Fixed-time Active Disturbance Rejection Consensus Tracking Control of A Multi-Agent System via Event-Triggered Mechanism

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Fixed-time Active Disturbance Rejection Consensus Tracking Control of A Multi-Agent System via Event-Triggered Mechanism

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Abstract A fixed-time active disturbance rejection control (FTADRC) consensus tracking strategy is proposed for a class of non-affine nonlinear multi-agent systems with an event-trigger-based communication. Non-affine followers are transformed into affine ones by combining the implicit function theorem with the mean value theorem. A distributed event-triggered estimator is introduced based on its neighbor output information. It is for estimation of a leader's signal for parts of followers, who are not able to access the leader signal in a direct manner. A distributed FTADRC control strategy is then developed via an event-triggered communication in the framework of backstepping technology. With the help of the fixed-time control, the settling time of an MAS is assignable and independent on initial conditions. Extended state observers and tracking differentiators are employed to compensate unknown dynamics of each follower in real time and estimate derivatives of virtual control laws, respectively. It is proven theoretically that the MAS achieves input-to-state practically stability and the consensus tracking error converges to a neighborhood around the origin in a fixed time. Also, Zeno behavior is excluded. Finally, two examples are performed to illustrate the effectiveness of the proposed strategy.

Keywords Consensus tracking control · active disturbance rejection control · fixed-time control · distributed event-triggered estimator

1 Introduction

Recently, considerable attentions have been paid to consensus control of multi-agent systems (MASs) due to its potential applications in various fields, for in-

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stance, mobile robots [1], unmanned aerial vehicles (UAVs) [2] and autonomous underwater vehicles [3]. Consensus tracking control is one of critical issues, and it steers agents to achieve output consensus with neighbors' information via topological communication.

Fruitful results associated to consensus tracking control of MASs are presented. A adaptive neural network controller is designed for nonlinear MASs to realize the consensus tracking in [4]. Zhao et al. propose a distributed adaptive consensus tracking control protocol for a class of nonlinear MASs, which is based on state observer and command filter in [6]. Liu et al. design a distributed adaptive output consensus tracking controller for a class of high order nonlinear MASs with output constraints, actuator failures and external disturbances [7]. A class of second-order MASs are studied in [8], and it proposes a consensus control protocol based on adaptive neural control. Fan et al. put forward a backstepping control scheme based on finite time command filter to achieve consensus tracking in [9]. The MASs in the previously mentioned references [4, 6–9] are in affine form.

Active disturbance rejection control (ADRC) [10] is one of key techniques for consensus tracking control. From the perspective of controller design, the results of ADRC are mainly divided into two categories. The first one is to design the active disturbance rejection controller of the system directly, and optimize the controller parameters by optimal algorithms [14–16]. For example, Tian et al. apply ADRC to a general nonlinear uncertain system with mismatched disturbances [14]. Chen et al. propose an active disturbance rejection controller to deal with measurement uncertainties [15]. A cascaded ADRC scheme is introduced to cope with external disturbances and model uncertainties in [16]. The second one is the controller design for nonlinear system based on ADRC. Unknown terms or/and external disturbances are estimated by extended state observers (ESOs), and tracking differentiators (TDs) are utilized to avoid computational explosion in the traditional backstepping method [17, 18]. For instance, Feng et al. design an ESO with high gains for offset disturbance in [17]. An ESO in [18] is also constructed to estimate states and total uncertainties, including uncertain internal dynamics and external disturbances. Nevertheless, the above results are based on time-triggered mechanism, and they do not consider limited communication resource.

Apart from time-triggered approaches, an event-triggered mechanism (ETM) is another alternative scheduling scheme. In order to take advantage of distributed MASs and reduce communication and computation as much as possible, event-triggered control is introduced. As the measurement error violates a triggered condition, one or more events will be triggered, which triggers a series of system actions such as controller update or communication among followers. The setting of triggered conditions can be broadly classified into two categories: one is a state-related triggered conditions [19–22], and the other is a state-independent triggered conditions. Also, they are divided into fixed threshold triggered conditions [25–27] and time-varying threshold ones [28]. In addition, several triggered locations are concerned including the following

several channels: from a sensor to a controller [19–22], from a controller to an actuator [23, 24, 29, 30], and two channels mentioned above [31]. Consequently, compared with traditional time-triggered control, event-triggered control effectively reduces the number of control tasks, thus significantly saving communication resources on the basis of ensuring the closed-loop system performance. Nevertheless, the communication burdens among followers have not been taken into account, and there still exist few triggered results for communication links in an MAS.

Many efforts have been made on fixed-time control (FTC) for nonlinear MASs. The concept of fixed-time stability is introduced firstly by Polyakov [32]. Due to the existence of exponential terms, this fixed-time controller ensures state variables converge to an equilibrium quickly. In [33], a fixed time fault tolerant controller is designed for MASs with directed communication topology, so that the followers can track the leader in a fixed time. A distributed control strategy based on fixed-time observer is provided [34], which can estimate the state of the leader for each follower. Similar to [34], Yang et al. also introduce a distributed fixed-time observer in [35], so that each agent can accurately receive the information of the leader in bounded time. Then, a time-varying formation fixed-time tracking control protocol is developed to ensure that all followers can achieve the required time-varying formation and track the leader at the same time. However, the control schemes [33–35] have not taken the limited communication resources in practical applications into account. On the basis of [35], Ni et al. combine the ETM with distributed fixed-time observer to estimate the leader's state in [36]. An event-based adaptive fuzzy fixed-time tracking control scheme is investigated in [23]. The scheme ensures the fixed-time stability of the system, and the ETM can save the network communication resources. Xiao et al. study the fixed-time tracking problem for uncertain nonlinear system with input saturation constraints in [24]. In order to reduce the communication burden between the controller and the actuator, the adaptive controller based on event-triggered is considered to ensure that the control input is updated only when the event-triggered condition is violated. The above literatures all are triggered for the channel from a controller to an actuator, and there is still a lack of theory to consider communication link among followers.

Although consensus tracking control of nonlinear MASs has achieved fruitful achievements, there still exist technical gaps. Many references only pay attention to stability of an MAS with ADRC approach [37–39], and these results rarely mention settling time. Even though authors in [40] introduce finite-time control for MASs, its convergence time is dependent on initial conditions. Most results are discussed for MASs with affine followers, and there is still a lack of theory to deal with FTC, as well as ADRC approach, for non-affine nonlinear MASs. The aforementioned control problems for MASs [23, 24] only focus on communication resource in a follower itself, and communication burden among followers are not taken into account. There is a lack of a feasible approach for reducing the burdens over communication links in an MAS.

To fill in these technical gaps, in this paper, we develop a FTC strategy for non-affine nonlinear MASs, where the settling time is independent of initial states, and introduce an ETM over communication channels among followers. The main contributions are summarized as follows.

1. A distributed fixed-time active disturbance rejection control(FTADRC) consensus tracking strategy is proposed. Generally, we take the directly available leader's signal as the input signal of ESOs to process the unknown dynamics in MASs. However, we consider the case where the leader's signal is not directly available, and this increase the difficulty of ESOs construction. On the basis of this, we introduce a distributed estimator which takes the output information of neighbor as input. The leader signal is obtained indirectly through the distributed estimator, which is then utilized as input to the ESOs. And the strategy contains not only estimation information from ESOs and TDs but also a fractional power term, as well as a integer power term greater than 1. Meanwhile, a traditional Lyapunov [40] is not feasible for stability analysis due to the existence of estimation error of ESOs and TDs. The ISpS theory is then introduced, and it is proven that the MAS is bounded within a fixed time.
2. Our distributed FTADRC is proposed for non-affine nonlinear MASs. The implicit function theorem and mean value theorem are utilized to transform a non-affine follower into an affine one, and the control input appears in affine form. Different from the control approaches for a strict feedback MAS in [41] and an affine one in [5–9], we extend the form of followers from an affine structure to a non-affine one. We treat linearly parameterized and affine pure-feedback ones as special cases, and it is a challenging and nontrivial task to be concerned on this extension.
3. A distributed event-triggered estimator is designed for constructing leader's signal. Due to the existence of the followers who cannot obtain leader signal directly, we design a distributed estimator for them based on neighbor output information. An ETM is introduced into the distributed estimator to avoid the waste of communication resources caused by real-time data transmission. In contrast to the distributed control method based on real-time communication in [41], the distributed event-triggered estimator proposed in this paper realizes intermittent communication between adjacent agents.

2 Problem Formulation and Preliminaries

2.1 Problem Formulation

Consider an MAS consisting of N followers and one leader. The dynamics of the i th follower in non-affine form are

$$\begin{cases} \dot{x}_{i,1} = f_{i,1}(x_{i,1}, x_{i,2}), \\ \dot{x}_{i,2} = f_{i,2}(\bar{x}_{i,2}, x_{i,3}), \\ \vdots \\ \dot{x}_{i,n_i} = f_{i,n_i}(\bar{x}_{i,n_i}, u_i), \\ y_i = x_{i,1}, \end{cases} \quad (1)$$

where $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \dots, x_{i,m}]^T$, $m = 2, \dots, n_i$, and $u_i \in \mathbb{R}$, $i = 1, \dots, N$ are the state vector and the control law of the i th follower, respectively, $f_{i,q}(\cdot)$ is an unknown and smooth function, $q = 1, 2, \dots, n_i$, and y_i is the output signal of the i th follower.

We define continuous functions $g_{i,q}(x_{i,q+1}) = [\partial f_{i,q}(\bar{x}_{i,q}, x_{i,q+1})] / [\partial x_{i,q+1}]$, $i = 1, \dots, N$, $q = 1, \dots, n_i$ and $x_{i,n_i+1} = u_i$.

The following preliminaries with assumptions and lemmas are necessary for this paper.

Assumption 1 *The leader's signal y_r , its first-order derivative $\dot{y}_r$ and the second-order derivative $\ddot{y}_r$ are all bounded.*

Assumption 2 ([42] [43]) *The signs of functions $g_{i,q}(\cdot)$ are known, and there exist unknown positive constants $\bar{g}_{i,q}$ and $\underline{g}_{i,q}$ such that $\underline{g}_{i,q} \leq |g_{i,q}(\cdot)| \leq \bar{g}_{i,q}$ is satisfied.*

Remark 1 Due to the fact that $g_{i,q}(\cdot)$ are continuous smooth functions, Assumption 1 indicates that $g_{i,q}(\cdot)$ are strictly either positive or negative. In this paper, we mainly consider the scenario that $g_{i,q}(\cdot) > 0$. From the mathematical perspective, $g_{i,q}(\cdot) < 0$ is treated as a simple transformation, and this scenario can be discussed essentially in the same way. Without loss of generality, we only take $g_{i,q}(\cdot) > 0$ into account.

Remark 2 The dynamics of the i th follower (1) is in a non-affine form, and it is more general compared with the plant in [44]. In fact, the dynamics of the i th follower (1) represent many practical systems, for instance, robotic manipulators [45], hypersonic flight vehicles [46] and marine surface vessels [47]. Consequently, the control strategy for an MAS with follower (1) is not only for academic improvement but also for these systems in the real world.

Lemma 1 ([43] [48]) *Supposed that continuous function $f(x, y) : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$ is differentiable and there exists a constant $d > 0$ satisfying $\partial f(x, y) / \partial y > d$, $\forall (x, y) \in \mathbb{R}^n \times \mathbb{R}$. Then, there exists a smooth continuous function $\mu^*(x)$ such that $f(x, \mu^*) = 0$.*

Lemma 2 ([42]) For any $x \in \mathbb{R}^n$, if function $f(x, y)$ is continuous over $[a, b]$ and differentiable over (a, b) , $\exists \lambda, 0 < \lambda < 1$, the following result

$$f(x, y) = \frac{\partial f(x, y)}{\partial y} \Big|_{y=y_\lambda} (b - a) + f(x, a)$$

holds with $y_\lambda = \lambda b + (1 - \lambda)a$.

2.2 Fixed Time Stable

Consider a dynamical system in the following form:

$$\dot{x}(t) = f(x(t)), x(0) = x_0, \quad (2)$$

where $x \in \mathbb{R}^n$ denotes a state vector, and $f(\cdot)$ is a continuous function.

Lemma 3 ([49–51]) Let $V(x)$ be a smooth function with $V(x) \geq 0$. If the following inequality holds: $\dot{V}(x) \leq -(\mu_1 V^\alpha(x) + \mu_2 V^\beta(x))^\gamma + \varrho$, where $\mu_1, \mu_2, \alpha, \beta, \gamma$ are the positive parameters with $\alpha\gamma \in (0, 1)$, $\beta\gamma \in (1, \infty)$ and $\varrho > 0$. Then, the origin $x = 0$ of the system (2) is practically fixed time stable. The settling time can be derived as $T_s \leq \frac{1}{\mu_1^\gamma \omega^\gamma (1 - \alpha\gamma)} + \frac{1}{\mu_2^\gamma \omega^\gamma (\beta\gamma - 1)}$, where $0 < \omega \leq 1$.

Lemma 4 ([52]) For any constant $\varepsilon > 0$ and a variable $y \in \mathbb{R}$, the following inequality $0 \leq |y| \leq \varepsilon + \frac{y^2}{\sqrt{y^2 + \varepsilon^2}}$ holds.

Lemma 5 ([51]) For $x_i \in \mathbb{R}, i = 1, \dots, n$, and $\eta \in [0, 1]$, $(|x_1| + \dots + |x_n|)^\eta \leq |x_1|^\eta + \dots + |x_n|^\eta$ holds.

Lemma 6 For $z_i \in \mathbb{R}, i = 1, \dots, n$, $(\sum_{i=1}^n z_i)^2 \leq n \sum_{i=1}^n z_i^2$ holds.

Lemma 7 ([53]) For $\varpi_1, \varpi_2 \in \mathbb{R}$, the following inequality $\varpi_1 \varpi_2 \leq \frac{\epsilon^m}{m} |\varpi_1|^m + \frac{1}{n\epsilon^n} |\varpi_2|^n$ holds with $\epsilon > 0, m > 1, n > 1$, and $(m - 1)(n - 1) = 1$.

Lemma 8 ([54]) For $\phi \in \mathbb{R}, \varphi \in \mathbb{R}$, and $m > 0, n > 0, z > 0$, we get $|\phi|^m |\varphi|^n \leq \frac{m}{m+n} z |\phi|^{m+n} + \frac{n}{m+n} z^{-\frac{m}{n}} |\varphi|^{m+n}$.

2.3 Input-to-state practically Stability (ISpS)

In what follows, some necessary knowledge about ISpS and ISpS-Lyapunov function are presented.

Definition 1 ([55, 56]) For any initial condition $x(0)$ and bounded input u , if there exist a class $\mathcal{KL}$ function ψ_1 , a class $\mathcal{K}$ function ψ_2 and a constant $d_1 > 0$ such that

$$\|x(t)\| \leq \psi_1(\|x(0)\|, t) + \psi_2(\|u\|_\infty) + d_1, \forall t \geq 0, \quad (3)$$

the nonlinear system $\dot{x} = f(x, u)$ is said to be input-to-state practically stable, where ψ_2 is a nonlinear L_∞ gain, and $\|\cdot\|$ and $\|\cdot\|_\infty$ are 2-norm and ∞ -norm of a vector.

Definition 2 ([56]) A C^1 function $V(x)$ is an ISpS-Lyapunov function of a nonlinear system $\dot{x} = f(x, u)$ if there exist $\mathcal{K}$ functions $\kappa_1(s)$, $\kappa_2(s)$, $\kappa_3(s)$, $\kappa_4(s)$, and a non-negative constant d_2 such that

$$\begin{cases} \kappa_1(\|x\|) \leq V(x) \leq \kappa_2(\|x\|), \forall x \in \mathbb{R}^n, \\ \frac{\partial V(x)}{\partial x} f(x, u) \leq -\kappa_3(\|x\|) + \kappa_4(\|u\|) + d_2. \end{cases} \quad (4)$$

Lemma 9 ([57]) A system $\dot{x} = f(x, u)$ is ISpS if there exists an ISpS-Lyapunov function. And, a nonlinear L_∞ gain can be chosen as $\psi_2(s) = \kappa_1^{-1} \circ \kappa_2 \circ \kappa_3^{-1} \circ \kappa_4(s)$.

2.4 Graph Theory

In this paper, we consider an MAS consisting of N followers and one leader. The communication topology is connected by a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$. $\mathcal{V} = \{0, 1, \dots, N\}$ denotes a node set. The edges set can be expressed as $\mathcal{E} = \{(j, i) \in \mathcal{V} \times \mathcal{V}\}$. In a directed graph, $(j, i) \in \mathcal{E}$ is that node i can receive the information from node j , directly. As for the neighborhood set of node i , namely, $\mathcal{N}_i = \{j | (j, i) \in \mathcal{E}\}$. The adjacency matrix $\mathcal{A} = [a_{ij}]_{[N+1] \times [N+1]}$ is

$$a_{ij} = \begin{cases} 1, & \text{if } (j, i) \in \mathcal{E}, \\ 0, & \text{otherwise.} \end{cases} \quad \text{A directed graph } \mathcal{G} \text{ is said that it contains a spanning}$$

tree if there exists one node, and this node has a directed path from itself to other nodes. For describing of interactive communication among followers, we define a subgraph $\bar{\mathcal{G}}(\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$. In this subgraph, $\bar{\mathcal{V}} = \{1, \dots, N\}$ denotes a set consisting all followers, and $\bar{\mathcal{E}} = \{(j, i) \in \bar{\mathcal{V}} \times \bar{\mathcal{V}}\}$ means the set of edges. The adjacency matrix of the subgraph is defined as $\bar{\mathcal{A}} = [\bar{a}_{ij}]_{N \times N}$, and if $(j, i) \in \bar{\mathcal{E}}$, $\bar{a}_{ij} = 1$, otherwise, $\bar{a}_{ij} = 0$. Define the Laplacian matrix of $\mathcal{G}$ as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1 \times N} \\ -c & \bar{\mathcal{H}} \end{bmatrix}$ with $c = [c_1, \dots, c_N]^T$ and $\bar{\mathcal{H}} = \bar{\mathcal{L}} + \mathcal{C}_0$, where $\bar{\mathcal{L}} = \bar{\mathcal{D}} - \bar{\mathcal{A}}$ is the Laplacian matrix of a subgraph $\bar{\mathcal{G}}$ with its in-degree matrix $\bar{\mathcal{D}} = \text{diag}\{d_1, \dots, d_N\}$ and $d_i = \sum_{j=1}^N \bar{a}_{ij}$, $\mathcal{C}_0 = \text{diag}\{c_1, \dots, c_N\}$, and $c_i = 1$ if the i th follower can directly access to the information from leader, otherwise $c_i = 0$.

Assumption 3 The leader is considered as a root node, and the directed graph $\mathcal{G}$ includes a directed spanning tree.

Lemma 10 ([58, 59]) If a subgraph $\bar{\mathcal{G}}$ contains a directed spanning tree, the real parts of all eigenvalues in $\bar{\mathcal{H}} = \bar{\mathcal{L}} + \mathcal{C}_0$ are positive, and $\bar{\mathcal{H}}$ is reversible.

2.5 Control Objective

For an MAS in (1), we aim for designing an event-triggered FTADRC consensus tracking control strategy. In this way, the outputs of followers track the

Fig. 1 Block diagram of the MAS.

trajectory of the leader y_r in a fixed time. That is, the consensus tracking error $\Lambda = y - (y_r \otimes I_N)$ converges to a compact set, where $y = [y_1, \dots, y_N]^T \in \mathbb{R}^N$, I_N is an identity matrix and $\otimes$ stands for the Kronecker product.

3 Event-triggered based FTADRC consensus tracking control strategy

In this section, an event-triggered based FTADRC consensus tracking control strategy is designed. A block diagram of the MAS is presented in Fig. 1, and the details can be found in the following process.

Due to the existence of the followers who cannot access information of the leader directly, we design a distributed estimator for each follower. Also, we introduce the ETM to realize intermittent communication between adjacent agents and save the communication resources.

$$\begin{cases} \dot{\hat{r}}_{i,1} = \hat{r}_{i,2}, \\ \dot{\hat{r}}_{i,2} = -\eta_{i,1} (\hat{r}_{i,1} - \zeta_i) - \eta_{i,2} \hat{r}_{i,2}, \end{cases} \quad (5)$$

where $\hat{r}_{i,1}$, and $\hat{r}_{i,2}$ are the estimation of the leader's signal y_r and its derivation $\dot{y}_r$, respectively, $\eta_{i,1}$ and $\eta_{i,2}$ are both positive design parameters, $i = 1, \dots, N$, $\zeta_i = \frac{1}{d_i + c_i} \left(\sum_{j \in \mathcal{N}_i} a_{ij} \check{y}_j + c_i y_r \right)$, $\check{y}_j$ is the event-triggered output signal of the j th follower

$$\check{y}_j(t) = \begin{cases} y_j(t_{j,k}), & \text{if } \Gamma_j(t) < 0, \\ y_j(t), & \text{otherwise,} \end{cases} \quad (6)$$

$\Gamma_j(t) = |e_{y,j}| - \vartheta_{y,j}|e_{j,1}| - \bar{\vartheta}_{y,j}$ is the event-triggered condition, $e_{y,j} = y_j(t) - y_j(t_{j,k})$ is a measurement error, $t_{j,k}$ is k th triggered instant, $k \in \mathbb{Z}^+$, and $\vartheta_{y,j}$ and $\bar{\vartheta}_{y,j}$ are positive constants. Only the event-triggered condition (6) is violated, the output of j th follower will be updated and transmitted to estimator (5) for estimating leader's signal, and this instant is recorded as $t_{j,k+1}$. Otherwise, the value of j th follower's output is maintained from the perspective of its neighbors during the period $t \in [t_{j,k}, t_{j,k+1})$.

The estimation errors are defined as $\tilde{r}_{i,1} = r_{i,1} - \hat{r}_{i,1}$, $\tilde{r}_{i,2} = r_{i,2} - \hat{r}_{i,2}$, $\tilde{r}_1 = [\tilde{r}_{1,1}, \dots, \tilde{r}_{N,1}]^T \in \mathbb{R}^N$, $\tilde{r}_2 = [\tilde{r}_{1,2}, \dots, \tilde{r}_{N,2}]^T \in \mathbb{R}^N$, and $\tilde{r} = [\tilde{r}_1, \tilde{r}_2]^T$. The dy-

namics of these estimation errors are
$$\begin{cases} \dot{\tilde{r}}_{i,1} = \tilde{r}_{i,2}, \\ \dot{\tilde{r}}_{i,2} = -\eta_{i,1}(\zeta_i - \hat{r}_{i,1}) - \eta_{i,2}\tilde{r}_{i,2} + \Upsilon_i, \end{cases}$$

where $\Upsilon_i = \eta_{i,2}\dot{y}_r + \ddot{y}_r$, $\zeta_i - \hat{r}_{i,1} = \frac{1}{d_i + c_i} \left(\sum_{j \in \mathcal{N}_i} a_{ij}e_{j,1} - \sum_{j \in \mathcal{N}_i} a_{ij}e_{y,j} + \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{r}_{j,1} - \hat{r}_{i,1}) + c_i(y_r - \hat{r}_{i,1}) \right)$, $e_{j,1} = y_j - \hat{r}_{j,1}$, and $e_{y,j} = y_j - \check{y}_j$.

Accordingly, $\dot{\tilde{r}} = E_1\tilde{r} - E_2e_1 + E_2e_y + E_3$ with $e_y = [e_{y,1}, \dots, e_{y,N}]^T$, $E_1 = \begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -\eta_1 \bar{\mathcal{H}}^{\mathcal{F}} & -\eta_2 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0_{N \times N} \\ \eta_1 \bar{\mathcal{A}}^{\mathcal{F}} \end{bmatrix}$, and $E_3 = \begin{bmatrix} 0_N \\ \Upsilon \end{bmatrix}$, where $\bar{\mathcal{H}}^{\mathcal{F}} = (\bar{\mathcal{D}} + \mathcal{C}_0)^{-1} \bar{\mathcal{H}}$, $\bar{\mathcal{A}}^{\mathcal{F}} = (\bar{\mathcal{D}} + \mathcal{C}_0)^{-1} \bar{\mathcal{A}}$, $\eta_1 = \text{diag}\{\eta_{1,1}, \dots, \eta_{N,1}\}$, $\eta_2 = \text{diag}\{\eta_{1,2}, \dots, \eta_{N,2}\}$, and $\Upsilon = [\Upsilon_1, \dots, \Upsilon_N]^T$. Due to the fact that $\eta_{i,1} > 0$, $\eta_{i,2} > 0$ and $\bar{\mathcal{H}} > 0$, E_1 is a stable matrix. Thus, for any given matrix $S > 0$, there exists a positive symmetric matrix P such that $E_1^T P + P E_1 = -S$.

On the basis of the triggered estimation information, we develop the control strategy in a recursive manner, and the procedure contains n_i steps.

Step $i, 1$: Define the error variable $e_{i,1}(t) = x_{i,1} - \hat{r}_{i,1}$, and its derivation is

$$\dot{e}_{i,1} = f_{i,1}(x_{i,1}, x_{i,2}) - \hat{r}_{i,2}. \quad (7)$$

In (7), since $g_{i,1} \geq \underline{g}_i$, $i = 1, \dots, N$, and according to the implicit function theorem in Lemma 1, one sees that there exists a continuous function vector $\alpha_{i,2}^*(\bar{x}_{i,2})$ such that $f_{i,1}(x_{i,1}, \alpha_{i,2}^*) = 0$. Then, from the mean value theorem, we have

$$f_{i,1}(x_{i,1}, x_{i,2}) = f_{i,1}(x_{i,1}, \alpha_{i,2}^*) + g_{\lambda_{i,1}}(x_{i,2} - \alpha_{i,2}^*), \quad (8)$$

where $g_{\lambda_{i,1}} = g_{i,1}(x_{i,1}, x_{\lambda_{i,1}})$ and $x_{\lambda_{i,1}} = \lambda_{i,1}x_{i,2} + (1 - \lambda_{i,1})\alpha_{i,2}^*$. Accordingly, the dynamics of $e_{i,1}$ are

$$\dot{e}_{i,1} = -g_{\lambda_{i,1}}\alpha_{i,2}^* + g_{\lambda_{i,1}}x_{i,2} - \hat{r}_{i,2}. \quad (9)$$

An item $F_{i,1} = -g_{\lambda_{i,1}}\alpha_{i,2}^* + (g_{\lambda_{i,1}} - b_{i,2})x_{i,2}$ is introduced here, where $b_{i,2}$ is a positive parameter to be designed, $i = 1, \dots, N$. Accordingly, (9) is transformed

$$\dot{e}_{i,1} = F_{i,1}(t) + b_{i,2}x_{i,2} - \hat{r}_{i,2}. \quad (10)$$

Due to the existence of $F_{i,1}$ in (10), an ESO is constructed to estimate it

$$\begin{cases} E_{i,1} = \phi_{i,1,1} - e_{i,1}, \\ \dot{\phi}_{i,1,1} = \phi_{i,1,2} - \rho_{i,1,1} E_{i,1} + b_{i,2} x_{i,2} - \hat{r}_{i,2}, \\ \dot{\phi}_{i,1,2} = -\rho_{i,1,2} |\phi_{i,1,1}|^{\sigma_{i,1,2}} \text{sgn}(E_{i,1}), \end{cases} \quad (11)$$

where $E_{i,1}$ is the estimation error, $\phi_{i,1,1}$ and $\phi_{i,1,2}$ are the state variables of the ESO (11), $\phi_{i,1,2}$ is the estimation value for $F_{i,1}(t)$, $\text{sgn}(\cdot)$ represents the signum function, and the design parameters $\rho_{i,1,1} > 0$, $\rho_{i,1,2} > 0$ and $\sigma_{i,1,2} \in (0, 1)$, $i = 1, \dots, N$.

As a matter of fact, with the Lemma 3 in mind, we design the virtual control law by the following equation. That is, $\alpha_{i,2} = -\frac{e_{i,1} \tilde{\alpha}_{i,2}^2}{b_{i,2} \sqrt{e_{i,1}^2 \tilde{\alpha}_{i,2}^2 + \varepsilon_{i,1}^2}}$, where $\tilde{\alpha}_{i,2} = k_{i,1,1} \left(\frac{1}{2}\right)^\alpha \frac{(e_{i,1}^2)^\alpha}{e_{i,1}} + k_{i,1,2} \left(\frac{1}{2}\right)^\beta e_{i,1}^{2\beta-1} + \phi_{i,1,2} - \hat{r}_{i,2}$. Inspired by the result in [60] and for convenience of derivation, the parameters in Lemma 3 are selected as $\alpha = \frac{3}{4}$, $\beta = 2$ and $\gamma = 1$.

Then, a virtual control law is designed in this step

$$\alpha_{i,2} = -\frac{e_{i,1} \tilde{\alpha}_{i,2}^2}{b_{i,2} \sqrt{e_{i,1}^2 \tilde{\alpha}_{i,2}^2 + \varepsilon_{i,1}^2}} \quad (12)$$

with

$$\tilde{\alpha}_{i,2} = k_{i,1,1} \left(\frac{1}{2}\right)^{\frac{3}{4}} \frac{(e_{i,1}^2)^{\frac{3}{4}}}{e_{i,1}} + k_{i,1,2} \left(\frac{1}{2}\right)^2 e_{i,1}^3 + \phi_{i,1,2} - \hat{r}_{i,2}, \quad (13)$$

where $k_{i,1,1} \left(\frac{1}{2}\right)^{\frac{3}{4}} \frac{(e_{i,1}^2)^{\frac{3}{4}}}{e_{i,1}}$ and $k_{i,1,2} \left(\frac{1}{2}\right)^2 e_{i,1}^3$ are the fixed-time control terms, $b_{i,2} > 0$, $k_{i,1,1} > 0$ and $k_{i,1,2} > 0$ are positive parameters.

Choose the following Lyapunov function

$$V_{i,1} = \tilde{r}^T P \tilde{r} + \frac{1}{2} e_{i,1}^2, \quad (14)$$

where P is a positive symmetric matrix. According to $\dot{\tilde{r}} = E_1 \tilde{r} - E_2 e_1 + E_2 e_y + E_3$ and (10), the time derivative of $V_{i,1}$ is

$$\begin{aligned} \dot{V}_{i,1} &= 2\tilde{r}^T P (E_1 \tilde{r} - E_2 e_1 + E_2 e_y + E_3) + e_{i,1} (F_{i,1} + b_{i,2} x_{i,2} - \hat{r}_{i,2}) \\ &= 2\tilde{r}^T P E_1 \tilde{r} - 2\tilde{r}^T P E_2 e_1 + 2\tilde{r}^T P E_2 e_y + 2\tilde{r}^T P E_3 + e_{i,1} F_{i,1} \\ &\quad + b_{i,2} e_{i,1} e_{i,2} + b_{i,2} e_{i,1} \alpha_{i,2} - e_{i,1} \hat{r}_{i,2}. \end{aligned} \quad (15)$$

From the event-triggered condition $\Gamma_j(t) = |e_{y,j}| - \vartheta_{y,j} |e_{j,1}| - \bar{\vartheta}_{y,j}$, there exists a matrix $E_r = \text{diag}\{\mu(t), \dots, \mu(t)\} \in \mathbb{R}^N$, $0 \leq |\mu(t)| \leq 1$, such that $e_y = E_r (\vartheta_y [|e_1|] + \bar{\vartheta}_y)$, where $\vartheta_y = \text{diag}\{\vartheta_{y,1}, \dots, \vartheta_{y,N}\}$, $\bar{\vartheta}_y = [\bar{\vartheta}_{y,1}, \dots, \bar{\vartheta}_{y,N}]^T$ and $[|e_1|] = [|e_{1,1}|, \dots, |e_{2,1}|, \dots, |e_{N,1}|]^T \in \mathbb{R}^N$. Furthermore, from the Young's inequality, we obtain $2\tilde{r}^T P E_1 \tilde{r} \leq -s_c \tilde{r}^T \tilde{r}$, $\|2\tilde{r}^T P E_2 e_1\| \leq \frac{s_c}{4} \tilde{r}^T \tilde{r} +$

$\frac{4}{s_c} \|P\|^2 \|E_2\|^2 e_1^T e_1, \|2\tilde{r}^T P E_2 e_y\| \leq \frac{s_c}{4} \tilde{r}^T \tilde{r} + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\vartheta_y\|^2 e_1^T e_1 + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\bar{\vartheta}_y\|^2$
 and $2\tilde{r}^T P E_3 \leq \frac{s_c}{4} \tilde{r}^T \tilde{r} + \frac{4}{s_c} \|P\|^2 \|E_3\|^2$, where s_c is the minimum eigenvalue of matrix S . With (12), (13), Lemma 4 and Lemma 7, we rewrite (15) as

$$\begin{aligned} \dot{V}_{i,1} \leq & -\frac{s_c}{4} \tilde{r}^T \tilde{r} + \left(\frac{4}{s_c} \|P\|^2 \|E_2\|^2 + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\vartheta_y\|^2 \right) e_1^T e_1 \\ & + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\bar{\vartheta}_y\|^2 + \frac{4}{s_c} \|P\|^2 \|E_3\|^2 - k_{i,1,1} \left(\frac{e_{i,1}^2}{2} \right)^{\frac{3}{4}} \\ & - (k_{i,1,2} - \epsilon_{i,1,1}^4) \left(\frac{e_{i,1}^2}{2} \right)^2 + \frac{3}{4\epsilon_{i,1,1}^{\frac{4}{3}}} |F_{i,1} - \phi_{i,1,2}|^{\frac{4}{3}} + b_{i,2} e_{i,1} e_{i,2} + \varepsilon_{i,1}, \end{aligned} \quad (16)$$

where $k_{i,1,1} > 0$, $k_{i,1,2} > 0$ and $\epsilon_{i,1,1} > 0$ satisfying $k_{i,1,2} - \epsilon_{i,1,1}^4 > 0$.

From the result in (16), $b_{i,2} e_{i,1} e_{i,2}$ cannot be processed in this current step. In order to counteract this item, it is necessary to design virtual control law in the next step.

Step i, q ($2 \leq q \leq n_i - 1$): Define $e_{i,q}(t) = x_{i,q} - \alpha_{i,q}$. Similar to the Step $i, 1$, we take derivative of $e_{i,q}$ yielding

$$\dot{e}_{i,q} = F_{i,q} + b_{i,q+1} x_{i,q+1} - \dot{\alpha}_{i,q}, \quad (17)$$

where $F_{i,q}(t) = -g_{\lambda_{i,q}} \alpha_{i,q+1}^* + (g_{\lambda_{i,q}} - b_{i,q+1}) x_{i,q+1}$, $g_{\lambda_{i,q}} = g_{i,q}(\bar{x}_{i,q}, x_{\lambda_{i,q}})$, $\lambda_{i,q} \in (0, 1)$, $x_{\lambda_{i,q}} = \lambda_{i,q} x_{i,q+1} + (1 - \lambda_{i,q}) \alpha_{i,q+1}^*$, $b_{i,q+1} > 0$ are design parameters and $i = 1, \dots, N$.

Similar to Step $i, 1$, a TD

$$\begin{cases} \dot{v}_{i,q-1,1} = v_{i,q-1,2}, \\ \dot{v}_{i,q-1,2} = -\lambda_{i,q}^2 \text{sgn}(v_{i,q-1,1} - \alpha_{i,q}) |v_{i,q-1,1} - \alpha_{i,2}|^{\sigma_{i,q,1}} - \lambda_{i,q} v_{i,q-1,2}, \end{cases} \quad (18)$$

and an ESO

$$\begin{cases} \dot{E}_{i,q} = \phi_{i,q,1} - e_{i,q}, \\ \dot{\phi}_{i,q,1} = \phi_{i,q,2} - \rho_{i,q,1} E_{i,q} + b_{i,q+1} x_{i,q+1} - v_{i,q,2}, \\ \dot{\phi}_{i,q,2} = -\rho_{i,q,2} |E_{i,q}|^{\sigma_{i,q,2}} \text{sgn}(E_{i,q}), \end{cases} \quad (19)$$

are utilized to estimate the derivative of virtual control law $\dot{\alpha}_{i,q}$ and the uncertain item $F_{i,q}$, respectively, where $\lambda_{i,q} > 0$ and $0 < \sigma_{i,q,l} < 1$ are constants to be designed and the states of TD $v_{i,q-1,1}$ and $v_{i,q-1,2}$ are the estimations of $\alpha_{i,q}$ and $\dot{\alpha}_{i,q}$, respectively, $E_{i,q}$ is the estimation error, $\phi_{i,q,1}$ and $\phi_{i,q,2}$ are the state variables, $\phi_{i,q,2}$ is the estimation value of $F_{i,q}(t)$, and the design parameters $\rho_{i,q,1} > 0$, $\rho_{i,q,2} > 0$ and $\sigma_{i,q,2} \in (0, 1)$, $i = 1, \dots, N$.

Then, the virtual control variable $\alpha_{i,q+1}$ is designed as

$$\alpha_{i,q+1} = -\frac{e_{i,q} \tilde{\alpha}_{i,q+1}^2}{b_{i,q+1} \sqrt{e_{i,q}^2 \tilde{\alpha}_{i,q+1}^2 + \varepsilon_{i,q}^2}} \quad (20)$$

with

$$\tilde{\alpha}_{i,q+1} = k_{i,q,1} \left(\frac{1}{2} \right)^{\frac{3}{4}} \frac{(e_{i,q}^2)^{\frac{3}{4}}}{e_{i,q}} + k_{i,q,2} \left(\frac{1}{2} \right)^2 e_{i,q}^3 + \phi_{i,q,2} - v_{i,q,2} + b_{i,q} e_{i,q-1}, \quad (21)$$

where $k_{i,q,1} \left(\frac{1}{2} \right)^{\frac{3}{4}} \frac{(e_{i,q}^2)^{\frac{3}{4}}}{e_{i,q}}$ and $k_{i,q,2} \left(\frac{1}{2} \right)^2 e_{i,q}^3$ are the fixed-time control terms, $b_{i,q+1} > 0$, $k_{i,q,1} > 0$, and $k_{i,q,2} > 0$ are design positive parameters, and $b_{i,q} e_{i,q-1}$ is used to deal with $b_{i,2} e_{i,1} e_{i,2}$ in the previous step.

Select Lyapunov function candidate

$$V_{i,q} = V_{i,q-1} + \frac{1}{2} e_{i,q}^2. \quad (22)$$

The derivative of $V_{i,q}$ is

$$\dot{V}_{i,q} = \dot{V}_{i,q-1} + e_{i,q} \dot{e}_{i,q}. \quad (23)$$

Combining (20), (21), Lemma 4 and Lemma 7, we have

$$\begin{aligned} e_{i,q} \dot{e}_{i,q} &\leq -k_{i,q,1} \left(\frac{e_{i,q}^2}{2} \right)^{\frac{3}{4}} - (k_{i,q,2} - \epsilon_{i,q,1}^4 - \epsilon_{i,q,2}^4) \left(\frac{e_{i,q}^2}{2} \right)^2 \\ &\quad + \frac{3}{4\epsilon_{i,q,1}^{\frac{4}{3}}} |F_{i,q} - \phi_{i,q,2}|^{\frac{4}{3}} + \frac{3}{4\epsilon_{i,q,2}^{\frac{4}{3}}} |v_{i,q-1,2} - \dot{\alpha}_{i,q}|^{\frac{4}{3}} \\ &\quad + b_{i,q+1} e_{i,q} e_{i,q+1} - b_{i,q} e_{i,q-1} e_{i,q} + \varepsilon_{i,q}, \end{aligned} \quad (24)$$

where $\epsilon_{i,q,1} > 0$ and $\epsilon_{i,q,2} > 0$ are the design parameters.

Substituting (24) into (23), one obtains

$$\begin{aligned} \dot{V}_{i,q} &\leq -\frac{s_c}{4} \tilde{r}^T \tilde{r} + \left(\frac{4}{s_c} \|P\|^2 \|E_2\|^2 + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\vartheta_y\|^2 \right) e_1^T e_1 \\ &\quad + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\bar{\vartheta}_y\|^2 + \frac{4}{s_c} \|P\|^2 \|E_3\|^2 - \sum_{j=1}^q k_{i,j,1} \left(\frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} \\ &\quad - \sum_{j=1}^q (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4) \left(\frac{e_{i,j}^2}{2} \right)^2 + \sum_{j=1}^q \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} \\ &\quad + \sum_{j=2}^q \frac{3}{4\epsilon_{i,j,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} + \sum_{j=1}^q \varepsilon_{i,j} + b_{i,q+1} e_{i,q} e_{i,q+1}, \end{aligned} \quad (25)$$

where $i = 1, \dots, N$, $k_{i,j,1} > 0$, $k_{i,j,2} > 0$, $\epsilon_{i,j,1} > 0$ and $\epsilon_{i,j,2} > 0$ such that $k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 > 0$.

Obviously, in (25), there exists an item $b_{i,q+1} e_{i,q} e_{i,q+1}$ and it cannot be processed in the current step. Thus we need to design virtual control law in the next step.

Step i, n_i : Recursively, the dynamic of defined error $e_{i,n_i}(t) = x_{i,n_i} - \alpha_{i,n_i}$ can be expressed as

$$\dot{e}_{i,n_i} = F_{i,n_i}(t) + b_{i,n_i+1}u_i - \dot{\alpha}_{i,n_i}, \quad (26)$$

where $F_{i,n_i}(t) = -g_{\lambda_{i,n_i}}\alpha_{i,n_i+1}^* + (g_{\lambda_{i,n_i}} - b_{i,n_i+1})x_{i,n_i+1}$, $g_{\lambda_{i,n_i}} = g_{i,n_i}(\bar{x}_{i,n_i}, x_{\lambda_{i,n_i}})$, $\lambda_{i,n_i} \in (0, 1)$, $x_{\lambda_{i,n_i}} = \lambda_{i,n_i}u_i + (1 - \lambda_{i,n_i})\alpha_{i,n_i+1}^*$, and $b_{i,n_i+1} > 0$ is a design parameter, $i = 1, \dots, N$.

Similar to previous steps, a TD

$$\begin{cases} \dot{v}_{i,n_i-1,1} = v_{i,n_i-1,2}, \\ \dot{v}_{i,n_i-1,2} = -\lambda_{i,n_i}^2 \text{sgn}(v_{i,n_i-1,1} - \alpha_{i,n_i-1}) |v_{i,n_i-1,1} - \alpha_{i,n_i-1}|^{\sigma_{i,n_i,1}} \\ \quad - \lambda_{i,n_i} v_{i,n_i-1,2}, \end{cases} \quad (27)$$

and an ESO

$$\begin{cases} E_{i,n_i} = \phi_{i,n_i,1} - e_{i,n_i}, \\ \dot{\phi}_{i,n_i,1} = \phi_{i,n_i,2} - \rho_{i,n_i,1}E_{i,n_i} + b_{i,n_i+1}u_i - v_{i,n_i-1,2}, \\ \dot{\phi}_{i,n_i,2} = -\rho_{i,n_i,2}|E_{i,n_i}|^{\sigma_{i,n_i,2}} \text{sgn}(E_{i,n_i}), \end{cases} \quad (28)$$

are introduced here to estimate the derivative of virtual control law $\dot{\alpha}_{i,n_i}$ and the uncertain item F_{i,n_i} , respectively, where $\lambda_{i,n_i} > 0$ and $0 < \sigma_{i,n_i,l} < 1$ are constants to be designed and the states of TD $v_{i,n_i-1,1}$ and $v_{i,n_i-1,2}$ are the estimations of α_{i,n_i} and $\dot{\alpha}_{i,n_i}$, respectively, E_{i,n_i} is the estimation error, $\phi_{i,n_i,1}$ and $\phi_{i,n_i,2}$ are the state variables, $\phi_{i,n_i,2}$ is the estimation value of $F_{i,n_i}(t)$, and $\rho_{i,n_i,1} > 0$, $\rho_{i,n_i,2} > 0$ and $\sigma_{i,n_i,2} \in (0, 1)$ are design parameters, $i = 1, \dots, N$.

Then, the actual control law u_i is designed as

$$u_i = -\frac{e_{i,n_i}\tilde{\alpha}_{i,n_i+1}^2}{b_{i,n_i+1}\sqrt{e_{i,n_i}^2\tilde{\alpha}_{i,n_i+1}^2 + \varepsilon_{i,n_i}^2}} \quad (29)$$

with

$$\tilde{\alpha}_{i,n_i+1} = k_{i,n_i,1}\left(\frac{1}{2}\right)^{\frac{3}{4}}\frac{(e_{i,n_i}^2)^{\frac{3}{4}}}{e_{i,n_i}} + k_{i,n_i,2}\left(\frac{1}{2}\right)^2 e_{i,n_i}^3 + \phi_{i,n_i,2} - v_{i,n_i-1,2} + b_{i,n_i}e_{i,n_i} \quad (30)$$

where $b_{i,n_i+1} > 0$, $k_{i,n_i,1} > 0$, $k_{i,n_i,2} > 0$ are design positive parameters,

$k_{i,n_i,1}\left(\frac{1}{2}\right)^{\frac{3}{4}}\frac{(e_{i,n_i}^2)^{\frac{3}{4}}}{e_{i,n_i}}$ and $k_{i,n_i,2}\left(\frac{1}{2}\right)^2 e_{i,n_i}^3$ are the fixed-time control terms.

In this recursive procedure, our event-triggered based FTADRC consensus tracking control strategy is developed for the i th follower, and its block diagram is summarized in Fig. 2. In what follows, we will analyze the stability of the MAS as well as the Zeno-free phenomenon.

Theorem 1 Consider a non-affine nonlinear MAS, including one leader and N followers in (1). With the virtual control laws (12), (20) and the actual control law (29), by selecting appropriate parameters, the MAS achieves ISpS stability, and the consensus tracking error $\Lambda = y - (y_r \otimes I_N)$ converges to a neighborhood around zero in a fixed time. Also, the triggered interval $\{t_{k+1} - t_k\} > 0$ is satisfied, $\forall k \in \mathbb{Z}^+$.

Fig. 2 Event-triggered based FTADRC consensus tracking control strategy for the i th follower.

Proof The proof consists of two parts. One is the ISpS stability of the MAS is proved, and Zeno-free phenomenon is also proved. The other is to prove that the MAS is fixed time stable.

Choose the overall Lyapunov candidate function

$$V(t) = \tilde{r}^T P \tilde{r} + \sum_{i=1}^N \sum_{j=1}^{n_i} \frac{1}{2} e_{i,j}^2, \quad (31)$$

where $\tilde{r} = [\tilde{r}_1, \tilde{r}_2]^T$, $\tilde{r}_l = [\tilde{r}_{1,l}, \dots, \tilde{r}_{N,l}]^T$, $l = 1, 2$, $i = 1, \dots, N$.

According to virtual control laws (12), (20) and the actual control law (29), we obtain

$$\begin{aligned} \dot{V}(t) \leq & -\frac{s_c}{4\lambda_{\max}(P)} \tilde{r}^T P \tilde{r} + \sum_{i=1}^N \left\{ -\sum_{j=1}^{n_i} k_{i,j,1} \left(\frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} \right. \\ & - \sum_{j=1}^{n_i} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1) \left(\frac{e_{i,j}^2}{2} \right)^2 + \sum_{j=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} \\ & \left. + \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,2,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} \right\} + \gamma_1^2 + \gamma_2 + \sum_{j=1}^{n_i} \varepsilon_{i,j}, \end{aligned} \quad (32)$$

where $\lambda_{\max}(P)$ is the maximum eigenvalue of the matrix P , $\gamma_1 = \frac{4}{s_c} \|P\|^2 \|E_2\|^2 + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\vartheta_y\|^2$, and $\gamma_2 = \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\bar{\vartheta}_y\|^2 + \frac{4}{s_c} \|P\|^2 \|E_3\|^2$. Then we prove the ISpS stability of the MAS in Part I.

Part I: The results in [12] [13] [11] have already proven that the estimation errors $F_{i,1} - \phi_{i,1,2}$, $F_{i,q} - \phi_{i,q,2}$, and $v_{i,q-1,2} - \dot{\alpha}_{i,q}$, $i = 1, \dots, N$, $q = 2, \dots, n_i$, are bounded via choosing parameter properly. Then $|F_{i,1} - \phi_{i,1,2}|^{\frac{4}{3}}$, $|F_{i,q} - \phi_{i,q,2}|^{\frac{4}{3}}$, and $|v_{i,q-1,2} - \dot{\alpha}_{i,q}|^{\frac{4}{3}}$, $i = 1, \dots, N$, $q = 2, \dots, n_i$ are also bounded. By the Young's inequality $ab \leq a^2 + \frac{b^2}{4}$, we have

$$\begin{aligned} \dot{V}(t) \leq & -\frac{s_c}{4\lambda_{\max}(P)} \tilde{r}^T P \tilde{r} + \sum_{i=1}^N \left\{ -\sum_{j=1}^{n_i} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)^2 \frac{e_{i,j}^2}{2} \right. \\ & + \sum_{j=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} + \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,2,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} \\ & \left. + \sum_{j=1}^{n_i} \varepsilon_{i,j} + \sum_{j=1}^{n_i} \frac{1}{4} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)^3 \right\} + \gamma_1^2 + \gamma_2. \end{aligned} \quad (33)$$

Then, we regard those variables $U_1 = [|F_{i,1} - \phi_{i,1,2}|^{\frac{2}{3}}, \dots, |F_{i,n_i} - \phi_{i,n_i,2}|^{\frac{2}{3}}, |v_{i,1,2} - \dot{\alpha}_{i,2}|^{\frac{2}{3}}, \dots, |v_{i,n_i-1,2} - \dot{\alpha}_{i,n_i}|^{\frac{2}{3}}]^T$ as the bounded inputs of the system

$$\sum_1: \begin{cases} \dot{\tilde{r}} = E_1 \tilde{r} - E_2 e_1 + E_2 E_r (\vartheta_y |e_1| + \bar{\vartheta}_y) + E_3, \\ \dot{e}_{i,1} = -k_{i,1,1} \left(\frac{1}{2} \right)^{\frac{3}{4}} \frac{(e_{i,1}^2)^{\frac{3}{4}}}{e_{i,1}} - k_{i,1,2} \left(\frac{1}{2} \right)^2 (e_{i,1})^3 + (F_{i,1} - \phi_{i,1,2}) \\ \quad + b_{i,2} e_{i,2}, \\ \vdots \\ \dot{e}_{i,n_i} = -k_{i,n_i,1} \left(\frac{1}{2} \right)^{\frac{3}{4}} \frac{(e_{i,n_i}^2)^{\frac{3}{4}}}{e_{i,n_i}} - k_{i,n_i,2} \left(\frac{1}{2} \right)^2 (e_{i,n_i})^3 + (F_{i,n_i} - \phi_{i,n_i,2}) \\ \quad + (v_{i,n_i-1,2} - \dot{\alpha}_{i,n_i}) - b_{i,n_i} e_{i,n_i-1}, \end{cases}$$

with the states $\Pi_1 = [\tilde{r}^T P \tilde{r}, \frac{e_{i,1}^2}{2}, \frac{e_{i,2}^2}{2}, \dots, \frac{e_{i,n_i}^2}{2}]^T$, $i = 1, \dots, N$. Select class

K functions $\kappa_1(s) = a_1 s$, $\kappa_2(s) = a_2 s$, $\kappa_3(s) = a_3 s$, $\kappa_4(s) = a_4 s^2$, $a_1 = a_2 = 1$, $a_3 = \min_{1 \leq i \leq N, 1 \leq j \leq n_i} \left\{ \frac{s_c}{4\lambda_{\max}(P)}, (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)^2 \right\}$, $a_4 = \max_{1 \leq i \leq N, 1 \leq j \leq n_i} \left\{ \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}}, \frac{3}{4\epsilon_{i,j,2}^{\frac{4}{3}}} \right\}$.

From Definition 2, $V(t)$ is an ISpS-Lyapunov function of the system $\sum_1$. Meanwhile, $\sum_1$ is ISpS, and nonlinear L_∞ gain $\psi_2(s) = \kappa_1^{-1} \circ \kappa_2 \circ \kappa_3^{-1} \circ \kappa_4(s) = \frac{a_2 a_4}{a_1 a_3} s^2$. From Definition 1 of ISpS, there exist a KL class function ψ_1 and nonlinear gain ψ_2 such that $\|I_1(t)\| \leq \psi_1(\|I_1(0), t\|) + \psi_2(\|U\|_\infty) \leq \psi_1(\|I_1(0), t\|) + \frac{a_2 a_4}{a_1 a_3} U^*$, and $U^* = \max\{|F_{i,j} - \phi_{i,j,2}|, |v_{i,j-1,2} - \dot{\alpha}_{i,j}|\}$, $i = 1, \dots, N$, $j = 1, \dots, n_i$. From $\lim_{t \rightarrow \infty} \psi_1 \rightarrow 0$, it is further observed that the signals of the system $\sum_1$ are all bounded ultimately. Then, the following inequality

$$\dot{V}(t) \leq -CV(t) + M \quad (34)$$

holds, where $C = \min_{1 \leq i \leq N, 1 \leq j \leq n_i} \left\{ \frac{s_c}{4\lambda_{\max}(P)}, (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)^2 \right\} > 0$, $M = \Delta + \sum_{i=1}^N \sum_{j=1}^{n_i} \epsilon_{i,j} + \sum_{i=1}^N \sum_{j=1}^{n_i} \frac{1}{4} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)^3 + \gamma_1^2 + \gamma_2$, $\Delta = \sup \left\{ \sum_{i=1}^N \sum_{q=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} + \sum_{i=1}^N \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,j,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} \right\}$, $\gamma_1 = \frac{4}{s_c} \|P\|^2 \|E_2\|^2 + \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\vartheta_y\|^2$, and $\gamma_2 = \frac{8}{s_c} \|P\|^2 \|E_2\|^2 \|\bar{\vartheta}_y\|^2 + \frac{4}{s_c} \|P\|^2 \|E_3\|^2$. The solution of (34) yields

$$\frac{1}{2} e_{i,1}^2 \leq V(t) \leq \left(V(0) - \frac{M}{C} \right) \exp(-Ct) + \frac{M}{C}. \quad (35)$$

Next, a proof of the non-Zeno phenomenon in MAS is given. According to the measurement error $e_{y,i} = y_i(t) - y_i(t_{i,k})$, for $t_{i,k} \leq t < t_{i,k+1}$, one has

$$\frac{d|e_{y,i}|}{dt} = \frac{d}{dt} (e_{y,i}^2)^{\frac{1}{2}} = \frac{e_{y,i} \dot{e}_{y,i}}{|e_{y,i}|} \leq \frac{|e_{y,i}| |\dot{e}_{y,i}|}{|e_{y,i}|} = |\dot{e}_{y,i}| = |\dot{y}_i|. \quad (36)$$

From the dynamics of the i th follower (1), one has $\dot{y}_i = F_{i,1}(t) + b_{i,2} x_{i,2}$. Since the boundness of $e_{i,q}$, $\alpha_{i,qd}$, $x_{i,q}$, $\phi_{i,q,2}$ and $\hat{r}_{i,2}$, there exists a positive constant Ξ_i such that $|\dot{y}_i| \leq \Xi_i$. Then, based on the event-triggered condition $\Gamma_i(t) = |e_{y,i}| - \vartheta_{y,i} |e_{i,1}| - \bar{\vartheta}_{y,i}$, $e_{y,i} = y_i(t) - y_i(t_{i,k})$, it can be observed that $e_{y,i}(t_{i,k}) = 0$ and $\lim_{t \rightarrow t_{i,k+1}^l} e_{y,i}(t) = \vartheta_{y,i} |e_{i,1}| + \bar{\vartheta}_{y,i}$. Furthermore, $t_{i,k+1} - t_{i,k} \geq \frac{\vartheta_{y,i} |e_{i,1}| + \bar{\vartheta}_{y,i}}{\Xi_i} \geq \frac{\bar{\vartheta}_{y,i}}{\Xi_i}$ can be obtained. The time interval between two triggered events are greater than zero, which means that there will be no infinite events occurred in a finite time. The Zeno behavior is ruled out.

The result (35) implies that the error variable $e_{i,1}$ converges to the neighborhood $\Omega_1 = \{e_{i,1} \mid |e_{i,1}| \leq \sqrt{2M/C}\}$. Consequently, $\tilde{r}_{i,1}$ and $\tilde{r}_{i,2}$, $i = 1, \dots, N$, are bounded. From $\Lambda_i = e_{i,1} - \tilde{r}_{i,1}$, the consensus tracking error vector $\Lambda = y - (y_r \otimes I_N)$ asymptotically converges to a compact set $\Omega_2 = \{\Lambda \mid \|\Lambda\| \leq 2\sqrt{2M/C}\}$.

Part II: It is proved that the MAS is stable in a fixed time. That is, the proposed adaptive FTC strategy is designed based on backstepping algorithm, ADRC and ETM, which make the practical output signal tracks the desired output signal well in a fixed time.

By utilizing Lemma 8, we denote $\phi = 1$, $\varphi = \frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r}$, $m = \frac{1}{4}$, $n = \frac{3}{4}$, and it is not difficult to arrive at

$$\left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^{\frac{3}{4}} \leq d_1 + \frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r}, \quad (37)$$

where $d_1 = 0.11 > 0$. Inserting (37) into (32) yields

$$\begin{aligned} \dot{V}(t) \leq & - \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^{\frac{3}{4}} - \tau \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^2 + \sum_{i=1}^N \left\{ - \sum_{j=1}^{n_i} \right. \\ & k_{i,j,1} \left(\frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} - \sum_{j=1}^{n_i} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1) \left(\frac{e_{i,j}^2}{2} \right)^2 + \sum_{j=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} \\ & \left. |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} + \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,2,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} \right\} + (1 + \tau) \\ & \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^2 + \gamma_1^2 + \gamma_2 + \sum_{j=1}^{n_i} \varepsilon_{i,j}. \end{aligned} \quad (38)$$

According to Part I, we obtain $(1 + \tau) \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^2$ is bounded, where τ is a constant. Also, the term $\sum_{j=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} + \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,2,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}}$ is also bounded. Consequently, we obtain

$$\begin{aligned} \dot{V}(t) \leq & - \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^{\frac{3}{4}} - \sum_{i=1}^N \sum_{j=1}^{n_i} k_{i,j,1} \left(\frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} - \tau \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^2 \\ & - \sum_{i=1}^N \sum_{j=1}^{n_i} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1) \left(\frac{e_{i,j}^2}{2} \right)^2 + \varrho, \end{aligned} \quad (39)$$

where $\varrho = (1 + \tau) \left(\frac{s_c}{4\lambda_{\max}(P)}\tilde{r}^T P \tilde{r} \right)^2 + \sum_{j=1}^{n_i} \frac{3}{4\epsilon_{i,j,1}^{\frac{4}{3}}} |F_{i,j} - \phi_{i,j,2}|^{\frac{4}{3}} + \sum_{j=2}^{n_i} \frac{3}{4\epsilon_{i,2,2}^{\frac{4}{3}}} |v_{i,j-1,2} - \dot{\alpha}_{i,j}|^{\frac{4}{3}} + \gamma_1^2 + \gamma_2 + \sum_{j=1}^{n_i} \varepsilon_{i,j}$.

Denote $\bar{\mu}_1 = \left(\frac{s_c}{4\lambda_{\max}(P)}\right)^{\frac{3}{4}}$, $\hat{\mu}_1 = \sum_{i=1}^N \sum_{j=1}^{n_i} k_{i,j,1}$, $\bar{\mu}_2 = \tau \left(\frac{s_c}{4\lambda_{\max}(P)}\right)^2$ and $\hat{\mu}_2 = \sum_{i=1}^N \sum_{j=1}^{n_i} (k_{i,j,2} - \epsilon_{i,j,1}^4 - \epsilon_{i,j,2}^4 - 1)$. It is easy to arrive at

$$\dot{V}(t) \leq -\bar{\mu}_1 (\tilde{r}^T P \tilde{r})^{\frac{3}{4}} - \hat{\mu}_1 \left(\sum_{i=1}^N \sum_{j=1}^{n_i} \frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} - \bar{\mu}_2 (\tilde{r}^T P \tilde{r})^2 - \hat{\mu}_2 \left(\sum_{i=1}^N \sum_{j=1}^{n_i} \frac{e_{i,j}^2}{2} \right)^2 + \varrho. \quad (40)$$

Here, we denote $\mu_1 = \min\{\bar{\mu}_1, \hat{\mu}_1\}$ and $\check{\mu}_2 = \min\{\bar{\mu}_2, \hat{\mu}_2\}$, and then the derivative of the $V(t)$ yields

$$\dot{V}(t) \leq -\mu_1 \left\{ (\tilde{r}^T P \tilde{r})^{\frac{3}{4}} + \left(\sum_{i=1}^N \sum_{j=1}^{n_i} \frac{e_{i,j}^2}{2} \right)^{\frac{3}{4}} \right\} - \check{\mu}_2 \left\{ (\tilde{r}^T P \tilde{r})^2 + \left(\sum_{i=1}^N \sum_{j=1}^{n_i} \frac{e_{i,j}^2}{2} \right)^2 \right\} + \varrho. \quad (41)$$

From Lemma 5, Lemma 6 and (41), the following inequality

$$\dot{V}(t) \leq -\mu_1 V^{\frac{3}{4}}(t) - \mu_2 V^2(t) + \varrho \quad (42)$$

holds, where $\mu_2 = \frac{\check{\mu}_2}{2}$. For the situation with the fixed-time convergence, via (42), the following formula is established

$$\dot{V}(t) \leq -\mu_1 V^{\frac{3}{4}}(t) - (1 - \omega) \mu_2 V^2(t) - \mu_2 \omega V^2(t) + \varrho, \quad (43)$$

where $0 < \omega < 1$. Therefore, it is easy to get $\dot{V}(t) \leq -\mu_1 V^{\frac{3}{4}}(t) - (1 - \omega) \mu_2 V^2(t)$ as $V^2(t) \geq \frac{\varrho}{\mu_2 \omega}$. Meanwhile, from Lemma 3, the settling time is given by

$$T_s \leq \frac{1}{4\omega\mu_1} + \frac{1}{\omega\mu_2}. \quad (44)$$

This completes the proof.

4 Simulation Illustration

Two simulation examples are presented to verify the theoretical claims. One is a numerical example, and the other is related to a multiple single-link robot manipulator system.

Fig. 3 The communication graph in example A.

4.1 Numerical Example

In this subsection, a non-affine nonlinear MAS with four followers and one leader is considered, where the communication graph is shown in Fig. 3, Node 0 is the leader, and Node 1–Node 4 are followers. The dynamics of the

follower are
$$\begin{cases} \dot{x}_{i,1} = f_{i,1}(x_{i,1}, x_{i,2}), \\ \dot{x}_{i,2} = f_{i,2}(\bar{x}_{i,2}, u_i), \\ y_i = x_{i,1}, \end{cases} \quad \text{where } x_{i,1} \text{ and } x_{i,2} \text{ are the states of } i\text{th}$$

follower, $f_{i,1}(x_{i,1}, x_{i,2}) = 2x_{i,2} + \sin(x_{i,1})x_{i,2}$, $f_{i,2}(\bar{x}_{i,2}, u_i) = x_{i,1}x_{i,2} + u_i + (2 - \sin(x_{i,1}) \cos(x_{i,2}))u_i$, $i = 1, 2, 3, 4$, are the unknown dynamics of i th follower, y_i is the i th follower's output, and the trajectory of the leader is $y_r = \sin(0.8t)$.

The initial values are set as $x_{1,1}(0) = 0.05$, $x_{1,2}(0) = 0$, $x_{2,1}(0) = 0.1$, $x_{2,2}(0) = 0.05$, $x_{3,1}(0) = 0.15$, $x_{3,2}(0) = 0.1$, $x_{4,1}(0) = 0.05$, and $x_{4,2}(0) = 0.05$. The initial conditions of distributed estimator, ESOs and TDs are all set as zero. The control parameters in the strategy are $k_{i,1,j} = 28$, $\varepsilon_{i,j} = 1$, $b_{i,n} = 1$, $\eta_{i,1} = 110$, and $\eta_{i,2} = 5$. The parameters of event-triggered condition are $\vartheta_{y,i} = 0.1$ and $\bar{\vartheta}_{y,i} = 0.02$. The parameters of ESOs and TDs are $\rho_{i,j,1} = 10$, $\rho_{i,j,2} = 50$, $\sigma_{i,j,2} = 0.9$, $\sigma_{i,2,1} = 0.4$, $\lambda_{i,2} = 10$, $i = 1, 2, 3, 4$, $j = 1, 2$, and $n = 2, 3$.

Fig. 4 and Fig. 15 reveal the effectiveness of the proposed control strategy. Fig. 4 gives the trajectories of output y_i of the followers and the leader signal y_r . We see that the output signal y_i of the follower tracks the leader signal y_r within a fixed time, which verifies that the proposed control strategy has satisfactory consensus control performance. The curves of consensus tracking control laws u_i are plotted in Fig. 5. Fig. 6 presents the consensus tracking error of followers. Obviously, the consensus tracking error converges to a small neighborhood of origin. Fig. 7 and Fig. 8 show the curves of the states in TDs and the virtual control laws. It can be clearly seen from Fig. 7 and Fig. 8 that the tracking differentiator holds a satisfactory performance. The curves of $x_{i,1}$ and $x_{i,2}$ are given in Fig. 9 and Fig. 10. Fig. 11 and Fig. 12 offer estimation behaviors of the leader's signal from the distributed event-triggered estimators. The triggered instants are recorded in Fig. 13. In the simulation, we take the 1st follower as an example and calculate the upper bound of settling time $T_s \leq 5.5s$. The blue curves in Fig. 14 and Fig. 15 show the actual settling time with different initial conditions. It can be observed, from Fig. 14 and Fig. 15, that the actual settling time is always lower than the upper bound of the settling time despite the initial value grows. Table 1 and Table 2 list the actual settling time corresponding to different initial values with different control gains. From the results in the two tables, we conclude that, with the

Fig. 4 Trajectories of four followers and the desired trajectory in Example A.

Fig. 5 Consensus tracking control laws u_i in Example A.

increase of control gains, the settling time of the system will become shorter and the control precision will be higher.

Table 1 The actual settling time corresponding to different initial values with control gain is 5.

Case	1	2	3	4	5	6	7	8	9
$x(0)$	-2	-1.5	-1	-0.5	0.5	1	1.5	1.8	2
T_s	0.147	0.154	0.148	0.128	0.094	0.277	0.320	0.346	0.365

Fig. 6 Consensus tracking error of followers $e_{i,1}$ in Example A.

Fig. 7 The state of $v_{i,1,1}$ in TD and virtual control $\alpha_{i,2}$, $i = 1, 2$, in Example A.

Fig. 8 The state of $v_{i,1,1}$ in TD and virtual control $\alpha_{i,2}$, $i = 3, 4$, in Example A.

Fig. 9 The curves of state $x_{i,1}$ and $x_{i,2}$, $i = 1, 2$, in Example A.

Fig. 10 The curves of state $x_{i,1}$ and $x_{i,2}$, $i = 3, 4$, in Example A.

Fig. 11 The curves of the distributed event-triggered estimator's estimation of the leader's signal in Example A.

Fig. 12 The curves of the distributed event-triggered estimator's estimation of the leader's signal in Example A.

Fig. 13 The record of triggered instants in Example A.

Table 2 The actual settling time corresponding to different initial values with control gain is 10.

Case	1	2	3	4	5	6	7	8	9
$x(0)$	-1.4	-1.2	-1	-0.5	0.2	0.5	1	1.2	1.5
T_s	0.097	0.088	0.188	0.087	0.052	0.224	0.275	0.293	0.281

4.2 A Multiple Single-Link Manipulator System

Four single-link manipulators are connected, and the communication graph is shown in Fig. 16. The dynamics of the i th manipulator are [61]

$$M_i \ddot{q}_i + B_i \dot{q}_i + N_i \sin(q_i) = I_i, i = 1, 2, 3, 4, \quad (45)$$

Fig. 14 The curve of the actual settling time with $k_{1,1,1} = k_{1,1,2} = k_{1,2,1} = k_{1,2,2} = 5$ in Example A.

Fig. 15 The curve of the actual settling time with $k_{1,1,1} = k_{1,1,2} = k_{1,2,1} = k_{1,2,2} = 10$ in Example A.

where $M_i = \frac{J_i}{K_{i,\tau}} + \frac{m_i L_{i,0}^2}{3K_{\tau,i}} + \frac{m_{i,0} L_{i,0}^2}{K_{i,\tau}} + \frac{2m_{i,0} R_i^2}{5K_{i,\tau}}$, $N_i = \frac{m_i L_{i,0} G_i}{2K_{i,\tau}} + \frac{m_{i,0} L_{i,0} G_i}{K_{i,\tau}}$, $B_i = \frac{B_{i,0}}{K_{i,\tau}}$, J_i is the rotor inertia, m_i is the link mass, $m_{i,0}$ is the load mass, $L_{i,0}$ is the link length, R_i is the radius of the load, G_i is the gravity coefficient, $B_{i,0}$ is the coefficient of viscous friction at the joint, q_i is the angular position, I_i is the armature current with the maximum value 29A and K_i is a coefficient.

Here, denote $x_{i,1} = q_i$, $x_{i,2} = \dot{q}_i$, and $u_i = I_i$. Then we have

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}, \\ \dot{x}_{i,2} = g_i u_i + f_{i,2}, \end{cases} \quad (46)$$

where $g_i = \frac{1}{M_i}$, and $f_{i,2} = -\frac{B_i}{M_i} x_{i,2} - \frac{N_i}{M_i} \sin(x_{i,2})$. The parameters are given by $M_1 = M_2 = M_3 = M_4 = 1$, $N_1 = N_2 = N_3 = N_4 = 10$ and $B_1 = B_2 =$

Fig. 16 The communication graph in Example B.

$B_3 = B_4 = 1$. The initial conditions are chosen as $x_{1,1}(0) = 0.05$, $x_{1,2}(0) = 0$, $x_{2,1}(0) = 0.01$, $x_{2,2}(0) = 0.05$, $x_{3,1}(0) = -0.05$, $x_{3,2}(0) = 0.1$, $x_{4,1}(0) = -0.01$, and $x_{4,2}(0) = 0$. The leader's output is $q_d = \sin(0.5t)$. The design parameters are selected as $k_{i,j,q} = 30$, $k_{2,j,q} = 35$, $k_{4,j,q} = 38$, $\varepsilon_{m,j} = 1$, $\eta_{m,1} = 120$, $\eta_{m,2} = 10$, $b_{m,n} = 1$, $i = 1, 3$, $j = 1, 2$, $q = 1, 2$, $m = 1, 2, 3, 4$ and $n = 2, 3$. The initial values of ESOs and TDs are all set to zero. The initial values of the distributed event-triggered estimator are $\hat{r}(0)_{1,1} = 0.1$, $\hat{r}(0)_{1,2} = 0$, $\hat{r}(0)_{2,1} = -0.1$, $\hat{r}(0)_{2,2} = 0.1$, $\hat{r}(0)_{3,1} = 0.5$, $\hat{r}(0)_{3,2} = 0$, $\hat{r}(0)_{4,1} = -0.3$, and $\hat{r}(0)_{4,2} = 0.1$. The parameters of event-triggered condition are $\vartheta_{y,i} = 0.1$, $\bar{\vartheta}_{y,i} = 0.02$, and $i = 1, 2, 3, 4$. The parameters of ESOs and TDs are $\rho_{a,b,1} = 10$, $\rho_{a,b,2} = 50$, $\sigma_{a,b,2} = 0.9$, $\sigma_{a,2,1} = 0.4$, $\lambda_{a,2} = 10$, $a = 1, 2, 3, 4$, and $b = 1, 2$.

Simulation results are given in Fig. 17–Fig. 21. Fig. 17 plots that follower's angular position signal q_i tracks the leader's angular position signal q_d within a fixed time driven by the control strategy, which verifies the effective of the proposed control strategy in this paper. The curves of the motor armature current I_i are shown in Fig. 18. As stated in the previous model description, the maximum armature current does not exceed 29A, which can be applied in practice. Fig. 19 is the curves of the consensus tracking error of followers. Obviously, the consensus tracking error can converge to a small neighborhood of origin. As observed from Fig. 20, the ESOs estimate the unknown dynamics well. Figs. 21 shows the record of the triggered events.

To show the advantage of the FTC designed herein, we compare the consensus tracking error by our FTC with that by a non-fixed-time one. The two strategies are with the same design parameters for the same follower (46). The curves of consensus tracking error with/without FTC strategy in 1st follower are depicted in Fig. 22. It is worth pointing out that the consensus tracking error with fixed time control is smaller and the control accuracy is improved by nearly 50%. Consequently, the FTC strategy proposed in this paper is very effective to ensure the tracking performance. Under the condition that the initial states and control parameters are the same, we also do the comparison simulation of consensus tracking error under FTC strategy and finite-time control strategy. The simulation result is shown in Fig. 23. It can be seen that the proposed fixed-time controller leads to satisfactory convergence speed as compared with the finite-time controller. Furthermore, in order to provide more fair comparison conditions, we utilize FTC strategy and finite-time control strategy respectively to stabilize the consensus tracking error within the origin neighborhood. The simulation results are plotted in Fig. 24–Fig. 25. When 10s, disturbance $\sin(t)$ is added to the desired signal, it can be seen that the

Fig. 17 The leader's angular position q_d and followers' angular position q_i in Example B.

Fig. 18 Current I_i in Example B.

consensus tracking errors under the two control strategies are affected by disturbance. The consensus tracking error under the FTC strategy takes 0.1s to stabilize again in the origin neighborhood, while the consensus tracking error under the control of finite-time takes 1s to stabilize again in the origin neighborhood. The comparison results show that the MAS by our FTC strategy performs better with shorter settling time.

5 Conclusion

In this paper, for a non-affine nonlinear MAS, a FTADRC consensus tracking control strategy has been proposed via an event-triggered mechanism. The implicit function theorem and mean value theorem are utilized to complete the transformation from non-affine MASs to affine ones. ESOs and TDs have been employed in a recursive procedure. For followers who cannot directly

Fig. 19 Consensus tracking error of four followers $e_{i,1}$ in Example B.

Fig. 20 Uncertainties and their estimations for the 1st follower in Example B.

Fig. 21 The record of triggered times in Example B.

Fig. 22 Comparison of consensus tracking error with/without FTC method for the 1st follower in Example B.

Fig. 23 Comparison of consensus tracking error under FTC method and finite-time control method for the 1st follower in Example B.

obtain the leader's signal, the distributed event-triggered estimator has been introduced to estimate the leader's signal. The consensus tracking error has been theoretically analyzed and proved to be bounded in a fixed time. Two simulation examples have been given to illustrate the effectiveness of the proposed strategy for the MAS. Our future research work will focus on formation control of nonlinear MASs.

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Fig. 24 The curve of consensus tracking error under FTC method for the 1st follower in Example B.

Fig. 25 The curve of consensus tracking error under finite-time control method for the 1st follower in Example B.

Data Availability Statements

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. Y Zuo, C Deng, M Lu, et al, Distributed output feedback consensus tracking control of multiple nonholonomic mobile robots with directed communication graphs and sensor faults, *Nonlinear Dynamics*, 105, 2327-2339 3 (2021).
2. A Jw, A Zz, A Cw, B Zd, et al, Cascade structure predictive observer design for consensus control with applications to UAVs formation flying, *Automatica*, DOI: 10.1016/j.automatica.2020.109200.
3. Z Peng, D Wang, H Wang, W Wang, et al, Distributed coordinated tracking of multiple autonomous underwater vehicles, *Nonlinear Dynamics*, 78, 1261-1276 2 (2014).
4. D Yao, C Dou, D yue, N Zhao, T Zhang, et al, Adaptive neural network consensus tracking control for uncertain multi-agent systems with predefined accuracy, *Nonlinear Dynamics*, 101, 2249-2262 4 (2020).
5. G Wang, C Wang, X Cai, L Li, et al, Distributed adaptive output consensus tracking of higher-order systems with unknown control directions, *Neurocomputing*, 203, 129-138 (2016).
6. L Zhao, J Yua, C Lin, et al, Distributed adaptive output consensus tracking of nonlinear multi-agent systems via state observer and command filtered backstepping, *Information Sciences*, 478, 355-374 (2019).
7. J Liu, C Wang, Y Xu, et al, Distributed adaptive output consensus tracking for high-order nonlinear time varying multi-agent systems with output constraints and actuator faults, *Journal of the Franklin Institute*, 357, 1090-1117 (2020).
8. L Zhang, B Chen, C Lin, et al, Adaptive neural consensus tracking control for a class of 2-order multi-agent systems with nonlinear dynamics, *Neurocomputing*, 404, 84-92 (2020).
9. J Fan, L Zhao, et al, Distributed finite-time adaptive consensus tracking control for multiple AUVs with state constraints, *Journal of the Franklin Institute*, 358, 9158-9177 (2021).
10. J Han, et al, From PID to active disturbance rejection control, *IEEE Transactions on Industrial Electronics*, 56, 900-906 3 (2009).
11. B Guo, Z Zhao, et al, Weak convergence of nonlinear high-gain tracking differentiator, *IEEE Transactions on Automatic Control*, 58, 1074-1080 4 (2013).
12. Y Huang, J Han, et al, Analysis and design for the second order nonlinear continuous extended states observer, *Chinese Science Bulletin*, 45, 1938-1944 21 (2000).
13. J Han, Active disturbance rejection control technique-the technique for estimating and compensating the uncertainties, National Defense Industry Press, Beijing (2008)
14. J Tian, S Zhang, Y Zhang, L Tong, et al, Active disturbance rejection control based robust output feedback autopilot design for airbreathing hypersonic vehicles, *ISA Transactions*, 74, 45-59 (2018).
15. S Chen, Z Chen, et al, On active disturbance rejection control for a class of uncertain systems with measurement uncertainty, *IEEE Transactions on Industrial Electronics*, 68, 1475-1485 2 (2020).
16. L Xu, H Ma, D Guo, A Xie, D Song, et al, Backstepping sliding-mode and cascade active disturbance rejection control for a quadrotor UAV, *IEEE/ASME Transactions on Mechatronics*, 25, 2743-2753 6 (2020).
17. H Feng, B Guo, et al, A new active disturbance rejection control to output feedback stabilization for a one-dimensional anti-stable wave equation with disturbance, *IEEE Transactions on Automatic Control*, 62, 3774-3786 8 (2017).
18. M Ran, Q Wang, C Dong, et al, Active disturbance rejection control for uncertain nonaffine-in-control nonlinear systems, *IEEE Transactions on Automatic Control*, 62, 5830-5836 11 (2017).
19. Y Li, G Yang, S Tong, et al, Event-triggered adaptive fuzzy tracking control of nonlinear MIMO systems, *International Journal of Systems Science*, 49, 2618-2629 12 (2018).
20. Y Li, S Tong, G Yang, et al, Observer-based adaptive fuzzy decentralized event-triggered control of interconnected nonlinear system, *IEEE Transactions on Cybernetics*, 50, 3104-3112 7 (2020).
21. C Zhu, C Li, X Chen, K Zhang, H Wei, et al, Event-triggered adaptive fault tolerant control for a class of uncertain nonlinear systems, *Entropy*, 22, 3-17 6 (2020).

22. C Zhang, G Yang, et al, Event-triggered control for a class of strict-feedback nonlinear systems, *International Journal of Robust and Nonlinear Control*, 29, 2112-2124 7 (2018).
23. X Hu, Y Li, Z Hou, et al, Event-triggered fuzzy adaptive fixed-time tracking control for nonlinear systems, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2020.3035779.
24. X Shi, Z Zhou, D Zhou, R Li, et al, Event-triggered fixed-time adaptive trajectory tracking for a class of uncertain nonlinear systems with input saturation, *IEEE Transactions on Circuits and Systems II: Express Briefs*, 68, 983-987 3 (2020).
25. W Wang, Y Li, S Tong, et al, Neural-network-based adaptive event-triggered consensus control of nonstrict-feedback nonlinear systems, *IEEE Transactions on Neural Networks and Learning Systems*, 32, 1750-1763 4 (2021).
26. Y Huang, J Wang, D Shi, L Shi, et al, Towards event-triggered extended state observer, *IEEE Transactions on Automatic Control*, 63, 1842-1849 6 (2018).
27. Y Huang, J Wang, D Shi, L Shi, et al, Event triggered control for a saturated nonlinear system with prescribed performance and finite time convergence, *International Journal of Robust and Nonlinear Control*, 28, 5312-5325 17 (2018).
28. Z Zuo, Q Luo, D Li, Y Wang, et al, Event-triggered control for stabilization of a class of nonlinear systems based on extended state observer, 2016 12th World Congress on Intelligent Control and Automation (WCICA) (2016).
29. J Liu, Y Zhang, Y Yu, C Sun, et al, Fixed-time event-triggered consensus for nonlinear multiagent systems without continuous communications, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 49, 2221-2229 11 (2018).
30. S Song, H Ju, B Zhang, X Song, et al, Event-triggered adaptive practical fixed-time trajectory tracking control for unmanned surface vehicle, *IEEE Transactions on Circuits and Systems*, 68, 436-440 1 (2020).
31. D Shi, J Xue, J Wang, Y Huang, et al, A high-gain approach to event-triggered control with applications to motor systems, *IEEE Transactions on Industrial Electronics*, 66, 6281-6291 8 (2018).
32. A Polyakov, et al, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Transactions on Automatic Control*, 57, 2106-2110 8 (2012).
33. Y Wu, Z Wang, et al, Fuzzy adaptive practical fixed-time consensus for second-order nonlinear multiagent systems under actuator faults, *IEEE Transactions on Cybernetics*, 51, 1150-1161 3 (2020).
34. Y Liu, G Yang, et al, Neural learning-based fixed-time consensus tracking control for nonlinear multiagent systems with directed communication networks, *IEEE Transactions on Neural Networks and Learning Systems*, 32, 639-651 2 (2020).
35. H Yang, D Ye, et al, Time-varying formation tracking control for high-order nonlinear multi-agent systems in fixed-time framework, *Applied Mathematics and Computation*, DOI: 10.1016/j.amc.2020.125119.
36. J Ni, P Shi, Y Zhao, Q Pan, S Wang, et al, Fixed-time event-triggered output consensus tracking of high-order multiagent systems under directed interaction graphs, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2020.3034013.
37. Y Yang, J Tan, et al, Distributed adaptive output consensus control of a class of uncertain nonlinear multiagents systems, *International Journal of Adaptive Control and Signal Processing*, 32, 1145-1161 8 (2018).
38. Y Zheng, Z Chen, Z Huang, M Sun, Q Sun, et al, Active disturbance rejection controller for multi-area interconnected power system based on reinforcement learning, *Neurocomputing*, 425, 149C159 (2021).
39. Y Zheng, Q Sun, Z Chen, M Sun, J Tao, H Sun, et al, Deep Q-Network based real-time active disturbance rejection controller parameter tuning for multi-area interconnected power systems, *Neurocomputing*, 460, 360C373 (2021).
40. C Yang, B Xi, D Xin, C Gw, et al, Command-filter-based adaptive finite-time consensus control for nonlinear strict-feedback multi-agent systems with dynamic leader, *Information Sciences*, 565, 17-31 (2021).
41. S Yoo, et al, Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph, *IEEE Transactions on Neural Networks and Learning Systems*, 24, 666-672 4 (2013).
42. L Chen, Q Wang, et al, Adaptive robust control for a class of uncertain MIMO non-affine nonlinear systems, *IEEE Journal of Automatica Sinica*, 3, 105-112 1 (2016).

43. W Wang, D Wang, Z Peng, et al, Distributed containment control for uncertain nonlinear multi-agent systems in non-affine pure-feedback form under switching topologies, *Neurocomputing*, 152, 1-10 25 (2015).
44. J Wang, X Ding, C Wang, L Li, H Han, et al, Affine formation control for multi-agent systems with prescribed convergence time, *Journal of the Franklin Institute*, 358, 7055-7072 (2021).
45. N Tan, P Yu, et al, Robust model-free control for redundant robotic manipulators based on zeroing neural networks activated by nonlinear functions, *Neurocomputing*, 438, 44-54 (2021).
46. R Zuo, Y Li, M Lv, Z Liu, et al, Realization of trajectory precise tracking for hypersonic flight vehicles with prescribed performances, *Aerospace Science and Technology*, 111, (2021).
47. R Zuo, Y Li, M Lv, Z Liu, et al, A nonlinear updated gain observer for MIMO systems: Design, analysis and application to marine surface vessels, *ISA Transactions*, 129-140, (2016).
48. M Wang, X Liu, P Shi, et al, Adaptive neural control of pure-feedback nonlinear time-delay systems via dynamic surface technique, *IEEE Transactions on Systems, Man, and Cybernetics*, 41, 1681-1692 6 (2011).
49. A Polyakov, et al, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Transactions on Automatic Control*, 57, 2106-2110 8 (2012).
50. B Jiang, Q Hu, M Friswell, et al, Fixed-time attitude control for rigid spacecraft with actuator saturation and faults, *IEEE Transactions on Control Systems Technology*, 24, 1892-1898 5 (2016).
51. Z Zheng, Y Xia, M Fu, et al, Attitude stabilization of rigid spacecraft with finite-time convergence, *International Journal of Robust and Nonlinear Control*, 21, 686-702 6 (2011).
52. C Wang, L Yan, et al, Decentralized adaptive tracking control for a class of interconnected nonlinear time-varying systems, *Automatica*, 54, 16-24 (2015).
53. H Wang, C Bing, L Chong, et al, Adaptive neural tracking control for a class of stochastic nonlinear systems, *International Journal of Robust and Nonlinear Control*, 24, 1262-1280 7 (2014).
54. L Wang, J Mendel, et al, Fuzzy basis functions, universal approximation, and orthogonal least-squares learning, *IEEE Transactions on Neural Networks*, 3, 807-814 5 (1992).
55. K Miroslav, I Kanellakopoulos, V Petar, *Nonlinear and adaptive control design*. Wiley, New York (1995).
56. E Sontag, et al, Further results about input-state stabilization, *IEEE Transactions on Automatic Control*, 35, 473-476 4 (1990).
57. E Sontag, et al, On the input-to-state stability property, *European Journal of Control*, 1, 24-36 1 (1995).
58. W Ren, Randal W, et al, Consensus seeking in multiagent systems under dynamically changing interaction topologies, *IEEE Transactions on Automatic Control*, 50, 655-661 5 (2005).
59. R Wei, R Beard, et al, Distributed consensus in multi-vehicle cooperative control, *Profiles of Drug Substances, Excipients and Related Methodology*, 3, 807-814 5 (1992).
60. J Xu, et al, Adaptive fixed-time control for MIMO nonlinear systems with asymmetric output constraints using universal barrier functions, *IEEE Transactions on Automatic Control*, 64, 3046-3053 7 (2019).
61. D Dawson, J Carroll, et al, Integrator backstepping control of a brush DC motor turning a robotic load, *IEEE Transactions on Control Systems Technology*, 2, 233-244 3 (1994).