

Iterative learning control for time-delay non-linear DPS hyperbolic type

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Iterative learning control for time-delay non-linear DPS hyperbolic type

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Abstract In this paper, we study the application of the iterative learning control to a class of nonlinear varying time-delay equations. The handled system is the distributed parameter system hyperbolic type. The selected plan is based on the PD-type iterative learning control with initial state learning. Initially, we present the system and the control law utilized. Subsequently, we presented some dilemmas. Then, sufficient conditions for monotone convergence of the error under the proper assumption are founded. Also, a detailed convergence analysis for the system is given based on once offered lemmas and Gronwall's inequality. Finally, we show the usefulness of the method utilizing a numerical model.

Keywords Control of non-linear DPS · PD-type ILC · initial learning state · time-delay systems

1 Introduction

The time delay arises in multiple applications since most phenomena do not depend on the present state only, but also the past behavior [1–4]. Delay is

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considered a basis of instability. It may destabilize the model unless we add a controller or more to the systems.

Lately, the control of the PDE with time delay, and the DPS governed by PDEs with time delay effects has become an attractive and active area of research. Numerous essays were published concerning this subject [5, 6, 9, 7, 8, 10]. In [5], the authors consider the state estimation approach over the mobile sensor-actuator networks for parabolic type distributed parameter systems with time delay. Likewise, the authors present the approached solution of the parabolic PDE with time delay and analysis its stability in [6]. The authors in [7] for the control of DPS with time-delay governed by parabolic partial differential equations presented an SDC algorithm. In [8], the authors presented a sampled-data controller to stabilize the time-delay PDE exponentially. Also, [9] considers the optimal control method for the time delay hyperbolic system. Likewise, in [10], the back-stepping method is used to introduce a feedback control law for distributed parameter systems with time delays in the boundary.

The ILC is comparatively a new enhancement to the engineering control methods. It's based mainly on the knowledge in practice, which represents its main advantage. After Arimoto introduced the theory of iterative learning control in 1984, the research of ILC has become a subject of focus in the field of controllers, conducive research advancement has been made in theory and application. ILC has been used to track a determined target in many systems [13, 11, 12] including distributed parameter systems or partial differential systems [14, 15], partial difference systems [16–18], and the delay systems [19–25].

In [19], the authors consider the PD-type iterative learning control method for a class of nonlinear time-delay systems. Further, the authors in [20] propose an iterative learning control (ILC) algorithm to periodicals processes with a variable time delay greater than one iteration of the process. In [21], for SISO processes with uncertainties in modeling and time delays, the authors proposed iterative learning control (ILC) techniques to improve tracking performance via repetition. Likewise, in [22], the authors are concerned with iterative learning control procedures for nonlinear systems with multiple delays, in which output stature noises and outer disturbances are concerned. In [23], the authors study the problem of iterative learning control for switched singular time-delay systems, where the system's rules are switched during a repetitive in a fixed time interval. In [24], the ILC issue is examined for a class of singular time-delay discrete systems. Furthermore, a new iterative learning control algorithm was proposed in [25] for nonlinear time-delay systems with unknown time-varying parameters.

However, few works consider the ILC for the PDE or DPS or PDS with time delay. In [26], the authors examined the iterative learning control problem for time-delay distributed parameter systems, based on the P-type iterative learning control method and utilizing the convenient norm and satisfactory conditions for the convergence of systems. In [27], the iterative learning control (ILC) method is applied to multi-input multi-output systems governed by parabolic partial difference equations with time delay ILC algorithm is intro-

duced for the input delay and state delay system. The authors in [28] offered an iterative learning control method for the distributed parameter systems ruled by parabolic PDEs.

From the emphasis overhead, here in this paper, we study the ILC problem of varying time-delay nonlinear distributed parameter systems. Because of the system complexity in this work, the discrete form (partial difference system [18]) is used. The proposed technique is an ILC algorithm based on the PD-controller and a new output function for the system. The convergence conditions are given utilizing discrete directions. It is demonstrated that deciding the learning gains value via the iterative learning process ensures the convergence of the error between the set desired output and the actual output. Compared with the recent works, the main parts of this work are recapitulated below.

1. Because of the method's high performance with the nonlinear hyperbolic partial difference systems [18], here, an extension of the technique to a class of nonlinear hyperbolic DPS with varying time delay is given, where the sufficient condition of the convergence are provided in the iteration domain.
2. Like in [14] and [18]. Here, we also propose an iterative updating law for the initial state. Which means the initial conditions can vary.
3. In this work, to achieve faster convergence and monotone, a new form of the output function is proposed.
4. The solution of this class of equations does not have a general form and steels a research topic. So we use some mathematical mechanisms, such as the Young inequality and the Gronwall inequality to establish the convergence of the suggested process.

The structural piece of this essay is arranged as follows. In Section 2, we introduce the controller as well as the system. Analysis of convergence is done in Section 3. Section 4, we provide an example to exemplify the usefulness of the proposed method. Finally, Section 5 summarizes this paper.

Notation: For a function $y(p, m) \in R$, the norm of y is: $\|y(\cdot, m)\|_{L^2} = \sqrt{\sum_{p=a}^b y(p, m)^2}$, and its λ -norm is given by: $\|y\|_\lambda = \sup_{j \in [0, J]} (\lambda^m y(\cdot, m))_{L^2}^2$.

2 Problem Statement

In this section, we present the system and the control law.

The nonlinear DPS with time-varying delay described by the hyperbolic partial difference system is considered as:

$$\begin{cases} \Delta_1^2 y(p, m-1) - A \Delta_2^2 y(p-1, m) = F(y(p, m), \bar{y}(p, m)) + Bu(p, m); \\ z(p, m) = Cy(p, m) + D \sum_{t=0}^m u(p, t) \end{cases} \quad (\text{PB.1})$$

where $p \in \{1, 2, \dots, I\}, m \in \{0, 1, \dots, J\}$ denote the space and time variables respectively and J, I are integers. $y(p, m), z(p, m), u(p, m) \in R$ are the sys-

tem state, the system output, and the system input, respectively. $\bar{y}(p, m) = y(p, m - \tau(m))$. Where the non-linearity term still satisfies Lipschitz condition

$$\|F(y_1(s, \mu), \bar{y}_1(s, \mu)) - F(y_2(s, \mu), \bar{y}_2(s, \mu))\| \leq f_0(\|y_1(s, \mu) - y_2(s, \mu)\| + \|\bar{y}_1(s, \mu) - \bar{y}_2(s, \mu)\|)$$

And $0 \leq \tau(m) \leq \tau$

The partial differences symbols Δ are defined as

$$\begin{cases} \Delta_1 y(p, m) = y(p, m + 1) - y(p, m) \\ \Delta_1^2 y(p, m - 1)^2 = y(p, m + 1) - 2y(p, m) + y(p, m - 1) \\ \Delta_2 y(p, m) = y(p + 1, m) - y(p, m) \\ \Delta_2^2 y(p - 1, m) = y(p + 1, m) - 2y(p, m) + y(p - 1, m) \end{cases} \quad (1)$$

where $p \in \{1, 2, \dots, I\}, m \in \{0, 1, \dots, J\}$

The learning system as the k^{th} iteration is given as follow

$$\begin{cases} \Delta_1^2 y_k(p, m - 1) - A \Delta_2^2 y_k(p - 1, m) = F(y_k(p, m), \bar{y}_k(p, m)) + B u_k(p, m); \\ z_k(p, m) = C y_k(p, m) + D \sum_{s=0}^m u_k(p, s) \end{cases} \quad (PB.2)$$

The considered controller in this paper is by:

$$\begin{cases} u_{k+1}(p, m) = u_k(p, m) + L_1 e_k(p, m) + L_2 \Delta_1 e_k(p, m - 1) \\ y_{k+1}(p, \epsilon) = y_k(p, \epsilon) + L_3 e_k(p, 0) \\ y_{k+1}(p, 1) = y_k(p, 1) + L_4 e_k(p, 1) \end{cases} \quad (2)$$

$k = 1, 2, \dots$ represent the iteration numbers. L_1, L_2, L_3 and L_4 are positive numbers (control gains). e_k represents the tracking error, and it is characterized as follows: $e_k(p, m) = z_d(p, m) - z_k(p, m)$, where $z_d(p, m)$ is the desired target (output function), $z_k(p, m)$ is the system k^{th} iteration output, respectively. The principal object of this writing is to design an ILC controller to follow the provided desired target $y_d(p, m)$ based on the measurable system output so that the tracking error tends to zero when the iteration time k tends to infinity, i.e.

$$\lim_{k \rightarrow +\infty} e_k(p, m) = \lim_{k \rightarrow +\infty} (y_d(p, m) - y_k(p, m)) = 0$$

Assumption 1 For the system (PB.2), we assume that there is a unique system input $u_d(p, m) \in R$ satisfying that

$$\begin{cases} \Delta_1^2 y_d(p, m - 1) - A \Delta_2^2 y_d(p - 1, m) = F(y_d(p, m), \bar{y}_d(p, m)) + B u_d(p, m); \\ z_d(p, m) = C y_d(p, m) + D \sum_{s=0}^l u_d(p, s) \end{cases}$$

Remark 1 Assumption 1 tells that exists conceivable pair of input and state such that the system (PB.2) can reach the desired results.

Remark 2 The boundary value conditions of the system (PB.2) are supposed to be identical for all the iteration and to be duplicate to the desired output boundary value $y_k(0, m) = y_d(0, m)$, $y_k(I+1, m) = y_d(I+1, m)$, $-\tau \leq m \leq J$. The initial condition when $-\tau(m) \leq t < 0$ is also supposed to be equal and the same as the desired value $y_{k+1}(p, t) = y_d(p, t) = \phi(p, t)$, $k = 1, 2, \dots$; $i \in \{1, 2, \dots, I\}$, and $-\tau(m) \leq t < 0$

3 Convergence Analysis

Lemma 1 Let A_k and B_k be a real non-negative sequence, such as, $A_{k+1} \leq \nu A_k + B_k$. If $0 \leq \nu < 1$ and $B_k \rightarrow 0$ when $k \rightarrow \infty$, we have $A_k \rightarrow 0$ when $k \rightarrow \infty$.

The proof is easy to complete.

Lemma 2 (Discrete Gronwall inequality)

Let the constant sequences $y(m)$, $c(m)$ and $d(m)$ be real sequences which satisfy $y(m+1) \leq c(m)y(m) + d(m)$, $0 \leq c(m)$, $0 \leq m$. Then, we have

$$y(j) \leq \prod_{m=0}^{j-1} c(m)y(0) + \sum_{m=1}^{j-1} d(m) \prod_{s=m+1}^{j-1} c(s) \quad (3)$$

For more detail refer to [29].

Lemma 3 For $H(p, m) \in R$ real function, there exist two constant $0 \leq R_1$ and $0 \leq R_2$ such that

$$\|H\|_\lambda \leq R_1 \|\Delta_1 H\|_\lambda + R_2 \|H(p, 0)\|_{L^2}^2$$

For more detail refer to [18].

Proposition 1 if $(1 - CL_3 - DL_1)^2 \leq 1$, then

$$\|e_k(\cdot, 0)\|_{L^2} \rightarrow 0 \text{ when } k \rightarrow \infty$$

and if $(1 - CL_4 - D(L_1 + L_2))^2 \leq 1$, then

$$\|e_k(\cdot, 1)\|_{L^2} \rightarrow 0 \text{ when } k \rightarrow \infty$$

Proof - In the first part we proof that $\|e_k(\cdot, 0)\|_{L^2} \rightarrow 0$ when $k \rightarrow \infty$

$$\begin{aligned} e_{k+1}(p, 0) &= z_d(p, 0) - z_{k+1}(p, 0) = e_k(p, 0) + z_k(p, 0) - z_{k+1}(p, 0) \\ (e_{k+1}(p, 0))^2 &\leq (1 - CL_3 - DL_1)^2 e_k(p, 0)^2 \\ \sum_{p=1}^I (e_{k+1}(p, 0))^2 &\leq (1 - CL_3 - DL_1)^2 \sum_{p=1}^I e_k(p, 0)^2 \end{aligned}$$

According to the Proposition 1 and to Lemma 1, $\|e_k(\cdot, 0)\|_{L^2} \rightarrow 0$ when $k \rightarrow \infty$
- Secondly, we proof that $\|e_k(\cdot, 1)\|_{L^2} \rightarrow 0$ when $k \rightarrow \infty$

$$\begin{aligned} e_{k+1}(p, 1) &= z_d(p, 1) - z_{k+1}(p, 1) = e_k(p, 1) + z_k(p, 1) - z_{k+1}(p, 1) \\ (e_{k+1}(p, 1))^2 &\leq (1 - CL_4 - D(L_1 + L_2))^2 e_k(p, 1)^2 + D^2(L_1 + L_2)^2 e_k(p, 0)^2 \\ \sum_{p=1}^I (e_{k+1}(p, 1))^2 &\leq (1 - CL_4 - D(L_1 + L_2))^2 \sum_{p=1}^I e_k(p, 1)^2 + D^2(L_1 + L_2)^2 \sum_{p=1}^I e_k(p, 0)^2 \end{aligned}$$

According to the Proposition 1, Lemma 1, and because $\|e_k(\cdot, 0)\|_{L^2} \rightarrow 0$ when $k \rightarrow \infty$ we get $\|e_k(\cdot, 1)\|_{L^2} \rightarrow 0$ when $k \rightarrow \infty$

For more details see [18]

Theorem 1 *Suppose that Proposition 1 and Assumption 1 are satisfied. If*

$$\rho = 4((1 - DL_2)^2 + 8BC^2L_2^2) \leq \epsilon < 1$$

under the effect of the control law (2), then the proposed method converge, i.e. $\|e_k\|_\lambda \rightarrow 0$ when $k \rightarrow \infty$

Proof To proof the theorem and according to Proposition 1 and Lemma 3, we need to prove that $\|\Delta_1 e_k\|_\lambda \rightarrow 0$ when $k \rightarrow \infty$ to prove the convergence of the method.

Let $v_k = u_k - u_{k+1}$, $w_k = y_k - y_{k+1}$, and $F_k(p, m) = F(y_k(p, m), \bar{y}(p, m))$

$$\begin{aligned} \Delta_1 e_{k+1}(p, m) &= \Delta_1(z_d(p, m) - z_{k+1}(p, m)) = \Delta_1(e_k(p, m) + z_k(p, m) - z_{k+1}(p, m)) \\ &= \Delta_1 e_k(p, m) + C\Delta_1 w_k(p, m) + D\Delta_1 \sum_{l=0}^m v_k(p, l) \\ &= (1 - DL_2)\Delta_1 e_k(p, m) + C\Delta_1 w_k(p, m) + DL_1 e_k(p, 0) - DL_1 e_k(p, m+1) \\ (\Delta_1 e_{k+1}(p, m))^2 &\leq 4(1 - DL_2)^2 (\Delta_1 e_k(p, m))^2 + 4(DL_1 e_k(p, m+1))^2 \\ &\quad + 4(C\Delta_1 w_k(p, m))^2 + 4(DL_1 e_k(p, 0))^2 \\ &= 4(1 - DL_2)^2 (\Delta_1 e_k(p, m))^2 + 4D^2 L_1^2 (e_k(p, m+1))^2 \\ &\quad + 4C^2 (\Delta_1 w_k(p, m))^2 + 4D^2 L_1^2 (e_k(p, 0))^2 \end{aligned}$$

Let $\rho = 4(1 - DL_2)^2$,

$$\begin{aligned} \|\Delta_1 e_{k+1}(\cdot, m)\|_{L^2}^2 &\leq \rho \|\Delta_1 e_k(\cdot, m)\|_{L^2}^2 + 4D^2 L_1^2 \|e_k(\cdot, m+1)\|_{L^2}^2 \\ &\quad + 4C^2 \|\Delta_1 w_k(\cdot, m)\|_{L^2}^2 + 4D^2 L_1^2 \|e_k(\cdot, 0)\|_{L^2}^2 \end{aligned}$$

$$\begin{aligned} \lambda^j \|\Delta_1 e_{k+1}(\cdot, m)\|_{L^2}^2 &\leq \lambda^j \rho \|\Delta_1 e_k(\cdot, m)\|_{L^2}^2 + 4\lambda^m D^2 L_1^2 \|e_k(\cdot, m+1)\|_{L^2}^2 \\ &\quad + 4\lambda^m C^2 \|\Delta_1 w_k(\cdot, m)\|_{L^2}^2 + 4\lambda^m D^2 L_1^2 \|e_k(\cdot, 0)\|_{L^2}^2 \end{aligned}$$

$$\begin{aligned} \|\Delta_1 e_{k+1}\|_\lambda &\leq \rho \|\Delta_1 e_k\|_\lambda + 4C^2 \|\Delta_1 w_k\|_\lambda \\ &\quad + 4D^2 L_1^2 \lambda \|e_k\|_\lambda + 4\lambda^J D^2 L_1^2 \|e_k(\cdot, 0)\|_{L^2}^2 \end{aligned}$$

from Lemma 3, we get

$$\begin{aligned} \|\Delta_1 e_{k+1}\|_\lambda &\leq \rho \|\Delta_1 e_k\|_\lambda + 4C^2 \|\Delta_1 w_k\|_\lambda + 4D^2 L_1^2 R_1 \|\Delta_1 e_k\|_\lambda \\ &\quad + 4D^2 L_1^2 R_2 \|e_k(\cdot, 0)\|_{L^2}^2 + 4\lambda^J D^2 L_1^2 \|e_k(\cdot, 0)\|_{L^2}^2 \end{aligned} \quad (4)$$

To finish the proof we need to estimate $\|\Delta_1 w_k\|_\lambda$,

$$\Delta_1 w_k(p, m) - \Delta_1 w_k(p, m-1) = A\Delta_2^2 w_k(p-1, m) + F_k(p, j) - F_{k+1}(p, m) + Bv_k(p, m)$$

$$\begin{aligned} \Delta_1 w_k(p, m)^2 &= (\Delta_1 w_k(p, m-1) + A\Delta_2^2 w_k(p-1, m) + F_k(p, m) - F_{k+1}(p, m) + Bv_k(p, m)) \\ &\quad (\Delta_1 w_k(p, m-1) + A\Delta_2^2 w_k(p-1, m) + F_k(p, m) - F_{k+1}(p, m) + Bv_k(p, m)) \\ &\leq 4\Delta_1 w_k(p, m-1)^2 + 4A\Delta_2^2 w_k(p-1, m)^2 + 4f_0 w_k(p, m)^2 + 4f_0 \bar{w}_k(p, m)^2 + 4Bv_k(p, m)^2 \\ \sum_{p=1}^I \Delta_1 w_k(p, m)^2 &\leq \sum_{p=1}^I (4\Delta_1 w_k(p, m-1)^2 + 4A\Delta_2^2 w_k(p-1, m)^2 + 4f_0 w_k(p, m)^2 + 4f_0 \bar{w}_k(p, m)^2 + 4Bv_k(p, m)^2) \end{aligned}$$

we also have,

$$\sum_{p=1}^I \Delta_2^2 w_k(p-1, m)^2 \leq 18 \sum_{p=1}^I w_k(p, m)^2 \quad (5)$$

then

$$\|\Delta_1 w_k(\cdot, m)\|_{L^2}^2 \leq 4\|\Delta_1 w_k(\cdot, m-1)\|_{L^2}^2 + 4f_0 \|w_k(\cdot, m)\|_{L^2}^2 + 4f_0 \|\bar{w}_k(\cdot, m)\|_{L^2}^2 + 72A \|w_k(\cdot, m)\|_{L^2}^2 + 4B \|v_k(\cdot, m)\|_{L^2}^2$$

$$\begin{aligned} \|\Delta_1 w_k(\cdot, m)\|_{L^2}^2 &\leq 4\|\Delta_1 w_k(\cdot, m-1)\|_{L^2}^2 \\ &\quad + (4f_0 + 72A) \|w_k(\cdot, m)\|_{L^2}^2 + 4f_0 \|\bar{w}_k(\cdot, m)\|_{L^2}^2 + 4B \|v_k(\cdot, m)\|_{L^2}^2 \end{aligned} \quad (6)$$

Using Lemma 2, we get

$$\begin{aligned} \|\Delta_1 w_k(\cdot, m-1)\|_{L^2}^2 &\leq \prod_{l=0}^{m-1} 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + \sum_{l=1}^{m-1} ((4f_0 + 72A) \|w_k(\cdot, l)\|_{L^2}^2 + 4f_0 \|\bar{w}_k(\cdot, l)\|_{L^2}^2 + 4B \|v_k(\cdot, l)\|_{L^2}^2) \\ &\leq 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + (4f_0 + 72A) \sum_{l=1}^{m-1} \|w_k(\cdot, l)\|_{L^2}^2 \\ &\quad + 4f_0 \sum_{l=1}^{m-1} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2 + 4B \sum_{l=1}^{m-1} \|v_k(\cdot, l)\|_{L^2}^2 \end{aligned} \quad (7)$$

We calculate $\sum_{l=1}^{m-1} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2$,

If $\tau < j$,

$$\begin{aligned} \sum_{l=1}^{m-1} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2 &= \sum_{l=1}^{\tau} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2 + \sum_{l=\tau+1}^{j-1} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2 \\ &= \sum_{l=\tau+1}^{m-1} \|w_k(\cdot, l - \tau(l))\|_{L^2}^2 \\ &\leq \sum_{l=1}^{m-1} \|w_k(\cdot, l)\|_{L^2}^2 \end{aligned} \quad (8)$$

Replace (8) in (7) we get,

$$\begin{aligned} \|\Delta_1 w_k(\cdot, m-1)\|_{L^2}^2 &\leq 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + (4f_0 + 72A) \sum_{l=1}^{m-1} \|w_k(\cdot, l)\|_{L^2}^2 \\ &\quad + 4f_0 \sum_{l=1}^{m-1} \|w_k(\cdot, l)\|_{L^2}^2 + 4B \sum_{l=1}^{m-1} \|v_k(\cdot, l)\|_{L^2}^2 \end{aligned}$$

$$\begin{aligned} \|\Delta_1 w_k(\cdot, m-1)\|_{L^2}^2 &\leq 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + (8f_0 + 72A) \|w_k\|_{\lambda} \sum_{l=1}^{m-1} \lambda^{-l} + 4B \|v_k\|_{\lambda} \sum_{l=1}^{m-1} \lambda^{-l} \\ \|\Delta_1 w_k\|_{\lambda} &\leq 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + (8f_0 + 72A) \frac{\lambda^J - 1}{\lambda - 1} \|w_k\|_{\lambda} + 4B \frac{\lambda^J - 1}{\lambda - 1} \|v_k\|_{\lambda} \end{aligned}$$

Using Lemma 3, we get

$$\begin{aligned} \|\Delta_1 w_k\|_{\lambda} &\leq 4\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + (8f_0 + 72A) \frac{\lambda^J - 1}{\lambda - 1} (R_1 \|\Delta_1 w_k\|_{\lambda} + R_2 \|w_k(\cdot, 0)\|_{L^2}^2) \\ &\quad + 4B \frac{\lambda^J - 1}{\lambda - 1} \|v_k\|_{\lambda} \\ \|\Delta_1 w_k\|_{\lambda} &\leq \frac{4}{\alpha_{\lambda}} \|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 + \frac{\beta_{\lambda}}{\alpha_{\lambda}} \|w_k(\cdot, 0)\|_{L^2}^2 + 4B \frac{\gamma_{\lambda}}{\alpha_{\lambda}} \|v_k\|_{\lambda} \end{aligned} \quad (9)$$

with $\alpha_{\lambda} = 1 - (8f_0 + 72A)R_1\gamma_{\lambda}$, $\beta_{\lambda} = (8f_0 + 72A)R_2\gamma_{\lambda}$, and $\gamma_{\lambda} = \frac{\lambda^J - 1}{\lambda - 1}$
replacing (9) in (4) we get,

$$\begin{aligned} \|\Delta_1 e_{k+1}\|_{\lambda} &\leq \rho \|\Delta_1 e_k\|_{\lambda} + 4C^2 \left(\frac{4}{\alpha_{\lambda}} \|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 \right) \\ &\quad + 4C^2 \left(\frac{\beta_{\lambda}}{\alpha_{\lambda}} \|w_k(\cdot, 0)\|_{L^2}^2 + 4B \frac{\gamma_{\lambda}}{\alpha_{\lambda}} \|v_k\|_{\lambda} \right) \\ &\quad + 4D^2 L_1^2 R_1 \|\Delta_1 e_k\|_{\lambda} + 4D^2 L_1^2 R_2 \|e_k(\cdot, 0)\|_{L^2}^2 + 4\lambda^J D^2 L_1^2 \|e_k(\cdot, 0)\|_{L^2}^2 \end{aligned}$$

or, $\|w_k(\cdot, 0)\|_{L^2}^2 = L_3^2 \|e_k(\cdot, 0)\|_{L^2}^2$
 now, we estimate $\|v_k\|_\lambda$ and $\|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2$

$$\begin{aligned} \|v_k\|_\lambda &\leq 2L_1^2 \|e_k\|_\lambda + 2L_2^2 \|\Delta_1 e_k\|_\lambda \\ \|\Delta_1 w_k(\cdot, 0)\|_{L^2}^2 &\leq L_3^2 \|e_k(\cdot, 0)\|_{L^2}^2 + L_4^2 \|e_k(\cdot, 1)\|_{L^2}^2 \end{aligned}$$

at that point, for quite small λ and using the Theorem 1 we get, $\|\Delta_1 e_k\|_\lambda \rightarrow 0$ when $k \rightarrow \infty$, then from Lemma 3 we can have, $\|e_k\|_\lambda \rightarrow 0$ when $k \rightarrow \infty$,

4 Numerical example

To demonstrate the method's effectiveness, we provide an example for the system. Let the desired system output be,

$$zd(p, j) = j * \sin(\pi - p) + \sin\left(\frac{p}{2}\right) * \cos(j)$$

with $A = B = C = D = 1$, $I = 14/100$, $J = 8/100$, $\tau(j) = j/2$, and $F(p, j) = y_k(p, j) * y_k(p, j - \tau(j))$

The simulation results of the proposed method are shown in Figures 1-3 and Table 1.

Figure 1 shows the desired output. Figure 2 presents the 6th iteration of the system output Figure 3 shows the λ -norm of the error-iteration number and the error-space-time. Table 1 represents iteration number and the norm value of the error.

4.1 Discussion

According to the previous analysis, the proposed scheme is successful in the case of second-order nonlinear time-delay DPS control in terms of performance and speed. This resounding success underscores the efficiency of the PD controller, the laws of iterative updating of initial states, and the new output function. Confirms that the proposed algorithm can reach the desired result in a limited number of iterations with monotonic convergence of the error towards zero. These are the advantages of the proposed method.

5 conclusion

In this essay, we have examined the distributed control of a type of nonlinear distributed parameters with state time-delay ruled by a hyperbolic partial difference equation, designing an ILC algorithm and confirming the error of provided system convergence to 0 in λ -norm meaning. We extend the ILC process to state time-delay nonlinear distributed parameters systems. We also use a simulated example, and we get the expected outcome.

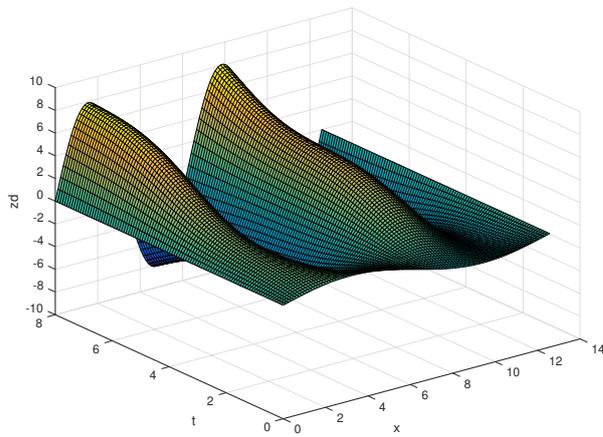


Fig. 1 The desired output

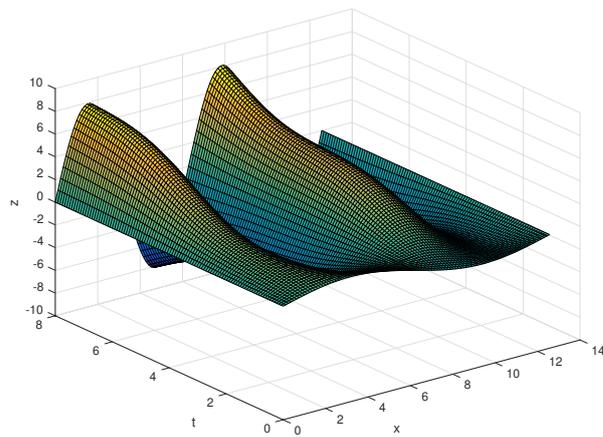


Fig. 2 The system output at $k = 6$

Table 1 Error-norm value

k	1	2	3	4	5	6
norm	0.6542	0.1480	0.0296	0.0059	0.0010	0.0001

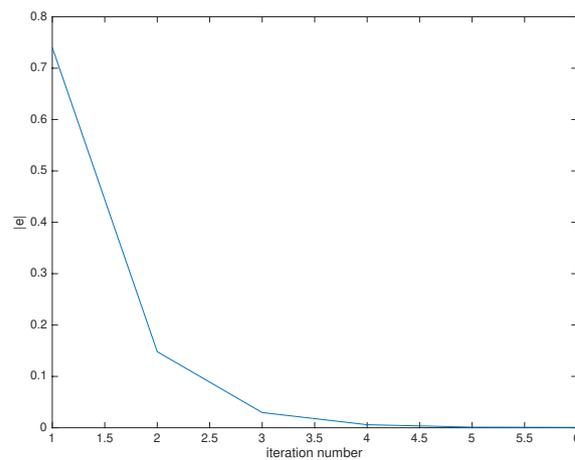


Fig. 3 The error-norm with iteration number

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Conflict of interest

The authors declare that they have no conflict of interest.

Data Availability Statements

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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