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Analysis of uniform film thickness and stationary points for the thin film flow of Carreau fluid model in the presence of surface tension gradient

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Abstract

This article addresses the analysis of the uniform film thickness and stationary points for the Carreau thin fluid film flow. The flow of fluid on a vertically upward moving cylinder takes place in the presence of a surface tension gradient. The resulting non-linear and inhomogeneous ordinary differential equation is solved for the series form solution using Adomian decomposition methods (ADM). Stokes number S_t , inverse capillary number C , Weissenberg number We and fluid behavior index n emerged as flow control parameters. The analysis showed that the positions of stationary points transferred towards the surface of the cylinder by the increase of S_t and C while towards the fluid-air interface by the increase of n . We delineated vice versa effects on positions of stationary points for the shear thickening fluid film and shear thinning fluid film. The width of uniform film thickness reduces by an increment in the S_t and C whereas it increases by an increment in the n . The width of shear thickening uniform film thickness increases whilst shear thinning uniform film thickness decreases as the We increases. A comparison between the linearly viscous fluid and Carreau fluid is also made.

Keywords. Carreau fluid model; Uniform film thickness; Stationary points; Thin film flow; Surface tension gradient.

1 Introduction

The thin film flow is the flow of fluid in the form of a fine layer. Thin film contains the region of fluid bounded by the solid body and fluid-air interface. Microchip construction, tertiary oil reformation,

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coatings on manufactured materials, condensate motion, paint films levelling, soldering, lubricating, mold filling, film cooling and boiling heat transfer are examples of such flows. Due to above mentioned real-world applications in physical sciences study of thin film flow have received considerable consideration [1–8].

Over the years, the investigation of the lifting and drainage of thin fluid film on a vertically upward moving cylinder has attracted the intentions of various scientists and researchers [9–13]. The frequent structure of this type of flow is such that when a vertical solid cylinder is raised from a container filled with a fluid with variable/constant velocity, it takes up a fluid film, however, the gravity and surface tension gradient try to drain it down. Because of the no-slip condition the fluid nearby the cylinder adheres and moves with the velocity of the cylinder. However, with the increment in horizontal distance from the cylinder, the impact of the fluid lifting reduces and there be a point where the lift velocity balance the drainage velocity. That certain point is called the “stationary point” and at this point, there is no velocity of a fluid. Due to the action of gravity and surface tension gradient at this point, drainage of the fluid film starts. The component of velocity normal to the cylinder is much lesser than the component of velocity, which occurs along with the vertically upwards moving cylinder. Consequently, flow is considered to be one-dimensional.

Siddiqui et al. [14–16] contributed to the literature by giving the idea of uniform film thickness and stationary points in the thin film flow phenomenon. Siddiqui et al. [14] considered the problem of lifting of the Sisko fluid film on a vertically upward moving belt with a free surface and solved the resulting non-linear ordinary differential equation using Adomian decomposition method ADM. Specified the conditions of uniform film thickness and stationary points. Observed the impacts of the emerging parameters on the positions of Sisko stationary points and width of the uniform Sisko film thickness. Siddiqui et al. [15] modeled the problem of horizontal Sisko thin fluid film flow induced by the motion of the plate and surface tension gradient. Solved the resulting non-linear ordinary differential equation using (ADM). Depicted the impacts of emerging parameters on the positions of stationary points within the domain. Later they [16] extended the work of Siddiqui et al. [14] for the PTT fluid model. Solved the resulting linear ordinary differential equations for the exact solutions. Inferred the impacts of emerging parameters on the positions of PTT stationary points and width of uniform PTT fluid film thickness. Also gave a comparison of the uniform film thickness and stationary points among the Maxwell, LPTT and EPTT fluid models. Ali [17] and Wakeel [18] extended the work of Siddiqui et al. [14–16] by using Jeffrey fluid model respectively for the upwards moving belt and cylinder. They solved the resulting linear ordinary differential equations to obtain the exact solutions. Discussed the impacts of emerging parameters on the positions of Jeffrey fluid stationary points within the domain as well as the width of uniform Jeffrey fluid film thickness.

In practical life, several substances express the mechanical properties of elasticity and viscosity called non-Newtonian fluids. Some examples of such fluids are molten polymers, salt solutions, custard, honey, toothpaste, starch suspension, corn starch, paint and blood. Shear rate is not linearly proportional to the corresponding shear stress. The non-Newtonian fluids flow has gained significant consideration due to its various applications in industry, technology and engineering. It is a notable reality that the flow features of such fluids are distinct from the linearly viscous fluid. It is not possible to describes completely the characteristics of non-Newtonian fluids by a single model [19]. The aforementioned fluids comprise shear-thickening, shear-thinning, viscoelasticity, viscoplasticity, etc., that are frequently examined by the power law fluid model. A few of these flows were not absolutely

depicted by this fluid model. A modern rheologist Pierre J. Carreau [20] proposed a fluid model which simultaneously possesses the shear thinning, shear thickening and elastic behaviours in view of this situation [21,22]. Pantokratoras [23] using the controlling parameter n to explain the Carreau fluid model. Carreau fluid exhibits shear thinning behavior for all $n \in (0, 1)$ and shear thickening behavior for all $n \in (0, 2)$. As a limiting case the linearly viscous fluid model can be recovered for $n = 1$.

Over the years, numerous techniques are being used by the engineers and scientists to obtain the series form solutions of nonlinear ordinary and partial differential equations. Adomian decomposition method (ADM) has been used to approximate the solutions in many branches of sciences and engineering [24–29]. In this paper, we will use ADM [30–32] to get the series form solution of resulting non-linear ordinary differential equation. In 1983 George Adomian was the first who developed this method to achieve the accurate solution of the nonlinear problematic systems of equations in space and time, which arise in the modeling of boundary value problems. It is an approximate and non-numerical technique that produces series form solutions. It does not need linearization, perturbation, any small parameter, or other limitations and hypotheses.

This article aims to theoretically provide the analysis of uniform film thickness and stationary points. We consider the Carreau thin fluid film flow on a vertically upward moving cylinder in the presence of a surface tension gradient. Series form solution of the resulting non-linear ordinary differential equation will be obtained using ADM. We will derive the series form expressions for the flow variables: velocity, volume flow rate, average film velocity, shear stress and surface tension. Conditions for the uniform film thickness and positions of stationary points will be attained and discussed. Quantitative effects of the flow control parameters on some of these flow variables will be analyzed. A comparison between the linearly viscous fluid and Carreau fluid will also be provided.

2 Basic Equations

The basic governing equations for the flow under consideration are stated as

$$\text{div} \mathbf{V} = 0, \quad (1)$$

$$\rho \frac{D\mathbf{V}}{Dt} = \rho \mathbf{f} - \nabla p + \nabla \cdot \mathbf{S}, \quad (2)$$

where we represent velocity vector by the $\mathbf{V}$, fluid constant density by the ρ , pressure by the p , body force per unit mass by the $\mathbf{f}$, extra stress tensor by the $\mathbf{S}$ and the material time derivative by the $\frac{D}{Dt}$. We now introduce the incompressible Carreau fluid model in the form of extra stress tensor:

$$\mathbf{S} = \mu \left[1 + \lambda_1^2 \left(\frac{\text{tr} \mathbf{A}_1^2}{2} \right) \right]^{\frac{n-1}{2}} \mathbf{A}_1, \quad (3)$$

where λ_1 is the relaxation time, μ is the dynamic viscosity, n is the fluid behaviour index and $\mathbf{A}_1$ is the first Rivlin-Ericksen tensor:

$$\mathbf{A}_1 = \text{grad} \mathbf{V} + (\text{grad} \mathbf{V})^T, \quad (4)$$

in which T is the transpose and

$$\text{grad}\mathbf{V} = \begin{bmatrix} \frac{\partial u}{\partial r} & \frac{1}{r} \frac{\partial u}{\partial \theta} - \frac{v}{r} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial r} & \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial r} & \frac{1}{r} \frac{\partial w}{\partial \theta} & \frac{\partial w}{\partial z} \end{bmatrix}. \quad (5)$$

Remark

- If we set $n = 1$ in Eq. (3), then we recovered the extra stress tensor for the linearly viscous fluid.
- When $n \in (0, 1)$, then Carreau fluid exhibits shear thinning behaviour and when $n \in (0, 2)$, then Carreau fluid exhibits shear thickening behaviour.

3 Problem formulation

We consider that an infinitely long cylinder of radius R moves vertically upward through a container filled with incompressible Carreau fluid. The cylinder moves upward with constant velocity U and takes up a small quantity of fluid in the form of the thin film. At fluid-air interface some non-isothermal gas exerts a surface tension gradient $\sigma' = \frac{d\sigma}{dz}$. Surface tension $\sigma(z)$ is increasing in the downward direction. Gravity and surface tension gradient oppose the lifting of fluid along the cylinder. Down the cylinder under the action of gravity and surface tension gradient the fluid film tends to drain. We choose a cylindrical coordinate system in which z -axis is taken along the cylinder, r -axis normal to the cylinder and origin at the mid-plane at $-r \leq R \leq r$ as shown in the schematic diagram (see Fig. 1).

Assume that the flow is steady, laminar and uniform, fluid film thickness h is uniform, pressure is atmospheric and flow takes place in the z -direction only. Velocity vector that describe, such type of flow is of the form:

$$\mathbf{V} = [0, 0, w(r)], \quad (6)$$

and also we have

$$\mathbf{S} = \mathbf{S}(r). \quad (7)$$

When we utilized the velocity vector (6) into equation of continuity (1) then it is identically satisfied. Eq. (3)-(5) upon using velocity vector (6) and Eq. (7) yield

$$\mathbf{S} = \mu \left[1 + \lambda_1^2 \left(\frac{dw}{dr} \right)^2 \right]^{\frac{n-1}{2}} \begin{bmatrix} 0 & 0 & \frac{dw}{dr} \\ 0 & 0 & 0 \\ \frac{dw}{dr} & 0 & 0 \end{bmatrix},$$

which, follows with the following non zero shear stress component:

$$S_{rz} = S_{zr} = \mu \left[1 + \lambda_1^2 \left(\frac{dw}{dr} \right)^2 \right]^{\frac{n-1}{2}} \frac{dw}{dr}. \quad (8)$$

Figure 1: The schematic diagram of the lifting and drainage of Carreau fluid film.

The equation of motion (2) upon using velocity vector (6), Eq. (7) and assumptions taken into consideration, will lead to

$$\frac{d(rS_{zr})}{dr} = \rho gr, \quad (9)$$

subject to boundary conditions

$$S_{zr}(R+h) = -\sigma'. \quad (\text{surface tension gradient condition}), \quad (10)$$

$$w(R) = U. \quad (\text{no slip condition}). \quad (11)$$

In order to nondimensionlize the Eqs. (7)-(11) we define following characteristic scales in terms of geometric and fluid parameters:

Characteristic length scale = R .

Characteristic velocity scale = U .

Characteristic shear stress scale = $\frac{\mu U}{R}$.

Characteristic surface tension scale = μU .

In view of afore-mentioned characteristic scales following dimensionless parameters are defined:

$$r^* = \frac{r}{R}, \quad w^* = \frac{w}{U}, \quad S_{zr}^* = \frac{S_{zr}}{\frac{\mu U}{R}}, \quad h^* = \frac{h}{R}, \quad \sigma^* = \frac{\sigma}{\mu U}. \quad (12)$$

Utilizing dimensionless parameters (12) into Eqs.(7)-(11) respectively, we have after dropping ‘*’:

$$S_{zr} = \left[1 + We^2 \left(\frac{dw}{dr} \right)^2 \right]^{\frac{n-1}{2}} \frac{dw}{dr}, \quad (13)$$

$$\frac{d}{dr}(rS_{zr}) = rS_t, \quad (14)$$

$$S_{zr}(\Lambda) = -C \quad (\text{surface tension gradient condition}), \quad (15)$$

$$w(1) = 1 \quad (\text{no slip condition}). \quad (16)$$

Using binomial expansion in Eq. (13), we arrive at

$$S_{zr} = \left[1 + \frac{We^2(n-1)}{2} \left(\frac{dw}{dr} \right)^2 + O(We^2)^2 \right] \frac{dw}{dr}. \quad (17)$$

where $S_t = \frac{\rho g R^2}{\mu U}$ is the Stokes number, $C = \frac{R\sigma'}{\mu U}$ is inverse capillary number and we let $\Lambda = 1 + h$. With respect to 'r' integration of Eq. (14) and subjection on the surface tension gradient boundary condition (15) yields

$$S_{zr} = \frac{S_t}{2} \left[r - \frac{\Lambda^2}{r} \right] - \frac{C\Lambda}{r}. \quad (18)$$

Now, invoking Eq. (18) into Eq. (13), we obtain

$$\frac{dw}{dr} + \frac{We^2(n-1)}{2} \left(\frac{dw}{dr} \right)^3 = \frac{S_t}{2} \left[r - \frac{\Lambda^2}{r} \right] - \frac{C\Lambda}{r}. \quad (19)$$

Eq. (19) is an inhomogeneous first order non-linear ordinary differential equation. The exact solution of this equation seems to be impossible, therefore, in the next section we will obtain the series form solution of Eq. (19) subject to the boundary condition (16) by using ADM.

4 Solution of the Problem

We re-write Eq. (19) in operator form as suggested by Adomian to solve it subject to the boundary condition (16) [30–32]:

$$L_r(w) = \frac{S_t}{2} \left[r - \frac{\Lambda^2}{r} \right] - \frac{C\Lambda}{r} - \frac{We^2(n-1)}{2} N(w), \quad (20)$$

where $L_r = \frac{d}{dr}$ is a linear one fold operator and $N(w) = \left(\frac{dw}{dr} \right)^n$ is a non-linear term. Since linear one fold operator L_r is invertible then its inverse operator is defined as

$$L_r^{-1}(*) = \int (*) dr.$$

Operating L_r^{-1} on both sides of Eq. (20), we have

$$w(r) = \frac{S_t}{2} \left[\frac{r^2}{2} - \Lambda^2 \ln r \right] - C\Lambda \ln r + C_1 - \frac{We^2(n-1)}{2} L_r^{-1} N(w), \quad (21)$$

where C_1 is the constant of integration.

The unknown $w(r)$ can be decomposed by the decomposition series given by

$$w(r) = \sum_{m=0}^{\infty} w_m(r), \quad (22)$$

and the non-linear term $N(w)$ can be expanded by an infinite series of Adomian polynomials A_m :

$$N(w) = \sum_{m=0}^{\infty} A_m, \quad (23)$$

where the components $w_m(r)$, $m \geq 0$ and A_m can easily be computed. Substituting Eq. (22) and Eq. (23) into Eq. (21), we have

$$\begin{aligned} w_0(r) + \sum_{m=1}^{\infty} w_m &= \frac{S_t}{2} \left[\frac{r^2}{2} - \Lambda^2 \ln r \right] - C\Lambda \ln r + C_1 \\ &\quad - \frac{We^2(n-1)}{2} L_r^{-1} \sum_{m=0}^{\infty} A_m, \end{aligned} \quad (24)$$

which follows with the following recursive relation

$$w_0(r) = \frac{S_t}{2} \left[\frac{r^2}{2} - \Lambda^2 \ln r \right] - C\Lambda \ln r + C_1, \quad (25)$$

$$w_{m+1}(r) = -\frac{We^2(n-1)}{2} L_r^{-1} A_m, \quad \text{for } m \geq 0. \quad (26)$$

Using decomposition series (22) into boundary condition (16), we get

$$w_0(1) = 1, \quad (27)$$

$$w_1(1) = 0, \quad (28)$$

$$w_2(1) = 0, \quad (29)$$

$$w_3(1) = 0. \quad (30)$$

Invoking boundary condition (27) into Eq. (25), we have

$$w_0(r) = 1 - \frac{S_t}{4} [(1 - r^2) + 2\Lambda^2 \ln r] - C\Lambda \ln r. \quad (31)$$

Utilizing Eq. (22) into Eq. (23), then in turn after making use of the binomial expansion, we acquire up to second order the following Adomian polynomials A_m :

$$A_0 = (w'_0(r))^n, \quad (32)$$

$$A_1 = n (w'_0(r))^{n-1} w'_1(r), \quad (33)$$

$$A_2 = n (w'_0(r))^{n-1} w'_2(r) + \frac{n^2 - n}{2} (w'_0(r))^{n-2} (w'_1(r))^2, \quad (34)$$

where we represent the derivative of w with respect to ' r ' by prime ' $'$ ' over w . Making use of Adomian polynomials (32)-(34) into the recursive relation (26) and then in turn utilizing the concern boundary

conditions (28)-(30), gives the following components of $w(r)$:

$$w_1(r) = We^2 \left[\frac{K_1}{4} (1 - r^4) + \zeta_1 (1 - r^2) + \zeta_2 \ln r + \zeta_3 \left(1 - \frac{1}{r^2} \right) \right], \quad (35)$$

$$w_2(r) = \frac{We^4}{12} \left[2K_5(1 - r^6) - \zeta_4(1 - r^4) + \zeta_5(1 - r^2) + \zeta_6 \ln r - \zeta_7 \left(1 - \frac{1}{r^2} \right) - \zeta_8 \left(1 - \frac{1}{r^4} \right) \right], \quad (36)$$

$$w_3(r) = We^6 \left[\frac{K_{11}}{2} (1 - r^8) + \zeta_9(1 - r^6) + \zeta_{10}(1 - r^4) + \zeta_{11}(1 - r^2) + \zeta_{12} \ln r + \zeta_{13} \left(1 - \frac{1}{r^2} \right) + \zeta_{14} \left(1 - \frac{1}{r^4} \right) + \zeta_{15} \left(1 - \frac{1}{r^6} \right) \right], \quad (37)$$

where

$$\begin{aligned} \zeta_1 &= -\frac{3K_1\Lambda^2}{2} - \frac{K_2\Lambda}{2}, \zeta_2 = -3K_1\Lambda^4 - 2K_2\Lambda^3 - K_3\Lambda^2, \\ \zeta_3 &= \frac{k_1\Lambda^6}{2} + \frac{k_2\Lambda^5}{2} + \frac{k_3\Lambda^4}{2} + K_4\Lambda^3, \zeta_4 = 15K_5\Lambda^2 + 5K_6\Lambda, \\ \zeta_5 &= 60K_5\Lambda^4 + 40K_6\Lambda^3 + 20K_7\Lambda^2, \\ \zeta_6 &= 120K_5\Lambda^6 + 120K_6\Lambda^6 + 120K_7\Lambda^4 + 40K_6\Lambda^3, \\ \zeta_7 &= 30K_5\Lambda^8 + 40K_6\Lambda^7 + 60K_7\Lambda^6 + 40K_8\Lambda^5 + 10K_9\Lambda^4, \\ \zeta_8 &= 3K_5\Lambda^{10} + 5K_6\Lambda^9 + 10K_7\Lambda^8 + 10K_8\Lambda^7 + 15K_9\Lambda^6 + K_{10}\Lambda^5, \\ \zeta_9 &= -\frac{14}{3}K_{11}\Lambda^2 - \frac{14}{3}K_{12}\Lambda, \zeta_{10} = 21K_{11}\Lambda^4 + 42K_{12}\Lambda^3 + 7K_{13}\Lambda^2, \\ \zeta_{11} &= -70K_{11}\Lambda^6 - 210K_{12}\Lambda^5 - 70K_{13}\Lambda^4, \\ \zeta_{12} &= -140K_{11}\Lambda^8 - 560K_{12}\Lambda^7 - 280K_{13}\Lambda^6 - \frac{560}{3}K_{14}\Lambda^5 - \frac{140}{3}K_{15}\Lambda^4, \\ \zeta_{13} &= 42K_{11}\Lambda^{10} + 210K_{12}\Lambda^9 + 140K_{13}\Lambda^8 + 140K_{14}\Lambda^7 + 70K_{15}\Lambda^6 + 14K_{16}\Lambda^5, \\ \zeta_{14} &= -7K_{11}\Lambda^{12} - 42K_{12}\Lambda^{11} - 35K_{13}\Lambda^{10} - \frac{140}{3}K_{14}\Lambda^9 - 35K_{15}\Lambda^8 \\ &\quad - 14K_{16}\Lambda^7 - 7K_{17}\Lambda^6, \\ \zeta_{15} &= \frac{2}{3}K_{11}\Lambda^{14} + \frac{14}{3}K_{12}\Lambda^{13} + \frac{14}{3}K_{13}\Lambda^{12} + \frac{70}{9}K_{14}\Lambda^{11} + \frac{70}{9}K_{15}\Lambda^{10} \\ &\quad + \frac{14}{3}K_{16}\Lambda^9 + \frac{14}{3}K_{17}\Lambda^8 + K_{18}\Lambda^7, \end{aligned}$$

in which

$$\begin{aligned}
K_1 &= \frac{(n-1)S_t^3}{16}, K_2 = \frac{3(n-1)S_t^2C}{8}, K_3 = \frac{3(n-1)S_tC^2}{4}, K_4 = \frac{(n-1)C^3}{4}, \\
K_5 &= \frac{3(n-1)^2S_t^5}{128}, K_6 = \frac{9(n-1)^2S_t^4C}{64}, K_7 = \frac{9(n-1)^2S_t^3C^2}{32}, \\
K_8 &= \frac{9(n-1)^2S_t^2C^3}{16}, K_9 = \frac{9(n-1)^2S_tC^4}{8}, K_{10} = \frac{3(n-1)^2C^5}{16}, \\
K_{11} &= \frac{3(n-1)^3S_t^7}{1024}, K_{12} = \frac{3(n-1)^3S_t^6C}{512}, K_{13} = \frac{9(n-1)^3S_t^5C^2}{256}, \\
K_{14} &= \frac{9(n-1)^3S_t^4C^3}{128}, K_{15} = \frac{9(n-1)^3S_t^3C^4}{64}, K_{16} = \frac{9(n-1)^3S_t^2C^5}{32}, \\
K_{17} &= \frac{3(n-1)^3S_tC^6}{16}, K_{18} = \frac{(n-1)^3C^7}{4}.
\end{aligned}$$

Inserting the components $w_0(r)$, $w_1(r)$, $w_2(r)$ and $w_3(r)$ from Eq. (31) and Eqs. (35)-(37) into the decomposition series (22), we get the velocity of the thin film flow of Carreau fluid on a vertically upwards moving cylinder in the presence of surface tension gradient up to third order:

$$\begin{aligned}
w(r) &= 1 - \frac{S_t}{4} [(1-r^2) + 2\Lambda^2 \ln r] - C\Lambda \ln r + We^2 \left[\frac{K_1}{4} \right. \\
&\quad \times (1-r^4) + \zeta_1 (1-r^2) + \zeta_2 \ln r + \zeta_3 \left(1 - \frac{1}{r^2} \right) \left. \right] + \frac{We^4}{12} \\
&\quad \times [2K_5(1-r^6) - \zeta_4(1-r^4) + \zeta_5(1-r^2) + \zeta_6 \ln r - \zeta_7 \\
&\quad \times \left(1 - \frac{1}{r^2} \right) - \zeta_8 \left(1 - \frac{1}{r^4} \right) \left. \right] + We^6 \left[\frac{K_{11}}{2}(1-r^8) + \zeta_9 \right. \\
&\quad \times (1-r^6) + \zeta_{10}(1-r^4) + \zeta_{11}(1-r^2) + \zeta_{12} \ln r \\
&\quad \left. + \zeta_{13} \left(1 - \frac{1}{r^2} \right) + \zeta_{14} \left(1 - \frac{1}{r^4} \right) + \zeta_{15} \left(1 - \frac{1}{r^6} \right) \right]. \tag{38}
\end{aligned}$$

Lifting of fluid film is a continuous process of lifting along the cylinder and drainage near the fluid-air interface. Lifting is due to the upward motion of the cylinder. The drainage is due to the gravity and surface tension gradient. The lifting effect is prominent closer to the cylinder, while the drainage effect is prominent closer to the fluid-air interface. Thus, we decompose the velocity (38) into two

segments:

$$w_L(r) = 1,$$

and

$$\begin{aligned} w_D(r) = & -\frac{S_t}{4} [(1 - r^2) + 2\Lambda^2 \ln r] - C\Lambda \ln r + We^2 \left[\frac{K_1}{4} (1 - r^4) \right. \\ & + \zeta_1 (1 - r^2) + \zeta_2 \ln r + \zeta_3 \left(1 - \frac{1}{r^2} \right) \left. \right] + \frac{We^4}{12} [2K_5(1 - r^6) \\ & - \zeta_4(1 - r^4) + \zeta_5(1 - r^2) + \zeta_6 \ln r - \zeta_7 \left(1 - \frac{1}{r^2} \right) - \zeta_8 \left(1 - \frac{1}{r^4} \right) \left. \right] \\ & + We^6 \left[\frac{K_{11}}{2} (1 - r^8) + \zeta_9(1 - r^6) + \zeta_{10}(1 - r^4) + \zeta_{11}(1 - r^2) + \zeta_{12} \ln r \right. \\ & + \zeta_{13} \left(1 - \frac{1}{r^2} \right) + \zeta_{14} \left(1 - \frac{1}{r^4} \right) + \zeta_{15} \left(1 - \frac{1}{r^6} \right) \left. \right], \end{aligned}$$

where we represent the lift velocity by the $w_L(r)$ and drainage velocity by the $w_D(r)$. During the process of lifting, there must exist a point r_0 within the domain, where the fluid film lifting balances the fluid film drainage. Mathematically, we have

$$\begin{aligned} w_L(r_0) = & -w_D(r_0), \\ 1 = & \frac{S_t}{4} [(1 - r_0^2) + 2\Lambda^2 \ln r_0] + C\Lambda \ln r_0 - We^2 \left[\frac{K_1}{4} (1 - r_0^4) \right. \\ & + \zeta_1 (1 - r_0^2) + \zeta_2 \ln r_0 + \zeta_3 \left(1 - \frac{1}{r_0^2} \right) \left. \right] - \frac{We^4}{12} [2K_5(1 - r_0^6) \\ & - \zeta_4(1 - r_0^4) + \zeta_5(1 - r_0^2) + \zeta_6 \ln r_0 - \zeta_7 \left(1 - \frac{1}{r_0^2} \right) - \zeta_8 \left(1 - \frac{1}{r_0^4} \right) \left. \right] \\ & - We^6 \left[\frac{K_{11}}{2} (1 - r_0^8) + \zeta_9(1 - r_0^6) + \zeta_{10}(1 - r_0^4) + \zeta_{11}(1 - r_0^2) + \zeta_{12} \ln r_0 \right. \\ & + \zeta_{13} \left(1 - \frac{1}{r_0^2} \right) + \zeta_{14} \left(1 - \frac{1}{r_0^4} \right) + \zeta_{15} \left(1 - \frac{1}{r_0^6} \right) \left. \right]. \end{aligned} \quad (39)$$

Inserting different values of h , S_t , C , We and n into Eq. (39) then solving the resulting equation for r_0 one can acquire the positions of Carreau fluid stationary points r_0 within the domain:

$$Dom(w) = \{r | r \in \Re \wedge 1 \leq r \leq \Lambda\}.$$

Putting $We = 0$ in Eq. (39), we obtain the positions of stationary points for the linearly viscous fluid film:

$$1 - \frac{S_t}{4} [(1 - r_0^2) + 2\Lambda^2 \ln r_0] - C\Lambda \ln r_0 = 0. \quad (40)$$

Invoking different values of h , S_t and C into Eq. (40) we get the positions of linearly viscous fluid. The dimensionless volume flow rate Q is given by

$$Q = 2\pi \int_1^\Lambda rw(r)dr. \quad (41)$$

Substituting Eq. (38) into Eq. (41), we have

$$\begin{aligned} Q = & \pi [\Lambda^2 - 1] - \frac{S_t\pi}{2}N_1 - \frac{C\pi}{2}N_2 + 2We^2\pi \left[\frac{K_1}{4}N_3 + \zeta_1N_4 \right. \\ & + \zeta_2N_5 + \zeta_3N_6] + \frac{We^4\pi}{6} [2K_5N_7 - \zeta_4N_3 + \zeta_5N_4 + \zeta_6N_5 \\ & - \zeta_7N_6 - \zeta_8N_8] + 2We^6\pi \left[\frac{K_{11}N_9}{2} + \zeta_9N_7 + \zeta_{10}N_3 + \zeta_{11}N_4 \right. \\ & \left. + \zeta_{12}N_5 + \zeta_{13}N_6 + \zeta_{14}N_8 + \zeta_{15}N_{10} \right], \end{aligned} \quad (42)$$

where

$$\begin{aligned} N_1 = & -\frac{1}{4} + \Lambda^2 + \Lambda^4 \left\{ \ln \Lambda - \frac{3}{4} \right\}, N_2 = \Lambda + \Lambda^3 \{2 \ln \Lambda - 1\}, \\ N_3 = & \frac{\Lambda^2}{2} - \frac{\Lambda^6}{6} - \frac{1}{3}, N_4 = \frac{\Lambda^2}{2} - \frac{\Lambda^4}{4} - \frac{1}{4}, \\ N_5 = & \frac{\Lambda^2 \ln \Lambda}{2} - \frac{\Lambda^2}{4} + \frac{1}{4}, N_6 = \frac{\Lambda^2}{2} - \frac{1}{2} - \ln \Lambda, \\ N_7 = & \frac{\Lambda^2}{2} - \frac{\Lambda^8}{8} - \frac{3}{8}, N_8 = \frac{\Lambda^2}{2} + \frac{1}{2\Lambda^2} - 1, \\ N_9 = & \frac{\Lambda^2}{2} - \frac{\Lambda^{10}}{10} - \frac{2}{5}, N_{10} = \frac{\Lambda^2}{2} + \frac{1}{4\Lambda^4} - \frac{3}{4}. \end{aligned}$$

Uniform fluid film thickness ' h ' can be computed by utilizing the “zero net flow condition”. According to this condition, as much quantity of the fluid film is being lifted by the upward motion of the cylinder, as much is being drained under the action of gravity and surface tension gradient. Mathematically, we have

$$Q = 0. \quad (43)$$

Making use of Eq. (42) into Eq. (43), we get

$$\begin{aligned} & \pi [\Lambda^2 - 1] - \frac{S_t\pi}{2}N_1 - \frac{C\pi}{2}N_2 + 2We^2\pi \left[\frac{K_1}{4}N_3 + \zeta_1N_4 \right. \\ & + \zeta_2N_5 + \zeta_3N_6] + \frac{We^4\pi}{6} [2K_5N_7 - \zeta_4N_3 + \zeta_5N_4 + \zeta_6N_5 \\ & - \zeta_7N_6 - \zeta_8N_8] + 2We^6\pi \left[\frac{K_{11}N_9}{2} + \zeta_9N_7 + \zeta_{10}N_3 + \zeta_{11}N_4 \right. \\ & \left. + \zeta_{12}N_5 + \zeta_{13}N_6 + \zeta_{14}N_8 + \zeta_{15}N_{10} \right] = 0. \end{aligned} \quad (44)$$

The average film velocity $\bar{V}$ is given by

$$\bar{V} = \frac{Q}{\pi(\Lambda^2 - 1)}, \quad (45)$$

which with the help of Eq. (42) becomes.

$$\begin{aligned} \bar{V} = & 1 - \frac{S_t N_1}{2(\Lambda^2 - 1)} - \frac{C N_2}{2(\Lambda^2 - 1)} + \frac{2W e^2}{\Lambda^2 - 1} \left[\frac{K_1}{4} N_3 + \zeta_1 N_4 \right. \\ & + \zeta_2 N_5 + \zeta_3 N_6 \left. \right] + \frac{W e^4}{6(\Lambda^2 - 1)} [2K_5 N_7 - \zeta_4 N_3 + \zeta_5 N_4 + \zeta_6 N_5 \\ & - \zeta_7 N_6 - \zeta_8 N_8] + \frac{2W e^6}{\Lambda^2 - 1} \left[\frac{K_{11} N_9}{2} + \zeta_9 N_7 + \zeta_{10} N_3 + \zeta_{11} N_4 \right. \\ & \left. + \zeta_{12} N_5 + \zeta_{13} N_6 + \zeta_{14} N_8 + \zeta_{15} N_{10} \right]. \end{aligned} \quad (46)$$

Using Eq. (38) into Eq. (13) we get the shear stress which the upward moving cylinder exert on the Carreau fluid film:

$$\begin{aligned} S_{zr} = & \frac{S_t}{2} \left(r - \frac{\Lambda^2}{r} \right) - \frac{C\Lambda}{r} + W e^2 \left[-K_1 r^3 - 2r\zeta_1 + \frac{\zeta_2}{r} + \frac{2\zeta_3}{r^3} \right] + \frac{W e^4}{12} \\ & \times \left[-12K_5 r^5 + 4r^3\zeta_4 - 2r\zeta_5 + \frac{\zeta_6}{r} - \frac{2\zeta_7}{r^3} - \frac{4\zeta_8}{r^5} \right] + W e^6 N_{12} [-4K_{11} r^7 \\ & - 6\zeta_9 r^5 - 4\zeta_{10} r^3 - 2\zeta_{11} r + \frac{\zeta_{12}}{r} + \frac{2\zeta_{13}}{r^3} + \frac{4\zeta_{14}}{r^5} + \frac{6\zeta_{15}}{r^7}] \\ & + \frac{W e^2(n-1)}{2} \left[\frac{S_t}{2} \left(r - \frac{\Lambda^2}{r} \right) - \frac{C\Lambda}{r} + W e^2 \left(-K_1 r^3 - 2r\zeta_1 + \frac{\zeta_2}{r} + \frac{2\zeta_3}{r^3} \right) \right. \\ & + \frac{W e^4}{12} \left(-12K_5 r^5 + 4r^3\zeta_4 - 2r\zeta_5 + \frac{\zeta_6}{r} - \frac{2\zeta_7}{r^3} - \frac{4\zeta_8}{r^5} \right) + W e^6 N_{12} (-4K_{11} r^7 \\ & \left. - 6\zeta_9 r^5 - 4\zeta_{10} r^3 - 2\zeta_{11} r + \frac{\zeta_{12}}{r} + \frac{2\zeta_{13}}{r^3} + \frac{4\zeta_{14}}{r^5} + \frac{6\zeta_{15}}{r^7} \right) \left. \right]^3. \end{aligned} \quad (47)$$

Shear stress at the Carreau fluid-air interface at $r=\Lambda$ is calculated as follows:

$$\begin{aligned} S_{zr}(\Lambda) = & -C + \frac{W e^2(n-1)C^3}{2} - \frac{3W e^4(n-1)^2 C^5}{4} + \frac{3W e^6(n-1)^3 C^7}{2} \\ & + \frac{W e^2(n-1)}{2} \left[-C + \frac{W e^2(n-1)C^3}{2} - \frac{3W e^4(n-1)^2 C^5}{4} \right. \\ & \left. + \frac{3W e^6(n-1)^3 C^7}{2} \right]. \end{aligned} \quad (48)$$

From dimensional surface tension gradient boundary condition (10), we have

$$\frac{d\sigma}{dz} = -S_{zr}(R + h), \quad (49)$$

subject to initial condition

$$\sigma(R) = -\sigma_0, \quad (50)$$

where σ_0 is the reference surface tension. Utilizing dimensionless parameters (12) into Eq. (49) and initial condition (50), we get

$$\frac{d\sigma}{dz} = -S_{zr}(\Lambda), \quad (51)$$

$$\sigma(0) = -1, \quad (52)$$

where $\sigma_0 = \mu U$.

Now, integrating Eq. (51) with respect to ' z ' and then in turn upon using initial condition (52) and Eq. (48) we finally arrive at

$$\begin{aligned} \sigma(z) = -1 + & \left[C - \frac{We^2(n-1)C^3}{2} + \frac{3We^4(n-1)^2C^5}{4} - \frac{3We^6(n-1)^3C^7}{2} \right. \\ & + \frac{We^2(n-1)}{2} \left(C - \frac{We^2(n-1)C^3}{2} + \frac{3We^4(n-1)^2C^5}{4} \right. \\ & \left. \left. - \frac{3We^6(n-1)^3C^7}{2} \right) \right] z, \end{aligned} \quad (53)$$

which represents the surface tension of the Carreau fluid film.

Remark:- When we set $We = 0$ in Eqs. (38), (42), (47) and (53), then respectively we get the velocity, volume flow rate, shear stress and surface tension of the linearly viscous fluid.

5 Results and Discussion

This section accords with the estimation of the effects of emerging flow control parameters: Stokes number S_t , inverse capillary number C , Weissenberg number We and fluid behaviour index n on the velocity and surface tension of the Carreau fluid film through graphs (see Figs. 1-9). Computer codes are developed in MATHEMATICA for the numerical evaluations of the analytical results obtained in the previous section. To compute the uniform Carreau fluid film thickness and positions of Carreau stationary points we performed numerical computation in MATHEMATICA. Numeric data are tabulated in Tables 1-10 to observe the effects of flow control parameters on width of the uniform Carreau fluid film thickness and positions of the Carreau stationary points.

5.1 Velocity Profile

In Figs. 2-6 velocity versus normal distance plots are displayed. It is noted from these figures that the lifting velocity of the Carreau fluid film diminishes with an increment in the normal distance r . The velocity diminishes to zero at stationary points. Drainage of Carreau fluid film initiates after this point, which increases with an increase in the normal distance r . Fig. 2 and Fig. 3 display the impacts of Stokes number S_t and inverse capillary number C on the velocity of the Carreau fluid film. In these figures, we observed that as the S_t and C are increased then the lifting velocity decreases while drainage velocity increases. Since $S_t = \frac{\rho g R^2}{\mu U}$ then increment in the S_t means increment in the

g , with the increment in g the lifting of Carreau fluid film decreases whilst drainage of Carreau fluid film increases. Since $C = \frac{R\sigma'}{\mu U}$, then increment in C means increment in the surface tension gradient. One can say that with the increment in the surface tension gradient the lifting of Carreau fluid film decreases whereas drainage of Carreau fluid film increases. From these figures, it is also noted that the effect of S_t and C on shear thinning fluid film is more prominent than the shear thickening fluid film. Fig. 4(a) elucidates the effect of Weissenberg number We on the velocity of the Carreau shear thinning fluid film. In this figure, we observed that as We is increased then the lifting velocity decreases while its drainage velocity increases. On the other hand in Fig. 4(b) Carreau shear thickening fluid behaves oppositely, i.e, the lifting velocity increases while the drainage decreases by the increase of We . Since $We = \frac{\lambda_1 U}{R}$ (the ratio of relaxation time to the characteristics time), then increase in We means increase in relaxation time. It means relaxation time has opposite effects on shear thinning fluid film and shear thickening fluid film. Fig. 5 displays the effects of fluid behaviour index n on the velocity of Carreau fluid film. In this figure, we observed that when n increases then the lifting velocity increases whilst drainage velocity decreases.

We display a comparison between the linearly viscous fluid and Carreau fluid's shear thinning fluid and shear thickening characteristics in Fig. 6 for velocity. This figure reveals that shear thinning fluid film drain down with greater velocity while shear thickening fluid film lift upwards with greater

Figure 2: Effects of S_t on velocity of the Carreau (a) shear thinning fluid film with $n = 0.5$ and (b) shear thickening fluid film with $n = 1.5$ when $We = 0.10$, $C = 1.90$, $h = 0.6$ are fixed.

Figure 3: Effects of C on velocity of the Carreau (a) shear thinning fluid film with $n = 0.5$ and (b) shear thickening fluid film with $n = 1.5$ when $We = 0.10$, $S_t = 2.4$, $h = 0.6$ are fixed.

Figure 4: Effects of We on velocity of the Carreau (a) shear thinning fluid film with $n = 0.5$ and (b) shear thickening fluid film with $n = 1.5$ when $C = 1.90$, $S_t = 2.4$, $h = 0.6$ are fixed.

Figure 5: Effects of n on velocity of the Carreau fluid film when $C = 1.90$, $S_t = 2.4$, and $h = 0.6$ are fixed.

Figure 6: Comparison of velocity between the linearly viscous fluid ($We = 0$) and Carreau fluid's shear thinning ($n = 0.5$) and shear thickening ($n = 1.5$) characteristics when $C = 1.90$, $S_t = 2.4$, and $h = 0.6$ are fixed.

5.2 Surface Tension

Fig. 7 shows the effects of Weissenberg number We on surface tension. In this figure, we noted a reduction in the surface tension by the increase of We . Fig. 8 displays the effects of inverse capillary number C on the surface tension. It is inferred from this figure that the surface tension increases by the increment of C . Fig. 9 is plotted to depict the effects of fluid behaviour index n on surface tension. It is depicted from this figure that the effect of n on Carreau shear thinning fluid film surface tension and shear thickening fluid film surface tension is vice versa. When n increases then surface tension of the shear thinning fluid film increases while decreases in case of shear thickening fluid film. Fig. 10 is plotted to have the comparison between the linearly viscous fluid and Carreau fluid's shear thinning and thickening characteristics for surface tension. From this graph, it is noted that the

linearly viscous fluid film has more surface tension as compared to the shear thinning fluid film and shear thickening fluid film. It means at fluid-air interface non-isothermal gas exerts a greater amount of surface tension on the linearly viscous fluid film than the Carreau fluid film.

Figure 7: Effects of We on surface tension of the Carreau (a) shear thinning fluid film with $n = 0.65$ and (b) shear thickening fluid film with $n = 1.45$ when $C = 1.90$ is fixed.

Figure 8: Effects of C on surface tension of the Carreau (a) shear thinning fluid film with $n = 0.65$ and (b) shear thickening fluid film with $n = 1.45$ when $We = 0.45$ is fixed.

Figure 9: Effects of n on surface tension of the Carreau fluid when $C = 1.90$ and $We = 0.45$, are fixed.

Figure 10: Comparison of surface tension between the linearly viscous fluid ($We = 0.0$) and Carreau fluid's shear thinning ($n = 0.65$) and shear thickening ($n = 1.45$) characteristics when $C = 1.90$ is fixed.

5.3 Uniform Film Thickness

In Tables 1-5, we tabulated numeric data to seek the effects of Stokes number S_t , inverse capillary number C , Weissenberg number We and fluid behaviour index n on uniform Carreau fluid film thickness. From Table 1 and Table 2, we inferred that the width of uniform Carreau fluid film thickness decreases by the increases of S_t and C . From Table 3, it is seen that the width of uniform Carreau fluid film thickness reduces when We increases in shear thinning fluid film case but show opposite behaviour in case of shear thickening fluid film. When Carreau fluid exhibits shear thickening behaviour then width of uniform film thickness increases by the increase of We . From these tables (see Tables 1-3) it is also elucidated that the width of uniform Carreau fluid film thickness in case of shear thickening fluid is greater than the width of shear thinning fluid film. The width of uniform Carreau fluid film thickness increases by the increment of n as is noted from Table 4.

Table 5 shows a comparison between the width of the uniform film thickness of the linearly viscous fluid and Carreau fluid's shear thinning and shear thickening characteristics. This comparison unveils that the width of shear thickening fluid film is greater than the width of linearly viscous fluid film and shear thinning fluid film.

Table 1: Variations of Stokes number S_t on uniform Carreau fluid film thickness when $C=1.90$ and $We=0.10$.

2ptheight	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
S_t	$h(S_t)$	$h(S_t)$
2.1	0.526809	0.584118
2.4	0.507063	0.580095
2.7	0.4868	0.578366
3.0	0.465857	0.578446

Table 2: Variations of inverse capillary number C on uniform Carreau fluid film thickness when $We=0.10$ and $S_t=2.4$.

C	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
	$h(C)$	$h(C)$
1.80	0.518423	0.594274
1.90	0.507063	0.580095
2.00	0.496146	0.566758
2.10	0.485648	0.554213

Table 3: Variations of Weissenberg number We on uniform Carreau fluid film thickness when $C=1.90$ and $S_t=2.4$.

We	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
	$h(We)$	$h(We)$
0.04	0.530659	0.541429
0.06	0.52457	0.549011
0.08	0.516628	0.561101
0.10	0.507063	0.580095

Table 4: Variations of n on uniform Carreau fluid film thickness when $We=0.10$, $S_t=2.4$ and $C=1.90$.

n	$h(n)$
0.1	0.486528
0.5	0.507063
1.1	0.542886
1.5	0.580095

Table 5: Comparison between the uniform film thickness of linearly viscous fluid and Carreau fluid's shear thinning and shear thickening characteristics when $S_t=2.4$ and $We=0.10$.

C	linearly viscous fluid film	Shear thinning fluid film	Shear thickening fluid film
	$We = 0$	$n = 0.5, We = 0.10$	$n = 1.5, We = 0.10$
	$h_0(C)$	$h_0(C)$	$h_0(C)$
1.80	0.549402	0.518423	0.594274
1.90	0.535862	0.507063	0.580095
2.00	0.52291	0.496146	0.566758
2.10	0.510513	0.485648	0.554213

5.4 Stationary Points

Tables 6-9 are tabulated to notice the variations of Stokes number S_t , inverse capillary number C , Weissenberg number We and fluid behaviour index n on the positions of the stationary points of Carreau fluid film. From Table 6 and Table 7 it is noted that as the S_t and C are increased then stationary points shift their positions towards the surface of the cylinder. Table 8 displays the variation of We on stationary points. In this case, we examined that as the We increases

then stationary points shifts their positions towards the surface of the cylinder when Carreau fluid exhibits shear thinning behaviour. On the other hand, the positions of stationary points shifts towards the interface when Carreau fluid exhibits shear thickening behaviour. Comparison between the stationary points of Carreau shear thinning fluid film and shear thickening fluid film can also be observed through Tables 6-8. This comparison reveals that shear thinning fluid stationary points lie more closely to the surface of the cylinder. Table 9 presents the variation of n on stationary points. From this table, we delineate that stationary points shift their positions towards the interface as the value of n increases.

Table 10 shows the comparison between the stationary points of the linearly viscous fluid and Carreau fluid's shear thinning and shear thickening characteristics. This comparison reveals that stationary points of shear thinning fluid film lie more closely to the surface of the cylinder than that of the linearly viscous fluid film and shear thickening fluid film. The order of the location of stationary points is

shear thinning fluid film < linearly viscous fluid film < shear thickening fluid film.

Table 6: Variations of Stokes number S_t on stationary points of the Carreau fluid film when $C=1.90$, $We=0.10$, and $h=0.6$.

2ptheight	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
S_t	$r_0(S_t)$	$r_0(S_t)$
2.1	1.2418	1.26659
2.4	1.22799	1.25408
2.7	1.21528	1.24268
3.0	1.20354	1.23228

Table 7: Variations of C on stationary points of the Carreau fluid film when $We=0.10$, $S_t=2.4$, and $h=0.6$.

2ptheight	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
C	$r_0(C)$	$r_0(C)$
1.80	1.23909	1.26442
1.90	1.22799	1.25408
2.00	1.21778	1.24467
2.10	1.20835	1.23605

Table 8: Variations of We on stationary points of the Carreau fluid film when $C=1.90$, $S_t=2.4$, and $h=0.6$.

2ptheight	Shear thinning fluid film	Shear thickening fluid film
	$n = 0.5$	$n = 1.5$
We	$r_0(We)$	$r_0(We)$
0.05	1.23898	1.2454
0.10	1.22799	1.25408
0.15	1.20515	1.26787
0.20	1.16399	1.29303

Table 9: Variations of n on stationary points of the Carreau fluid film when $We=0.10$, $S_t=2.4$, $C=1.90$ and $h=0.6$.

2ptheightn	$r_0(n)$
0.1	1.21406
0.5	1.22799
1.1	1.24479
1.5	1.25408

Table 10: Comparison between the stationary points of linearly viscous fluid and Carreau fluid's shear thinning and shear thickening characteristics when $S_t=2.4$ and $We=0.10$.

2ptheight	Linearly viscous fluid film	Shear thinning fluid film	Shear thickening fluid film
	$We = 0$	$n = 0.5, We = 0.10$	$n = 1.5, We = 0.10$
C	$r_0(C)$	$r_0(C)$	$r_0(C)$
1.80	1.25287	1.23909	1.26442
1.90	1.24227	1.22799	1.25408
2.00	1.23258	1.21778	1.24467
2.10	1.22369	1.20835	1.23605

6 Concluding Remarks

In this paper analysis of uniform film thickness and stationary points for the thin film flow of the Carreau fluid model on a vertically upward moving cylinder in the presence of surface tension gradient is presented. Modeling of the problem gave a non-linear ordinary differential equation. Analytical series form solution has been obtained by using ADM. From the discussion of graphs and tables we summarize the conclusion of this work as follows:

- On the upward moving cylinder, the lift velocity of the Carreau fluid film diminishes to zero at the stationary point. The drainage of the Carreau fluid film initiates from this point, which increases with increasing C and S_t . Carreau film lift velocity in shear thinning fluid film case decreases while in case of shear thickening fluid film increases as the We increases. On the other hand, C , S_t and We on Carreau fluid film drainage velocity has a vice versa effects. With the increment in n lift velocity increases whereas drainage velocity decreases.

We and n have opposite effects on the surface tension of the shear thinning fluid film and shear thickening fluid film. Surface tension increases with an increase in the C .

- The width of the uniform fluid film thickness decreases by the increase of S_t and C . It reduces in shear thinning fluid film case whereas increases in shear thickening fluid film case as the values of We are increased. An increase in width with the increase in n is noted.
- Stationary points shift their positions towards the surface of the cylinder with an increase in the S_t and C . When the values of We are increased, then shear thinning fluid stationary points shift their positions towards the surface of the cylinder while shear thickening fluid stationary points towards the interface. Stationary points shift their positions towards the interface by the increment of n .
- Comparison between the linearly viscous fluid and Carreau fluid is also provided.

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