fig. 1 Output $y(t)$ and desired reference $y_r(t)$.

5 Conclusions

In this paper, a general framework of tracking control is established for high-order uncertain nonlinear systems with unknown time-varying external disturbances. Given any initial system condition and any desired output tracking

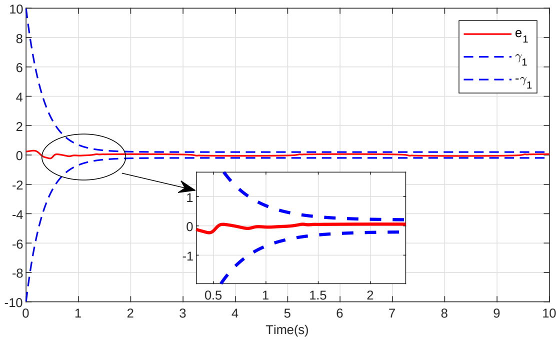

Fig. 2 Output error e_1 and prescribed performance function $\gamma_1(t)$.

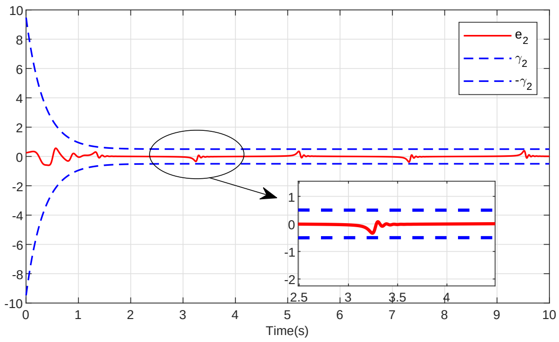

Fig. 3 State error e_2 and prescribed performance function $\gamma_2(t)$.

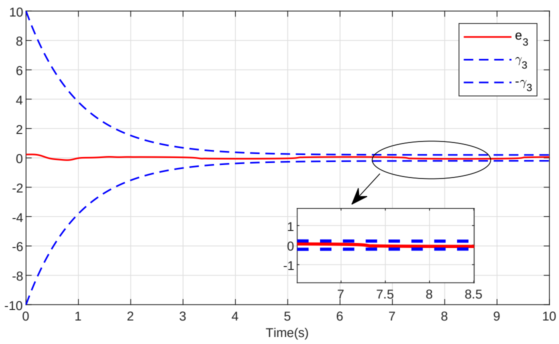

Fig. 4 State error e_3 and prescribed performance function $\gamma_3(t)$.

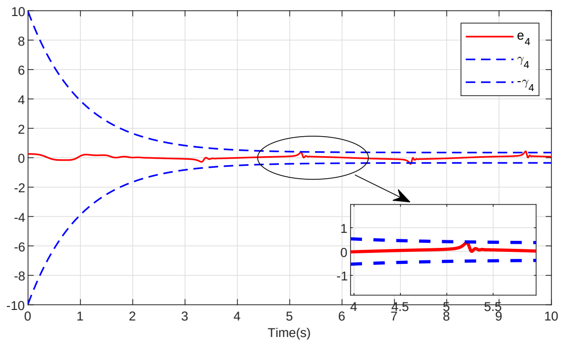

Fig. 5 State error e_4 and prescribed performance function $\gamma_4(t)$.

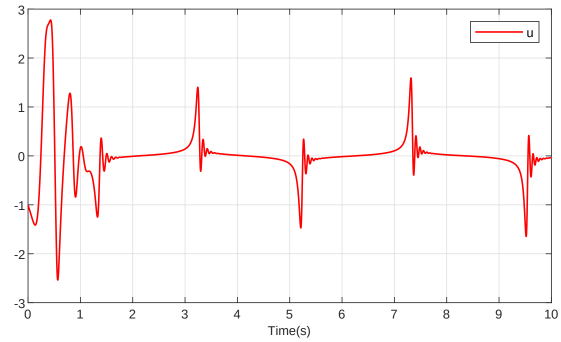

Fig. 6 The control input u.

trajectory signal, a state feedback controller of low-complexity is designed to obtain prescribed performance with respect to trajectory oriented metrics. Contrary to the related results in the literature, the restrictions on powers and system uncertainties are relaxed, thus the robustness of the proposed control scheme is extended. Additionally, our design scheme can deal with serious uncertainties without usage of the adaptive and function approximation technique, and the computation of the repeated derivatives of certain signals is avoided. Thus, the explosion of complexity is totally overcome without resorting to filter.

It should be noticed that the developed scheme requires knowledge of the signs of control coefficients (Assumption 1). Relaxing the aforementioned requirements, while maintaining the low-complexity of the control design will be the objective of our future study.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Data Availability Statement

The simulation data that support the findings of this study are available within the article.

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