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On the Exact Soliton Solutions of Radhakrishnan-Kundu-Lakshmanan Equation

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Abstract

Using an efficient modified extended tanh expansion method, we establish robust solutions for the Radhakrishnan-Kundu-Lakshmanan equation in this work. We want to utilize the effective scheme to solve the problem for the governing model. Bright and mixed dark–bright optical solitons are successfully exposed using the Maple symbolic package. On the 3-dimensional and 2-dimensional graphs, the physical features of the presented data are easily visible.

Keywords: Radhakrishnan-Kundu-Lakshmanan Equation, soliton solutions, modified extended tanh method.

1. Introduction

There has been significant and thorough research into the generation of soliton waves and their applications in recent decades. Optical solitons, which are made up of soliton transmission technology and soliton molecules, are one sort of this research. They can transport information across intercontinental distances all over the world. Optical solitons are confined electromagnetic waves with constant intensity due to dispersion and nonlinearity effects [1-7].

Nonlinear complex physical processes are vital in physics, chemistry, biology, plasma, fibers, nonlinear optics, geochemistry, illumination, energy transmission, and communications, among other fields of science. There has been a review of nonlinear Schrödinger equations with low group velocity dispersion, dispersion terms, Kerr nonlinearities, spatiotemporal dispersion, self-steepening, and other solutions. Optical solitons are seen in nonlinear models such as nano-fibers, optical fibers, quantum electronics, optoelectronics, and photonics [1-7].

Because of the major challenges of breather synchronization linked with their self-oscillating properties, breather solutions to NPDEs, in addition to solitons, are important models for exploring nonlinear wave solutions. Phase-sensitive breather interactions are now widely studied from this perspective [8–18]. When the group velocity and temporal phase of breathers are appropriately synced, breather molecules can be created for co-propagative breathers; however, the phase-sensitive collision approach exhibits different dynamic features for counter propagating breathers. There are many various types of solutions that have been proposed in the literature, but there are still many more that have yet to be investigated. As a result, the goal of this paper is to investigate the following perturbed and generalized Radhakrishnan-Kundu-Lakshmanan (RKL) equations defined as [19].

The perturbed and generalized RKL equations is given:

$$i \frac{\partial u}{\partial t} + \alpha \frac{\partial^2 u}{\partial x^2} + \beta |u|^2 u = i\sigma \frac{\partial u}{\partial x} + i\lambda \frac{\partial}{\partial x} (|u|^2 u) + i\mu \frac{\partial}{\partial x} (|u|^2) u - i\delta \frac{\partial^3 u}{\partial x^3}, \quad i = \sqrt{-1} \quad (1)$$

$$u(x,t) = e^{i\theta} u(\eta) \quad , \quad \eta = x - vt \quad , \quad \theta = -kx + \omega t + \varphi \quad (2)$$

In this study, the modified extended tanh expansion method (METEM) enhanced with new Riccati solutions has been applied for the first time in order to construct some soliton solutions of RKL equation.

We used the Maple computer algebra system for this study [20-21]. The computer algebra system has played an important role to solve very difficult problems until now [22-23].

2. The modified extended tanh function method enhanced with the new Riccati solutions

Step 1: Consider the following nonlinear partial differential equation (NLPDE) and the wave travelling equations, respectively:

$$F\left(M, \frac{\partial M}{\partial t}, \frac{\partial M}{\partial x}, \frac{\partial^2 M}{\partial t^2}, \frac{\partial^2 M}{\partial x^2}, \frac{\partial^2 M}{\partial x \partial t}, M^2 \frac{\partial^2 M}{\partial x^2}, \dots\right) = 0 \quad (3)$$

$$M(x, y, t) = M(\eta), \quad \eta = p_1 x + p_2 y + p_3 t \quad (4)$$

Substituting Eq.(4) into Eq.(3), we obtain the following nonlinear ordinary differential equation (NODE):

$$P(M(\eta), M'(\eta), M''(\eta), M^2(\eta)M'(\eta), \dots) = 0 \quad (5)$$

In Eq.(4) P is a polynomial in $M(\eta)$ and the prime denotes the ordinary derivative of $M(\eta)$ with respect to η and Eq.(3) satisfies the Eq.(5).

Step 2: In order to get the NODE form of Eq.(5), we seek the solutions in the form:

$$M(\eta) = A_0 + \sum_{i=1}^m A_i \kappa^i(\eta) + \sum_{i=1}^m B_i \kappa^{-i}(\eta) \quad , \quad A_m, B_m \neq 0 \quad (6)$$

In Eq.(6), $A_0, \dots, A_m, B_1, \dots, B_m$ are real constants to be computed later. We obtain positive integer m which is the balancing term by balancing the highest derivative term and the highest power nonlinear term in Eq.(5). Furthermore, the function $\kappa(\eta)$ satisfies the following the Riccati differential equation:

$$\frac{d\kappa(\eta)}{d\eta} = w^2 + [\kappa(\eta)]^2 \quad (7)$$

where w is a real constant.

Step 3: According to well known modified extended tanh method it is possible to take full advantage of Eq.(7) and use it's general solutions as:

$$\kappa(\eta) = \begin{cases} -\sqrt{-w} \tanh(\sqrt{-w}\eta) \\ -\sqrt{-w} \coth(\sqrt{-w}\eta) \end{cases} \quad w < 0 ; \quad \kappa(\eta) = \begin{cases} \sqrt{w} \tan(\sqrt{w}\eta) \\ -\sqrt{w} \cot(\sqrt{w}\eta) \end{cases} \quad w > 0 ; \quad \kappa(\eta) = -\frac{1}{\eta}, \quad w = 0 \quad (8)$$

Step 4: In this study METEM is studied with new exact solutions of Eq.(7) [XX] which given in Table 1.

Table 1. The new exact solutions of Riccati Eq.(7) with $\mu = \mp 1$.

$\kappa(\eta)$ for $w < 0$	$\kappa(\eta)$ for $w > 0$
$\kappa_1(\eta) = -\sqrt{-w} \left(\tanh(2\sqrt{-w}\eta) + \operatorname{sech}(2\sqrt{-w}\eta) \right)$	$\kappa_7(\eta) = \sqrt{w} \left(\tan(2\sqrt{w}\eta) + \mu \operatorname{sec}(2\sqrt{w}\eta) \right)$
$\kappa_2(\eta) = -\sqrt{-w} \left(\coth(2\sqrt{-w}\eta) + \mu \operatorname{csch}(2\sqrt{-w}\eta) \right)$	$\kappa_8(\eta) = -\sqrt{w} \left(\cot(2\sqrt{w}\eta) + \mu \operatorname{csc}(2\sqrt{w}\eta) \right)$
$\kappa_3(\eta) = -\frac{1}{2}\sqrt{-w} \left(\tanh\left(\frac{\sqrt{-w}}{2}\eta\right) + \coth\left(\frac{\sqrt{-w}}{2}\eta\right) \right)$	$\kappa_9(\eta) = \frac{1}{2}\sqrt{w} \left(\tan\left(\frac{\sqrt{w}}{2}\eta\right) - \cot\left(\frac{\sqrt{w}}{2}\eta\right) \right)$
$\kappa_4(\eta) = \frac{w - \sqrt{-w} \tanh(\sqrt{-w}\eta)}{1 + \sqrt{-w} \tanh(\sqrt{-w}\eta)}$	$\kappa_{10}(\eta) = \frac{-\sqrt{w} (1 - \tan(\sqrt{w}\eta))}{1 + \tan(\sqrt{w}\eta)}$
$\kappa_5(\eta) = \frac{\sqrt{-w} (5 - 4 \cosh(2\sqrt{-w}\eta))}{3 + 4 \sinh(2\sqrt{-w}\eta)}$	$\kappa_{11}(\eta) = \frac{\sqrt{w} (4 - 5 \cos(2\sqrt{w}\eta))}{3 + 5 \sin(2\sqrt{w}\eta)}$
$\kappa_6(\eta) = \frac{-2\sqrt{-w}}{\tanh\left(\frac{\sqrt{-w}}{2}\eta\right) + \coth\left(\frac{\sqrt{-w}}{2}\eta\right)}$	$\kappa_{12}(\eta) = \frac{-2\sqrt{w}}{\tan\left(\frac{\sqrt{w}}{2}\eta\right) - \cot\left(\frac{\sqrt{w}}{2}\eta\right)}$

Step 5: Substituting of Eqs.(6) and (7) into Eq.(5) gives a polynomial in powers of $\kappa(\eta)$. Summing the coefficients of $\kappa(\eta)$ with the same power and subsequently setting each summation to zero, we get a set of over-determined algebraic equations for $A_0, \dots, A_m, B_1, \dots, B_m$ and p, q and c by using the Maple software.

Step 6: Solving the set of algebraic equations in Step 5, we obtain the suitable solution set(s) of parameters. Selecting the suitable solution sets and substituting the into Eqs.(6)-(7) and by using the Eq.(4) the solution(s) of NLPDE Eq.(3) is found.

3. Application of the method

In this section, the application of the METEM enhanced with new Riccati solutions is given.

3.1 Implementation to Radhakrishnan-Kundu-Lakshmanan equation

Let suppose the NLSFL [XX] given in Eq.(1). Substituting the travelling wave transformation Eq.(2) into Eq.(1) we obtain the following equation from the real and the imaginary parts.

$$(-k\lambda + \beta)(M(\eta))^3 + (-\delta k^3 - \alpha k^2 - k\sigma - \omega)M(\eta) + (3\delta k + \alpha)\frac{d^2}{d\eta^2}M(\eta) = 0 \quad (9)$$

$$\delta \frac{d^3}{d\eta^3}M(\eta) + \left((-3\lambda - 2\mu)(M(\eta))^2 - 3\delta k^2 - 2\alpha k - \nu - \sigma \right) \frac{d}{d\eta}M(\eta) = 0 \quad (10)$$

Integrating the Eq.(10) once and taking the integration constant as zero we can write the following equation for imaginary part;

$$(-3\lambda - 2\mu)(M(\eta))^3 + (-9\delta k^2 - 6\alpha k - 3\sigma - 3\nu)M(\eta) + 3\delta \frac{d^2}{d\eta^2} M(\eta) = 0 \quad (11)$$

Since the function $M(\eta)$ satisfies both Eqs. (9) and (11), according the homogeny balance principle we can write the constraint conditions between the Eqs.(9) and (11) and solving these equations we can write following constraint equations:

$$\beta = -\frac{6\delta k\lambda + 6\delta k\mu + 3\alpha\lambda + 2\alpha\mu}{3\delta} \quad (12)$$

$$\nu = -\frac{8\delta^2 k^3 + 8\alpha\delta k^2 + 2\alpha^2 k + 2\delta k\sigma + \alpha\sigma - \delta\omega}{3\delta k + \alpha} \quad (13)$$

So, we can write the NLODE form of Eq.(1) by taking Eq.(9) (or Eq.(11)) as follow:

$$(\beta - k\lambda)M^3 - (\delta k^3 + k^2 + k\sigma + \omega)M + (\alpha + 3k\delta)M'' = 0 \quad (14)$$

Considering the terms M'' and M^3 in Eq.(14) results $m=1$ which is the balancing constant.

According to the Eq.(6) the solution function of Eq.(14) will be in Eq.(15,

$$M(\eta) = A_0 + A_1\kappa(\eta) + B_1 \frac{1}{\kappa(\eta)}, \quad A_1, B_1 \neq 0 \quad (15)$$

Substituting the Eqs.(15) and (7) into Eq.(14) we get the polynomial form respect to $\kappa(\eta)$. Collecting the each term according to power of $\kappa^i(\eta)$ and equating the each coefficients to zero we get the over-determined algebraic system as:

$$\begin{aligned} \kappa^{-3}(\eta) : & \frac{B_1(3\delta k + \alpha)(6\delta\omega^2 - 3\lambda B_1^2 - 2\mu B_1^2)}{3\delta} = 0 \\ \kappa^{-2}(\eta) : & -\frac{B_1^2(3\delta k + \alpha)(2\mu + 3\lambda)A_0}{\delta} = 0 \\ \kappa^{-1}(\eta) : & -\frac{B_1}{\delta} \left(k(k^2 - 6\omega)\delta^2 + (k^2\alpha + (6A_1(\mu + 3/2\lambda)B_1 + 6\mu A_0^2 + 9\lambda A_0^2 + \sigma)k - 2\omega\alpha + \omega)\delta \right) \\ & + 2(\mu + 3/2\lambda)(A_0^2 + A_1B_1)\alpha \Big) = 0 \\ \kappa^0(\eta) : & -\frac{A_0}{\delta} \left(\delta^2 k^3 + (k^2\alpha + ((2\mu + 3\lambda)A_0^2 + 12A_1(\mu + 3/2\lambda)B_1 + \sigma)k + \omega)\delta + 4\alpha(\mu + 3/2\lambda) \right) \\ & + (A_1B_1 + 1/6 A_0^2) \Big) = 0 \\ \kappa^1(\eta) : & -\frac{A_1}{\delta} \left(k(k^2 - 6\omega)\delta^2 + (k^2\alpha + (6A_1(\mu + 3/2\lambda)B_1 + 6\mu A_0^2 + 9\lambda A_0^2 + \sigma)k - 2\omega\alpha + \omega)\delta \right) \\ & + 2(\mu + 3/2\lambda)(A_0^2 + A_1B_1)\alpha \Big) = 0 \\ \kappa^2(\eta) : & -\frac{A_1^2(3\delta k + \alpha)(2\mu + 3\lambda)A_0}{\delta} = 0 \\ \kappa^3(\eta) : & -2\frac{(\delta k + \alpha/3)A_1((\mu + 3/2\lambda)A_1^2 - 3\delta)}{\delta} = 0 \end{aligned} \quad (16)$$

In Eq.(16) $A_0, A_1, B_1, \alpha, \beta, \omega, k, \sigma$ and δ are unknown parameters to be calculated. By solving this system we get the following possible solution sets:

$$\begin{aligned}
\text{Rset}^1 &= \left\{ \omega = -\delta k^3 - \alpha k^2 + (-12\delta w - \sigma)k - 4w\alpha, A_0 = 0, A_1 = \frac{\sqrt{6}}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta}, B_1 = \frac{\sqrt{6}w}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta} \right\} \\
\text{Rset}^2 &= \left\{ \omega = -\delta k^3 - \alpha k^2 + (24\delta w - \sigma)k + 8w\alpha, A_0 = 0, A_1 = \frac{\sqrt{6}}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta}, B_1 = -\frac{\sqrt{6}w}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta} \right\} \\
\text{Rset}^3 &= \left\{ \omega = -\delta k^3 - \alpha k^2 + (24\delta w - \sigma)k + 8w\alpha, A_0 = 0, A_1 = -\frac{\sqrt{6}}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta}, B_1 = \frac{\sqrt{6}w}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta} \right\} \\
\text{Rset}^4 &= \left\{ \omega = -\delta k^3 - \alpha k^2 + (-12\delta w - \sigma)k - 4w\alpha, A_0 = 0, A_1 = -\frac{\sqrt{6}}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta}, B_1 = -\frac{\sqrt{6}w}{2\mu + 3\lambda} \sqrt{(2\mu + 3\lambda)\delta} \right\}
\end{aligned} \tag{17}$$

Substituting the $\kappa_i(\eta)$ in Table.1 into Eq.(15) and using the Eq.(2),(12) and (13) the following solution functions for NLPDE Eq.(1) are obtained.

$$\begin{aligned}
\Omega_1(\eta) &= -\sqrt{-w} \left(\tanh(2\sqrt{-w}\eta) + \text{sech}(2\sqrt{-w}\eta) \right) : \\
M_1(x, t) &= e^{i(-kx + \omega t + \phi)} \left(A_0 - A_1 \sqrt{-w} \left(\tanh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \right. \\
&\quad \left. \left. + \text{sech} \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \right) \\
&\quad \left(-B_1 \frac{1}{\sqrt{-w}} \left(\tanh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \right. \\
&\quad \left. \left. + \text{sech} \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \right) \right)^{-1}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\Omega_2(\eta) &= -\sqrt{-w} \left(\coth(2\sqrt{-w}\eta) + \mu \text{csch}(2\sqrt{-w}\eta) \right) : \\
M_2(x, t) &= e^{i(-kx + \omega t + \phi)} \left(A_0 - A_1 \sqrt{-w} \left(\coth \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \right. \\
&\quad \left. \left. + \text{csch} \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \right) \\
&\quad \left(-B_1 \frac{1}{\sqrt{-w}} \left(\coth \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \right. \\
&\quad \left. \left. + \text{csch} \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \right) \right)^{-1}
\end{aligned} \tag{19}$$

$$\Omega_3(\eta) = -\frac{1}{2} \sqrt{-w} \left(\tanh \left(\frac{\sqrt{-w}}{2} \eta \right) + \coth \left(\frac{\sqrt{-w}}{2} \eta \right) \right) :$$

$$M_3(x, t) = e^{i(-kx + \omega t + \phi)} \left(A_0 - \frac{A_l}{2} \sqrt{-w} \left(\tanh \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) + \coth \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) - 2 \frac{B_l}{\sqrt{-w}} \left(\tanh \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) + \coth \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \right)^{-1} \quad (20)$$

$$\Omega_4(\eta) = (w - \sqrt{-w} \tanh(\sqrt{-w} \eta)) (1 + \sqrt{-w} \tanh(\sqrt{-w} \eta))^{-1} :$$

$$M_4(x, t) = e^{i(-kx + \omega t + \phi)} \left(A_0 + A_l \left(w - \sqrt{-w} \tanh \left(\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \left(1 + \sqrt{-w} \tanh \left(\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right)^{-1} + B_l \left(1 + \sqrt{-w} \tanh \left(\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \left(w - \sqrt{-w} \tanh \left(\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right)^{-1} \right) \quad (21)$$

$$\Omega_5(\eta) = \sqrt{-w} (5 - 4 \cosh(2\sqrt{-w} \eta)) (3 + 4 \sinh(2\sqrt{-w} \eta))^{-1} :$$

$$M_5(x, t) = e^{i(-kx + \omega t + \phi)} \left(A_0 + A_l \sqrt{-w} \left(5 - 4 \cosh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \left(3 + 4 \sinh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right)^{-1} + B_l \left(3 + 4 \sinh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \left(\frac{1}{\sqrt{-w}} \left(5 - 4 \cosh \left(2\sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \right)^{-1} \right) \quad (22)$$

$$\Omega_6(\eta) = -2\sqrt{-w} \left(\tanh \left(\frac{\sqrt{-w}}{2} \eta \right) + \coth \left(\frac{\sqrt{-w}}{2} \eta \right) \right)^{-1} :$$

$$M_6(x, t) = e^{i(-kx + \omega t + \phi)} \left(A_0 - 2A_l \sqrt{-w} \left(\tanh \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) + \coth \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) - \frac{B_l}{2} \left(\tanh \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) + \coth \left(\frac{1}{2} \sqrt{-w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega) t}{3k\delta + \alpha} \right) \right) \right) \frac{1}{\sqrt{-w}} \right) \quad (23)$$

$$\Omega_7(\eta) = \sqrt{w} \left(\tan(2\sqrt{w}\eta) + \mu \sec(2\sqrt{w}\eta) \right):$$

$$M_7(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 + A_I \sqrt{w} \left(\tan \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. + \sec \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ + B_I \frac{1}{\sqrt{w}} \left(\tan \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. + \sec \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \end{array} \right)^{-1} \quad (24)$$

$$\Omega_8(\eta) = -\sqrt{f} \left(\cot(2\sqrt{f}\eta) + \mu \csc(2\sqrt{f}\eta) \right):$$

$$M_8(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 - A_I \sqrt{w} \left(\cot \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. + \csc \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ - B_I \frac{1}{\sqrt{w}} \left(\cot \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. + \csc \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \end{array} \right)^{-1} \quad (25)$$

$$\Omega_9(\eta) = \frac{1}{2} \sqrt{w} \left(\tan \left(\frac{\sqrt{w}}{2} \eta \right) - \cot \left(\frac{\sqrt{w}}{2} \eta \right) \right):$$

$$M_9(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 + \frac{A_I}{2} \sqrt{w} \left(\tan \left(\frac{1}{2} \sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. - \cot \left(\frac{1}{2} \sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ + 2 \frac{B_I}{\sqrt{w}} \left(\tan \left(\frac{1}{2} \sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right. \\ \left. - \cot \left(\frac{1}{2} \sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \end{array} \right)^{-1} \quad (26)$$

$$\Omega_{10}(\eta) = -\sqrt{w} \left(1 - \tan(\sqrt{w}\eta) \right) \left(1 + \tan(\sqrt{w}\eta) \right)^{-1}:$$

$$M_{10}(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 - A_I \sqrt{w} \left(1 - \tan \left(\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ \left(1 + \tan \left(\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right)^{-1} \\ -B_I \left(1 + \tan \left(\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ \frac{I}{\sqrt{w}} \left(1 - \tan \left(\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right)^{-1} \end{array} \right) \quad (27)$$

$$\Omega_{11}(\eta) = \sqrt{w} \left(4 - 5 \cos \left(2\sqrt{w}\eta \right) \right) \left(3 + 5 \sin \left(2\sqrt{w}\eta \right) \right)^{-1} :$$

$$M_{11}(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 + A_I \sqrt{w} \left(4 - 5 \cos \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ \left(3 + 5 \sin \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right)^{-1} \\ +B_I \left(3 + 5 \sin \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right) \\ \frac{I}{\sqrt{w}} \left(4 - 5 \cos \left(2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \right)^{-1} \end{array} \right) \quad (28)$$

$$\Omega_{12}(\eta) = -2\sqrt{w} \left(\tan \left(\frac{\sqrt{w}}{2} \eta \right) - \cot \left(\frac{\sqrt{w}}{2} \eta \right) \right)^{-1} :$$

$$M_{12}(x, t) = e^{i(-kx + \omega t + \phi)} \left(\begin{array}{l} A_0 - 2A_I \sqrt{w} \left(\begin{array}{l} \tan \left(1/2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \\ -\cot \left(1/2\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \end{array} \right) \\ -\frac{B_I}{2} \left(\begin{array}{l} \tan \left(\frac{1}{2}\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \\ -\cot \left(\frac{1}{2}\sqrt{w} \left(x + \frac{(8\delta^2 k^3 + 8\alpha \delta k^2 + 2\alpha^2 k + 2\delta k \sigma + \alpha \sigma - \delta \omega)t}{3k\delta + \alpha} \right) \right) \end{array} \right) \frac{I}{\sqrt{w}} \end{array} \right) \quad (29)$$

Fig. 1 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real, (f) imaginary of $M_1(x,t)$ in Eq.(18) by selecting set $Rset^1$ in Eq.(17) and for the parameters $w = -0.4, \alpha = 0.5, \sigma = -0.5, \lambda = 0.5, \mu = 0.5, \delta = 0.5, k = 1, \varphi = 1$

Fig. 2 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real, (f) imaginary of $M_1(x,t)$ in Eq.(18) by selecting set Rset^2 in Eq.(17) and for the parameters $w = -0.4, \alpha=0.5, \sigma=-0.5, \lambda=0.5, \mu=0.5, \delta=0.5, k=1, \varphi=1$

Fig. 3 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real, (f) imaginary of $M_5(x,t)$ in Eq.(22) by selecting set Rset^1 in Eq.(17) and for the parameters $w = -0.4, \alpha=0.5, \sigma=-0.5, \lambda=0.5, \mu=0.5, \delta=0.5, k=1, \varphi=1$

Fig.4 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real, (f) imaginary of $M_s(x,t)$ in Eq.(22) by selecting set $Rset^3$ in Eq.(17) and for the parameters $w = -0.4, \alpha=0.5, \sigma=-0.5, \lambda=0.5, \mu=0.5, \delta=0.5, k=1, \varphi=1$

Fig.5 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real, (f) imaginary of $M_{11}(x,t)$ in Eq.(24) by selecting set $Rset^4$ in Eq.(17) and for the parameters $w = 0.4, \alpha=0.5, \sigma=0.5, \lambda=0.5, \mu=0.5, \delta=0.5, k=1, \varphi=1$

Fig.6 The 3D (a) modulus, (b) real, (c) imaginary and 2D plots (d) modulus, (e) real , (f) imaginary of $M_7(x,t)$ in Eq.(24) by selecting set $Rset^1$ in Eq.(17) and for the parameters $w = 0.41, \alpha=0.5, \sigma=0.5, \lambda=0.5, \mu=0.5, \delta=0.5, k=0.1, \varphi=1$

4. Conclusion

We have utilized an efficient modified extended tanh expansion method to establish robust solutions for the Radhakrishnan-Kundu-Lakshmanan equation in this work. The effective scheme have been shown to be effective for the governing model. Bright and combined dark–bright optical solitons are successfully exposed using the Maple symbolic package. On the 3-dimensional and 2-dimensional graphs, the physical features of the presented results have been depicted.

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