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Abdellatif Selmi (✉ selmi_fr2016@yahoo.com)

Prince Sattam bin Abdulaziz University

Research

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Vibration of FG shell rested on Winkler foundation

Abdellatif Selmi^{1,2,*}

¹Prince Sattam bin Abdulaziz University, College of Engineering, Civil Engineering
Department, Alkharj 11942, Saudi Arabia

²University of Tunis El Manar, National Engineering School of Tunis, Civil Engineering
Laboratory, Tunis, Tunisia

*Corresponding author:

Prince Sattam bin Abdulaziz University, College of Engineering, Civil Engineering
Department, Alkharj 11642, Saudi Arabia

Email: Selmi_fr2016@yahoo.com

Abstract

Love's first approximation theory is employed with the combination of Winkler term for the vibration of functionally graded cylindrical shell. MATLAB software is utilized for the vibration of functionally graded cylindrical shell with elastic foundation of Winkler and the results are verified with the open literature. For isotropic materials, the physical properties are same everywhere where the laminated and functionally graded materials, they vary from point to point. Here the shell material has been taken as functionally graded material. The influence of the elastic foundation, wave number, length- and height-to-radius ratios is investigated with different boundary conditions. The frequencies of length-to-radius and height-to-radius ratio are counter part of each other. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down for the variations of wave number. It is found that due to inducting the elastic foundation of Winkler, the frequencies increases.

Keywords: FGM, Stainless steel, Winkler foundation, natural frequency.

1. Introduction

The shell material is organized by various techniques and their applications are seen in dynamical elements such as plates, beams and shells. Moreover, these materials are also observed in space crafts, nuclear reactors and missiles technology etc. During the recent years, study of cylindrical shell with elastic foundations has gained the attention of researchers doing work on their vibration characteristics. Advanced composite materials keep extreme particular stiffness, strength and are resistant to corrosion. The elastic foundation equation is applied to influence on the shell vibrations. Study of vibration characteristics of cylindrical shells is a widely area of research in applied mathematics and theoretical mechanics. Analytical investigation of vibrations of these shell are performed to estimate the

probable dynamical response. Variations in the shell physical parameters are induced to enhance their strength and stability. Vibration of shell problems occur in industrial engineering fields. Their vibration analysis predicts to approximate their experimental results. More the shell material sustains a load due to physical situations, more the shell is stable. Any predicted fatigue due to burden of vibrations is evaded by estimating their dynamical aspects. Addition of more physical parameters may give rise more instability in a system of a submerged cylindrical shell (CSs). More than one type of materials is used to structure the functionally graded (FG) materials and their physical properties vary from one surface to the other surface. In these surfaces, one has highly heat resistance property while other may preserve great dynamical perseverance and differs mechanically and physically in regular manner from one surface to other surface, making them of dual physical appearance. All these materials have changeable outer and inner sides and their physical properties greatly differ from each other [1].

Loy and Lam (1997) investigated shell vibrations with ring supports that restricted the motion of cylindrical shells in the transverse direction [2]. This influence was induced by the polynomial functions. Chung et al. (1981) investigated the vibrations of CSs and presented an analysis of experimental and analytical investigation [3]. Jiang and Olson (1994) recommended the characteristics of analysis of stiffened shell using finite element method to diminish large computational efforts which are required in the conventional finite element analysis [4].

A large use of shell structures in practical applications makes their theoretical analysis an important field of structural dynamics. Since a shell problem is a physical one, so their vibrational behaviors are distorted by variations of physical and material parameters. To elude any complications which may risk a physical system their analytical investigation was done. Pankaj et al. (2019) studied the functionally graded material using sigmoid law distribution

under hygrothermal effect [5]. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Ergin and Temarel (2002) did a vibration study of cylindrical shells [6]. The shells lied in a horizontal direction and contained fluid and submerged in it. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods [7]. The shells were strengthened with longitudinal stiffeners. Wang et al. (1997) scrutinized the vibrations of ring-stiffened CSs using Ritz polynomial functions [8]. Materials of both shells and rings were of isotropic nature. These shells were stiffened with isotropic rings having three types of locations on the shell outer surface. To increase the stiffness of CSs was stabilized by ring-stiffeners. Isotropic materials are the constituents of these rings. Ansari et al. (2015) performed nonlocal model for the frequencies of multi-walled carbon nanotubes with small effects subject to various boundary conditions (BCs) using Rayleigh-Ritz technique [9]. The governing equation was formulated based on Flügge's and nonlocal shell theory. Some new resonant frequencies were identified with the association of vibrational modes and circumferential modes into shell model. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order [10]. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. FG material parameters were changed step by step. Shah et al. (2009) and Sofiyev and Avcar (2010) studied stability of CSs based on Rayleigh - Ritz and Galerkin technique using elastic foundations [11, 12]. The structures of cylindrical shell are tackled under the exponential law and axial load. Rouhi et al. (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory [13]. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for

nonlocal parameter. Naeem et al. (2013) conducted the vibrational behavior of submerged FG-CSs. The problem of submerged cylindrical shells were frequently met where fluid envelopes a structure [14]. The present problem consists of a CSs submerged in a fluid and surrounded by ring supports. Benguidiab et al. (2014) explored the features of zigzag double-walled CNT [15]. A comprehensive research presented by Salvatore Brischetto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model [16]. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Recently some researcher used different methods for nonlinear modeling [17-21] and for other structures [22-31].

This current paper describes the vibration characteristics of FG-CSs with Winkler foundation using Love's first approximation theory. The frequency behavior is investigated versus circumferential wave number, length-to-radius and height-to-radius ratios. Moreover, frequency pattern is found for the various values of Winkler foundation. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down for wave number

2. Theoretical formulation

The shell is assumed to have length L , thickness h and the radius R for cylindrical shell with its coordinate system (x, θ, z) as shown in Fig. 1. The x, θ co-ordinate are assumed to be along longitudinal and circumferential direction, respectively and z - co-ordinates are taken in its radial directions. The shell dynamical expressions based on Love's first approximation theory are defined as:

$$A_{11} \frac{\partial^2 u}{\partial x^2} + \frac{A_{66}}{R^2} \frac{\partial^2 u}{\partial \theta^2} + \left(\frac{A_{12} + A_{66}}{R} + \frac{B_{11} + 2B_{66}}{R^2} \right) \frac{\partial^2 v}{\partial x \partial \theta} + \frac{A_{12}}{R} \frac{\partial w}{\partial x} - B_{11} \frac{\partial^3 w}{\partial x^3} - \left(\frac{B_{12} + 2B_{66}}{R^2} \right) \frac{\partial^3 w}{\partial x \partial \theta^2} = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{11}+2B_{66}}{R^2}\right)\frac{\partial^2 u}{\partial x \partial \theta}+\left(\frac{A_{22}}{R^2}+\frac{2B_{66}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^2 v}{\partial \theta^2}+\left(A_{66}+\frac{4B_{66}}{R^2}+\frac{4D_{66}}{R^2}\right)\frac{\partial^2 v}{\partial x^2}+\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)\frac{\partial w}{\partial \theta}-\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^3 w}{\partial \theta^3}-\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)\frac{\partial^3 w}{\partial x^2 \partial \theta}=\rho_t \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$\begin{aligned} & \frac{A_{12}}{R} \frac{\partial u}{\partial x}-B_{11} \frac{\partial^3 u}{\partial x^3}-\left(\frac{B_{11}+2B_{66}}{R^2}\right) \frac{\partial^3 u}{\partial x \partial \theta^2}+\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right) \frac{\partial v}{\partial \theta}-\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right) \frac{\partial^3 v}{\partial \theta^3} \\ & -\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}}{R^2}+\frac{4D_{66}}{R^2}\right) \frac{\partial^3 v}{\partial x^2 \partial \theta}+\frac{A_{22}}{R^2} w-\frac{B_{12}}{R} \frac{\partial^2 w}{\partial x^2} \\ & -\frac{B_{22}}{R^3} \frac{\partial^2 w}{\partial \theta^2}+\left(\frac{D_{22}}{R^4}\right) \frac{\partial^4 w}{\partial \theta^4}+D_{11} \frac{\partial^4 w}{\partial x^4}+\left(\frac{2D_{12}}{R^2}+\frac{4D_{66}}{R^2}\right) \frac{\partial^4 w}{\partial x^2 \partial \theta^2}=\rho_t \frac{\partial^2 w}{\partial t^2}+K w \end{aligned} \quad (3)$$

where K , denotes the ‘Winkler’ foundation modulus .

Fig.1. Geometry of CS

Here, ρ_t denotes density and expressed as

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz \quad (4)$$

Where A_{ij} , B_{ij} and D_{ij} ($i, j=1, 2, 6$) states the membrane, coupling and flexural stiffness, respectively are described as:

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij} dz \quad (5)$$

$$B_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij} z dz \quad (6)$$

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij} z^2 dz \quad (7)$$

On mixing two or more than two materials like ceramic and metal, functionally graded materials are obtained. This type of material are working in high-temperature dependence material goods. So the Young's modulus E_{fgm} , Poisson ratio ν_{fgm} and mass density ρ_{fgm} are defined as:

$$E_{fgm} = (E_1 - E_2) \left(\frac{z}{h} + 0.5 \right)^N + E_2 \quad (8)$$

$$\nu_{fgm} = (\nu_1 - \nu_2) \left(\frac{z}{h} + 0.5 \right)^N + \nu_2 \quad (9)$$

$$\rho_{fgm} = (\rho_1 - \rho_2) \left(\frac{z}{h} + 0.5 \right)^N + \rho_2 \quad (10)$$

Q_{ij} is reduced stiffness for isotropic materials with conjunction of E and ν are written as:

$$Q_{22} = \frac{E}{1 - \nu^2} \quad (11)$$

$$Q_{12} = \frac{\nu E}{1 - \nu^2} \quad (12)$$

$$Q_{66} = \frac{E}{2(1 - \nu)} \quad (13)$$

The modal displacement fields of shell may be written as:

$$u = \alpha_1 e^{-ik_m x} \cos(n\theta + \omega t) \quad (14)$$

$$v = \alpha_2 e^{-ik_m x} \cos(n\theta + \omega t) \quad (15)$$

$$w = \alpha_3 e^{-ik_m x} \cos(n\theta + \omega t) \quad (16)$$

The deformation of longitudinal, tangential and transverse direction are denoted as u , v and w , respectively. The coefficients α_1 , α_2 and α_3 represent the vibration amplitudes in the x , θ and z directions, correspondingly. Circumferential wavenumber is indicated by n and ω (rad /sec) describes the angular frequency of the shell. The subscript m denotes the axial wave number.

Substituting Eqs 14-16 into Eqs. 1-3, the following equations can be obtained

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rho_t \omega^2 + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rho_t \omega + \begin{pmatrix} \zeta_{11} & \zeta_{12} & \zeta_{13} \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ \zeta_{31} & \zeta_{32} & \zeta_{33} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \right\} = 0 \quad (17)$$

The element of matrix is tabulated in Appendix-I.

3. Results and discussions

Some numerical results are evaluated for isotropic shell for comparing with existing results found in the literature. The present model can be easily reduced to the isotropic one by considering suitable material parameter for isotropic shell. Hence the present model holds good agreement with the existing results [32-34] for isotropic shell as seen in Tables 1-3. A FG-CS consisting of two constituent materials. In these categories Nickel and Stainless steel are used as the interior surfaces and the exterior surface respectively, but their arrangement has profound influence on the formation of FG-CSs. The order of the FG constituent materials is reversed as Type-I and Type-II. At temperature 300K, for stainless steel and nickel, the material properties for FG-CS are: E , ν , ρ for Stainless steel are $2.07788 \times 10^{11} \text{ N/m}^2$, 0.317756 and 8166 Kg/m^3 and Nickel are $2.05098 \times 10^{11} \text{ N/m}^2$, 0.3100, and 8900 Kg/m^3 .

Table 1. Convergence of present method frequencies [33]

	Method	n			
		8	9	10	11
Shell	[33]	280.940	288.71	318.4	363.33
	Present	279.145	287.54	317.43	362.87

Table 2. Convergence of present model frequencies with [34]

L/R	m	Method	n			
			1	2	3	4
20	1	[34]	0.016101	0.00545	0.00504	0.008534
		Present	0.016103	0.00545	0.00505	0.008536
0.25	1	[34]	0.95193	0.93446	0.90673	0.87076
		Present	0.95194	0.93446	0.90674	0.87078

Table 3. Comparison of present model frequencies with [32]

L/R	h/R	Method	n					
			1	2	3	4	5	6
20	0.01	[32]	0.016102	0.009382	0.022105	0.042095	0.06801	0.09973
		Present	0.016101	0.009378	0.022103	0.042094	0.04209	0.09973

Table 4. Convergence of present model frequencies for Type-I and-II with [35]

N	Method	n			
		1	2	3	4
Type-I	[35]	13.560	4.599	4.265	7.225
	Present	12.8941	4.3688	4.0485	6.8575
Type-II	[35]	12.908	4.376	4.058	6.859
	Present	13.5477	4.5916	4.2628	7.2246

Table 5. Convergence of present model frequencies for Type-I and-II with [32]

N=0.7			
	n	[32]	Present
Type-I	1	13.269	13.266
	2	4.4994	4.4913
	3	4.1749	4.1705
	4	7.0691	6.9776
	5	11.29	11.279
Type-II	1	13.154	13.151
	2	4.455	4.4521
	3	4.1309	4.1249
	4	7.0026	6.9928
	5	11.189	11.173

Fig. 2. Winkler elastic foundation for FG-CSs

The frequencies of functionally graded cylindrical shell are compared with Xiang et al. (2012) and Loy et al. (1999) as shown Tables 4-5. The proposed model is applied in order to accurately predict the acquired results of material data point. The frequencies are taken for circumferential modes $n = 1 \sim 5$ and $m = 1$. Table 6 indicates that the frequency values versus circumferential wave number. It is observed that the frequencies first increases and after decreases and pronounces again on enhancing wave number. It is due membrane and flexural stiffness of the shell. In Table 7, natural frequencies (Hz) with thickness to radius (h/R) for Winkler elastic foundation $K = 2 \times 10^6$ (N-m) for Type and Type-II. With increase in values of h/R , the frequency increases fast in the beginning but gets slower as the shell gets thicker. It is noted that with Winkler foundation, on increases h/R frequencies increases as for other cases. In Table 7, natural frequencies (Hz) with thickness to radius (h/R) for Winkler elastic foundation $K = 1 \times 10^6$ (N-m) for Type and Type-II. It is noted that with Winkler foundation, on increases h/R frequencies increases as for other cases.

Table 6. Frequency variation of wave number n ($m=1$, $L=2$, $h=0.001$, $R=1$, $N=0.5$, $K=2 \times 10^6$).

n	Type – I	Type – II
1	13.57331	482.7318
2	4.420372	275.6446
3	2.356774	163.2589
4	2.432712	103.8762
5	3.511083	70.75681
14	2.829642	2.651830
15	3.251456	2.950367

Table 7. Frequency variation of h/R . ($m=1$, $L=5$, $n=1$, $n=1$, $N=1$, $K=2 \times 10^6$)

h/R	Type – I	Type – II
0.001	154.8270	154.8247
0.002	154.8516	154.8471
0.003	154.8763	154.8694
0.004	154.9009	154.8918
0.005	154.9255	154.9142
0.006	154.9501	154.9365
0.007	154.9748	154.9589
0.008	154.9994	154.9813
0.009	155.0240	155.0036
0.01	155.0487	155.0260

Fig. 3. Frequency variation of Type I shell with L/R ($m=1, n=1, h=0.02, N=1, K=2 \times 10^6$).

Fig. 4. Frequency variation of Type-II shell with L/R ($m=1, n=1, h=0.02, N=1, K=2 \times 10^6$).

Table 8. Frequency variation of Type-I and-II shell with K for SS-SS condition (m=1, n=1, h=0.001, N=1, R=1, L=2).

K (N-m)	Type I	Type II
1 x10 ⁶	613.6066	613.6059
2 x10 ⁶	627.0181	627.0174
3 x10 ⁶	639.7247	639.7239
4 x10 ⁶	651.7641	651.7634
5 x10 ⁶	663.1712	663.1703
6 x10 ⁶	673.9777	673.9769
7 x10 ⁶	684.2137	684.2127
8 x10 ⁶	693.9071	693.9061
9 x10 ⁶	703.0848	703.0838
10 x10 ⁶	711.7723	711.7712

Table 9. Frequency variation of Type-I and- II shell with K for C-S condition (m=1, n=1, h=0.001, N=1, R=1, L=2).

K(N-m)	Type – I	Type – II
1 x10 ⁶	657.1725	657.1717
2 x10 ⁶	668.2951	668.2942
3 x10 ⁶	678.8313	678.8304
4 x10 ⁶	688.8103	688.8093
5 x10 ⁶	698.8259	698.2584
6 x10 ⁶	707.2049	707.2038
7 x10 ⁶	715.6717	715.6706
8 x10 ⁶	723.6839	723.6828
9 x10 ⁶	731.2648	731.2636
10 x10 ⁶	738.4367	738.4355

Fig. 5. Frequency variation of Type-I and- II C-F shell versus K ($m=1$, $n=1$, $h=0.001$, $N=1$, $R=1$, $L=2$).

Fig. 6. Frequency variation of Type-I and-II C-C shell versus K ($m=1$, $n=1$, $h=0.001$, $N=1$, $R=1$, $L=2$).

Figs. 3 and 4 shows the variations of C-C, C-S, SS-SS and C-F boundary conditions versus L/R for FG-CS with Winkler elastic foundation $K = 1 \times 10^6, 2 \times 10^6$ (N-m) for Type and Type-II. There is a little bit changed when the value of L/R goes to higher step by step in both Types (I & II). It is observed that from these Figs, the frequencies of Type-I is greater than Type-II with Winkler foundation K . The frequencies increases on increasing the value of foundation K , from 1×10^6 (N-m) (See Fig. 4) to 2×10^6 (N-m) (Fig. 5). It is noted that shell frequencies lower down as L/R is enhanced i.e., as the shell becomes longer. The C-F conditions have low frequencies than other conditions. Tables 8 and 9 shows the frequencies with the variation of Winkler foundation $K = 1 \times 10^6 \sim 10 \times 10^6$ of FG-CSs with BCs C-S and SS-SS. The frequencies in Table 8 are tabulated with Type-I and Type-II. These variations of frequencies are drawn with two types of end conditions. In these Tables, the SS-SS (See Table 9) are lower than that of C-F (See Table 8). These frequencies gain maximum value with the increase of Winkler Foundation K . The frequencies of Type-II is bit less from Type-I due to the constitute materials. It is noted that the frequency have great impact on pacing the FG-CS on the Winkler foundation. The frequencies are visible and increases for C-C and C-F boundary conditions for first three Winkler foundation values $K = 1 \times 10^6 \sim 3 \times 10^6$ as shown in Figs 5 and 6. These frequencies first increases and gain maximum value with the increase of Winkler foundation. For clamped-clamped conditions, variations of frequencies are higher than that of C-F conditions. For $K = 4 \times 10^6 \sim 10 \times 10^6$, a symmetrical behavior for natural frequencies is seen with proposed boundary conditions. It can be seen that Type-II frequencies are smaller than that of Type-I. It is due to the inducting of material in the shell vibration. The frequencies are effected on inhaling the foundation in the cylinder.

4. Conclusion

Love's first approximation theory is utilized for vibrations of functionally graded cylindrical shells with Winkler elastic foundation. The frequency behavior is investigated for circumferential wave number, height and length-to-radius ratios. Also the variations have been plotted against the different values of Winkler foundation. The frequency pattern is found for the increasing and decreasing for height and length-to-radius ratios. The frequency first increases and gain maximum value with the increase for circumferential wave mode. It has been investigated that the frequencies get higher on implicating the elastic foundation of Winkler. For future concerns, the present model can be done for investigating the rotating FG-shells with Winkler model.

Declarations

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Authors' information: **Dr. Abdellatif Selmi.** Prince Sattam bin Abdulaziz University, College of Engineering, Civil Engineering Department, Alkharj 11642, Saudi Arabia
Email: Selmi_fr2016@yahoo.com

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Appendix-I

$$\zeta_{11} = -k_m^2 A_{11} - n^2 \left(\frac{A_{66}}{R^2} + \rho_t \right) + \rho_t \omega^2$$

$$\zeta_{12} = -ink_m \left(\frac{A_{12} + A_{66}}{R} + \frac{B_{12} + 2B_{66}}{R^2} \right)$$

$$\zeta_{13} = -ik_m^3 B_{11} - ik_m \left(\frac{A_{12}}{R} - \rho_t R \right) - in^2 k_m \left(\frac{B_{12} + 2B_{66}}{R^2} \right)$$

$$\zeta_{21} = ink_m \left(\frac{A_{12} + A_{66}}{R} + \frac{B_{12} + B_{66}}{R^2} + \rho_t R \right)$$

$$\zeta_{22} = -k_m^2 \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2} \right) - n^2 \left(\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4} \right) + \rho_t \rho_t \omega^2$$

$$\zeta_{23} = -n \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3} \right) - n^3 \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4} \right) - nk_m^2 \left(\frac{B_{12} + 2B_{66}}{R} + \frac{D_{12} + 2D_{66}}{R^2} \right) + 2\rho_t \omega$$

$$\zeta_{31} = i k_m \frac{A_{12}}{R} + i k_m^3 B_{11} + i n^2 k_m \left(\frac{B_{12} + 2B_{66}}{R^2} \right)$$

$$\zeta_{32} = -n \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3} + \rho_t \right) - n^3 \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4} \right) - n k_m^2 \left(\frac{B_{12} + 2B_{66}}{R} + \frac{D_{12} + 4D_{66}}{R^2} \right) + 2\rho_t \omega$$

$$\zeta_{33} = -\frac{A_{22}}{R^2} + \rho_t - k_m^2 \frac{2B_{12}}{R} - n^2 \left(\frac{2B_{22}}{R^3} + \rho_t \right) - k_m^4 D_{11} - 2n^2 k_m^2 \left(\frac{D_{12} + 2D_{66}}{R^2} \right) - n^4 \frac{D_{22}}{R^4}$$

Figure 1

The diagram illustrates a mechanical system. On the left, a circular disk is shown in cross-section, mounted on a central shaft. The disk is connected to a horizontal assembly on the right. This assembly consists of a central shaft passing through a series of parallel plates or layers. The shaft is supported by a series of springs and dampers (represented by zigzag lines) arranged in a row. A dimension line labeled R indicates the radius of the disk. A dimension line labeled h indicates the thickness of the plates. A downward arrow labeled K points to the central shaft area.

Figure 2

Winkler elastic foundation for FG-CSs

Figure 3

Frequency variation of Type I shell with /LR ($m=1, n=1, h=0.02, N=1, K=2 \times 10^6$).

Figure 4

Frequency variation of Type-II shell with L/R ($m=1$, $n=1$, $h=0.02$, $N=1$, $K=2 \times 10^6$).

Figure 5

Frequency variation of Type-I and- II C-F shell versus K ($m=1$, $n=1$, $h=0.001$, $N=1$, $R=1$, $L=2$).

Figure 6

Frequency variation of Type-I and-II C-C shell versus K ($m=1, n=1, h=0.001, N=1, R=1, L=2$).