Combinatorial Design of the MAUT and PAMSSEM II Methods for Multiple Attributes Group Decision Making with Probabilistic Linguistic Information

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Abstract

This paper considers the application of probabilistic linguistic term sets (PLTS) in multiple-attribute group decision-making (MAGDM) when the weights can't be determined. First, as an improvement of the PROMETHEE method, the PAMSSEM method can not only handle missing evaluations, but also proposes a rejection threshold to calculate the overall consistency of the plan, so as to rank the plan more reasonably. At the same time, the MAUT uses the marginal utility function to reallocate the attribute values of the alternatives in the interval $[0,1]$, and then calculate the total utility to sort them. Because the utility function is beneficial in expressing consumer satisfaction, we combine the MAUT method and PAMSSEM II method and apply it to solve decision-making problems under probabilistic linguistic environment. Secondly the coefficient of variation method, entropy method and analytic hierarchy process are used to calculate the weights in a combination. In the process of data processing, we use the transfer function to convert the PLTS into the hesitant probabilistic fuzzy set (HPFS) for calculation. Finally, the PL-MAUT-PAMSSEM II method, PROMTHER method, TOPSIS method and ARAS method are compared with each other.

Keywords Probabilistic linguistic term set · Hesitant probabilistic fuzzy set · PL-MAUT-PAMSSEM II method · Multiple attributes group decision making

1 Introduction

Decision-making activities are pervasive in all aspects of our lives. The decision-making process includes asking questions, determining alternatives, choosing alternatives, and making decisions (Xu and Zhang 2021). Due to the diversity of decision-making problems and the ambiguity of human thinking, it is not easy for us to use specific numbers to accurately describe the value of each alternative under different standards when making assessments

(Saleem and NoorUl 2014). Therefore, obtaining accurate assessments requires high costs. In the MAGDM problem, the decision makers (DMs) are required to provide the evaluation value of different alternatives under multiple attributes. Because it is difficult to provide accurate numbers to express preferences, linguistic terms such as "medium" or "good" are used as evaluation information (He et al. 2016; Ji and Yu 2017; Lin et al. 2019; Qu et al. 2018; Sun et al. 2019; Xi et al. 2019; Xi et al. 2018). Therefore, more and more scholars are aware of the importance of probability information in decision-making compared with digital information and devote themselves to related research (Wang et al. 2018b).

Based on the above situation, Zadeh (1975) introduced the Linguistic Term Set (LTS), which expresses DM's preferences through one or more linguistic information, and uses fuzzy linguistic to model qualitative evaluation information. Since then, scholars have carried out further research based on LTS, including linguistic cloud information (Herrera et al. 2000), two-tuple linguistic model (Xu 2004), uncertain linguistic structure (Herrera et al. 2008), unbalanced linguistic information (Xu 2008), and unbalanced uncertain linguistic information (Wang et al. 2014).

With the enrichment of human practical activities, the difficulty of decision-making is getting bigger, and some models that have been proposed cannot completely and accurately express the preferences of DMs in the actual decision-making problem (Wang et al. 2018b). For example, when a DM evaluates alternative plans for inbound tourist attractions, he cannot be 100% sure that the plan is "good", or he may think it is "medium", "good" or "very good". To help DMs make more reasonable decisions, Rodriguez et al. (2012) proposed the concept of hesitating fuzzy linguistic term set (HFLTS), and Liao et al. (2015) introduced the mathematical form of HFLTS, which allows the DMs to hesitate between several linguistic terms.

However, HFLTS believes that different linguistic terms have the same importance or weight, which is likely to be inconsistent with the true wishes of the DMs. For example, suppose there are 10 DMs evaluating alternatives

for inbound tourist attractions, one DM may think it is "good", and the other 9 experts may think it is "medium". Then the evaluation information represented by HFLTS is $\{\text{good}, \text{medium}\}$. Obviously, the result that does not consider the importance or weight of different linguistic terms may misrepresent the preferences of DMs. In order to avoid the above problems, Pang et al. (2016) introduced the probabilistic linguistic term set (PLTS) by adding the corresponding probability to the HFLTS, which reduces the information loss in the evaluation process and increases the DM's Express the applicability and accuracy of its linguistic assessment. Use PLTS to indicate that the result of the above example is $\{\text{good} (0.1), \text{medium} (0.9)\}$. PLTS can reflect the true weights of different attributes corresponding to the alternative and is suitable for group decision-making problems with distributed information (Wang et al. 2018a). Therefore, research on PLTS is necessary.

The rationality and flexibility of PLTS have attracted the attention of more and more scholars, and scholars have studied PLTS from different angles (Liao et al. 2019; Mi et al. 2020), including probabilistic linguistic vector item sets promote the application of multi-granular linguistic information (Zhai et al. 2016), MADM problem (Gou et al. 2016), the operation of PLTS, the distance measurement of PLTS (Lin et al. 2019; Lin and Xu 2018; Wang et al. 2018b; Zhang et al. 2020), the probability formula of PLTS (Bai et al. 2017; Feng et al. 2019; Xian et al. 2019; Yu et al. 2019a; Yu et al. 2019b), probabilistic linguistic preference relations (Gao et al. 2018; Song and Jun 2019; Tian et al. 2019; Zhang et al. 2018; Zhang et al. 2016), etc.

Actually, the process of determining attribute weights is equally important. The subjective weight given by the DMs is based on the relative importance of the standard (Lootsma 2010; Rodríguez et al. 2017), while the objective weight is obtained by evaluating the standard itself. Given the correlation between attributes, Diakoulaki et al. (1995) proposed an objective method of determining weight, which gives less weight to the more relevant attributes. In addition, some scholars combine subjective and objective weights to determine standard weights (Hafezalkotob and Hafezalkotob 2015). However, the above-mentioned weight determination techniques have two shortcomings

(Wu et al. 2018): (1) It is difficult for them to express all the subjective preferences of DMs in a single linguistic term, and they cannot integrate the evaluations of all DMs without complete information. (2) There are relatively few studies on the determination of the objective weight of attributes in a probabilistic linguistic environment. In order to maintain objectivity, when studying the application of MAUT-PAMSSEM II in PLTS, this article uses the entropy method to determine the weight. And finally, we compare this new method with PROMETHEE, TOPSIS, and ARAS to show its applicability.

In summary, the focus of this article includes the following points:

(a) We provide a new decision-making method when the attribute weight is unknown, called the PL-MAUT-PAMSSEM II method. It merges the MAUT and PAMSSEM methods into two calculation stages in a probabilistic linguistic environment, which not only provides more method options for DMs but also improves the accuracy of evaluation considering the different importance of linguistic terms.

(b) The PL-MAUT-PAMSSEM II method combines PLTS and HPFS through the equivalent conversion functions and compares the alternatives in pairs according to the marginal utility score to obtain the ranking of the alternatives.

(c) Compare and analyze the method proposed in this article with existing methods.

(d) We apply the methods provided to solve the problem of inbound tourist destination selection.

The rest of this article is as follows: Section 2 reviews some basic knowledge of PLTS and HPFS. Section 3 introduces the MAUT method and PAMSSEM method respectively and includes the entire calculation process of PL-MAUT-PAMSSEM II. Section 4 introduces the entropy method and the whole process of determining attribute weights. Section 5 shows a case of inbound tourism destination selection and compares several methods to prove the effectiveness of the proposed PL-MAUT-PAMSSEM II method. Section 6 is the conclusion of this article.

2 Preliminaries

In this section, we will give some basic concepts and formulas of PLTS, HPFS and the equivalent transfer functions.

2.1 The probabilistic linguistic term set

Definition 1 (Pang et al. 2016) The probabilistic linguistic term set (PLTS) on LTS L can be defined as:

$$PL(p) = \left\{ l^{(k)} \left(p^{(k)} \right) \middle| l^{(k)} \in L, p^{(k)} \geq 0, k = 1, 2, \dots, \#PL(p), \sum_{k=1}^{\#PL(p)} p^{(k)} \leq 1 \right\},$$

Where $l^{(k)}(p^{(k)})$ is made up of linguistic term $l^{(k)}$ and its associated probability $p^{(k)}$, and the cardinality of $PL(p)$ is $\#PL(p)$. If $k = p^{(k)} = 1$, then the PLTS reduces to a linguistic term.

The size of PLTS can be compared by the following scoring function and variance function:

Definition 2 Let $\sigma(l^{(k)})$ is the membership degree of the linguistic term $l^{(k)}$ in PLTS $PL(p)$, then the score function of $PL(p)$ is defined as:

$$S(PL(p)) = \sum_{k=1}^{\#PL(p)} \sigma(l^{(k)}) p^{(k)} \bigg/ \sum_{k=1}^{\#PL(p)} p^{(k)}, \quad (1)$$

Definition 3 Suppose that $\sigma(l^{(k)})$ is the membership degree of the linguistic term $l^{(k)}$ in PLTS $PL(p)$, and $S(PL(p))$ is the score value of the PLTS $PL(p)$. Then the variance function of $PL(p)$ is defined as:

$$D(PL(p)) = \left(\sum_{k=1}^{\#PL(p)} \left(\left(p^{(k)} \left(\sigma(l^{(k)}) - S(PL(p)) \right) \right)^2 \right)^{0.5} \right) \bigg/ \sum_{k=1}^{\#PL(p)} p^{(k)}, \quad (2)$$

For two any PLTSs $PL_1(p)$ and $PL_2(p)$ with $S(PL_1(p)) \neq S(PL_2(p))$,

(1) If $S(PL_1(p)) > S(PL_2(p))$, then $PL_1(p) > PL_2(p)$;

(2) If $S(PL_1(p)) < S(PL_2(p))$, then $PL_1(p) < PL_2(p)$;

(3) If $S(PL_1(p)) = S(PL_2(p))$, then

(i) If $D(PL_1(p)) > D(PL_2(p))$, then $PL_1(p) < PL_2(p)$

(ii) If $D(PL_1(p)) < D(PL_2(p))$, then $PL_1(p) > PL_2(p)$

(iii) If $D(PL_1(p)) = D(PL_2(p))$, then $PL_1(p) \approx PL_2(p)$

2.2 The hesitant probabilistic fuzzy set

Definition 4 (Torra 2010; Torra and Narukawa 2009) Let X be a reference set. Accordingly, an HFS A on X is

defined in terms of a function $h_A(x)$ that when applied to X returns a finite subset of $[0, 1]$:

$$A = \{ \langle x, h_A(x) \rangle \mid x \in X \}, \quad (3)$$

Where $h_A(x)$ is a set of multiple different values on $[0, 1]$, which represents the membership degree of the element x in X on A , and $h_A(x)$ represents HFE (Xia and Xu 2011).

Example 1 Let $X = \{x_1, x_2\}$ be a reference set, and $h_A(x_1) = \{0.1, 0.3\}$ and $h_A(x_2) = \{0.2, 0.3, 0.5\}$ be two HFEs of x_i ($i = 1, 2$) to a set A . Then, A can be expressed as an HFS as follows:

$$A = \{ \langle x_1, \{0.1, 0.3\} \rangle, \langle x_2, \{0.2, 0.3, 0.5\} \rangle \}.$$

Definition 5 (Xu and Zhou 2016) Let X be a reference set, then an HPFS A_p on X can be expressed by as:

$$A_p = \{ \langle x, h(p_x) \rangle \mid x \in X \}; \quad (4)$$

Where $h(p_x)$ is a set of multiple different values on $[0, 1]$, which described by a probability distribution p_x .

Among them, p_x represents the membership degree of the elements x on the set X on A_p . $h(p_x)$ can be expressed as an HPFE as follows:

$$h(p_x) = \{ \gamma_i(p_i) \mid i = 1, 2, \dots, k \}; \quad (5)$$

Among them, p_i represents probability of the possible value γ_i , and it satisfies $\sum_{i=1}^k p_i = 1$, k represents the number of $\gamma_i(p_i)$ in $h(p_x)$.

Example 2 Let $X = \{x_1, x_2\}$ be a reference set, and $h(p_1) = \{0.1(0.6), 0.2(0.4)\}$ and $h(p_2) = \{0.6(0.2), 0.2(0.8)\}$ be two HPFS as follows:

$$E_p = \{ \langle x_1, \{0.1(0.6), 0.2(0.4)\} \rangle, \langle x_2, \{0.6(0.2), 0.2(0.8)\} \rangle \}.$$

The size of HPFS can be compared by the following scoring function and variance function:

Definition 6 (Zhang et al. 2017) Let $h(p_x) = \{ \gamma_i(p_i) \mid i = 1, 2, \dots, k \}$ be an HPFE. Its expected mean can be defined as follows:

$$E(h(p_x)) = \sum_{i=1}^k \gamma_i p_i; \quad (6)$$

Based on the expected mean, the variance of an HPFE can be defined as follows.

Definition 7 Let $h(p_x) = \{\gamma_i(p_i) | i = 1, 2, \dots, k\}$ be an HPFE. Its variance can be defined as follows:

$$D(h(p_x)) = \sum_{i=1}^k (\gamma_i - E(h(p_x)))^2 p_i; \quad (7)$$

Definition 8 (Xu and Zhou 2016) Let $\bar{h}_i (i = 1, 2, \dots, I)$ be a collection of HPFS, $\omega = (\omega_1, \omega_2, \dots, \omega_I)^T$ be the weight vector of $\bar{h}_i$ with $\omega_i \in [0, 1]$ and $\sum_{i=1}^I \omega_i = 1$, p_i be the probability of γ_i in the HPFE $\bar{h}_i$, $\gamma_{\sigma(i)}$ be the i th largest of $\bar{h}_i$, $p_{\sigma(i)}$ be the probability of $\gamma_{\sigma(i)}$ in the HPFE $\bar{h}_i$, and $\omega_{\sigma(i)}$ be the i th largest of ω , then the aggregation operator can be defined as follows:

$$HPFOWA(\bar{h}_1, \bar{h}_2, \dots, \bar{h}_I) = \bigoplus_{i=1}^I (\omega_i \bar{h}_{\sigma(i)}) \bigcup_{\gamma_{\sigma(1)} \in \bar{h}_{\sigma(1)}, \gamma_{\sigma(2)} \in \bar{h}_{\sigma(2)}, \dots, \gamma_{\sigma(i)} \in \bar{h}_{\sigma(i)}} \left\{ 1 - \prod_{i=1}^I (1 - \gamma_{\sigma(i)})^{\omega_i} \mid p_{\sigma(1)} p_{\sigma(2)} \dots p_{\sigma(i)} \right\} \quad (8)$$

2.3 The equivalent transformation functions

Definition 9 (Gou and Xu 2016; Zhang and Qi 2013) Let $h_L = \{l_t | t \in [-\delta, \delta]\}$ be a HFLE on LTS $L = \{l_t | t = -\delta, \dots, -1, 0, 1, \dots, \delta\}$, and $H = \{\varepsilon | \varepsilon \in [0, 1]\}$ be a HFS. Then the linguistic variable l_t and membership degree ε can be transformed into each other by the function σ and σ^{-1} given as:

$$\begin{aligned} \sigma: [-\delta, \delta] &\rightarrow [0, 1], \\ \sigma(h_L) &= \left\{ \sigma(l_t) = \varepsilon = \frac{t}{2\delta} + \frac{1}{2} \mid t \in [-\delta, \delta] \right\} = h_\varepsilon; \\ \sigma^{-1}: [0, 1] &\rightarrow [-\delta, \delta], \\ \sigma^{-1}(h_\varepsilon) &= \left\{ \sigma^{-1}(\varepsilon) = l_t = l(2\varepsilon - 1)\delta \mid \varepsilon \in [0, 1] \right\} = h_L. \end{aligned} \quad (9)$$

2.4 Some operations of PLTSs

Definition 10 (Yue et al. 2020) Let $PL(p)$, $PL_1(p)$ and $PL_2(p)$ be three adjusted PLTSs on LTS L , and λ be a positive real number. Then

$$\begin{aligned} (1) PL_1(p) \oplus PL_2(p) &= \bigcup_{\xi_1^{(k)} \in \sigma(PL_1), \xi_2^{(k)} \in \sigma(PL_2)} \left\{ \sigma^{-1}(\xi_1^{(k)} + \xi_2^{(k)} - \xi_1^{(k)} \xi_2^{(k)}) (p^{(k)}) \right\}, k = 1, 2, \dots, \# PL_1(p) = \# PL_2(p); \\ (2) PL_1(p) \otimes PL_2(p) &= \bigcup_{\xi_1^{(k)} \in \sigma(PL_1), \xi_2^{(k)} \in \sigma(PL_2)} \left\{ \sigma^{-1}(\xi_1^{(k)} \xi_2^{(k)}) (p^{(k)}) \right\}, k = 1, 2, \dots, \# PL_1(p) = \# PL_2(p); \\ (3) PL_1(p) \ominus PL_2(p) &= \bigcup_{\xi_1^{(k)} \in \sigma(PL_1), \xi_2^{(k)} \in \sigma(PL_2)} \left\{ \sigma^{-1}(\Pi) (p^{(k)}) \right\}; \end{aligned}$$

where

$$\Pi = \begin{cases} \frac{\xi_1^{(k)} - \xi_2^{(k)}}{1 - \xi_2^{(k)}} & \text{if } \xi_1^{(k)} \geq \xi_2^{(k)} \text{ and } \xi_2^{(k)} \neq 1 \\ 0 & \text{if otherwise,} \end{cases}$$

and $k = 1, 2, \dots, \# PL_1(p) = \# PL_2(p)$;

$$(4) PL_1(p) \odot PL_2(p) = \bigcup_{\xi_1^{(k)} \in \sigma(PL_1), \xi_2^{(k)} \in \sigma(PL_2)} \{ \sigma^{-1}(\Pi)(p^{(k)}) \}$$

where

$$\Pi = \begin{cases} \frac{\xi_1^{(k)}}{\xi_2^{(k)}} & \text{if } \xi_1^{(k)} \leq \xi_2^{(k)} \text{ and } \xi_2^{(k)} \neq 1 \\ 0 & \text{if otherwise,} \end{cases}$$

and $k = 1, 2, \dots, \# PL_1(p) = \# PL_2(p)$.

3 Combinatorial of the MAUT and PAMSSEM II methods

3.1 The PAMSSEM method

The PAMSSEM methods were introduced by Martel (1998) and is applied to cloud resource integration management (Yazir et al. 2012). The PAMSSEM method includes stages I and II. The first stage gives a partial ranking of the alternatives by calculating entering and leaving flow, and the second stage gives a complete ranking of the alternatives by calculating net flow. The purpose of PAMSSEM is to establish a preference relationship (Bélanger et al. 2000a; Bélanger et al. 2000b; Bélanger and Martel 2012). After DMS are given a decision matrix, the indifference threshold parameters(q), the preference threshold(p), and the veto threshold(v), the alternatives are ranked by establishing a local preference model (Alinezhad and Khalili 2019).

The PROMETHEE method is proposed to deal with the incomparable situation when comparing targets (Figueira et al. 2005). This method is simple, clear, and stable (Brans et al. 1986). As an improvement of this method, PAMSSEM also has the above advantages. In addition, the advantages of the PAMSSEM method also

include the ability to handle missing evaluations, the discrimination threshold allows A_i better than A_i , while A_i is better than A_i and the rejection threshold can prevent A_i from being better than A_i . Among them, when the threshold value is biased, it may have an impact on the ranking results, but this does not conflict with the stability of the method. Changes in parameters will inevitably lead to changes in rankings, which just shows that the method considers the particularity of each scheme.

In mathematics, the probability density function of a continuous random variable is a function that describes the probability that the output value of this random variable is near a certain value point. The probability that the value of a random variable falls within a certain region is integral to the probability density function in this region. The decision matrix given by DMS represents the distribution of random variables (or statistical variables) X_{ij} with probability density or quality function $f_{ij}(x_{ij})$. A clear assessment can be seen as the degradation of the distribution, the distribution is reduced to only a point ($(p(X_{ij} = e_{ij}) = 1)$), and the probability distribution function is as follows (Guitouni et al. 1999):

$$\int_{-\infty}^{\infty} f_{ij}(x_{ij}) dx_{ij} = 1 \quad (10)$$

3.2 The MAUT method

The Multi-Attribute Utility Theory (MAUT) method was introduced by Keeney and Raiffa in 1976 (Emovon et al. 2016; Figueira et al. 2005). The hypothesis of this method is that each DM tries to consciously or implicitly optimize an aggregate function that integrates the opinions of all DMs. This function may not be known before the start of the decision, so DMs need to construct it first (Nemery and Ishizaka 2013; Wallenius et al. 2008). The utility function in economics is a function used to express the quantitative relationship between the utility obtained by consumers in consumption and the combination of commodities consumed, to measure the degree of satisfaction that consumers obtain from the consumption of a given combination of commodities. Compared with the previous method, this method is more convenient in the operation of solving MAGDM problems, which is one of its

advantages. This method gives DMs more initiative, and makes the evaluation results more real and accurate, and can better reflect the wishes of DMs.

3.3 The PL-MAUT-PAMSSEM II method

Based on the advantages of the MAUT method and the PAMSSEM method proposed above, we combine the two methods and apply them to PLTSs. PAMSSEM is a method of ranking programs based on preference. Based on the 2004-2019 inbound tourism data provided by the official website and statistical yearbook, we invite multiple DMs to evaluate multiple popular inbound tourist destinations and let the DMs set specific thresholds p , q , and v . Through the threshold p , q , v to remove some attribute values, the attribute value closest to the threshold p can reflect the preference of DMs. MAUT is a method of ranking alternatives based on the utility function. A utility function is also a tool that reflects the preferences or intentions of decision makers. Therefore, this article combines the two methods to optimize the decision-making process, make the decision-making results more real and accurate, and provide DMs with better travel plans.

The decision-making processes based on PL-MAUT-PAMSSEM II method are as follows:

For a MAGDM problem in PLTS, let $A = \{A_1, A_2, \dots, A_m\}$ be a finite set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be the set of attributes and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of attributes C_j ($j = 1, 2, \dots, n$), with $\omega_j \in [0, 1]$, $j = 1, 2, \dots, n$ and $\sum_{j=1}^n \omega_j = 1$. Suppose that $R_y = [LS(p)_{ij}]_{m \times n}$ is the decision matrix for year y , where $LS(p)_{ij} = \{LS_{ij}^{(t)}(p_{ij}^{(t)}) \mid t = 1, 2, \dots, \#LS(p)_{ij}\}$ is a PLTS, which is an evaluation value of alternative A_i about attribute C_j . Then the goal is to rank the alternatives.

Step1 Receive information from decision makers and form the PL decision-making matrixes

$$R_y = [LS(P)_{ij}]_{m \times n}, \quad y = 1, 2, \dots, Y.$$

Step2 Calculate the decision matrix $R = [LS(P)_{ij}]_{m \times n}$ after aggregation by formulas (8) and (9).

$$R = \begin{bmatrix} LS(p)_{11} & \cdots & LS(p)_{1j} & \cdots & LS(p)_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ LS(p)_{i1} & \cdots & LS(p)_{ij} & \cdots & LS(p)_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ LS(p)_{m1} & \cdots & LS(p)_{mj} & \cdots & LS(p)_{mn} \end{bmatrix}_{m \times n} ; i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (11)$$

Step3 Normalized the decision matrix $R^* = [LS(P)^*_{ij}]_{m \times n}$.

Step4 Use formula (1) to get the score matrix of the alternatives under different attributes S :

$$S = \begin{bmatrix} s(LS(p)^*_{11}) & \cdots & s(LS(p)^*_{1j}) & \cdots & s(LS(p)^*_{1n}) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s(LS(p)^*_{i1}) & \cdots & s(LS(p)^*_{ij}) & \cdots & s(LS(p)^*_{in}) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s(LS(p)^*_{m1}) & \cdots & s(LS(p)^*_{mj}) & \cdots & s(LS(p)^*_{mn}) \end{bmatrix}_{m \times n} ; i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (12)$$

In order to calculate the utility function, the DMs needs to first standardize the score value matrix to readjust the original performance between 0 and 1. The rescaling or normalization step is usually based on the minimum and maximum performance of the alternatives on each attribute:

$$s'_j(a_i) = \frac{s_j(a_i) - \min(s_j)}{\max(s_j) - \min(s_j)} \quad (13)$$

When maximizing the attribute, or

$$s'_j(a_i) = 1 + \left(\frac{\min(s_j) - s_j(a_i)}{\max(s_j) - \min(s_j)} \right) \quad (14)$$

When minimizing the attribute.

Step5 Calculate the weight of each attribute ω_j by formula (28)-(32).

Step6 Calculate the **marginal utility score** as follows:

$$u_{ij} = \frac{e^{(s'(LS(p)^*_{ij}))^2} - 1}{1.71} ; i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (15)$$

Step7 Calculate the **local outranking index** $\delta_j(A_i, A_i)$ by the following expression:

$$\delta_j(A_i, A_{i'}) = \sum_{A_{i'}} \left(\sum_{A_i} \bar{\delta}_j(A_i, A_{i'}) \cdot f_j(A_i) \right) \cdot f_j(A_{i'}); \quad i, i' \in \{1, 2, \dots, m\} \quad (16)$$

$f_j(A_{i'})$ and $f_j(A_i)$ are respectively the probability density functions (discrete) which are assumed to be equal one. $\bar{\delta}_j(A_i, A_{i'})$ is index computed according to the following formula:

$$\bar{\delta}_j(A_i, A_{i'}) = \begin{cases} 0 & \text{if } \Delta_j \leq -p_j \\ \frac{\Delta_j + p_j}{p_i - q_j} & \text{if } -p_j < \Delta_j - q_j; \quad p_j \geq q_j \geq 0 \\ 1 & \text{if } \Delta_j \geq -q_j \end{cases} \quad (17)$$

Where $\Delta_j = u_{ij} - u_{i'j}$, which indicates the difference between the level, and threshold q, p is the values determined by the DMs for each attribute. When the number of ordinal levels is greater than 3, after modification, we can get the same equivalent function as ordinal evaluation local outranking index:

$$\bar{\delta}_j(A_i, A_{i'}) = \begin{cases} 0 & \text{if } \Delta_j \leq -1 \\ \frac{1}{2} & \text{if } -1 < \Delta_j < 0; \quad i, i' \in \{1, 2, \dots, m\} \\ 1 & \text{if } \Delta_j \geq 0 \end{cases} \quad (18)$$

Step8 Calculate the **concordance index** $C(A_i, A_{i'})$ for each pair of alternatives as follows:

$$C(A_i, A_{i'}) = \sum_{j=1}^n \delta_j(A_i, A_{i'}) \cdot \omega_j; \quad i, i' \in \{1, 2, \dots, m\} \quad (19)$$

Where ω_j represent the weight of each attribute and $\sum_{j=1}^n \omega_j = 1$.

Step9 Calculate the **local discordance index** $D(A_i, A_{i'})$ as follows:

$$D_j(A_i, A_{i'}) = \sum_{A_{i'}} \left(\sum_{A_i} \bar{D}_j(A_i, A_{i'}) \cdot f_j(A_{i'}) \right) \cdot f_j(A_i) \quad (20)$$

$f_j(A_{i'})$ and $f_j(A_i)$ are respectively the probability density functions (discrete) which are assumed to be equal one. $\bar{D}_j(A_i, A_{i'})$ is index computed according to the following formula:

$$\bar{D}_j(A_i, A_{i'}) = \begin{cases} 0 & \text{if } \Delta_j \leq -v_j \\ -\frac{(\Delta_j + p_j)}{v_j - p_j} & \text{if } -v_j < \Delta_j < -p_j; \quad v_j > p_j \\ 1 & \text{if } \Delta_j \geq -p_j \end{cases} \quad (21)$$

Where the threshold v is determined by DMs. In addition, $\bar{D}_j(A_i, A_{i'})$ for the cardinal attributes is determined by following formula:

$$\bar{D}_j(A_i, A_{i'}) = \begin{cases} \min \left[1, -\xi(\omega_j) \cdot \left(\Delta_j + \frac{\alpha_j + 1}{2} \right) \right] & \text{if } \Delta_j < -\left(\frac{\alpha_j + 1}{2} \right) \\ 0 & \text{if } \Delta_j \geq -\left(\frac{\alpha_j + 1}{2} \right) \end{cases} \quad (22)$$

Where α_j is the number of measurement scale levels of the j -th attribute ($\alpha_j > 3$) and $\xi(\omega_j)$ is a non-decreasing function of the relative importance of the j -th criterion. This function can be calculated by following formula:

$$\xi(\omega_j) = 0.2 \left(1 + \frac{\omega_j}{2} \right); \quad j = 1, 2, \dots, n \quad (23)$$

Step10 Calculate the **outranking degree** $\varphi(A_i, A_{i'})$ as follows:

$$\varphi(A_i, A_{i'}) = C(A_i, A_{i'}) \cdot \prod_{j=1}^n [1 - \bar{D}^3(A_i, A_{i'})]; \quad 0 \leq \varphi(A_i, A_{i'}) \leq 1 \quad (24)$$

Step11 Calculate the entering and leaving flows of each alternative:

$$\varphi^+(A_i) = \sum_{A_{i'} \in A} \varphi(A_i, A_{i'}); \quad i, i' \in \{1, 2, \dots, m\} \quad (25)$$

$$\varphi^-(A_i) = \sum_{A_{i'} \in A} \varphi(A_{i'}, A_i); \quad i, i' \in \{1, 2, \dots, m\} \quad (26)$$

Step12 Calculate the net flow of each alternative:

$$\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i) \quad (27)$$

Step13 Rank all alternatives according to the value of $\varphi(A_i)$.

4 The Entropy method

Definition 11 (He et al. 2016; Wang et al. 2015; Xu et al. 2021; Yang and Cui 2011; Zou et al. 2006). The entropy method is an objective weighting method, which determines the weight of the attribute according to the information provided by the observation value of each attribute. This article combines the weights obtained by objective calculations and the decision-making information obtained by subjective evaluation to give the ranking of some popular inbound tourism areas. According to m alternatives and n attributes, the DMs give the decision matrix, and the steps of the entropy method to determine the weight are as follows:

Step1 Calculate the proportion of the i-th alternative under the j-th attribute as follows:

$$q_{ij} = \frac{s'(LS(p)_{ij}^*)}{\sum_{i=1}^m s'(LS(p)_{ij}^*)} (i = 1, 2, \dots, m, j = 1, 2, \dots, n), \quad \sum_{i=1}^n q_{ij} = 1 \quad (28)$$

Step2 Calculate the entropy value of the j-th index by following formula:

$$e_j = -k \times \sum_{i=1}^n q_{ij} \ln(q_{ij}); \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (29)$$

Where $k > 0$, $\ln$ is the natural logarithm, and $e_j \geq 0$. Generally let $k = \frac{1}{\ln m}$, then $0 \leq e \leq 1$. When $q_{ij} = 0$, define $\lim_{q_{ij} \rightarrow 0} (q_{ij} \ln q_{ij}) = 0$.

Step3 Calculate the difference coefficient of the j-th index by following formula:

$$g_j = 1 - e_j; \quad j = 1, 2, \dots, n \quad (30)$$

Step4 Calculate the weights by following formula:

$$\omega_j = \frac{g_j}{\sum_{j=1}^n g_j}; \quad j = 1, 2, \dots, n \quad (31)$$

Step5 The final weight vector Ω is:

$$\Omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) \quad (32)$$

5 Case Study: Choice of tourist attractions

In this section, we will use the PL-MAUT-PAMSSEM II method proposed above to rank the alternatives, so as to

provide decision-makers with selection reference.

5.1 Case presentation

After a long period of continuous research on the influencing factors of regional inbound tourism development, a relatively complete attribute index system has been formed. This article selects economic level, openness, hotel supply, transportation facilities, and tourism resources as the attribute indicators for the evaluation of inbound tourism competitiveness (Wu et al. 2020). The number of inbound tourists is selected as a reference index to test the validity and accuracy of the competitiveness evaluation. Among them, the number of inbound tourists is measured by the number of inbound overnight tourists, the economic level is measured by GDP, and the degree of openness is measured by the volume of imports and exports. At the same time, the fluctuation of domestic price and consumption level needs to be eliminated. The direct economic effect of the exchange rate change of the RMB against the US dollar and the fluctuation of the domestic prices and consumption level need to eliminate the direct economic effect of the import and export volume, so as to ensure the annual volume comparability of the GDP and the import and export volume; hotel supply passed comprehensive measurement of one-star, two-star, three-star, four-star and five-star hotels (houses); transportation facilities are comprehensively measured by inland river network density, highway network density, railway network density, and air transportation employment density; Tourism resources are comprehensively measured through A-level scenic spots, historical and cultural cities, world natural heritage and world cultural heritage.

The data required for this article comes from the official websites of the National Bureau of Statistics of the People's Republic of China, the People's Bank of China, the Ministry of Culture and Tourism of the People's Republic of China, 2001-2019 "Inbound Tourist Sampling Survey Data", "China Tourism Statistical Yearbook", "Fujian Statistical Yearbook" "Xinjiang Statistical Yearbook" and other provincial-level regional statistical yearbooks. According to relevant statistics in 2007, 2011, 2015 and 2019(these four years of data have the same

importance), 10 DMs were invited to A_1 (Guangdong), A_2 (Yunnan), A_3 (Shaanxi), A_4 (Sichuan), A_5 (Inner Mongolia) five inbound tourist hotspots in C_1 (economic level), C_2 (openness), C_3 (hotel supply), C_4 (transportation facilities), C_5 (tourism resources) scores under five standards to obtain decision-making information. The weight vector of criteria is $\Omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$, which can be calculated by formula (35), where $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^5 \omega_j = 1$ and LTS

$L = \{l_{-3} = \text{terrible}, l_{-2} = \text{very bad}, l_{-1} = \text{bad}, l_0 = \text{medium}, l_1 = \text{good}, l_2 = \text{very good}, l_3 = \text{fantastic}\}$. In order to obtain the decision matrixes, we collected the scores of 10 DMs in 2007, 2011, 2015 and 2019 and then convert them into PLTS through linguistic scale. Then, we listed the final assessments of these tourist attractions in Table 1-4, and form the probabilistic linguistic decision matrixes $(PL_{ij}(p))_{5 \times 5}$.

5.2 The Decision Steps

Step 1 Split the matrix, take attribute C_1 as an example, we can get Table 5.

Step 2 Calculate the hesitant probabilistic fuzzy decision matrix by formula (9), take attribute C_1 as an example, we can get Table 6.

Step 3 Calculate the aggregated hesitant probabilistic fuzzy decision matrix by formula (8), and we can get Table 7.

Step 4 Calculate the probabilistic linguistic decision matrix by formula (9), we can get Table 8.

Step 5 Using formula (1), we can obtain the score as follows:

$$S = \begin{bmatrix} 2.896 & 3 & 3 & 2.565 & 2.983 \\ 2.832 & 2.903 & 3 & 3 & 2.765 \\ 3 & 2.486 & 2.318 & 2.900 & 2.707 \\ 3 & 2.779 & 2.792 & 2.516 & 2.863 \\ 1.020 & 0.542 & 2.336 & 2.395 & 2.224 \end{bmatrix}$$

Step 6 Readjust the score matrix as follows:

$$S' = \begin{bmatrix} 0.948 & 1 & 1 & 0.281 & 1 \\ 0.915 & 0.961 & 1 & 1 & 0.713 \\ 1 & 0.791 & 0 & 0.835 & 0.636 \\ 1 & 0.910 & 0.695 & 0.200 & 0.842 \\ 0 & 0 & 0.026 & 0 & 0 \end{bmatrix}$$

Step 7 Using formula (28)-(32), we can obtain the attribute weight as follows:

$$\omega = (0.142, 0.144, 0.304, 0.260, 0.150)^T$$

Step 8 Calculate the marginal utility score by formula (15), and we can get:

$$u_{ij} = \begin{bmatrix} 0.852 & 1.005 & 1.005 & 0.048 & 1.005 \\ 0.766 & 0.888 & 1.005 & 1.005 & 0.387 \\ 1.005 & 0.508 & 0 & 0.590 & 0.292 \\ 1.005 & 0.754 & 0.363 & 0.024 & 0.603 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 9 Calculate the concordance index $C(A_i, A_r)$ by formula (16)-(19), and Table 9 indicates the parameters q, p,

and v of each attribute, and we can get:

$$C(A_i, A_r) = \begin{bmatrix} / & 1 & 1 & 1 & 1 \\ 1 & / & 1 & 1 & 1 \\ 0.856 & 0.856 & / & 1 & 1 \\ 1 & 1 & 1 & / & 1 \\ 0.784 & 0.856 & 1 & 1 & / \end{bmatrix}$$

Step 10 Calculate the local discordance index $D(A_i, A_r)$ by formula (20)-(23), and we can get:

$$D(A_i, A_r) = \begin{bmatrix} / & 0 & 0 & 0 & 0 \\ 0 & / & 0 & 0 & 0 \\ 0 & 0 & / & 0 & 0 \\ 0 & 0 & 0 & / & 0 \\ 0 & 0 & 0 & 0 & / \end{bmatrix}$$

Step 11 Calculate the outranking degree $\varphi(A_i, A_r)$ by formula (24), and we can get:

$$\varphi(A_i, A_r) = \begin{bmatrix} / & 1 & 1 & 1 & 1 \\ 1 & / & 1 & 1 & 1 \\ 0.856 & 0.856 & / & 1 & 1 \\ 1 & 1 & 1 & / & 1 \\ 0.784 & 0.856 & 1 & 1 & / \end{bmatrix}$$

Step 12 Calculate the entering and leaving flows of each $A_i (i = 1, 2, 3, 4, 5)$ by formula (25)-(26), we can get:

$$\varphi^+(A_i) = (4, 4, 3.712, 4, 3.640)$$

$$\varphi^-(A_i) = (3.640, 3.712, 4, 4, 4)$$

Step 13 Calculate the net flows of each $A_i (i = 1, 2, 3, 4, 5)$ by formula (27), and we can get:

$$\varphi(A_i) = (0.360, 0.288, -0.288, 0, -0.360)$$

Step 14 Rank the alternatives.

According to the value of $\varphi(A_i)$, the ranking is $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$

Table 1 The probabilistic linguistic decision matrix for the year 2007.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{l ₂ (0.5), l ₃ (0.5)}	{l ₃ (1)}	{l ₂ (0.2), l ₃ (0.8)}	{l ₁ (0.2), l ₂ (0.8)}	{l ₂ (0.2), l ₃ (0.8)}
A ₂	{l ₂ (0.9), l ₃ (0.1)}	{l ₂ (0.8), l ₃ (0.2)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₃ (1)}	{l ₁ (0.5), l ₂ (0.5)}
A ₃	{l ₀ (0.2), l ₁ (0.8)}	{l ₁ (1)}	{l ₀ (0.2), l ₁ (0.8)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₂ (0.8), l ₃ (0.2)}
A ₄	{l ₃ (1)}	{l ₁ (0.7), l ₂ (0.3)}	{l ₁ (0.6), l ₂ (0.4)}	{l ₁ (0.7), l ₂ (0.3)}	{l ₁ (0.3), l ₂ (0.7)}
A ₅	{l ₂ (0.3), l ₁ (0.7)}	{l ₁ (0.2), l ₀ (0.8)}	{l ₂ (0.1), l ₀ (0.9)}	{l ₀ (0.9), l ₁ (0.1)}	{l ₁ (1)}

Table 2 The probabilistic linguistic decision matrix for the year 2011.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{l ₂ (0.8), l ₃ (0.2)}	{l ₃ (1)}	{l ₂ (0.1), l ₃ (0.9)}	{l ₁ (0.8), l ₂ (0.2)}	{l ₂ (0.1), l ₃ (0.9)}
A ₂	{l ₂ (0.5), l ₃ (0.5)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₂ (0.8), l ₃ (0.2)}	{l ₃ (1)}	{l ₂ (1)}
A ₃	{l ₁ (0.7), l ₂ (0.3)}	{l ₁ (0.6), l ₂ (0.4)}	{l ₀ (0.3), l ₁ (0.7)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₁ (0.5), l ₂ (0.5)}
A ₄	{l ₃ (1)}	{l ₂ (0.8), l ₃ (0.2)}	{l ₁ (0.5), l ₂ (0.5)}	{l ₀ (0.6), l ₂ (0.4)}	{l ₂ (0.8), l ₃ (0.2)}
A ₅	{l ₀ (0.5), l ₁ (0.5)}	{l ₁ (1)}	{l ₂ (0.5), l ₁ (0.5)}	{l ₁ (0.9), l ₀ (0.1)}	{l ₀ (0.3), l ₁ (0.7)}

Table 3 The probabilistic linguistic decision matrix for the year 2015.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{l ₂ (1)}	{l ₂ (0.3), l ₃ (0.7)}	{l ₂ (0.4), l ₃ (0.6)}	{l ₁ (0.6), l ₂ (0.4)}	{l ₂ (0.5), l ₃ (0.5)}
A ₂	{l ₁ (0.5), l ₂ (0.5)}	{l ₂ (0.9), l ₃ (0.1)}	{l ₃ (1)}	{l ₃ (1)}	{l ₁ (0.5), l ₂ (0.5)}
A ₃	{l ₃ (1)}	{l ₁ (0.8), l ₂ (0.2)}	{l ₁ (0.8), l ₂ (0.2)}	{l ₂ (0.4), l ₃ (0.6)}	{l ₁ (0.8), l ₂ (0.2)}
A ₄	{l ₂ (0.8), l ₃ (0.2)}	{l ₁ (0.4), l ₂ (0.6)}	{l ₂ (0.9), l ₂ (0.1)}	{l ₁ (0.2), l ₂ (0.8)}	{l ₂ (0.9), l ₃ (0.1)}
A ₅	{l ₁ (0.2), l ₀ (0.8)}	{l ₂ (1)}	{l ₃ (0.2), l ₂ (0.8)}	{l ₂ (0.1), l ₀ (0.9)}	{l ₀ (0.8), l ₁ (0.2)}

Table 4 The probabilistic linguistic decision matrix for the year 2019.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{l ₂ (0.5), l ₃ (0.5)}	{l ₃ (1)}	{l ₂ (0.2), l ₃ (0.8)}	{l ₁ (0.2), l ₂ (0.8)}	{l ₂ (0.2), l ₃ (0.8)}
A ₂	{l ₂ (0.9), l ₃ (0.1)}	{l ₂ (0.8), l ₃ (0.2)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₃ (1)}	{l ₁ (0.5), l ₂ (0.5)}
A ₃	{l ₀ (0.2), l ₁ (0.8)}	{l ₁ (1)}	{l ₀ (0.2), l ₁ (0.8)}	{l ₂ (0.5), l ₃ (0.5)}	{l ₂ (0.8), l ₃ (0.2)}
A ₄	{l ₃ (1)}	{l ₁ (0.7), l ₂ (0.3)}	{l ₁ (0.6), l ₂ (0.4)}	{l ₁ (0.7), l ₂ (0.3)}	{l ₁ (0.3), l ₂ (0.7)}

A ₅	{l ₂ (0.3), l ₁ (0.7)}	{l ₁ (0.2), l ₀ (0.8)}	{l ₂ (0.1), l ₀ (0.9)}	{l ₀ (0.9), l ₁ (0.1)}	{l ₁ (1)}
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Table 5 The probabilistic linguistic decision matrix after being split for C₁ (Li et al. 2020).

	A ₁	A ₂	A ₃	A ₄	A ₅
2007	{l ₁ (0.3), l ₂ (0.2), l ₂ (0.3), l ₂ (0.2)}	{l ₁ (0.5), l ₁ (0.3), l ₂ (0.1), l ₂ (0.1)}	{l ₀ (0.2), l ₀ (0.3), l ₁ (0.2), l ₁ (0.3)}	{l ₂ (0.2), l ₃ (0.6), l ₃ (0.2)}	{l ₁ (0.2), l ₁ (0.1), l ₁ (0.1), l ₀ (0.1), l ₀ (0.5)}
2011	{l ₂ (0.3), l ₂ (0.2), l ₂ (0.3), l ₃ (0.2)}	{l ₂ (0.5), l ₃ (0.3), l ₃ (0.1), l ₃ (0.1)}	{l ₁ (0.2), l ₁ (0.3), l ₁ (0.2), l ₂ (0.3)}	{l ₃ (0.2), l ₃ (0.6), l ₃ (0.2)}	{l ₀ (0.2), l ₀ (0.1), l ₀ (0.1), l ₀ (0.1), l ₁ (0.5)}
2015	{l ₂ (0.3), l ₂ (0.2), l ₂ (0.3), l ₂ (0.2)}	{l ₁ (0.5), l ₂ (0.3), l ₂ (0.1), l ₂ (0.1)}	{l ₃ (0.2), l ₃ (0.3), l ₃ (0.2), l ₃ (0.3)}	{l ₂ (0.2), l ₂ (0.6), l ₃ (0.2)}	{l ₁ (0.2), l ₀ (0.1), l ₀ (0.1), l ₀ (0.1), l ₀ (0.5)}
2019	{l ₂ (0.3), l ₂ (0.2), l ₃ (0.3), l ₃ (0.2)}	{l ₂ (0.5), l ₂ (0.3), l ₂ (0.1), l ₃ (0.1)}	{l ₀ (0.2), l ₁ (0.3), l ₁ (0.2), l ₁ (0.3)}	{l ₃ (0.2), l ₃ (0.6), l ₃ (0.2)}	{l ₂ (0.2), l ₂ (0.1), l ₁ (0.1), l ₁ (0.1), l ₁ (0.5)}

Table 6 The hesitant probabilistic fuzzy decision matrix calculated by formula (8) for C₁.

	A ₁	A ₂	A ₃	A ₄	A ₅
2007	{2/3(0.3), 2/3(0.2), 5/6(0.3), 5/6(0.2)}	{2/3(0.5), 2/3(0.3), 5/6(0.1), 5/6(0.1)}	{1/2(0.2), 1/2(0.3), 2/3(0.2), 2/3(0.3)}	{5/6(0.2), 1(0.6), 1(0.2)}	{1/3(0.2), 1/3(0.1), 1/3(0.1), 1/2(0.1), 1/2(0.5)}
2011	{5/6(0.3), 5/6(0.2), 5/6(0.3), 1(0.2)}	{5/6(0.5), 1(0.3), 1(0.1), 1(0.1)}	{2/3(0.2), 2/3(0.3), 2/3(0.2), 5/6(0.3)}	{1(0.2), 1(0.6), 1(0.2)}	{1/2(0.2), 1/2(0.1), 1/2(0.1), 1/2(0.1), 1/2(0.5)}
2015	{5/6(0.3), 5/6(0.2), 5/6(0.3), 5/6(0.2)}	{2/3(0.5), 5/6(0.3), 5/6(0.1), 5/6(0.1)}	{1(0.2), 1(0.3), 1(0.2), 1(0.3)}	{5/6(0.2), 5/6(0.6), 1(0.2)}	{1/3(0.2), 1/2(0.1), 1/2(0.1), 1/2(0.1), 1/2(0.5)}
2019	{5/6(0.3), 5/6(0.2), 1(0.3), 1(0.2)}	{5/6(0.5), 5/6(0.3), 5/6(0.1), 1(0.1)}	{1/2(0.2), 2/3(0.3), 2/3(0.2), 2/3(0.3)}	{1(0.2), 1(0.6), 1(0.2)}	{1/6(0.2), 1/6(0.1), 1/3(0.1), 1/3(0.1), 1/3(0.5)}

Table 7 The hesitant probabilistic fuzzy decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{0.961(0.3), 0.972(0.2), 1(0.5)}	{1(1)}	{1(1)}	{0.889(0.2), 0.921(0.4), 0.944(0.2), 0.961(0.2)}	{0.972(0.1), 1(0.9)}
A ₂	{0.944(0.5), 1(0.5)}	{0.961(0.2), 0.972(0.3), 1(0.5)}	{1(1)}	{1(1)}	{0.944(0.5), 0.972(0.4), 1(0.1)}
A ₃	{1(1)}	{0.889(0.6), 0.944(0.2), 0.961(0.2)}	{0.833(0.2), 0.864(0.1), 0.889(0.5), 0.944(0.2)}	{0.944(0.2), 0.972(0.2), 1(0.6)}	{0.921(0.4), 0.944(0.1), 0.961(0.3), 1(0.2)}
A ₄	{1(1)}	{0.944(0.4), 0.961(0.3), 0.972(0.1), 1(0.2)}	{0.944(0.5), 0.961(0.1), 0.972(0.1), 1(0.3)}	{0.864(0.2), 0.904(0.4), 0.944(0.1), 0.961(0.1), 0.972(0.2)}	{0.961(0.3), 0.972(0.4), 1(0.3)}
A ₅	{0.570(0.2), 0.627(0.1), 0.667(0.1), 0.711(0.6)}	{0.444(0.2), 0.627(0.8)}	{0.741(0.1), 0.876(0.1), 0.897(0.3), 0.917(0.5)}	{0.519(0.1), 0.938(0.8), 0.969(0.1)}	{0.833(0.3), 0.864(0.5), 0.944(0.2)}

Table 8 The probabilistic linguistic decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	{l _{2.766} (0.3), l _{2.832} (0.2), l ₃ (0.5)}	{l ₃ (1)}	{l ₃ (1)}	{l _{2.334} (0.2), l _{2.526} (0.4), l _{2.664} (0.2), l _{2.766} (0.2)}	{l _{2.832} (0.1), l ₃ (0.9)}
A ₂	{l _{2.664} (0.5), l ₃ (0.5)}	{l _{2.766} (0.2), l _{2.832} (0.3), l ₃ (0.5)}	{l ₃ (1)}	{l ₃ (1)}	{l _{2.664} (0.5), l _{2.832} (0.4), l ₃ (0.1)}
A ₃	{l ₃ (1)}	{l _{2.334} (0.6), l _{2.664} (0.2), l _{2.766} (0.2), l _{2.832} (0.2)}	{l _{1.998} (0.2), l _{2.184} (0.1), l _{2.334} (0.2), l _{2.526} (0.2)}	{l _{2.664} (0.2), l _{2.832} (0.2), l _{2.998} (0.2), l _{3.184} (0.1)}	{l _{2.526} (0.4), l _{2.664} (0.1), l _{2.766} (0.2), l _{2.832} (0.2)}

		$l_{2.766}(0.2)\}$	$l_{2.334}(0.5), l_{2.664}(0.2)\}$	$l_3(0.6)\}$	$l_{2.766}(0.3), l_3(0.2)\}$
A ₄	$\{l_3(1)\}$	$\{l_{2.664}(0.4), l_{2.766}(0.3),$	$\{l_{2.664}(0.5), l_{2.766}(0.1),$	$\{l_{2.184}(0.2), l_{2.424}(0.4),$	$\{l_{2.766}(0.3), l_{2.832}(0.4),$
		$l_{2.832}(0.1), l_3(0.2)\}$	$l_{2.832}(0.1), l_3(0.3)\}$	$l_{2.664}(0.1), l_{2.766}(0.1), l_{2.832}(0.2)\}$	$l_3(0.3)\}$
A ₅	$\{l_{0.42}(0.2), l_{0.762}(0.1),$	$\{l_{-0.336}(0.2), l_{0.762}(0.8)\}$	$\{l_{1.446}(0.1), l_{2.256}(0.1),$	$\{l_{0.114}(0.1), l_{2.628}(0.8),$	$\{l_{1.998}(0.3), l_{2.184}(0.5),$
	$l_{1.002}(0.1), l_{1.226}(0.6)\}$		$l_{2.382}(0.3), l_{2.502}(0.5)\}$	$l_{2.814}(0.1)\}$	$l_{2.664}(0.2)\}$

Table 9 Parameters values.

Parameter	C ₁	C ₂	C ₃	C ₄	C ₅
q	1.1	1	0.9	1.2	0.9
p	1.2	1.5	1.1	1.4	1.3
v	1.3	2	1.3	1.6	1.5

Table 10 Experimental results of the four methods

Methods	Ranking
PL-MAUT-PAMSSEM II	$A_1 > A_2 > A_4 > A_3 > A_5$
MAUT	$A_2 > A_1 > A_4 > A_3 > A_5$
PAMSSEM II	$A_4 > A_5 > A_1 > A_2 > A_3$
PROMETHEE II (Brans et al. 1986; Figueira et al. 2005)	$A_1 > A_3 = A_4 > A_2 > A_5$
TOPSIS (Zhao et al. 2016)	$A_2 > A_1 > A_4 > A_3 > A_5$
ARAS (Zhu et al. 2012)	$A_2 > A_1 > A_4 > A_3 > A_5$

The same parameters values and weight are applied to the MAUT, PAMSSEM II, PROMETHEE, TOPSIS, and ARAS methods to get the different ranking. We show all the ranking results in Table 10. It can be seen from the table that when different decision-making methods are applied to MAGDM problems, they do not always get the same ranking results. Judging from the actual development of Sichuan, Chongqing, Guizhou, Yunnan, and Tibet in recent years, when all evaluation indicators are objective, the decision-making results given by the PL-MAUT-PAMSSEM II method are more realistic. Therefore, the comparison of various methods can more highlight the objectivity and rationality of the PL-MAUT-PAMSSEM II method proposed in this paper in solving practical problems.

6 Conclusions

PLTS is composed of several possible linguistic terms and their relative distribution, and it is an important means of expressing DM's wishes or evaluation in a linguistic environment. Due to its complex structure, PLTS is suitable

for expressing qualitative evaluation information in the actual decision-making process, but its application in the calculation is limited. Therefore, we combine PLTS, HPFS, and scoring functions. In this article, the PL-MAUT-PAMSSEM II method is based on different criteria to compare the marginal utility scores of the alternatives in pairs, combining MAUT and PAMSSEM II and applying them to the selection of inbound tourist attractions. It is the advantage of the MAUT method to express the preference of the DMs by introducing a utility function. The utility function is composed of various attributes, which can evaluate the total utility of alternatives. For each attribute, the DMs will give an evaluation value. After the evaluation value is standardized to the interval $[0,1]$, the score value calculated by the marginal utility function is called the marginal utility score. After the calculation of the marginal utility score, we added the PAMSSEM II method as the second stage of data processing. The purpose of this method is to build a priority relationship and use the marginal utility score to establish a local preference model. We calculate the consistency index and the inconsistency index by comparing the alternatives in pairs, and then perform a weighted summary on them, and obtain the final ranking of the scheme by calculating the net dominant flow. The PAMSSEM method has significant advantages in dealing with mixed data (cardinal and ordinal evaluation values) and missing data (irrelevant evaluation values or evaluation values that are related but difficult to obtain). In addition, this method can not only distinguish between different attributes Thresholds (preference threshold and irrelevant threshold) represent the degree of preference of the DM, and the rejection threshold can also be used to represent the compensation limit that the DM is willing to accept. Therefore, the results calculated by the combined application of MAUT and PAMSSEM are more convincing.

We will broaden the application areas of the MAUT-PAMSSEM II method, such as applying the method to project renovation selection, contractor selection, investment evaluation, and risk evaluation in the future. At the same time, we will further study other special cases of MAUT-PAMSSEM II in a probabilistic language environment. Of course, we will combine MAUT-PAMSSEM II with some information aggregation operators and

apply them to other fuzzy environments, such as dual hesitation fuzzy sets (Farhadinia and Xu 2016; Meng et al. 2015), language interval hesitation fuzzy sets (Lin et al. 2021), and weighted ordered hesitation fuzzy set (Zavadskas et al. 2010).

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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