Degree-of-Freedom Analysis and Structural Synthesis of a Class of Host–Parasite Multi-Loop Mechanisms

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Research Article

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Degree-of-freedom analysis and structural synthesis of a class of host–parasite multi-loop mechanisms

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A B S T R A C T

The method of analyzing the mechanism in series, parallel and hybrid modes can no longer meet the requirements of analyzing multi-loop mechanisms (MLMs), especially multi-loop mechanisms with passive degrees of freedom(P-DOFs).This study presents an approach to analyzing sub-degree-of-freedom (sub-DOF) relations in a class of MLMs with P-DOFs (P-DOFs) as well as structurally synthesizing these mechanisms. First,the DOFs of mechanisms with P-DOFs are decomposed and combined,and two methods—multi-loop serial connection and multi-loop stacking—are formulated to establish MLMs with P-DOFs.Second, a DOF space (DOF-S) model is generated.Host–parasite (H–P) MLMs are proposed, and various types of parasitism are analyzed. Finally, various DOF distribution patterns in H–P MLMs are analyzed based on real-world examples. The results show the following. H–P mechanisms are a class of MLMs with P-DOFs. For an H–P mechanism, its DOFs can be longitudinally and centrally, transversely and centrally, or comprehensively optimally distributed in the DOF-S by selecting a suitable type of parasitism. The H–P-type palletizing robot prototype developed in this study is able to self-balance. This demonstrates that the comprehensive optimization of DOF distribution is effective. This study enriches the theoretical knowledge on MLMs, which are extensively applied in fields such as aerospace, industrial robotics, and numerical-control machine tools.

Keywords: Passive degree of freedom; Degree-of-freedom space; Host–parasite mechanism; Multi-loop mechanism; Structural synthesis

1. Introduction

Nowadays, DOFs are understood in depth. When calculating DOFs, P-DOFs cannot be ignored, but most research on the DOFs of mechanisms is focused on those without P-DOFs,or it is considered that the P-DOFs are dispensable.The value of mechanisms with P-DOFs have not been effectively developed and utilized.

The degrees of freedom (DOFs) (also referred to as mobility) of a mechanism is the basis for studying its motion and force transfer.[1] The number of DOFs, F, is the number of independent parameters required to completely define the configuration of a mechanism at any time. As mechanisms have gradually evolved from simple planar mechanisms to complex spatial coupling mechanisms, F calculation methods have also developed from the Grübler–Kutzbach (G–K) equation to modified G–K equations.[2–4] A more in-depth understanding of DOFs has been developed based on the screw theory,[5, 6] the displacement group theory,[7, 8] and virtual loop methods.[9, 10] Passive DOFs (P-DOFs) cannot be overlooked when calculating F. P-DOFs are DOFs that have no effects on the motion of the output link of a mechanism.[11] Bagci[12] provided the general form of the F equation for mechanisms that account for P-DOFs. For type synthesis of closed mechanisms, Lu et al.[13] systematically examined the relationships between associated linkages, redundant constraints, P-DOFs, and DOFs. Zhao et al.[14] presented a more stringent and complete theory to analyze the DOFs of mechanisms. This theory adequately takes into consideration overconstraints and P-DOFs. Kim and Yoon[15] developed a closed mechanism with P-DOFs for vehicle suspensions.

Multi-loop mechanisms (MLMs) are a class of complex chain-coupling spatial mechanisms. Correct structural decomposition is the key to DOF analysis of MLMs. Liu et al.[16] equaled an MLM to a parallel kinematic mechanism (PKM) using splitting and equivalent transformation methods and subsequently calculated its F. Li et al.[17] proposed a cell division method to analyze the DOFs of MLMs. Cao et al.[18] performed mobility analysis and structural synthesis on a class of spatial mechanisms with coupling chains. Hu et al.[19] proposed to use topological analysis combined with the screw theory to analyze MLMs. Huang and Zheng[20] determined the

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overconstraints between branch-chain loops in a mechanism based on the screw theory and subsequently calculated the F of the whole mechanism. In addition, they also noted that overconstrained conditions would not occur. This approach is an inspiration for P-DOF analysis in this study. In addition to DOF analysis, researchers have also proposed a number of type synthesis methods for MLMs. Zhang et al.[21] proposed a new method to synthesize MLMs based on the virtual loop theory and the Assur group theory. Zhang et al.[22] transformed the type synthesis of MLMs into the type synthesis of corresponding serial kinematic mechanisms (SKMs) and PKMs based on the topology splitting method and the DOF splitting principle. Xun et al.[23] presented a novel rhombohedral multi-loop coupling mechanism with three DOFs. Ding et al.[24] formulated a general type synthesis method for two-layer and two-loop mechanisms. Parallel mechanism also belongs to MLMs, and the analysis method of parallel mechanism can provide valuable reference for the analysis of MLMs.[25]

In summary,there has been relatively little systematic research into the P-DOFs of complex spatial mechanisms and especially into constructing P-DOFs actively.This study aims to develop a DOF space (DOF-S) model as well as analyze and synthesize a class of MLMs with P-DOFs (referred to as host–parasite (H–P) MLMs). This class of mechanism expresses the hierarchical relationship between the DOFs of each sub-mechanism of an MLM as a host–parasite (H–P) relationship similar to that in biology, thereby giving rise to the term H–P mechanism.This class of MLMs improves the analysis of the DOFs and type synthesis of MLMs from a one-dimensional (1D) perspective to a multi-dimensional perspective. This study involves the following tasks: (A) develop a method for establishing P-DOFs in MLMs; (B) derive calculation criteria for P-DOFs and establish a DOF-S model; (C) analyze the H–P characteristics of MLMs with P-DOFs; (D) synthesize multiple classes of H–P MLMs based on the spatial distribution pattern of DOFs and H–P patterns as well as develop a robot prototype based on an improved mechanism and analyze its performance.

2. Decomposition and combination of P-DOFs

2.1. DOF characteristics of mechanisms

The F equation for a planar mechanism is

$$F = 3m - 2P_L - P_H \,, \tag{1}$$

where m is the number of movable links, P_L is the number of lower pairs, and P_H is the number of higher pairs.

The general form of the F equation for mechanisms in 3D space [11] is

$$F = 6(n - g - 1) + \sum_{j=1}^{g} f_j + \mu \,, \tag{2}$$

where n is the number of links, g is the number of joints, f_j is the number of DOFs of joint j, and μ is the number of overconstraints in the mechanism and consists of the number of common constraints, λ, and the number of redundant constraints (i.e., virtual constraints), ν:

$$\mu = \nu - \lambda(n - g - 1) \,, \tag{3}$$

$$d = 6 - \lambda \,, \tag{4}$$

where d is the order of the mechanism. By substituting Eqs. (3) and (4) into Eq. (2), a modified G–K equation for the F of mechanisms is obtained [11], namely

$$F = d(n - g - 1) + \sum_{j=1}^{g} f_j + \nu \,. \tag{5}$$

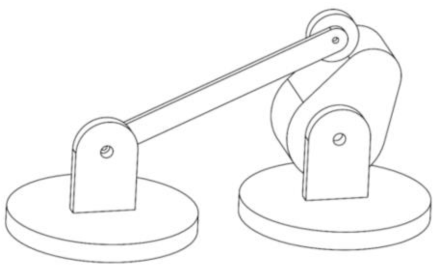


Fig. 1. Roller-containing cam mechanism.

With the framework of a mechanism as the reference system, the F of the output link varies with the link selected as the output link. Huang et al. [11] referred to the DOFs of a mechanism's output link as the nominal DOFs of the mechanism. A few mechanisms have P-DOFs that do not affect the motion of their output links. For example, the rollers in the roller-containing cam mechanism in Fig. 1 have a rotational P-DOF. Thus, the number of nominal DOFs, F_H, of the output link of the mechanism is

$$F_{\mathrm{H}} = F - F_{\mathrm{P}}\,, \tag{6}$$

where F_P is the number of P-DOFs. Eq. (6) can also be written in the form [12]

$$F = F_{\mathrm{H}} + F_{\mathrm{P}}\,. \tag{7}$$

If there are multiple P-DOFs, then Eq. (7) can also be written in the form

$$F = F_{\mathrm{H}} + \sum F_{\mathrm{P}}\,. \tag{8}$$

P-DOFs do not increase the number of overconstraints in the whole mechanism and can also be stacked. The following shows the condition for establishing a P-DOF. If a sub-DOF, DOF$_i$, does not generate new overconstraints on the nominal DOFs, then DOF$_i$ is a P-DOF. By contrast, if a new overconstraint is generated, then the nominal DOFs change. Consequently, the DOFs of the mechanism cannot be decomposed using Eq. (7).

2.2. Decomposition and combination of DOFs of mechanisms with P-DOFs

In this study, all mechanisms with P-DOFs are treated as spatial mechanisms, and their F values are calculated using Eq. (2). For a mechanism with $k-1$ P-DOFs, because each sub-DOF does not generate new overconstraints in the whole mechanism, all the variables in Eq. (2) can be decomposed uniformly into k sub-variables. Let

$$n = \sum_{i=1}^{k} n_i\,,\quad g = \sum_{i=1}^{k} g_i\,,\quad \sum f_j = \sum_{i=1}^{k}\sum f_{ij}\,,\ \text{and}\quad \mu = \sum_{i=1}^{k}\mu_i\,. \tag{9}$$

The equation for calculating the F of a mechanism with P-DOFs [i.e., Eq. (2)] can be written in the form

$$\begin{aligned} F &= 6(n-g-1)+\sum f_j+\mu \\ &= 6\left(\sum_{i=1}^{k} n_i - \sum_{i=1}^{k} g_i - 1\right)+\sum_{i=1}^{k}\sum f_{ij}+\sum_{i=1}^{k}\mu_i\,. \end{aligned} \tag{10}$$

Eq. (10) is decomposed further by adding constant terms:

$$\begin{aligned} F &= 6\left(n_1+\sum_{i=2}^{k} n_i + k - 1 - \sum_{i=1}^{k} g_i - (1+k-1)\right)+\sum_{i=1}^{k}\sum f_{ij}+\sum_{i=1}^{k}\mu_i \\ &= 6\left(n_1+\sum_{i=2}^{k}(n_i+1) - \sum_{i=1}^{k} g_i - (1+k-1)\right)+\sum_{i=1}^{k}\sum f_{ij}+\sum_{i=1}^{k}\mu_i \\ &= \left[6(n_1-g_1-1)+\sum f_{1j}+\mu_1\right]+6\left(\sum_{i=2}^{k}(n_i+1)-\sum_{i=2}^{k} g_i-(k-1)\right)+\sum_{i=2}^{k}\sum f_{ij}+\sum_{i=2}^{k}\mu_i \\ &= \left[6(n_1-g_1-1)+\sum f_{1j}+\mu_1\right]+\sum_{i=2}^{k}\left\{6\left[(n_i+1)-g_i-1\right]+\sum f_{ij}+\mu_i\right\}\,. \end{aligned} \tag{11}$$

To show the decomposition in Eq. (11) more succinctly, let

$$F_1 = 6(n_1 - g_1 - 1) + \sum f_{1j} + \mu_1 \tag{12}$$

where

$$\mu_1 = \nu_1 - \lambda_1\left(n_1 - g_1 - 1\right), \tag{13}$$

and let

$$F_i = 6\left[(n_i+1) - g_i - 1\right] + \sum f_{ij} + \mu_i = 6(n_i - g_i) + \sum f_{ij} + \mu_i \tag{14}$$

where

$$\mu_i = \nu_i - \lambda_i\left((n_i+1) - g_i - 1\right) = \nu_i - \lambda_i\left(n_i - g_i\right). \tag{15}$$

In Eq. (14), we have $1 < i \le k$, and Eq. (14) is for calculating F_p. By substituting Eqs. (12) and (14) into Eq. (11), we have

$$F = F_1 + \sum_{i=2}^{k} F_i = \sum_{i=1}^{k} F_i \qquad (F_i \geq 0). \tag{16}$$

Note that the precondition for DOF decomposition is that the number of sub-DOFs, F_i, is equal to or greater than zero (i.e., $F_i \geq 0$). The DOFs of a mechanism with P-DOFs can be decomposed into k sub-DOFs. If the DOFs of a mechanism are treated as a set, then each sub-DOFs is a corresponding DOF subset. The sub-DOFs can similarly be arranged and combined according to a certain connecting condition, namely

$$\sum_{i=1}^{a} F_i + \sum_{i=a+1}^{b} F_i + \cdots + \sum_{i=x+1}^{k} F_i = \sum\sum F_i = \sum_{i=1}^{k} F_i = F \qquad (F_i \geq 0). \tag{17}$$

Eqs. (12)–(17) are the decomposition and combination models for the DOFs of mechanisms with P-DOFs.

3. MLMs with P-DOFs

3.1. MLMs and loop closures

MLMs constitute a class of mechanisms formed by various types of mechanism branch-chains in the form of closed loops. The main types of loop closure include a single-loop closure in one step, a multi-parallel loop closure in one step, a multi-serial loop closure in one step, and a self-loop closure, as shown in Fig. 2 (wherein C_i represents one loop closure).

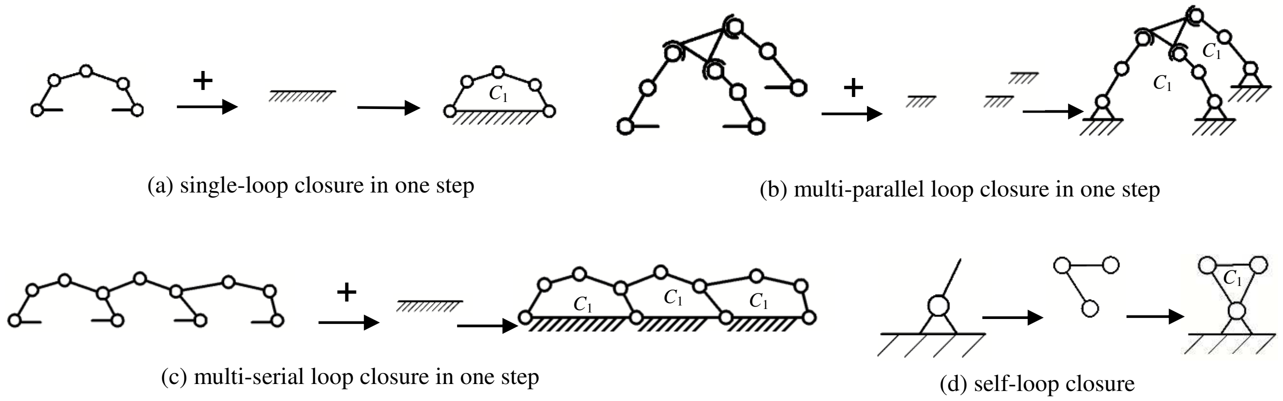
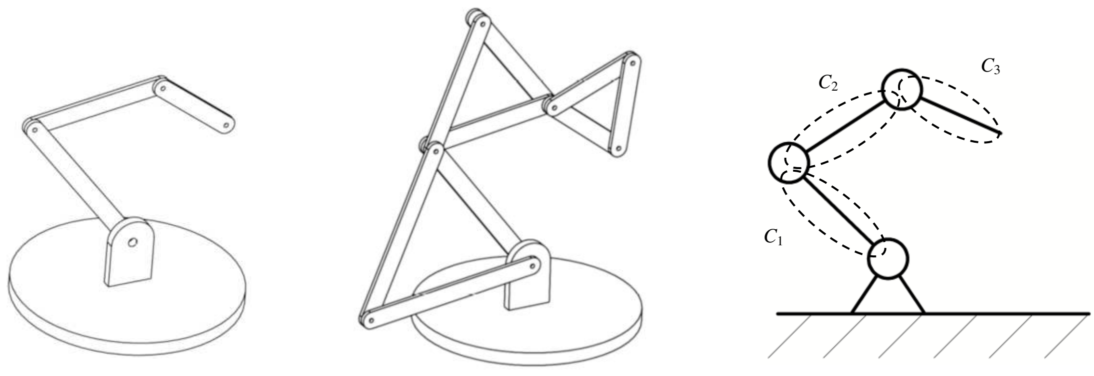


Fig. 2. Main types of loop closure.

Loop closures constitute a basis for forming MLMs. As a special type of loop closure, self-loop closures (i.e., loop closures formed by branch-chains composed of links and joints by themselves) do not result in overconstraints, and they form SKMs. To help understand self-loop closures in SKMs, a triangular-loop SKM, which is formed by modifying an SKM by closing the two ends of each of its link with a two-bar branch-chain with zero DOFs, is presented as shown in Fig. 3.

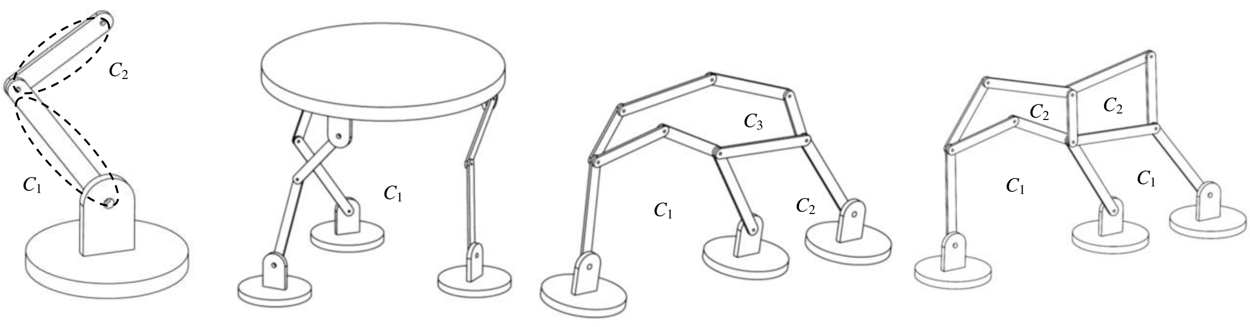


(a) SKM (b) triangular-loop SKM (c) self-loop closures of SKM

Fig. 3. Serial kinematic mechanism (SKM) and triangular-loop SKM.

As shown in Fig. 3, for the SKM we have the parameters $n = 4$, $g = 3$, $v = 0$, and $\lambda = 3$. Based on Eq. (3), we have $\mu = 0 - 3 \times (4 - 3 - 1) = 0$. Based on Eq. (2), the F of SKM is calculated as $F_1 = 6 \times (4-3-1)+3+0=3$. For the triangular-loop SKM we have the parameters $n = 10$, $g = 12$, $v = 0$, and $\lambda = 3$. Based on Eq. (3), we have $\mu = 0 - 3 \times (10 - 12 - 1) = 9$. Based on Eq. (2), the F of SKM is calculated as $F_2 = 6 \times (10-12-1)+12+9=3$. Evidently, we have $F_1 = F_2$, i.e., the F of the SKM is the same as that of the triangular-loop SKM. The triangular-loop SKM exhibits notable multi-loop closure characteristics and is an equivalent mechanism to the SKM. If the loop closures in the triangular-loop SKM are treated as explicit loop closures, then the self-loop closures in the SKM are implicit loop closures. These two types of loop closure both conform to the loop-closure characteristics of MLMs. From this perspective, SKMs are MLMs.

As shown in Fig. 4, C_i represents one loop closure. MLMs can be formed by four types of mechanism branch-chain, namely (i) SKMs, (ii) PKMs, (iii) multi-sub-loop kinematic coupling mechanisms (LKMs), and (iv) multi-loop chain kinematic coupling mechanisms (CKMs). An SKM is an MLM formed by multiple self-closed loops connected successively in series. A PKM is an MLM formed by simultaneous closures of multiple loops connected in parallel onto a framework in one step. An LKM is an MLM formed by successive closures of multiple individual sub-loops onto the preceding one or multiple sub-loops. A CKM is an MLM formed by successive closures of multiple loops connected in series onto the preceding one or multiple loop chains.

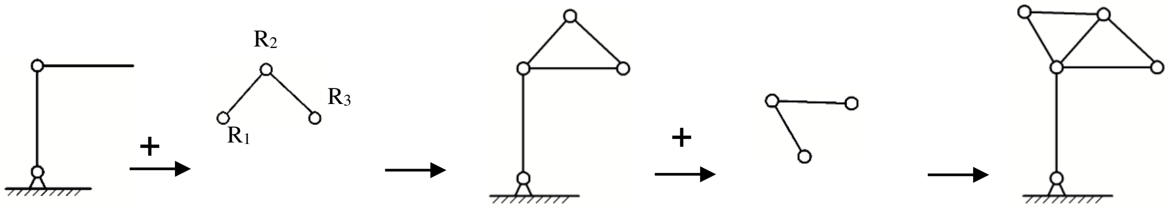


(a) SKM (b) PKM (c) LKM (d) CKM

Fig. 4. Four types of mechanism branch-chain.

Loop closures are the basis for forming MLMs. Overconstraints can occur only when loops are closed. However, new overconstraints do not necessarily occur when loops are closed. If closing a certain loop does not generate new overconstraints in the whole mechanism and this loop is not the output link of the mechanism, then the new DOF formed as a result of closing this loop is a P-DOF.

An arbitrary number of zero P-DOFs can be established without altering the DOFs of the original mechanism using unit branch-chains with zero P-DOFs and without overconstraints (e.g., the RRR branch-chain in Fig. 5). In particular, note that zero P-DOFs and an absence of P-DOFs have different physical meanings. The equation for calculating the number of DOFs that consist of multiple zero P-DOFs is $F = F_{\mathrm{H}} + \sum 0$.

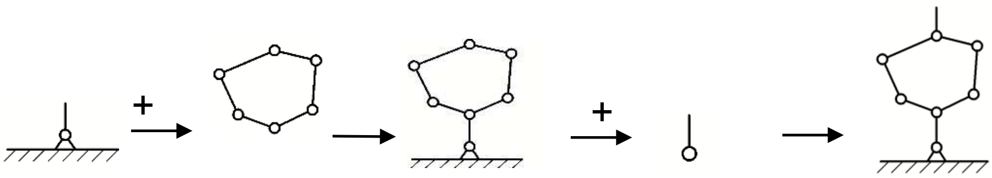


(a) single zero P-DOF (b) two zero P-DOFs

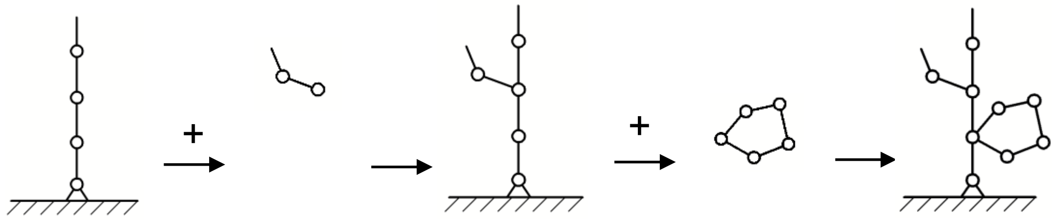
Fig. 5. Zero passive degrees of freedom (P-DOFs) of multi-loop mechanisms

3.2. Decomposition and combination of serial method

An SKM has an open-chain structure. As a result, adding each new DOF does not generate new overconstraints in an SKM. If multiple sub-output links are defined, then the DOFs of an SKM consist of one nominal DOF and i P-DOFs. Clearly, according to their establishing condition, P-DOFs can be established repeatedly by serial connection. The serial method for establishing P-DOFs connects two adjacent loops in series by i ($i = 1$) common joints. The serial method for establishing P-DOFs involves two techniques, namely single-chain serial connection and branch serial connection, as shown in Fig. 6. The single-chain serial connection technique connects multiple links successively in series via moving joints. The branch serial connection technique, which is based on the single-chain serial connection technique, produces multiple branch-chains with P-DOFs at local joints and thereby constructs a configuration similar to that of the branches of a tree.



(a) single-chain serial connection

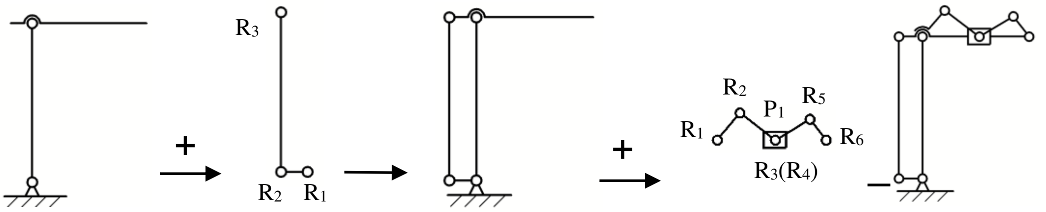


(b) branch serial connection

Fig. 6. Serial method for establishing P-DOFs.

3.3. Decomposition and combination of stacking method

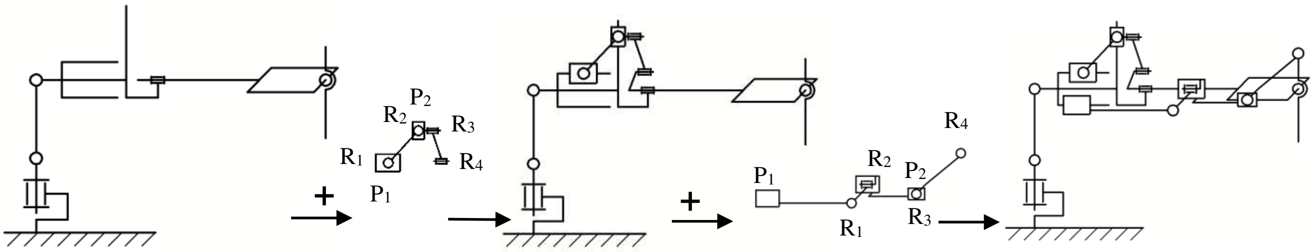
The key to using the stacking method is to choose suitable unit branch-chains with P-DOFs. These unit branch-chains do not generate new overconstraints in the whole mechanism. Any arbitrary complex branch-chains can be formed with multiple unit branch-chains. The stacking method successively stacks two adjacent mechanism branch-chains (e.g., PKMs, single-loop mechanisms, LKMs, and CKMs) via i ($i > 1$) common joints. The multi-loop stacking method involves three techniques, namely same-level serial stacking, same-level parallel stacking, and multi-level stacking. In same-level serial stacking (Fig. 7), multiple branch-chains are connected in a continuous series or a piecewise manner and are also all stacked onto the branch-chain of the previous level.



(a) single-branch stacked parasitism (b) two-branch stacked parasitism

Fig. 7. Same-level piecewise serial stacked parasitism.

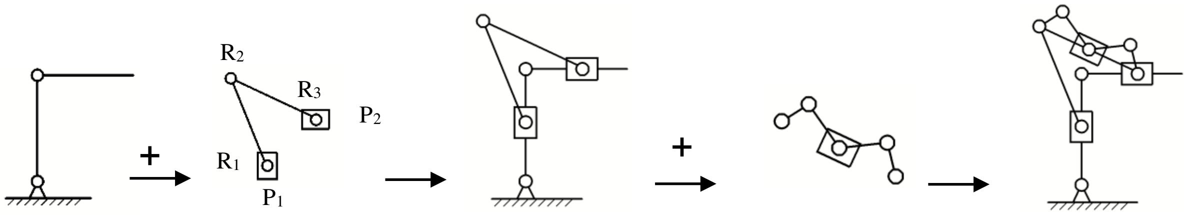
In same-level parallel stacking (Fig. 8), branch-chains are connected in parallel and are also all stacked onto the branch-chain of the previous level.



(a) single-branch (b) two-branch stacked parasitism

Fig. 8. Same-level parallel stacked parasitism.

In multi-level stacking (Fig. 9), multiple branch-chains are stacked successively and level-by-level onto the mechanism branch-chain of the previous level.



(a) single-branch stacked parasitism (b) two-branch stacked parasitism

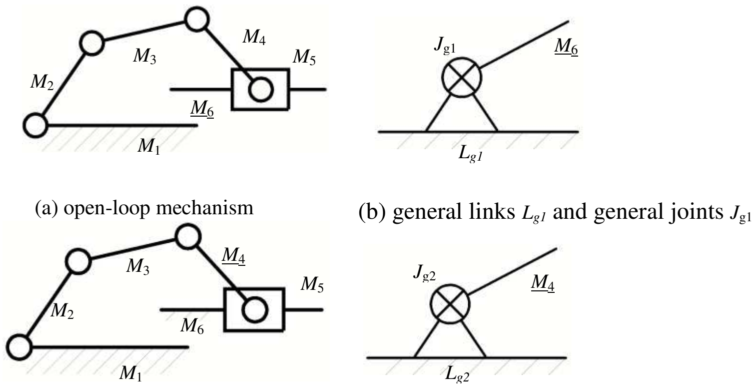
Fig. 9. Multi-level stacked parasitism.

4. DOF-S modeling for MLMs with P-DOFs

4.1. Basic concepts relating to DOF-S of mechanisms

This study introduces several concepts, namely general links, general joints, and the DOF-S of mechanisms. The DOF-S principle is employed to analyze the interrelations within the DOFs of MLMs with P-DOFs as well as the process by which these MLMs are formed.

Denoted by L_g, general links are defined as an abstract set of all the links with zero DOFs and "framework" characteristics in a certain DOF subset. In Eq. (14) (i.e., the equation for calculating F_i) of the DOF decomposition model, the "1" in $n_i + 1$ refers to one common general link corresponding to the DOF subset. Frameworks (e.g., L_{g1} and L_{g2} in Fig. 10) are typical general links. A framework is an ensemble of links with various forms that are fixed onto the ground. A framework is a general link with no fixed form and zero DOFs. In particular, note that a general link in a certain DOF subset has zero DOFs, but the links that comprise this general link do not necessarily have zero DOFs in another DOF subset.



(c) closed-loop mechanism (d) general links L_{g2} and general joints J_{g2}

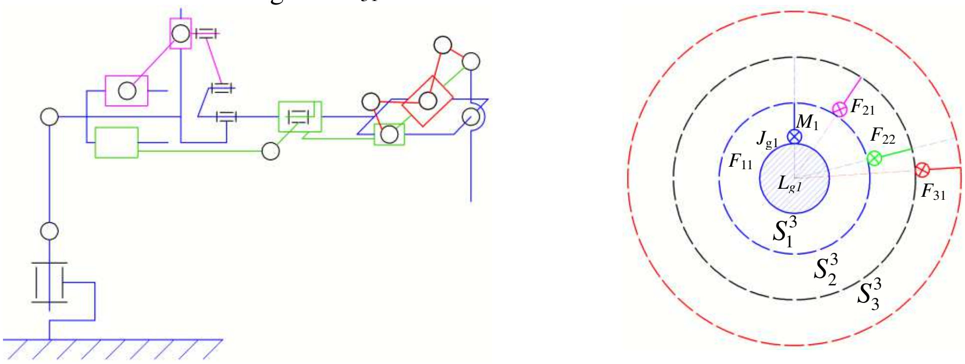
Fig. 10. General links and general joints.

Denoted by J_g, a general joint is the simplest linkage that allows independent, stable relative DOFs between any two links in an MLM. Based on Eq. (16) of the mechanism decomposition model, a mechanism with F DOFs can be decomposed into k general joints with F_i sub-DOFs. In Fig. 10, J_{g1} and J_{g2} are general joints, which are joints abstracted from the mechanism branch-chains based on DOF characteristics. An SKM, a PKM, a LKM, and an CKM branch-chain can all be equated to general joints. Links connected to general joints can be referred to as sub-output links. One general joint can connect with only one sub-output link and one general link.

4.2. Mathematical DOF-S model for mechanisms

An attempt is made to establish a mathematical DOF-S model for MLMs with P-DOFs based on their DOF decomposition and combination models. The DOF-S model can be used to transversely and longitudinally decompose and analyze various processes by which the DOFs of MLMs with P-DOFs are formed.

The DOF-S model is a set of multiple DOF subsets that are in level-wise inclusion relations and consist of sub-output links, general joints, and general links. A k-dimensional DOF-S is denoted by S^k. The DOF-S of a group of mechanisms is a complete sphere. A sub-DOF-S contains the sub-DOF-S that precedes it. A sub-DOF-S is denoted by S_i^k .As shown in Fig. 11(a), An MLM with P-DOFs consists of a main chain and three branch chains. The main chain is blue and the three branch chains are purple, green, and red. Fig. 11(b) shows the DOF-S of the MLM with P-DOFs. The S^k of this mechanism consists of F_{11}, F_{21}, F_{22}, and F_{31}, which are distributed between S_1^k and S_3^k. F_{11} consists of M_1, J_{g1}, and L_{g1}. The angle between F_{22} and F_{11} is larger than the angle between F_{21} and F_{11}, indicating that the branch of F_{22} is longer than that of F_{21}. The small angle between F_{31} and F_{22} indicates that the branch chain length of F_{31} is short.



(a) an MLM with P-DOFs (b) DOF-S of the MLMs with P-DOFs

Fig. 11. DOF-space (DOF-S) model.

The DOF-S of the first dimension, S_1^k, is the blue dotted circle core in the center. The S_1^k consists of a general link that contains the framework, a general joint, and a sub-output link. Based on Eq. (12) of the DOF decomposition model, the F of the mechanism in S_1^k, F_1, is calculated as

$$F_1 = 6(n_1 - g_1 - 1) + \sum f_1 + \mu_1 \tag{18}$$

where

$$\mu_1 = \nu_1 - \lambda_1 (n_1 - g_1 - 1). \tag{19}$$

The DOF-S of dimension i ($1 < i \le k$), S_i^k, is a subset of DOFs formed by multiple sub-output links via various general joints relative to one common general link. This common general link is formed by the consolidation of all the links in the DOF-Ss of the first dimension to dimension $i-1$ as a whole, i.e., the “framework” of the DOF-S of dimension i ($1 < i \le k$), L_{gi}. S_i^k consists of multiple sub-DOF sets. The mechanisms corresponding to these sub-DOFs are formed by simultaneous closures onto the common general link L_{gi} in one step. Mechanism branch-chain j in the DOF subspace of dimension i ($1 < i \le k$) is denoted by B_{ij}. The F of B_{ij}, F_{ij}, is calculated using Eq. (14) of the DOF decomposition model for mechanisms, namely

$$F_{ij} = 6\left[(n_{ij}+1) - g_{ij} - 1\right] + \sum f_{i(jr)} + \mu_{ij} = 6(n_{ij} - g_{ij}) + \sum f_{i(jr)} + \mu_{ij} \qquad (F_{ij} \ge 0) \tag{20}$$

where

$$\mu_{ij} = \nu_{ij} - \lambda_{ij}\left((n_{ij}+1) - g_{ij} - 1\right) = \nu_{ij} - \lambda_{ij}\left(n_{ij} - g_{ij}\right), \tag{21}$$

where n_{ij} is the number of links in B_{ij}, the “1” in $n_{ij}+1$ represents the general link L_{gi} in S_i^k, $f_{i(jr)}$ is the number of

DOFs of joint r of B_{ij}, and μ_{ij}, v_{ij}, and λ_{ij} are the overconstraint, redundant constraint, and common constraint, respectively, on B_{ij}. The F of the mechanism in S_i^k, F_i, is

$$F_i = \sum F_{ij} \qquad (F_{ij} \geq 0), \tag{22}$$

and the total F of the MLM with P-DOFs is

$$F = \sum_{i=1}^{k} F_i \qquad (F_i \geq 0). \tag{23}$$

The mathematical model for the k-dimensional DOF-S consists of Eqs. (18)–(23). There are four types of mechanism branch-chains, namely SKMs, PKMs, LKMs, and CKMs. An MLM robot with P-DOFs consisting of these four types of mechanisms can be represented with symbols as

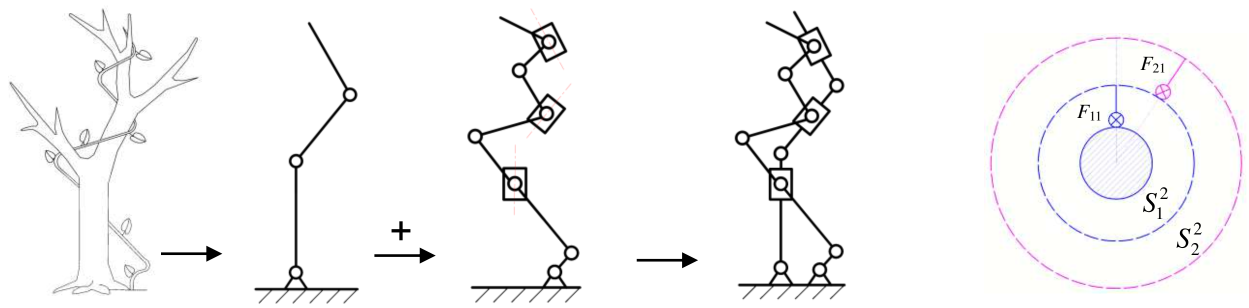
$$S_{B^F}^k \left(B^F = \sum \left(P_{k_i}^{F_i} (\mathrm{SKM}^{F_{ij}} + \mathrm{PKM}^{F_{ij}} + \mathrm{LKM}^{F_{ij}} + \mathrm{CKM}^{F_{ij}}) \right) \right), \tag{24}$$

where $S_{B^z}^k$ is a robot with a k-dimensional ($\sum k_i = k$) DOF-S and F DOFs, and $P_{k_i}^{F_i}$ is the mechanism branch-chain with F_i sub-DOFs in the DOF subspace of dimension i ($1 \leq i \leq k$). Assuming that a mechanism branch-chain is composed of all four types of branch-chains, this mechanism branch-chain is composed of four types of branch-chains with F_{ij} DOFs. In addition, we have $F_i = \sum F_{ij}$ and $F = \sum_{i=1}^{k} F_i$.

The DOF-S model can be used to analyze the distribution of DOFs in MLMs from longitudinal (dimensional feature) and transverse (DOF subspace feature) perspectives.

4.3. P-DOFs and H–P relationship

Symbiotic H–P relationships are ubiquitous in both the animal and plant kingdoms [26,27]. This also reflects biodiversity in nature. In an H–P relationship, the parasite lives on or in the host and seizes the host's nutrients. The parasite is unable to survive without the host. The host is harmed by the parasite but does not lose its original capacity to live because of the presence of the parasite [28]. Trees and vines as shown in Figure 12(a), there is also a stacked relationship between the host and the parasite.



(a)tree and rattan (b)tree mechanism (c)rattan mechanism (d)H-P mechanism (e) DOF-S of H-P mechanism

Fig. 12. H-P mechanism

Based on biological H–P relationships, an attempt is made to establish an H–P relationship for mechanisms. For an MLM with P-DOFs, its sub-mechanisms without P-DOFs are defined as host mechanisms (HMs), while its sub-mechanisms with P-DOFs are defined as parasite mechanisms (PMs). As shown in Figure 12, the tree mechanism and the rattan mechanism constitute a H-P mechanism, and have their own DOF-S.In a relationship between an HM and a PM, the PM lives off the HM via joints or links. The PM benefits from the HM by obtaining P-DOFs and a parasitic carrier from the HM, while the HM is limited by the singularity of the PM and therefore harmed. Nevertheless, the HM does not lose its original DOFs because of the presence of the PM. Spatially, there are stacked and serial relationships between the PM and the HM. MLMs composed of HMs and PMs can also be referred to as H–P MLMs.

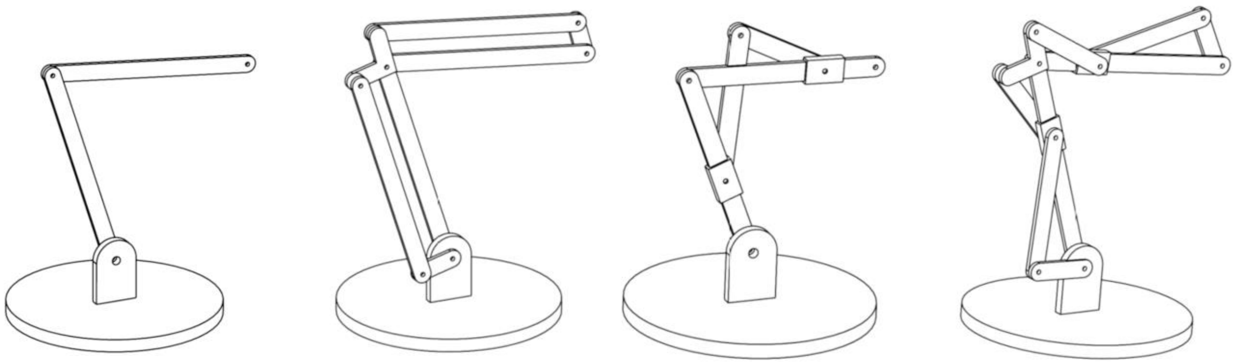
4.4. H–P mechanisms

The basic principle of parasitism in mechanisms is as follows. For an MLM with P-DOFs, the mechanisms in the DOF-S of dimension i ($i \geq 2$) are connected to the mechanism links or joints in the DOF-S of dimension j ($j = i - 1$) via joints. This is the process by which a mechanism with an i-dimensional ($i \geq 2$) DOF-S and H–P characteristics is formed.

As for the definition of H–P mechanisms, MLMs with P-DOFs and i-dimensional ($i \geq 2$) DOF-S characteristics are referred to as H–P mechanisms.The mathematical model for the DOF-S of H–P mechanisms consists of Eqs. (18)–(23).

As for the definition of HMs, mechanisms belonging to the DOF subspace of the first dimension of an i-dimensional ($i \geq 2$) DOF-S are referred to as HMs. The DOF-S of the first dimension is the DOF-S of the hosts.The DOF of HMs are calculated by equation (18).

As for the definition of PMs, mechanisms belonging to the DOF subspaces of an i-dimensional ($i \geq 2$) DOF-S other than the DOF subspace of the first dimension are referred to as PMs. The DOF subspaces other than the DOF-S of the first dimension are referred to as the DOF-S of the parasites. The DOF-S of the parasites can be divided further into various secondary DOF subspaces of the parasites. Secondary parasitism can also occur between PMs.The DOF of PMs are calculated by equation (20).



(a) a host mechanism (b) joint parasitism (c) link parasitism (d) mixed parasitism

Fig. 13. Three basic types of parasitism.

There are three basic types of parasitism, namely (i) joint parasitism, (ii) link parasitism, and (iii) mixed parasitism, as shown in Fig. 13. In joint parasitism, PMs parasitize HMs via joints alone. In link parasitism, PMs parasitize HMs via links alone. In mixed parasitism, H–P relationships among mechanisms are formed through a mixed combination of joint and link parasitism.

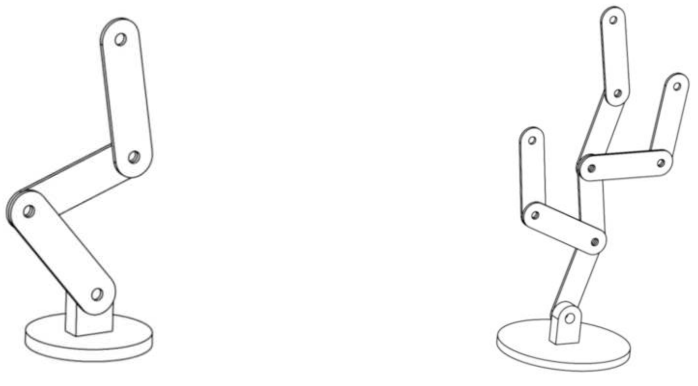
4.5. Classification of parasitism among branch-chains

Various types of parasite branch-chains (PBCs) can be formed by the above three basic types of parasitism. Table 1 summarizes the types and characteristics of branch-chain parasitism in H–P MLMs, which are determined based on the methods for establishing P-DOFs in MLMs as well as the relationships among PBCs and between PBCs and host branch-chains (HBCs). Specifically, the main types of branch-chain parasitism are serial parasitism, stacked parasitism, communal parasitism, zero-DOF parasitism, and twining parasitism.

Table 1
Modes of parasitism.

Serial number	Mode of parasitism	Sub-mode of parasitism	PBCs	Characteristic of DOF distribution	Structural compactness
1	Stacked parasitism	Same-level piecewise seria stacked parasitism	CKM, LKM	Concentrated	Average
		Same-level parallel stackec parasitism	CKM, LKM	Concentrated	Average
		Multi-level stacked parasitism	CKM, SKM, PKM, LKM	Scattered	Low
2	Serial parasitism	Single-chain serial parasitisı	SKM, PKM	Scattered	Average
		In branched serial parasitisr	SKM, PKM	Scattered	Low
3	Communal parasitism	Communal parasitism	SKM	Scattered	High
4	Zero-DOF parasitism	Zero-DOF parasitism	CKM, LKM	Scattered	Average
5	Twining parasitism	Twining parasitism	CKM	Concentrated	Average

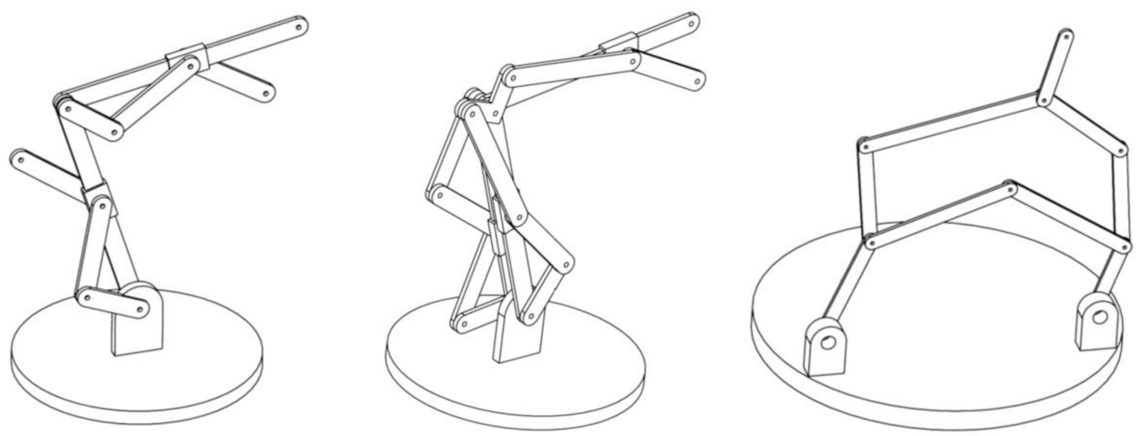
There are two types of serial parasitism, namely, single-chain and branched serial parasitism. Single-chain serial parasitism [as shown in Fig. 14(a)] involves ordinary mechanisms connected in series. In branched serial parasitism [as shown in Fig. 14(b)], various numbers of branch-chains connected in series are connected in the branched, serial manner to a single sub-joint of the HM or the PM of the previous level, forming a pattern similar to the branches of a tree.



(a) single-chain serial parasitism (b) branched serial parasitism

Fig. 14. Serial parasitism.

Stacked parasitism can be classified further into three types, namely (i) same-level piecewise serial stacked parasitism, (ii) same-level parallel stacked parasitism, and (iii) multi-level stacked parasitism, as shown in Fig. 15. In same-level piecewise serial stacked parasitism, multiple PBCs in the same DOF subspace parasitize an HM in a piecewise serial manner. In same-level parallel stacked parasitism, multiple PBCs in the same DOF subspace parasitize an HM in a parallel manner. In multi-level stacked parasitism, multiple PBCs in adjacent DOF subspaces of various dimensions form a stacked parasitic pattern.

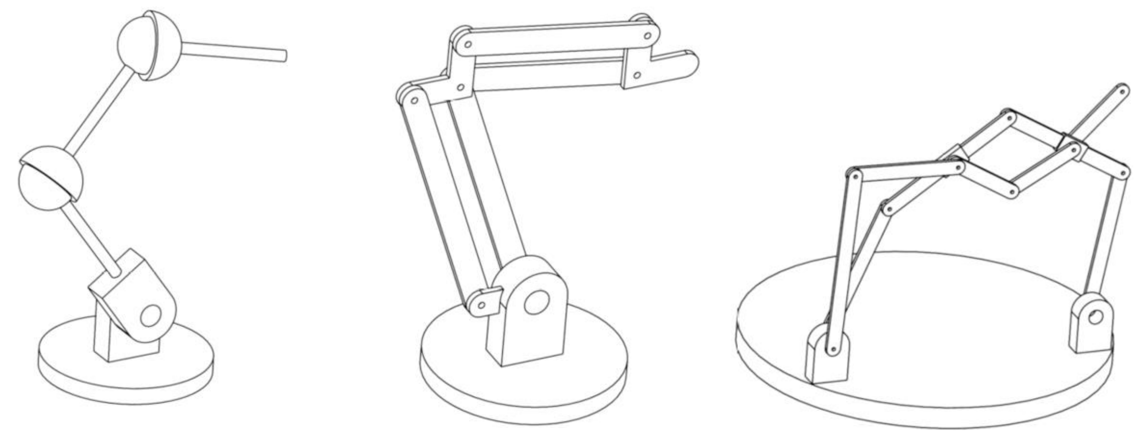


(a) same-level piecewise serial stacked (b) same-level parallel stacked (c) multi-level stacked

Fig. 15. Stacked parasitism.

Fig. 16(a) shows communal parasitism. The tongue-eating louse is a parasite that parasitizes the tongue of a fish. A fish so infested uses the tongue-eating lice as its tongue and is not particularly affected by them. This communal parasitism also exists in mechanisms. In communal parasitism, the PM and the HM share communal links and joints in local areas, and the P-DOFs corresponding to the PM are realized without increasing the numbers of links and joints in the whole mechanism.

Fig. 16(b) shows zero-DOF parasitism, in which a PBC with zero DOFs is relied on to maintain the output link in a certain state. Fig. 16(c) shows twining parasitism, in which a PBC takes full advantage of the remaining space to parasitize a mechanism by twining around it.



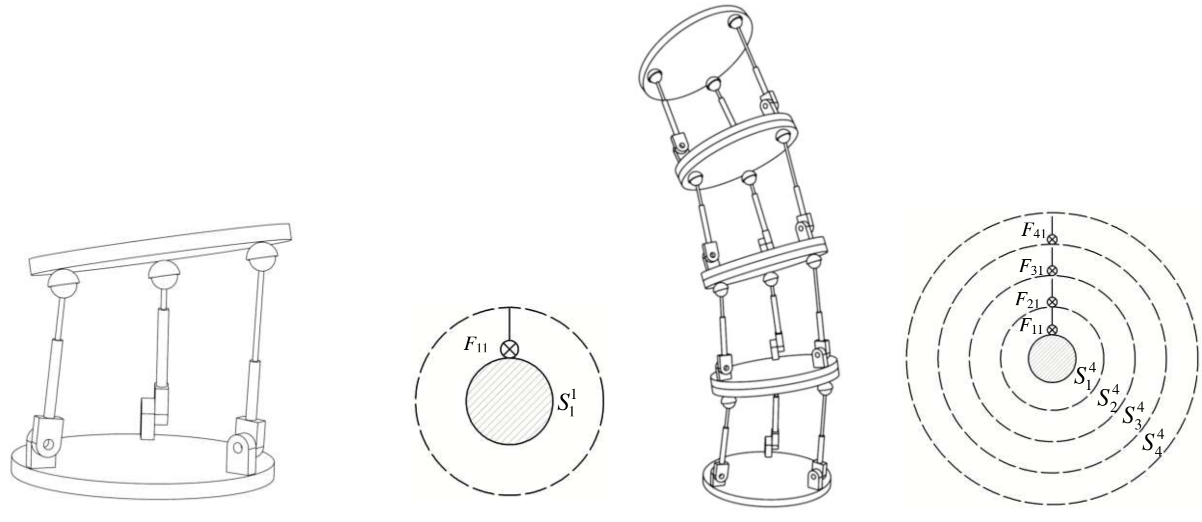
(a) communal parasitism (b) zero-DOF parasitism (c) twining parasitism

Fig. 16. Various types of parasitism.

5. Configuration analysis of H–P MLM examples

5.1. Examples with longitudinally and centrally distributed DOFs

Mechanism A is a 3-RPS PKM with the parameters n = 8, g = 9, v = 0, and λ = 0. Based on Eq. (3), we have $\mu = 0 - 0 \times (8 - 9 - 1) = 0$. Based on Eq. (2), the F of mechanism A is calculated as $F_A = 6 \times (8-9-1)+15+0=3$. Mechanism A is a mechanism with a one-dimensional (1D) DOF-S and no P-DOFs. Fig. 17(a) and (b) show the configuration and DOF-S, respectively, of mechanism A. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism A is $S^1_{B^3}(B^3 = P^3_{k1}(\mathrm{PKM}^3))$.



(a) mechanism A (b) DOF-S of mechanism A (c) mechanism B (d) DOF-S of mechanism B

Fig. 17. Mechanisms A and B.

Mechanism B is an H–P MLM formed by multiple mechanisms A in a stacked parasitic pattern. Table 2 summarizes the calculation of the F of mechanism B. Mechanism B has only a four-dimensional (4D) DOF-S. Fig. 17(c) and (d) show the configuration and DOF-S, respectively, of mechanism B. Based on Eq. (23), the F of mechanism B is calculated as $F_B = 3+3+3+3=12$. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism B is $S^4_{B^{12}}(B^{12} = P^3_{k1}(\mathrm{PKM}^3) + P^3_{k2}(\mathrm{PKM}^3) + P^3_{k3}(\mathrm{PKM}^3) + P^3_{k4}(\mathrm{PKM}^3))$.

Table 2
Summary of calculation of F of mechanism B.

Dimension i	Branch-chains	Sub-branched-chain F_{ij}	n	g	u	v	λ	Equations used for calculation	F_{ij}

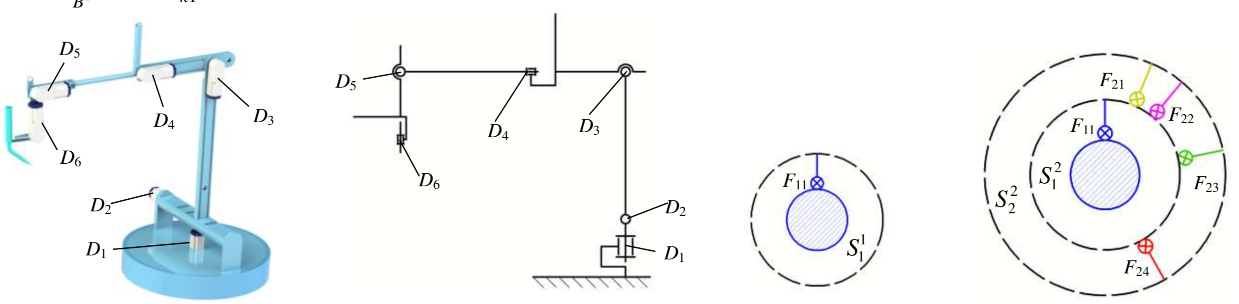
$i = 1$	P^3_{k1}(SKM^3)	F_{11}	8	9	0	0	0	(18) and (19)	3
$i = 2$	P^3_{k2}(CKM^3)	F_{21}	7	9	0	0	0	(20) and (21)	3
$i = 3$	P^3_{k3}(CKM^3)	F_{31}	7	9	0	0	0	(20) and (21)	3
$i = 4$	P^3_{k4}(CKM^3)	F_{41}	7	9	0	0	0	(20) and (21)	3

A comparative analysis of mechanisms A and B finds the following. Mechanism B is similar to the LX4 robot, which is a highly redundant parallel manipulator produced by stacking parallel robots [29]. Compared to mechanism A with only a 1D DOF-S, four branch-chains of mechanism B are longitudinally and centrally distributed in a 4D DOF-S, as shown in Fig. 17(d). The DOFs of mechanism B are notably longitudinally and centrally distributed. Mechanisms with longitudinally and centrally distributed DOFs are often predominantly characterized by high redundancy, a large working space, and a large load mass. Also, these mechanisms are difficult to control, which is their disadvantage.

5.2. Examples with transversely and centrally distributed DOFs

Mechanism C is a 6R SKM with the parameters $n = 7$, $g = 6$, $v = 0$, and $\lambda = 0$. Based on Eq. (3), we have $\mu = 0 - 0 \times (7-6-1) = 0$. Based on Eq. (2), the F of mechanism C is calculated as $F_C = 6 \times (7-6-1)+6+0=6$. Mechanism C can be decomposed in different ways by selecting different output links. To facilitate comparison with mechanism D, the end link farthest from the framework is selected as the output link of mechanism C. Thus, mechanism C has only a 1D DOF-S and no P-DOFs. Fig. 18(a)–(c) show the configuration, schematic, and DOF-S, respectively, of mechanism C. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism C is

$$S^1_{B^6}(B^6 = P^6_{k1}(\text{SKM}^6)).$$



(a) mechanism C (b) schematic of mechanism C (c) DOF-S of mechanism C (d) DOF-S of mechanism D

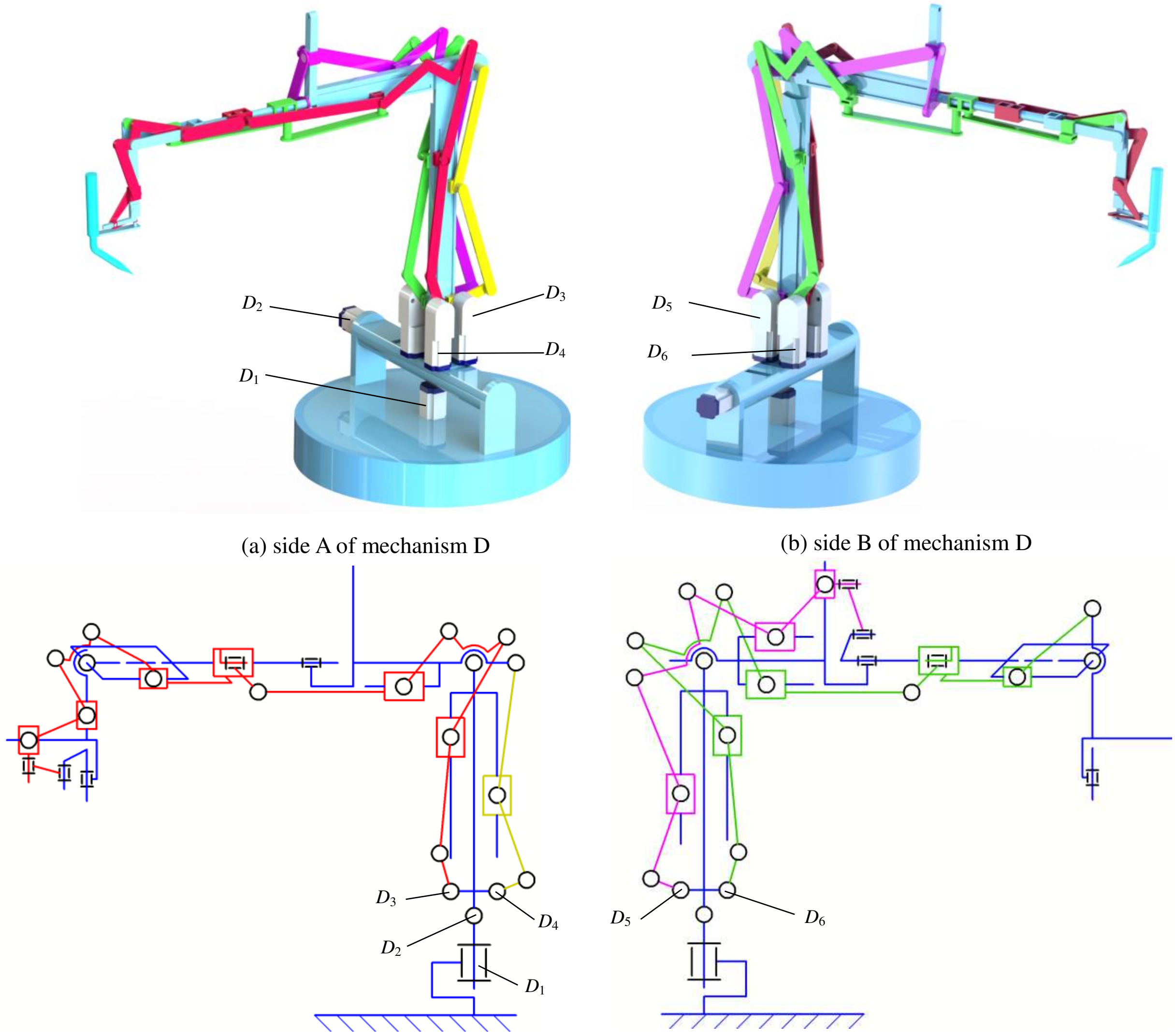
Fig. 18. Mechanism C and DOF-S of mechanism D.

Mechanism D is a brand-new H–P-mechanism-type welding robot design that has been granted a Chinese patent (no.2017105393674). Mechanism D is formed by stacking four CKMs [as the parasite branch-chains (PBCs)] onto mechanism C [as the host branch-chain (HBC)] in the same-level parallel, zero-DOF, and twining stacking patterns, as shown in Fig. 19. Table 3 summarizes the calculation of the F of mechanism D, and Fig. 18(d) shows the DOF-S of mechanism D. Based on Eq. (23), the F of mechanism D is calculated as $F_D = 6+0+0+0+0=6$. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism D is

$$S^2_{B^6}(B^6 = P^6_{k1}(\text{SKM}^6) + P^0_{k2}(\text{CKM}^0 + \text{CKM}^0 + \text{CKM}^0 + \text{CKM}^0)).$$

Table 3
Summary of calculation of F of mechanism D.

Dimension i	Branch-chains	Sub-branched-chain F_{ij}	Color	n	g	u	v	λ	Equations used for calculation	F_{ij}
$i = 1$	P^6_{k1}(SKM^6)	F_{11}	Blue	7	6	0	0	3	(18) and (19)	6
$i = 2$	P^0_{k2}(CKM^0)	F_{21}	Red	16	24	24	0	3	(20) and (21)	0
$i = 2$	P^0_{k2}(CKM^0)	F_{22}	Yellow	4	6	6	0	3	(20) and (21)	0
$i = 2$	P^0_{k2}(CKM^0)	F_{23}	Purple	10	15	15	0	3	(20) and (21)	0
$i = 2$	P^0_{k2}(CKM^0)	F_{24}	Green	10	15	15	0	3	(20) and (21)	0



(a) side A of mechanism D
(b) side B of mechanism D
(c) side A of mechanism D
(d) side B of mechanism D

Fig. 19. Mechanism D.

A comparative analysis of mechanisms C and D finds the following. Mechanism C has the typical mechanism configuration of a serial robot. In addition, mechanism C has six actuators, D_1–D_6, which are distributed successively at six rotational joints. The configuration of a serial robot consists of multiple cantilever beams connected in series. The disadvantages of serial robots include low mechanical efficiency, low stiffness, and proneness to joint error accumulation. Compared to mechanism C with only a 1D DOF-S, the HBC and four PBCs of mechanism D are longitudinally distributed in a two-dimensional DOF-S. In addition, the four PBCs with zero DOFs are centrally distributed in the DOF-S of the second dimension, as shown in Fig. 19. The DOFs of mechanism D are notably transversely and centrally distributed. Mechanism D has six actuators, D_1–D_6, all of which are centrally distributed near the framework. The mass of the motors and decelerators accounts for 40–60% of the total mass of the moving parts of a robot. Therefore, compared to the mass distribution of mechanism C, that of mechanism D allows it to effectively reduce the rotational inertia of the main joints of a robot. Mechanism D with transversely and centrally distributed DOFs can be used to effectively improve the dynamic performance of a serial robot while maintaining its large working space.

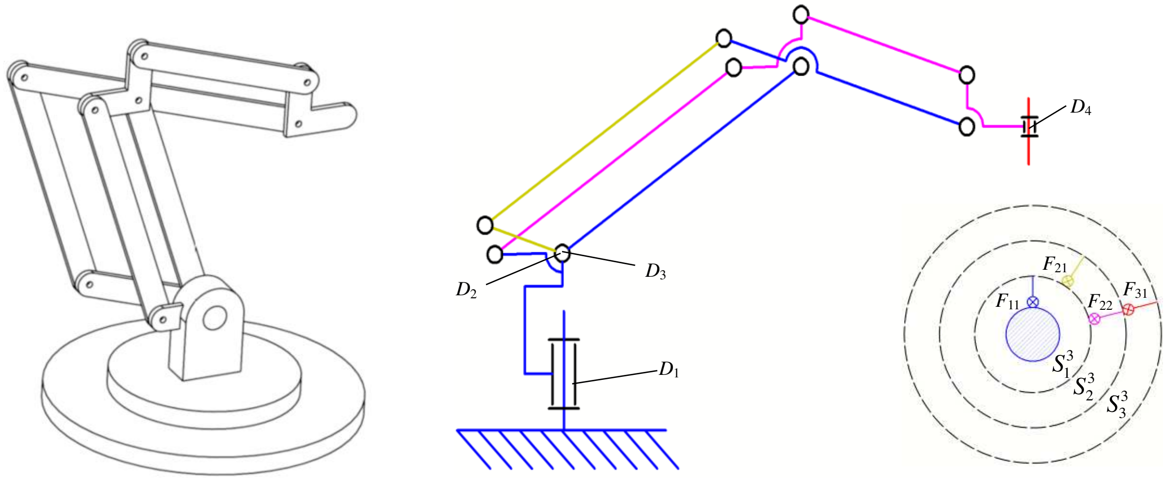
5.3. Examples with comprehensively optimally distributed DOFs

Mechanism E is a common configuration for handling robots. Mechanism E is an H–P MLM. Table 4 summarizes the calculation of the F of mechanism E, and Fig. 20 shows the configuration, schematic, and DOF-S of mechanism E. Based on Eq. (23), the total F of mechanism E is calculated as $F_E = 3+0+0+1=4$. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism E is

$$S^3_{B^4}(B^4 = P^3_{k1}(\mathrm{SKM}^3) + P^0_{k2}(\mathrm{LKM}^0 + \mathrm{CKM}^0) + P^1_{k3}(\mathrm{SKM}^1)).$$

Table 4
Summary of calculation of F of mechanism E.

Dimension i	Branch-chains	F_{ij}	Color	n	g	u	v	λ	Equations used for calculation	F_i
$i = 1$	$P^3{}_{S1}(SKM^3)$	F_{11}	Blue	4	3	0	0	3	(18) and (19)	3
$i = 2$	$P^0{}_{S2}(LKM^0)$	F_{21}	Yellow	2	3	3	0	3	(20) and (21)	0
$i = 2$	$P^0{}_{S2}(CKM^0)$	F_{22}	Purple	4	6	6	0	3	(20) and (21)	0
$i = 3$	$P^1{}_{S3}(SKM^1)$	F_{31}	Green	1	1	0	0	0	(20) and (21)	1



(a) mechanism E (b) schematic of mechanism E (c) DOF-S of mechanism E

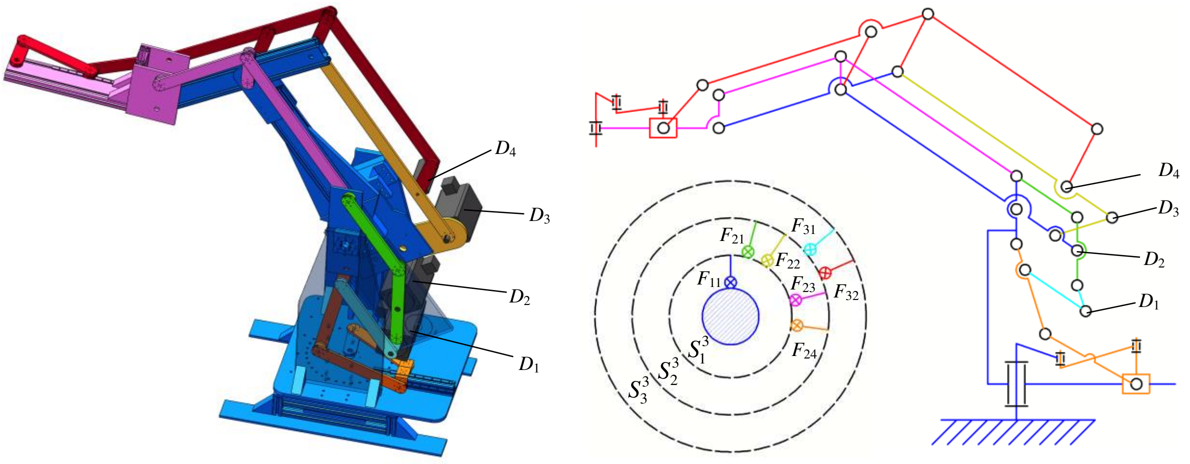
Fig. 20. Mechanism E.

Mechanism G is a brand-new H–P-mechanism-type palletizing robot design, for which a Chinese patent (no. 2019108332883) has been applied. This mechanism consists of one HM and six PBCs, as shown in Fig. 21. Table 5 summarizes the calculation of the F of mechanism G, and Fig. 21(b) shows the DOF-S of mechanism G. Based on Eq. (23), the total F of mechanism G is calculated as $F_G = 3+0+0+0+0+1+0=4$. The symbolic expression obtained based on Eq. (24) for the DOF-S of mechanism G is

$$S^3_{B^4}(B^4 = P^3_{k1}(\mathrm{SKM}^3) + P^0_{k2}(\mathrm{CKM}^0 + \mathrm{LKM}^0 + \mathrm{LKM}^0 + \mathrm{CKM}^0) + P^1_{k3}(\mathrm{CKM}^1 + \mathrm{LKM}^0)).$$

Table 5
Summary of calculation of F of mechanism G.

Dimension i	Branch-chains	Sub-branched-chain F_{ij}	Color	n	g	u	v	λ	Equations used for calculation	F_{ij}
$i = 1$	$P^3{}_{S1}(SKM^3)$	F_{11}	Blue	7	6	0	0	3	(18) and (19)	3
$i = 2$	$P^0{}_{S2}(CKM^0)$	F_{21}	Orange	4	6	6	0	3	(20) and (21)	0
$i = 2$	$P^0{}_{S2}(LKM^0)$	F_{22}	Yellow	2	3	3	0	3	(20) and (21)	0
$i = 2$	$P^0{}_{S2}(LKM^0)$	F_{23}	Green	2	3	3	0	3	(20) and (21)	0
$i = 2$	$P^0{}_{S2}(CKM^0)$	F_{24}	Purple	4	6	6	0	3	(20) and (21)	0
$i = 3$	$P^1{}_{S3}(CKM^1)$	F_{31}	Red	9	13	12	0	3	(20) and (21)	1
$i = 3$	$P^1{}_{S3}(LKM^0)$	F_{32}	Cyan	2	3	3	0	3	(20) and (21)	0

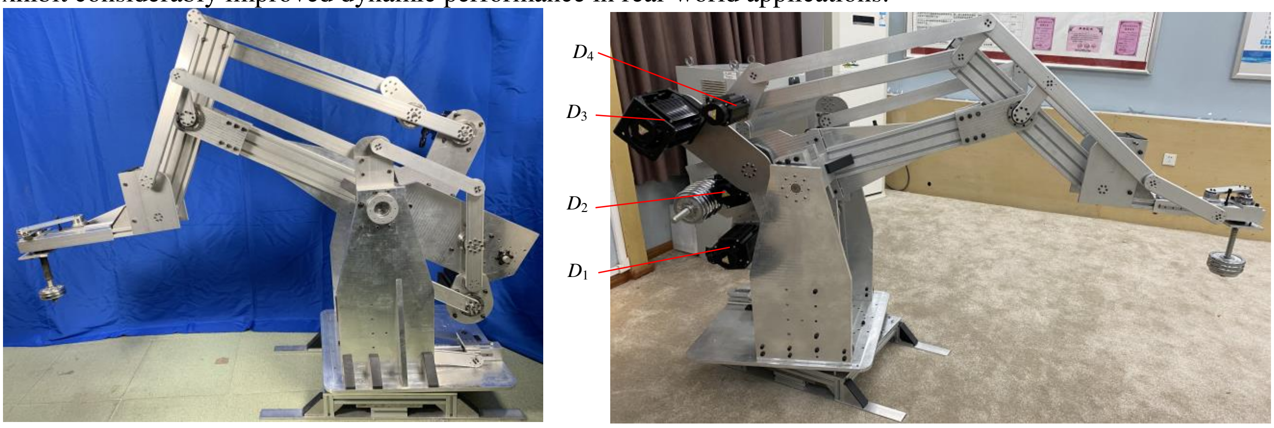


(a) mechanism G (b) DOF-S of mechanism G (c) schematic of mechanism G

Fig. 21. Mechanism G.

A comparative analysis of mechanisms E and G finds the following. Mechanism E has the typical configuration of a transfer robot. Mechanism E has four actuators, D_1–D_4, which are distributed successively at four rotational joints, as shown in Fig. 20. Mechanism E consists of one HM and three PBCs, with DOFs distributed in a 3D DOF-S. Actuator D_4 is installed at the end of mechanism E, whereas the other three actuators are installed near the framework. Compared to serial robots, mechanism E exhibits better dynamic performance.

Mechanism G is a new self-balancing palletizing robot designed by further optimizing mechanism E. Fig. 22 shows a photograph of a prototype of mechanism G. Compared to mechanism E, there are an additional LKM branch-chain and one CKM branch-chain in the DOF-S of the second dimension in mechanism G. In addition, instead of an SKM branch-chain in mechanism E, there is a CKM branch-chain in the DOF-S of the third dimension in mechanism G. Moreover, compared to mechanism E, there is an additional LKM branch-chain in the DOF-S of the third dimension in mechanism G. The DOF-S of mechanism G is considerably better than that of mechanism E, both longitudinally and transversely. Compared to mechanism E, the four actuators in mechanism G are installed at its lower bottom. The four actuators of the new robot, D_1–D_4, are distributed at four rotational joints, as shown in Figs. 21 and 22. By taking full advantage of their relatively large masses, all the motors and decelerators are designed creatively as counterweights in mechanism G. This design allows the center of mass of the whole new robot to remain free of any spatial displacements during transfer motion at any position and location in its working space. Similar to a rotating sphere, the center of mass of the new robot will rotate but will not undergo displacements. None of the four actuators of the new robot are equipped with a brake function. During the transfer process, the new robot can self-balance. The new robot can transfer a heavy load at its end with its hand alone to any location in its working space. In addition, the new robot can suspend a heavy load at its end for a long time and does not require each actuator to output any torque. Mechanism G has been found to exhibit considerably improved dynamic performance in real-world applications.



(a) side A of mechanism G (b) side B of mechanism G

Fig. 22. Photograph of a prototype of mechanism G

6. Conclusions

Although there are more and more mechanisms with P-DOFs, P-DOFs are often regarded as dispensable, and the value of P-DOFs have not been effectively developed and utilized.This study presents an analysis of MLMs with P-DOFs formed by multi-loop serial connection and multi-look stacking as well as a DOF-S model established based on the decomposition and combination of the DOFs of MLMs with P-DOFs. In addition, drawing inspiration from the characteristics of biological parasitism, this study also proposes H–P mechanisms based on the DOF-S model. Moreover, this study elucidates three basic types of parasitism and eight types of parasitism in mechanism branch-chains. The analysis of mechanism examples finds the following. The DOFs of an MLM with P-DOFs can be longitudinally and centrally, transversely and centrally, and comprehensively optimally distributed in its DOF-S through parasitism. By optimizing the distribution of DOFs, a self-balancing H–P-type transfer robot prototype has been developed. In particular, all the actuators of this new robot are designed as counterweights. This design allows the center of mass of the whole new robot to remain free of any spatial displacements during transfer motion at any position and location in its working space. This characteristic is similar to that of a rotating sphere. This study provides a theoretical approach and basis for DOF analysis and structural synthesis of MLMs and can facilitate effective application of MLMs in more fields.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research,authorship,and /or publication of this article.

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References

[1] G. Gogu, Mobility of mechanisms: a critical review, Mech. Mach. Theory 40 (2005) 1068-1097.

[2] K. Kutzbach, Mechanical manifold, its laws and applications, Mech. Eng 8 (1929) 710-716.

[3] H. Zhen,Q. Li,H. Ding, Theory of Parallel Mechanisms, Springer, Netherlands, 2013.

[4] Z. Huang,Q.C. Li, Type Synthesis of Symmetrical Lowermobility Parallel Mechanisms Using the Constraint-synthesis Method, Int. J. Robot. Res. 22 (2003) 59-82.

[5] Z. Huang,J.F. Liu,D.X. Zeng, A general methodology for mobility analysis of mechanisms based on constraint screw theory, Science in China Series E: Technological Sciences 52 (2009) 1337-1347.

[6] S. Liu,X.J. Wang, Parallel kinematics：Types，kinematics and optimal design, Springer-Verlag, Berlin Heidelberg, 2014.

[7] J.M. Rico,L.D. Aguilera,J. Gallardo,R. Rodriguez, A More General Mobility Criterion for Parallel Platforms, J. Mech. Design (2006).

[8] J.M. Rico,J. Gallardo,B. Ravani, Lie Algebra and the Mobility of Kinematic Chains, Journal of Robotic Systems 20 (2003) 477-499.

[9] Y.T. Zhang,D.J. Mu, New concept and new theory of mobility calculation for multi-loop mechanisms, Science China(Technological Sciences) (2010).

[10] Y.T. Zhang,W.J. Lu,D.J. Mu,Y.D. Yang,L.J. Zhang,D. Zeng, Novel Mobility Formula for Parallel Mechanisms Expressed with Mobility of General Link Group, Chinese Journal of Mechanical Engineering (2013) 24-32.

[11] Z. Huang,Y. Zhao,T. Zhao, Advanced space mechanism, Higher Education Press, Beijin, 2014.

[12] Bagci,Cemil, Degrees of Freedom of Motion in Mechanisms, Journal of Engineering for Industry 93 (1971) 140.

[13] Y. Lu,N.J. Ye,L.J. Zhang, Analysis and Determination of Associated Linkage, Redundant Constraint, and Degree of Freedom of Closed Mechanisms With Redundant Constraints and/or Passive Degree of Freedom, J. Mech. Design 134 (2012) 61002.

[14] J.S. Zhao,K. Zhou,Z.J. Feng, A theory of degrees of freedom for mechanisms, Mech. Mach. Theory 39 (2004) 621-643.

[15] J.G. Kim,Y.S. Yoon, Systematic development of vehicle suspensions having passive DOF and/or redundant constraint, Int. J. Vehicle Des. 38 (2005) 275-289.

[16] J. Liu, Equivalent Method of Output Mobility Calculation for a Novel Multi-loop Coupled Mechanism, Journal of Mechanical Engineering 50 (2014).

[17] C. Li,H. Guo,D. Tang,R. Liu,S.I. ZAHIN, Cell division method for mobility analysis of multi-loop mechanisms, Mech. Mach. Theory 141 (2019) 67-94.

[18] W.A. Cao,H.F. Ding,Z.M. Chen,S.P. Zhao, Mobility analysis and structural synthesis of a class of spatial mechanisms with coupling chains, Robotica (2015) 1-19.

[19] F. Hu,Y.P. Song,Z.R. Huang,W.L. Liu,W. Li, Malleability and optimization of tetrahedral metamorphic element for deployable truss antenna reflector, AIP Adv. 8 (2018) 55217.

[20] Z.X. Huang Z, Computation of Mechanisms DOF: Principle and Method, Higher Education Press, Beijin, 2016.

[21] X. Zhang,D.J. Mu,Y.T. Zhang,H.H. You,H.R. Wang, Type Synthesis of Multi-Loop Spatial MechanismsWith Three Translational Output Parameters Based on Virtual-Loop Theory and Assur Groups, Robotica 37 (2019) 1104-1119.

[22] S. Zhang,J. Liu,H. Ding, Research for a novel class of translational spatial multi-loop coupled mechanisms: Type synthesis and kinematic analysis, Advances in Mechanical Engineering 10 (2018).

[23] Z. Xun,Y.A. Yao,Y. Li,Y. Tian,X. Sun,X. Liu, A novel rhombohedron rolling mechanism, Mech. Mach. Theory 105 (2016) 285-303.

[24] H.F. Ding,W.A. Cao,Z.M. Chen, Structural synthesis of two-layer and two-loop spatial mechanisms with coupling chains, Mech. Mach. Theory 92 (2015) 289-313.

[25] Ma K, Ma H, Tian H. Kinematic analysis of a novel 2-PrRS-PR(P)S metamorphic parallel mechanism[J]. Advances in Mechanical Engineering. 2019, 11(11): 2072158065.

[26] X.F. Zheng,D.S. Guo,Y.L. Ding, Supperparasitic Evolution of Parasitic Shrubs on Trees, Acta Scientiarum Naturalium Universitatis Sunyatseni 42 (2003) 114-117.

[27] D. Sharma,N.K. Singh,H. Singh, Copro-prevalence and risk factor assessment of gastrointestinal parasitism in Indian domestic pigs, Helminthologia 57 (2020) 28-36.

[28] X. Cheng, Human parasitology, Fudan University Press, Shanghai, 2015.

[29] S. Charentus, Mod´elisation et commande d’ un robot manipulateur redondant compos´e de plusieurs plate-formes, Universit´e Paul Sabatier, Toulouse, 1990.