

Energy of Flipping A Spin

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ABSTRACT We found that the physics of using a spin's orientation to store data fundamentally differs from that of using a particle's position as a (classical) bit of information: the former is quantum dynamic and independent of temperature (if the temperature is below the Curie point), whereas the latter is thermodynamic and thereby dependent on temperature. The formula to calculate the minimum energy of flipping a spin should be the Bohr magneton times the magnetic field. Obviously, the key to calculating such a minimum energy is to find a minimum magnetic field that should not be zero; otherwise, spin-flipping will not take place. Our conclusion is that the energy limit of storing data in a modern way (using a spin's orientation) is $1.64E-36 J$, 15 orders of magnitude lower than that of storing data in a classical way (using a particle's position), which implies that spin electronics in data storage is fundamentally superior to classical charge-based methods in terms of energy efficiency and computational reversibility. We also verified this new limit based on a spin-spin interaction experiment.

INDEX TERMS Energy efficiency, information processing, thermodynamics, quantum dynamics, spintronics.

I. ENERGY OF PROCESSING A CLASSICAL BIT

A trapped particle moving upwards and downwards between impenetrable barriers at either end of a one-dimensional nanotube [Fig. 1(a)] can be treated as an ideal gas whose interactions with the ends are perfectly elastic collisions. It obeys the ideal gas law as follows:

$$PV = k_B T, \quad (1)$$

where P is the pressure, V is the volume, k_B is the Boltzmann constant and T is the (absolute) temperature.

In Fig. 1(a), the work to push the info electron to the desired half is

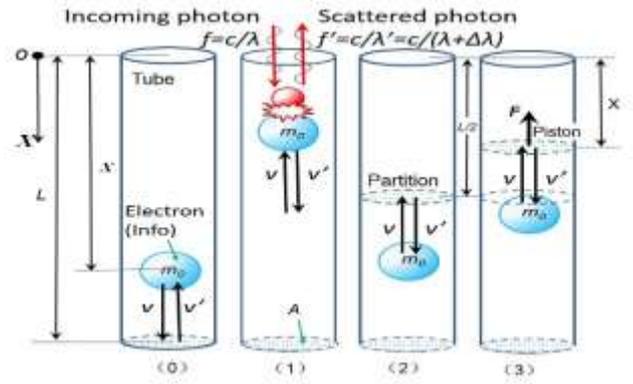
$$W = \int_0^{L/2} P A dx = \int_{L/2}^L \frac{k_B T A}{Ax} dx = k_B T \ln(2). \quad (2)$$

The erasure of a bit of (classical) information requires the same amount of (heat) energy that is approximately $3 \times 10^{-21} J$ at room temperature (300 K). This energy limit is called the Landauer bound [1].

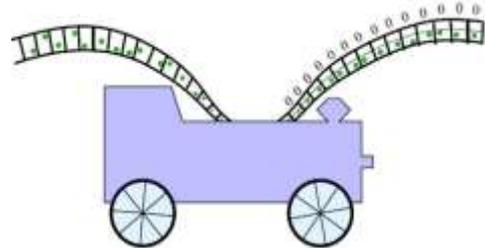
As pointed out by Landauer [1], this heat generation inevitably results in physical irreversibility (making a signal independent of its history) associated with logical irreversibility (a logic function that does not have a single-valued inverse). Such an erasure of a bit of information as an irreversible operation can also be described by the second law of thermodynamics in Clausius's work [2] and a minimal entropy production of $k_B \ln(2)$ in Szilard's work [3].

In 2012, the Landauer bound ($3 \times 10^{-21} J$) was experimentally verified using a system of a single colloidal particle trapped in a modulated double-well potential [4]. The authors observed that the mean dissipated heat saturates at the Landauer bound in the limit of long erasure cycles [4].

This work defines the ultimate physical limit of irreversible computation [4].



(a) A tiny tube with a trapped electron;



(b) An information engine "burning" information.

Fig. 1 A one-dimensional nanotube with a trapped electron. In (0), the electron only moves upwards and downwards, v and v' denote the velocities before and after the collision, respectively. Creating info is a two-step process: (1) emit a photon to push the info electron to the desired half; (2) add the partition to confine the info electron. In (3), if the partition is a frictionless piston, the info particle can push it to exert useful work in exchange for losing that written info (which side of the partition is occupied). This is a so-called "information engine" [Fig. 1(b)].

In 2014, reducing the exerted work to the Landauer limit during the erasure (at the expense of slow operation) was demonstrated using small particles in traps [5].

In 2016, a laser probe was used to measure the amount of energy dissipation (4.2 zeptojoules) when a nanomagnetic bit was flipped from off to on [6].

In 2018, an array of giant-spin ($S = 10$) quantum molecular magnets was reported [7]. The authors concluded that the erasure is still governed by the Landauer bound but can be performed at a high speed (because tunneling provides a high-speed path for spin reversal) even in the quantum realm [7]. In this article, we will prove that, in principle, the Landauer bound should no longer be used in the quantum case and that the researchers did not break the Landauer bound purely because of a numerical coincidence under their experimental conditions at that time. On the other hand, the authors used giant spins ($S=10$) [7] rather than a single spin.

II. STORING DATA IN A SPIN

An electron has a charge and a spin, which are inseparable. In classical information storage (Fig. 1), a charge is stored to save information. As a modern paradigm, a spin could replace a charge for the storage of information, allowing faster, low-energy operations [8].

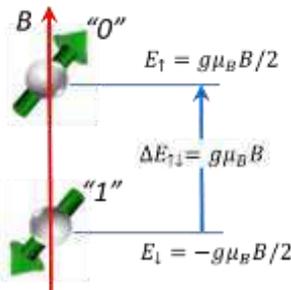


Fig. 2 Energy of a spin in a magnetic field B . The spin angular momentum is quantized with only two possible z -components and is independent of temperature [the ordered magnetic moments of ferromagnetic materials change and become disordered (paramagnetic) at the Curie temperature, which is normally well above room temperature]. The decoupling between the (temperature-dependent) spatial degree of freedom and the (temperature-independent) spin degree of freedom means that the Landauer bound (which is dependent on temperature T) should no longer be used in this case.

As shown in Fig. 2, the energy of flipping a spin in a magnetic field B can be expressed by:

$$\Delta E_{\uparrow\downarrow} = g\mu_B B \approx 2\mu_B B, \quad (3)$$

where μ_B is the Bohr magneton and the value of the electron spin g -factor is roughly equal to 2.002319. The reason g is not precisely 2 can be explained by quantum electrodynamics calculations of the anomalous magnetic dipole moment [9].

The minimum energy of using a spin's orientation to store data is $2\mu_B B$, regardless of which energy (electrical, magnetic, optical, chemical or even mechanical) is input. Analogous to the temperature T (as an environmental parameter) in the Landauer bound, the magnetic field B (as

another environmental parameter) is essential to determine this new energy bound for "nonclassical" information. We will calculate the strength of the magnetic field in the following cases: 1. An internal magnetic field due to the electron orbit in an isolated atom or ion; 2. A magnetic field in the spin-spin interaction; and 3. Various ambient magnetic field sources outside matter.

Note that the spin carrier's spatial degrees of freedom are in thermal equilibrium (conservation of energy) with a much larger heat bath (a thermal reservoir) at temperature T . In contrast, the internal, intrinsic spin degrees of freedom are in an independent equilibrium state [10]. The decoupling between the (temperature-dependent) spatial degree of freedom and the (temperature-independent) spin degree of freedom may be ensured by requiring the internal spin states of each spin to be degenerate in energy [10]. The temperature and spatial degrees of freedom are already described in the traditional Landauer bound [1], and such a decoupling means that the Landauer bound (which is dependent on temperature T) should not be used in the energy limit calculation (which has nothing to do with temperature) for a spin as a bit of information.

III. INTERNAL MAGNETIC FIELD IN AN ATOM OR ION

An electron normally exists in two ways: it can be either bound to the nucleus of an atom by the attractive Coulomb force (in the spin-spin interaction experiment to be illustrated in the next section, the ground state valence electron of a trapped 88Sr^+ ion can be treated as having only spin degrees of freedom) or promoted to free space (outside matter) by absorbing energy equal or greater than the work function. An electron in metals also behaves as if it was free. Let us first calculate the energy of flipping a spin in an atom.

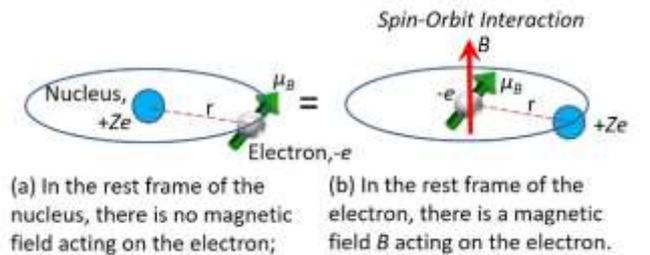


Fig. 3 Although in the rest frame of the nucleus, there is no magnetic field acting on the electron, there is a magnetic field in the rest frame of the electron. B in (b) is such a magnetic field seen from the electron perspective. Z is the effective central charge, which includes all other charges of the nucleus and inner electrons. e is the elementary charge.

As shown in Fig. 3, there is a magnetic field B acting on the electron in the rest frame of the electron. The energy of flipping this spin in the B that the electron experiences can be found in Eq.3.

Directly, the energy of flipping the spin between two states (spin up and spin down) can be expressed in its quantum dynamical form [9][11] as follows:

$$\Delta E = \frac{mc^2\alpha^4}{2} \cdot \frac{Z^4}{n^3l(l+1)}, \quad (4)$$

where m is the mass of an electron, c is the speed of light, α is the fine-structure constant, Z is the effective central charge, n is the principal quantum number and ℓ is the orbital angular momentum quantum number. These two states (spin up and spin down) exist inside the same " n ", and their total angular momentum number " j " is different by one. As one can see in Eq.4, this value is proportional to $\frac{Z^4}{n^3}$.

For example, first, we obtain the energy difference between $2p_{3/2}$ and $2p_{1/2}$ of the hydrogen atom. The principal quantum number " n " is "2" in both states. $j = l + s = 3/2$ in $2p_{3/2}$ (the spin quantum number associated with the spin angular momentum $s=1/2$), and $j = l - s = 1/2$ in $2p_{1/2}$. As shown in Fig. 4, the hydrogen fine structure (= doublet) between $2p_{3/2}$ and $2p_{1/2}$ is approximately 0.000045 eV, which is experimentally verified [7].

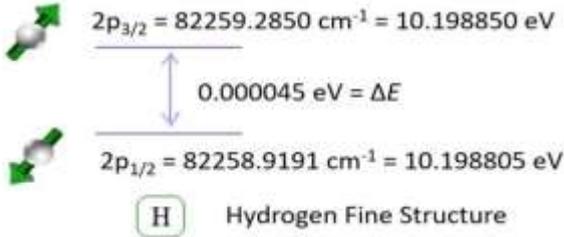


Fig. 4 Hydrogen $2p_{3/2} - 2p_{1/2}$ fine structure. It is the spin-orbit interaction that splits the hydrogen atomic energy levels and gives rise to a fine structure in the spectra of the hydrogen atom. As shown in the figure, the energy separation (0.000045 eV) between $2p_{3/2}$ and $2p_{1/2}$ corresponds to 0.365 cm^{-1} ($= 82259.2850 \text{ cm}^{-1} - 82258.9191 \text{ cm}^{-1}$), which has been verified by an absorption spectrum experiment for hydrogen: the

$2s_{1/2}(=2p_{1/2}) \rightarrow 2p_{3/2}$ transition requires a wavelength of 2.74 cm ($= \frac{1}{0.365}$) to be absorbed) [7].

Some typical values of the internal magnetic field experienced by a hydrogen atom electron in different states are listed in Table 1. As seen in the table, there exists a strong magnetic field inside an atom [9][11]. Since the energy is proportional to $\frac{Z^4}{n^3}$, the chemical elements ($Z>1$) after hydrogen require more energy to flip their spins. In other words, a hydrogen atom tends to have the lowest energy to flip its spin. For the same chemical element, the larger the orbit is of an electron ($n>2$), the less energy needed to flip its spin (note that the s state has no orbital angular momentum and that there is no spin-orbit splitting for this state).

In this work, what we want to work out is a minimum energy (the energy limit) to flip a spin. Therefore, we need to calculate the energy values of a hydrogen atom electron with a very high principal quantum number n . In fact, an excited atom with one or more electrons that have a very high n is called a Rydberg atom [9][12]. While being excited into a high energy level, those outer electrons of an atom may enter a spatially-extended orbital (far outside the ones of the other electrons). From the standpoint of an excited electron, it sees an "equivalent" atomic core that consists of the nucleus and all the inner electrons. Hence, this core has a net charge of $+e$, which is the same as that of a hydrogen nucleus. Therefore, the excited electron behaves as if it belongs to a hydrogen atom. Indeed, in many respects, a Rydberg atom behaves like a hydrogen atom [12].

TABLE 1 Some typical values of the internal magnetic field experienced by a hydrogen (or hydrogen-like) atom electron (a highly excited atom has an electronic structure roughly similar to that of atomic hydrogen) and the corresponding energy in the form of Z^4/n^3 .

The principal quantum number n	The orbital angular momentum quantum number l	The orbital radius $r = \frac{n^2 \hbar^2}{k e^2 m}$	$\Delta E_{\uparrow\downarrow} = E_{\uparrow} - E_{\downarrow}$ $\propto \frac{Z^4}{n^3}$	$B = \frac{\Delta E_{\uparrow\downarrow}}{2\mu_B}$
1	0	$5.3 \times 10^{-11} \text{ m}$ (Bohr radius)	0	0
2	0	$0.00021 \mu\text{m}$	0	0
2	1	$0.00021 \mu\text{m}$	$4.5 \times 10^{-5} \text{ eV}$	0.388 T
3	0	$0.00048 \mu\text{m}$	0	0
3	1	$0.00048 \mu\text{m}$	$4.56 \times 10^{-5} \text{ eV}$	0.393 T
3	2	$0.00048 \mu\text{m}$	$4.48 \times 10^{-5} \text{ eV}$	0.386 T
...
137 (Rydberg atom)	1	$1 \mu\text{m}$ (close to the size of bacteria!)	$1.75 \times 10^{-11} \text{ eV}$	$1.51 \times 10^{-7} \text{ T}$
290 (Rydberg atom, highest achieved in lab) [9]	1	$4.5 \mu\text{m}$	$1.82 \times 10^{-12} \text{ eV}$	$1.58 \times 10^{-8} \text{ T}$
630 (acre atoms, highest observed in interstellar space) [9]	1	$21.2 \mu\text{m}$	$1.80 \times 10^{-13} \text{ eV}$	$1.55 \times 10^{-9} \text{ T}$

So far, a Rydberg atom with $n=290$ has been achieved experimentally in the laboratory, and acre atoms whose outer electrons are in states with $n=630$ have been observed by radio astronomical methods in interstellar space [9]. Accordingly, we calculated both the energy required to flip the spin of its outermost electron and the internal magnetic field experienced by that electron, as presented in the bottom two rows of Table 1. Here, we chose $1.82 \times 10^{-12} \text{ eV}$ ($= 2.91 \times 10^{-31} \text{ J}$) as the (minimum) energy limit if one wants to use the electron spin in an atom to store a bit of information (in his/her human-made devices on Earth). Obviously, this energy limit is much lower than the Landauer bound ($3 \times 10^{-21} \text{ J}$).

Classically, an electron in a circular orbit of radius r , about a hydrogen nucleus of charge $+e$, obeys Newton's second law: $F = ma \Rightarrow \frac{ke^2}{r^2} = \frac{mv^2}{r}$, where $k = \frac{1}{4\pi\epsilon_0}$. Orbital momentum is quantized in units of the reduced Planck constant \hbar : $mvr = n\hbar$. Combining these two equations leads to Bohr's expression for the orbital radius in terms of the principal quantum number n : $r = \frac{n^2\hbar^2}{ke^2m}$. It is now apparent why the radius of the orbit of a Rydberg atom scales as n^2 (as shown in Table 1, the $n = 137$ state of hydrogen has an atomic radius $r = \frac{n^2\hbar^2}{ke^2m} = \frac{137^2 \times (1.05 \times 10^{-34} \text{ J}\cdot\text{s})^2}{(1.60 \times 10^{-19} \text{ C})^2 \times 9.11 \times 10^{-31} \text{ kg}} \approx 1 \mu\text{m}$, even

close to the size of bacteria!). Thus, Rydberg atoms are extremely large, with an extremely small internal magnetic field, and their spins can easily be flipped and ideally be used for information storage.

IV. MAGNETIC FIELD IN THE SPIN-SPIN INTERACTION

The reason why we favor using a spin as an energy/angular momentum source to drive another spin (as a bit of information) is twofold: on the one hand, the magnetic interaction between two electronic spins can impose a change in their orientation and thereby alter the stored information if the spins are used for data storage; on the other hand, the key to calculating a minimum energy to flip a spin is to find a minimum magnetic field B in Eq.3, and a spin is the smallest magnet (Bohr magneton) one can use to flip another spin. As shown in Fig. 5, the magnetic field of one Bohr magneton ($\vec{\mu}_B$) applied on the other magneton is given as

$$\vec{B} = \frac{\mu_0}{4\pi r^3} \left[\frac{3(\vec{\mu}_B \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu}_B \right], \quad (5)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$ is the vacuum permeability constant [9]. If $\vec{\mu}_B$ is colinear with \vec{r} , the generated magnetic field is also along \vec{r} , and its strength is

$$B = \frac{\mu_0}{4\pi} \frac{2\mu_B}{r^3}. \quad (6)$$

In 2014, the magnetic interaction between the two ground-state spin-1/2 valence electrons of two $^{88}\text{Sr}^+$ ions across a separation ($d = 2.18 \sim 2.76 \mu\text{m}$) was reportedly observed and measured [13]. The experimental setup is shown in Fig.

6(a). For $^{88}\text{Sr}^+$, the two spin states of the valence electron are $|\uparrow\rangle = |5s_{1/2}, J = \frac{1}{2}, M_J = 1/2\rangle$ and $|\downarrow\rangle = |5s_{1/2}, J = \frac{1}{2}, M_J = -1/2\rangle$. As mentioned above, an s -state electron has no orbital angular momentum, so it does not experience any internal magnetic field due to spin-orbit coupling since there is no spin-orbit splitting for this state.

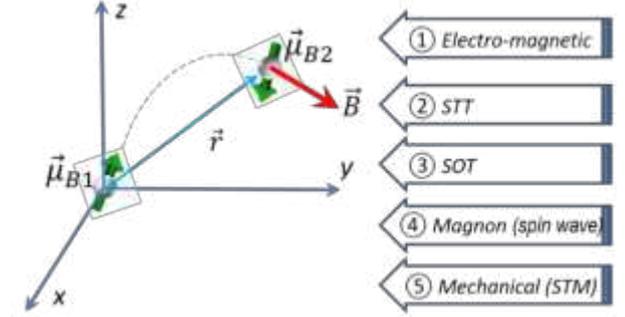


Fig. 5 One Bohr magneton applies a magnetic field B on another magneton. Note that the magnetic field experienced by one magneton may not be the same as the field experienced by the other considering their possible orientations. In this work, we have covered five different energy-inputting means to manipulate the spin-spin interactions (see main text for details).

As shown in Fig. 6(b), the four eigenstates of the two-ion Hamiltonian (the spin part) are $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ and the two entangled Bell states $|\psi_{\pm}\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$. The energy splitting between the last two is $\frac{\mu_0}{4\pi} \frac{4\mu_B^2}{hd^3}$ (note that this is independent from the external magnetic field B_{ext}), where h is the Planck constant. The spin-spin interaction within the decoherence-free subspace can be described by the geometric Bloch sphere [Fig. 6(c)], in which, starting from the north pole $|\uparrow\downarrow\rangle$, the system rotates through the fully entangled state $|\chi+\rangle = (|\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle)/\sqrt{2}$ and towards the south pole $|\downarrow\uparrow\rangle$. The apparatus of the authors enabled them to place the electronic spins at a controlled distance (via the trap voltage against the Coulomb repulsion) from one another, as well as to initialize the system state to $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$ (by optical pumping), manipulate (perform all possible collective spin rotations by pulsing a resonant radio-frequency magnetic field) and detect their internal spin state (by state-selective fluorescence) with high fidelity [13].

Their measurements showed that the spin-spin interaction depends on the interelectron distance, with a $d^{-3.0(4)}$ distance dependence [13], obeying the inverse-cube law as illustrated in Eq.6. Therefore, as shown in Fig. 6(a), one electron applies a magnetic field B on the other (colinear with the line linking them), and the strength of this field can be calculated based on Eq.6 as follows:

$$B = \frac{\mu_0}{4\pi} \frac{2\mu_B}{d^3} = \frac{10^{-7} \text{ T}\cdot\text{m}/\text{A} \times 2 \times 9.27 \times 10^{-24} \text{ A}\cdot\text{m}^2}{(2.76 \times 10^{-6} \text{ m})^3} = 8.82 \times 10^{-14} \text{ T}, \quad (7)$$

where $d = 2.76 \mu\text{m}$ is the largest separation for which the spin-spin interaction is still effective with a fidelity of above 98% [13]. Note that the used separation ($d = 2.76 \mu\text{m}$)

between the two spins is five orders of magnitude larger than the Bohr radius ($5.29 \times 10^{-11} \text{ m}$, which is equal to the most likely distance between the nucleus and the electron in a hydrogen atom in its ground state). This separation is close to the radius ($4.5 \text{ }\mu\text{m}$) of the largest human-made Rydberg atom ($n=290$) considering the likelihood that the radius of the largest (human-made) Rydberg atom may represent the largest distance for which the attractive Coulomb force (between an electron and a proton) is still effective on Earth.

According to Eq.3, the energy to flip a spin via the spin-spin interaction in the presence of the magnetic field B is:

$$\Delta E = 2\mu_B B \left(= 2\mu_B \frac{\mu_0 2\mu_B}{4\pi d^3} \approx \frac{\mu_0 4\mu_B^2}{4\pi d^3} \right) = 2 \times 9.27 \times 10^{-24} \text{ J/T} \times 8.82 \times 10^{-14} \text{ T} = 1.64 \times 10^{-36} \text{ J}. \quad (8)$$

In this new energy bound (Eq.8) for storing data in a spin, an environmental parameter (magnetic field B in which the spin immerses itself) is analogous to another environmental parameter (temperature T) in the Landauer bound (Eq.2) in the sense that neither the magnetic field B nor temperature T should be zero.

Although $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are indistinguishable, as illustrated in Fig. 6(b), the energy limit of flipping a spin can still be found by the above equation since it is proportional to the aforementioned energy splitting in frequency ($\frac{\mu_0 4\mu_B^2}{4\pi \hbar d^3} = 2 - 5 \text{ mHz}$) between the two entangled Bell states $|\psi_{\pm}\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$. In fact, our calculated new energy bound ($\Delta E = 1.64 \times 10^{-36} \text{ J}$) has been verified by this spin-spin interaction experiment since $\frac{\Delta E}{\hbar} = \frac{\mu_0 4\mu_B^2}{4\pi \hbar d^3} = \frac{1.64 \times 10^{-36} \text{ J}}{6.63 \times 10^{-34} \text{ J}\cdot\text{s}} = 2.47 \text{ mHz}$, which matches the lower end of the measured frequency range (2-5 mHz) [13].

This energy bound ($1.64 \times 10^{-36} \text{ J}$) is much lower than the Landauer bound ($3 \times 10^{-21} \text{ J}$). A good example taking advantage of such spin-spin interactions is the spin-transfer torque (STT) [14] and spin-orbit torque (SOT) [15], in which a spin-polarized current is directed into a magnetic layer, and the angular momentum can be transferred to change its spin orientation. STT/SOT can be used to flip the storage elements in magnetic random-access memory with no volatility and near-zero leakage power consumption; however, it is normally associated with a charge flow (an electron-mediated spin current) whose areal density is $\sim 3.4 \times 10^5 \text{ A/cm}^2$ for SOT switching [15] (two orders of magnitude smaller than that for STT switching) and thereby suffers from substantial energy dissipation caused by Joule heating.

In a recent work published in November 2019, magnon-torque-induced magnetization switching was experimentally demonstrated, in which a magnon current carries spin angular momentum (to flip spins) without involving moving electrons [16]. This experiment introduced another way to use the spin-spin interaction to flip spins in an energy-efficient way and will facilitate magnon-based memory/logic devices. Nevertheless, the utilized

antiferromagnet/insulator/ferromagnet interface is as thin as 25 nanometers [16], which is much smaller than the spin-spin separation ($d = 2.76 \text{ }\mu\text{m}$) that we used to estimate the minimum energy to flip a spin. In other words, our calculated new energy limit appears to be a reasonable minimum so far.

Note that this new energy limit represents purely the (magnetostatic) potential energy in a magnetic field and does not include any heat dissipation. It is worth mentioning, linking information and thermodynamics, the Landauer bound itself is actually the heat generated in erasing a (classical) bit of information, which inevitably results in physical and logical irreversibilities.

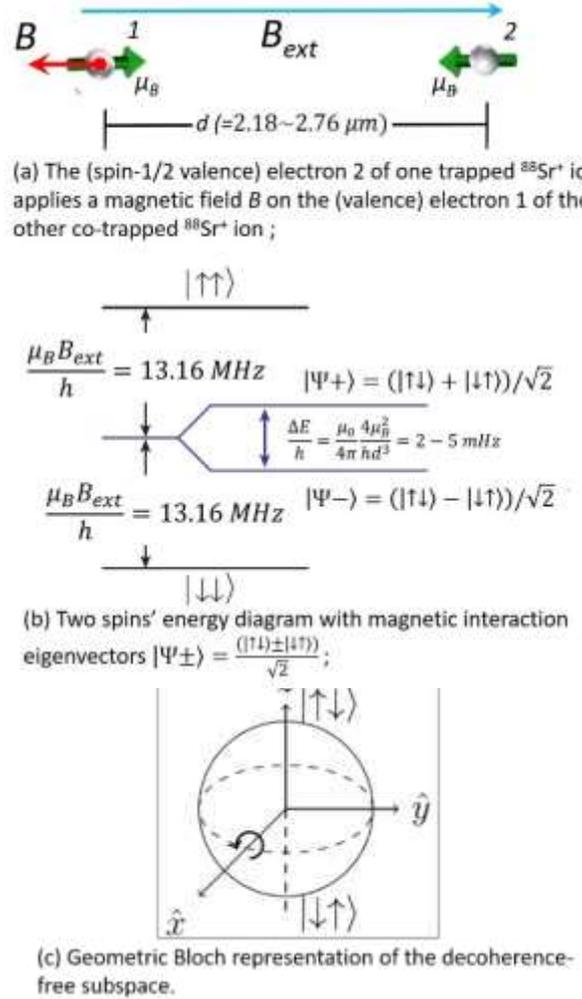


Fig. 6 In 2014, the weak, millihertz-scale magnetic interaction between the two ground-state spin-1/2 valence electrons of two $^{88}\text{Sr}^+$ ions against relatively strong magnetic noise across a separation ($d = 2.18 \sim 2.76 \text{ }\mu\text{m}$) was observed and measured [13]. It was found that the two ions can be tightly confined and laser-cooled to their mechanical ground state [13]. Note that an external magnetic field $B_{\text{ext}} (\neq B)$ was applied along the line (\hat{z}) connecting the two ions during the measurements. In a quantum system, such a magnetic field promotes the quantum mixing of the 'up' and 'down' spin orientations. One can initialize the system to $|\uparrow\downarrow\rangle$ along that direction and measure the state projections on the $(|\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle)/\sqrt{2}$ basis along \hat{y} and rotations of the system state around the \hat{x} axis (the system dynamics are nearly ideal $(|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle)$ rotation). The energy splitting $\Delta E = \hbar f = \frac{\hbar c}{\lambda}$, where \hbar is the Planck constant, f is the frequency, c is the speed of light in vacuum and λ is the wavelength.

In 2011, an information erasure scheme was proposed in which the energy cost can, at least in principle, be ambitiously reduced to zero by paying a cost in angular momentum or any other conserved quantity [10]. The scheme of the authors entails putting the memory spin in spin-exchange contact with a reservoir of spins (if this reservoir contains only one spin, the setup is similar to that of the spin-spin interaction experiment), letting the combined reservoir-memory spin system come to equilibrium (conservation of angular momentum), and then separating the memory spin from the reservoir [10]. An extremely vital assumption of their claim is that there are no residual magnetic fields in the vicinity of the spins that would remove their energy degeneracy and produce an energy cost associated with the above operations. Obviously, this assumption is not affirmed at all in the spin-spin interaction experiment, in which there exists a minimal magnetic field of $8.82 \times 10^{-14} T$, which is indispensable and inescapable in terms of effecting the spin-spin interaction as elaborated above.

V. COMPARISON OF THE ENERGY LIMITS WITH DIFFERENT MAGNETIC FIELD SOURCES

Table 2 lists examples of magnetic field B produced by various sources, ordered by orders of magnitude [17], as well as the corresponding energies needed to flip a spin with different magnetic field sources. Note that the magnetic field drops off as the cube of the distance from a dipole source, as illustrated in Eq.5.

From this table, it can be seen that the energy ($1.64 \times 10^{-36} J$) in the spin-spin interaction is much lower (4-5 orders of magnitude) than that ($2.91 \times 10^{-31} J$ for $n=290$ or $2.87 \times 10^{-32} J$ for $n=630$) for a spin that is bound in an atom. Therefore, it is reasonable for us to take $1.64 \times 10^{-36} J$ as the minimum energy limit to flip a spin if we shield our data storage devices well from those external magnetic field noises from various sources (labeled "Outside Matter" in Table 2).

An s -state electron (bound in an atom) does not experience any internal magnetic field in the atom (as mentioned above) but still needs a minimum energy ($1.64 \times 10^{-36} J$) to flip it.

TABLE 2 Energy to flip a spin with different magnetic field sources, grouped by inside/outside matter, in contrast to the Landauer bound ($3 \times 10^{-21} J$) for classical data storage.

Magnetic Field Sources		Magnetic Field B	Energy to Flip a Spin in B ($\Delta E = 2\mu_B B$)
Inside matter	The spin is bound in an atom (s state, $l=0$ for all n values).	0	$1.64 \times 10^{-36} J$ *
	The spin is bound in an atom ($n=2, l=1$).	0.388 T	$7.2 \times 10^{-24} J$
	The spin is bound in an atom ($n=290, l=1$).	$1.58 \times 10^{-8} T$	$2.91 \times 10^{-31} J$
	The spin is bound in an atom ($n=630, l=1$).	$1.55 \times 10^{-9} T$	$2.87 \times 10^{-32} J$
	The spin interacts with another spin.	$8.82 \times 10^{-14} T$	$1.64 \times 10^{-36} J$
Outside matter	Human brain magnetic field [17].	100 fT to 1 pT [17]	$1.85 \times 10^{-36} \sim 1.85 \times 10^{-35} J$
	In 2006, NASA found "potholes" in the magnetic field in the heliosheath around our solar system that are 10 picoteslas, as reported by Voyager 1 [18].	10 pT [18]	$1.85 \times 10^{-34} J$
	Magnetic field produced by a toaster, in use, at a distance of 30 cm [19].	60 nT to 700 nT [19]	$1.11 \times 10^{-33} \sim 1.30 \times 10^{-31} J$
	Magnetic field produced by residential electric distribution lines (34.5 kV) at a distance of 30 cm [17].	100 nT to 500 nT [17]	$1.85 \times 10^{-33} \sim 9.25 \times 10^{-33} J$
	Magnetic field produced by a microwave oven, in use, at a distance of 30 cm [17].	4 μ T to 8 μ T [17]	$7.4 \times 10^{-32} \sim 1.48 \times 10^{-31} J$
	Strength of magnetic tape near tape head [17].	24 μ T [17]	$4.44 \times 10^{-31} J$
	Earth's magnetic field at 0° latitude (on the equator) [17].	31 μ T [17]	$5.74 \times 10^{-31} J$
	Earth's magnetic field at 50° latitude [17].	58 μ T [17]	$1.07 \times 10^{-30} J$
	A typical refrigerator magnet [17].	5 mT [17]	$9.27 \times 10^{-26} J$
	The magnetic field strength of a sunspot [17].	150 mT [17]	$2.78 \times 10^{-24} J$
	Inside the core of a modern 50/60 Hz power transformer [20].	1 T to 2 T [20]	$1.85 \times 10^{-23} \sim 3.71 \times 10^{-23} J$
	Modern high-resolution research magnetic resonance imaging system; field strength of a 400 MHz NMR spectrometer [21].	9.4 T [21]	$1.74 \times 10^{-22} J$

An *s*-orbital electron with zero orbital angular momentum has spherical orbitals, which does not mean it moves around on the boundary of that sphere (if it was a classical particle); rather, it means that it has some probability of being found anywhere in that sphere.

This new energy bound ($1.64 \times 10^{-36} \text{ J}$) should be universal for a spin to be flipped regardless of whichever form of energy (electrical, magnetic, optical, chemical or even mechanical) is input unless new experimental evidence (e.g., the spin-spin interaction can still be observed while $d > 2.76 \mu\text{m}$) appears in the future proving that a lower energy is possible to flip a spin. This endeavor will no doubt be a major challenge since even the current measurement ($d \leq 2.76 \mu\text{m}$) was carried out in the presence of magnetic noise that was six orders of magnitude larger than the magnetic fields that the electrons apply on each other [13].

In 2010, the manipulation of the spin direction of an individual Co atom was performed by moving it with the aid of an approaching STM (scanning tunneling microscopy) tip along the in-plane antiferromagnetic Mn spin spiral (Mn magnetic moment up and down, alternatively) [22]. The authors found that the Co spin directions had been changed by $\sim 173^\circ$ due to exchange coupling between the Co spin and the (parallel/antiparallel) Mn spins [22]. This is a good example of taking advantage of the spin-spin interaction and using other energy forms (where it is the mechanical energy in spite of its inefficiency and heat dissipation in this case) to flip a spin, but we should bear in mind that the equilibrium distance between the Co and Mn atoms amounts to 2.286 \AA [22], which is (again) much smaller than the spin-spin separation ($d = 2.76 \mu\text{m}$) that we used to estimate the minimum energy to flip a spin. That is, our calculated energy limit still appears to be a reasonable minimum so far.

VI. CONCLUSION

In this work, we found that a minimum energy of using a spin's orientation to store data should be expressed by $\Delta E = 2\mu_B B$, in contrast to the well-known Landauer formula, $\Delta E = k_B T \ln(2)$, for classical data storage (Fig. 7). These two formulas are different because the physics of using a spin's orientation to store a bit of information is fundamentally different from that of using a particle's position as a (classical) bit of information: the former is quantum dynamic (independent of temperature below the Curie point), whereas the latter is thermodynamic (dependent on temperature). The decoupling between the (temperature-dependent) spatial degree of freedom and the (temperature-independent) spin degree of freedom means that the Landauer bound (which is dependent on temperature T) should no longer be used in the quantum case. The new energy limit is estimated to be approximately $1.64 \times 10^{-36} \text{ J}$, 15 orders of magnitude lower than the Landauer bound ($3 \times 10^{-21} \text{ J}$).

We stress that this new energy limit does not include any heat dissipation, which implies that it is possible to realize reversible computation that is highly energy efficient. We conclude that spin electronics in data storage is fundamentally

superior to classical charge-based methods in terms of energy efficiency and computational reversibility. We also verified the above limits based on a number of experiments including the Rydberg atom [12] and spin-spin interactions [13][16][22].

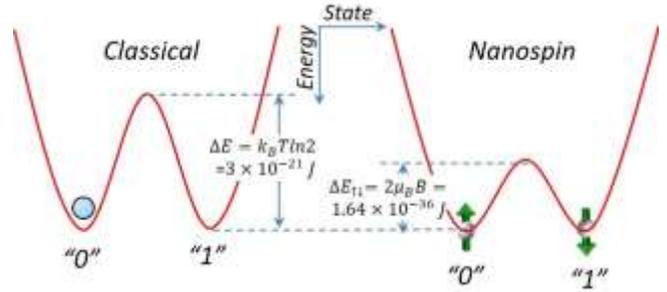


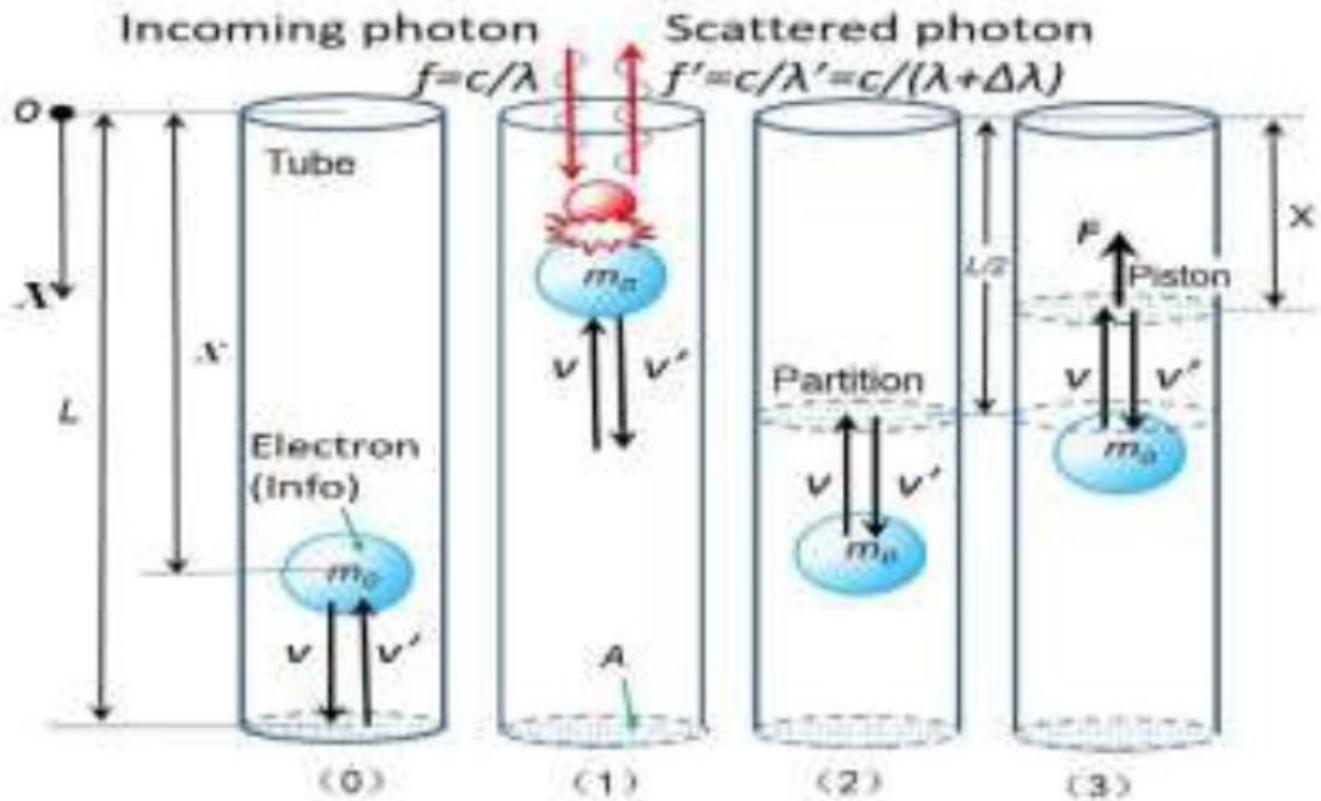
Fig. 7 Schematic comparison between the (classical) energy bound and our new energy bound for nanospin. To erase a classical bit of information, the barrier in the bistable potential well separating the two bistable states can be lowered (as illustrated in Fig. 1), and a possible thermal fluctuation allows the system to fluctuate between the two quasidegenerate states (Note that Landauer's principle is valid even if the gap is zero.). As mentioned in the main text, quantum tunneling between quasidegenerate $\pm S_z$ states can be promoted, which speeds up spin reversal.

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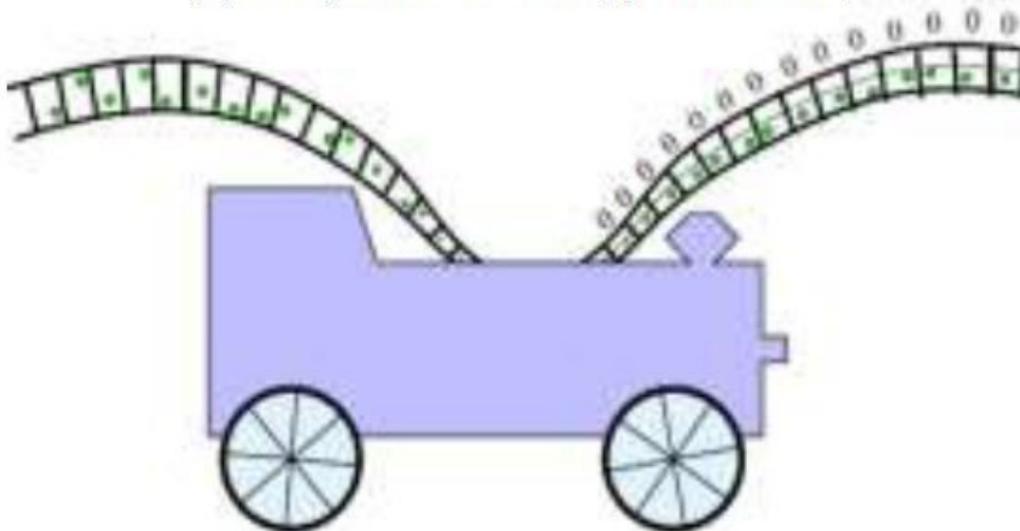
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Figures



(a) A tiny tube with a trapped electron;

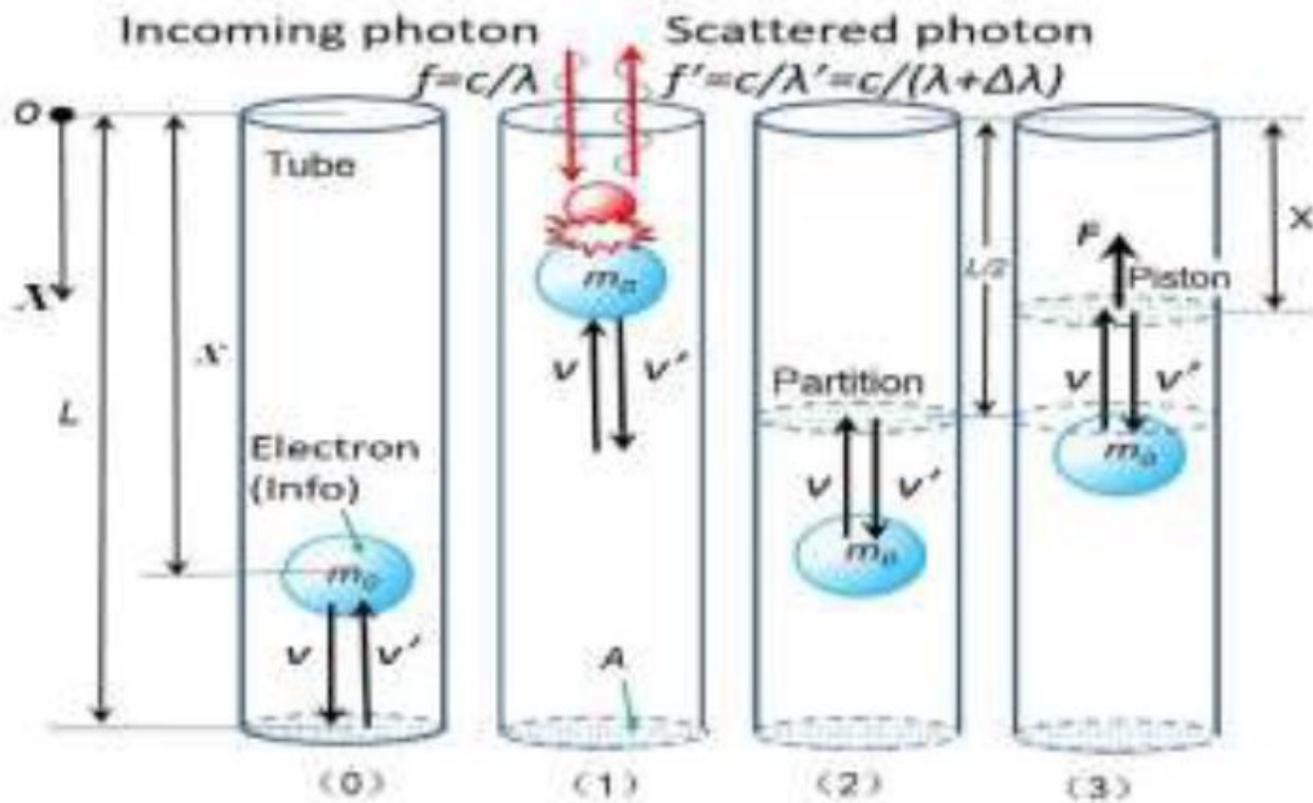


(b) An information engine "burning" information.

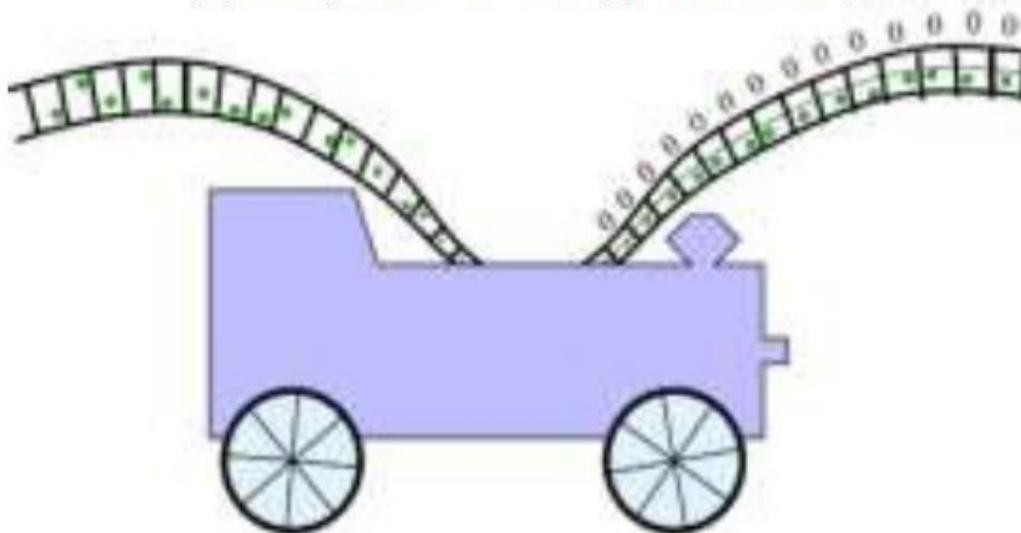
Figure 1

A one-dimensional nanotube with a trapped electron. In (0), the electron only moves upwards and downwards, v and v' denote the velocities before and after the collision, respectively. Creating info is a twostep process: (1) emit a photon to push the info electron to the desired half; (2) add the partition to

confine the info electron. In (3), if the partition is a frictionless piston, the info particle can push it to exert useful work in exchange for losing that written info (which side of the partition is occupied). This is a so-called “information engine” [Fig. 1(b)].



(a) A tiny tube with a trapped electron;



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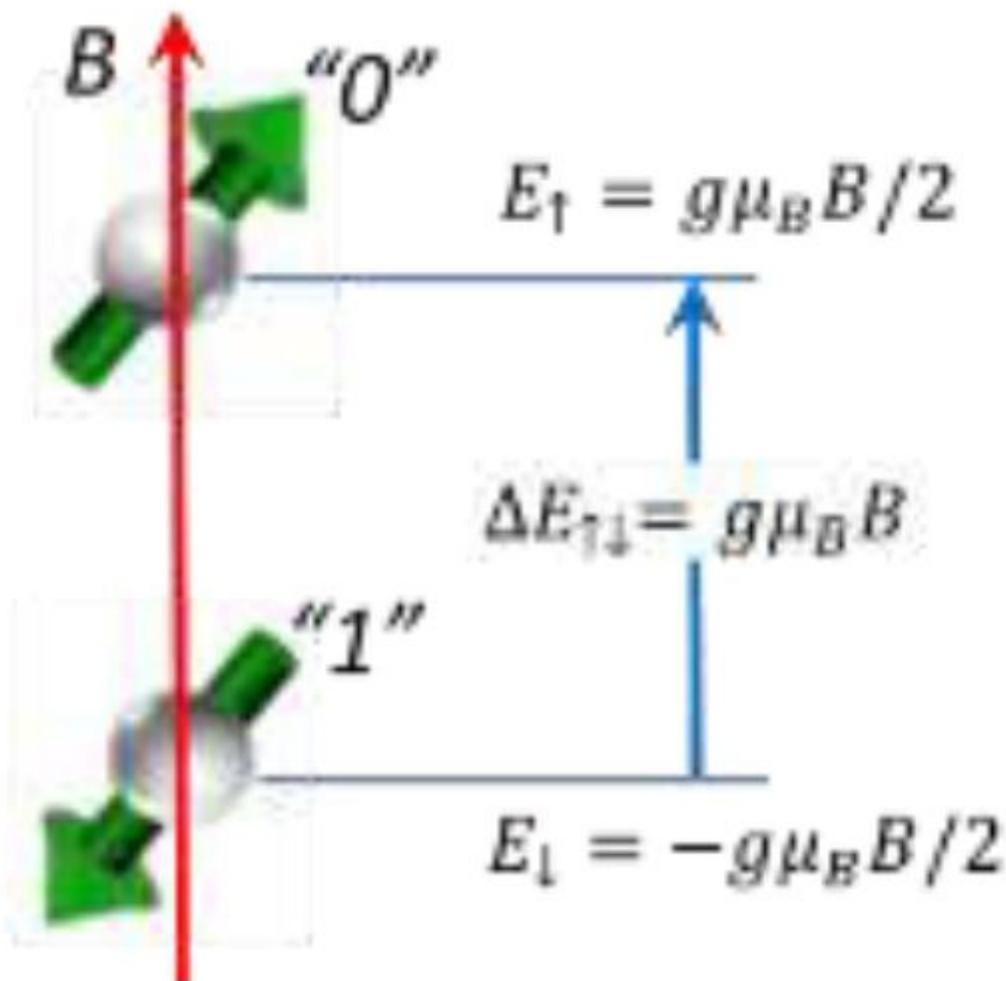


Figure 2

Energy of a spin in a magnetic field B . The spin angular momentum is quantized with only two possible z -components and is independent of temperature [the ordered magnetic moments of ferromagnetic materials change and become disordered (paramagnetic) at the Curie temperature, which is normally well above room temperature]. The decoupling between the (temperature-dependent) spatial degree of freedom and the (temperature-independent) spin degree of freedom means that the Landauer bound (which is dependent on temperature T) should no longer be used in this case.

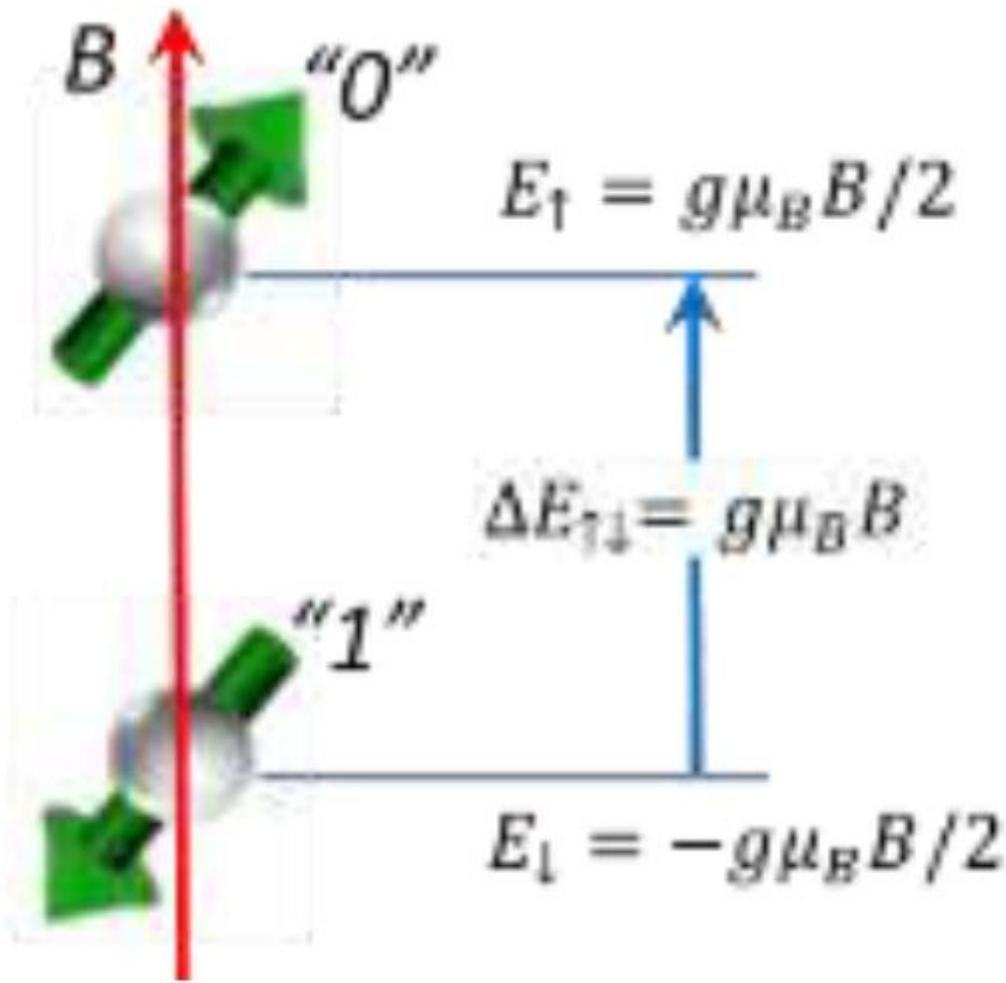


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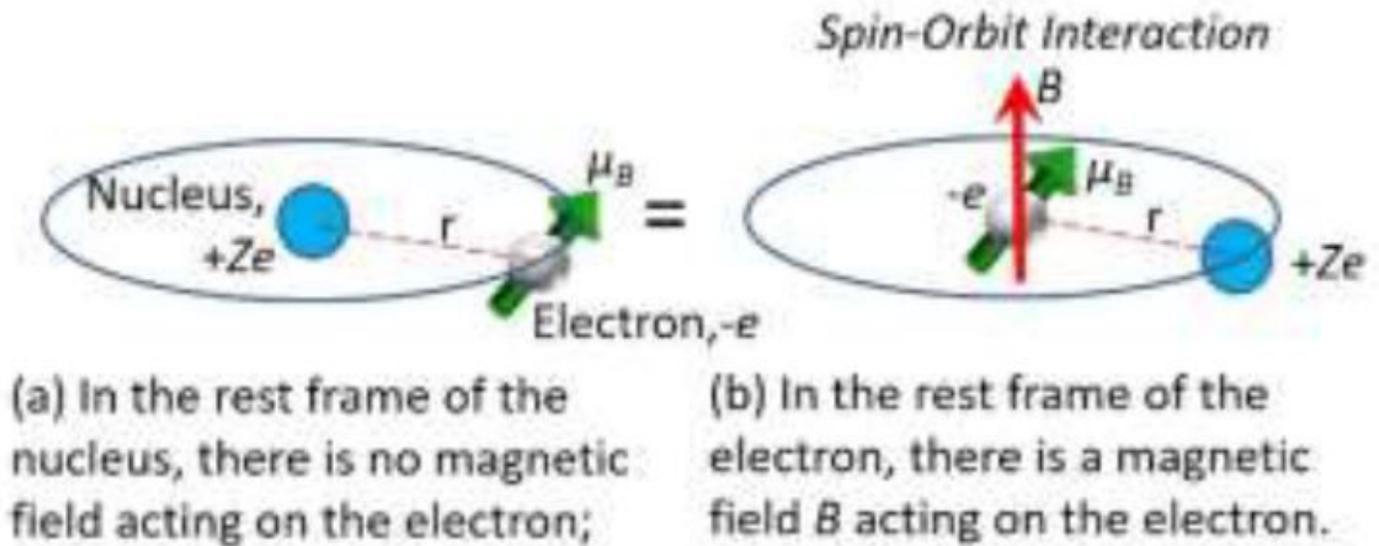


Figure 3

Although in the rest frame of the nucleus, there is no magnetic field acting on the electron, there is a magnetic field in the rest frame of the electron. B in (b) is such a magnetic field seen from the electron perspective. Z is the effective central charge, which includes all other charges of the nucleus and inner electrons. e is the elementary charge.

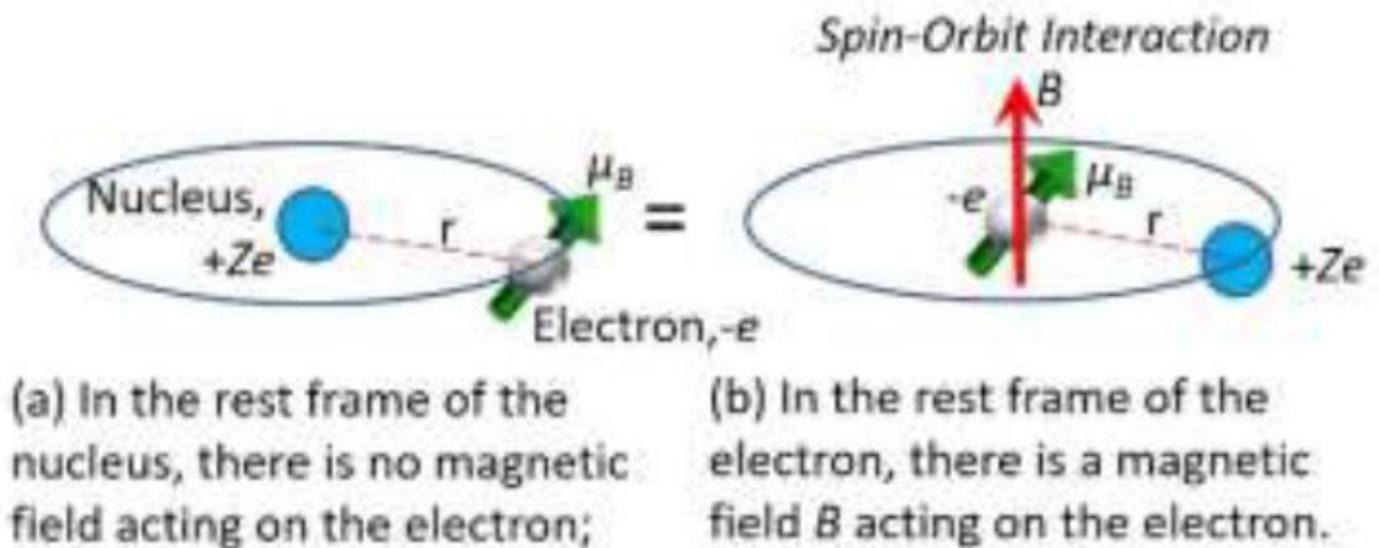


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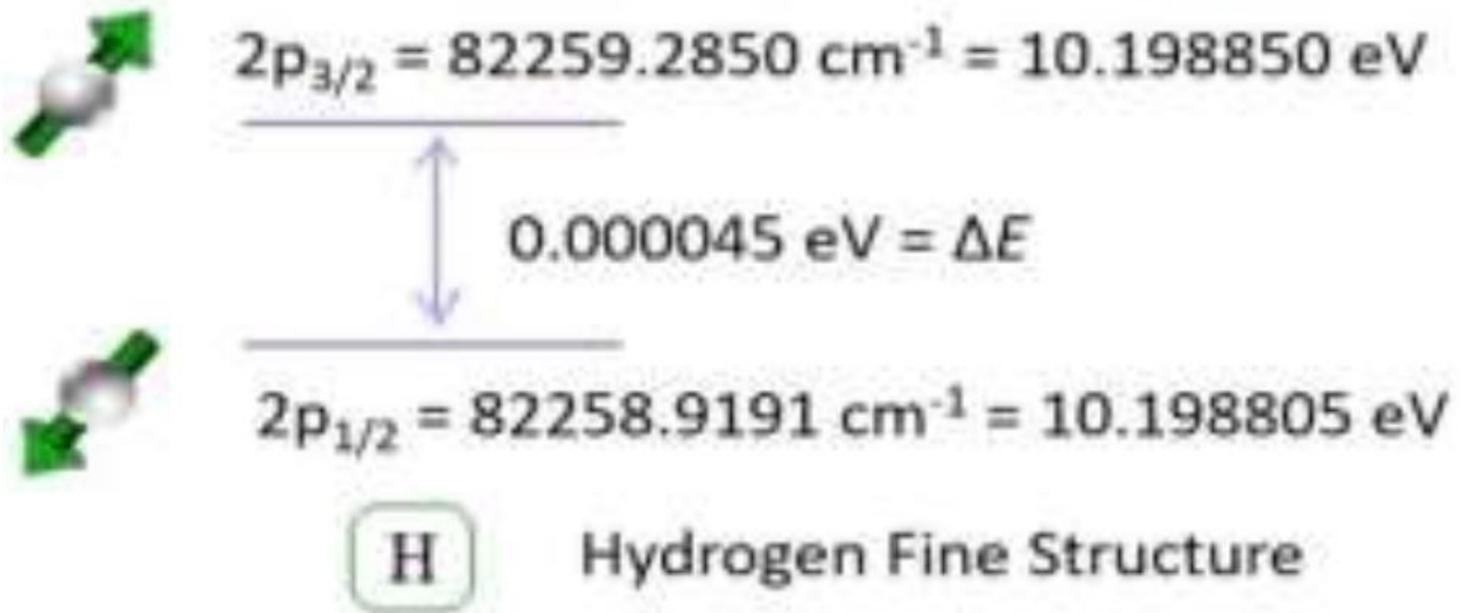


Figure 4

Hydrogen $2p_{3/2}$ - $2p_{1/2}$ fine structure. It is the spin-orbit interaction that splits the hydrogen atomic energy levels and gives rise to a fine structure in the spectra of the hydrogen atom. As shown in the figure, the energy separation (0.000045 eV) between $2p_{3/2}$ and $2p_{1/2}$ corresponds to $\lambda = 2.74 \text{ cm} = 2.74 \times 10^{-2} \text{ m}$ ($= \frac{hc}{\Delta E}$), which has been verified by an absorption spectrum experiment for hydrogen: the $2p_{3/2} \rightarrow 2p_{1/2}$ transition requires a wavelength of 2.74 cm ($= \frac{hc}{\Delta E}$) to be absorbed) [7].

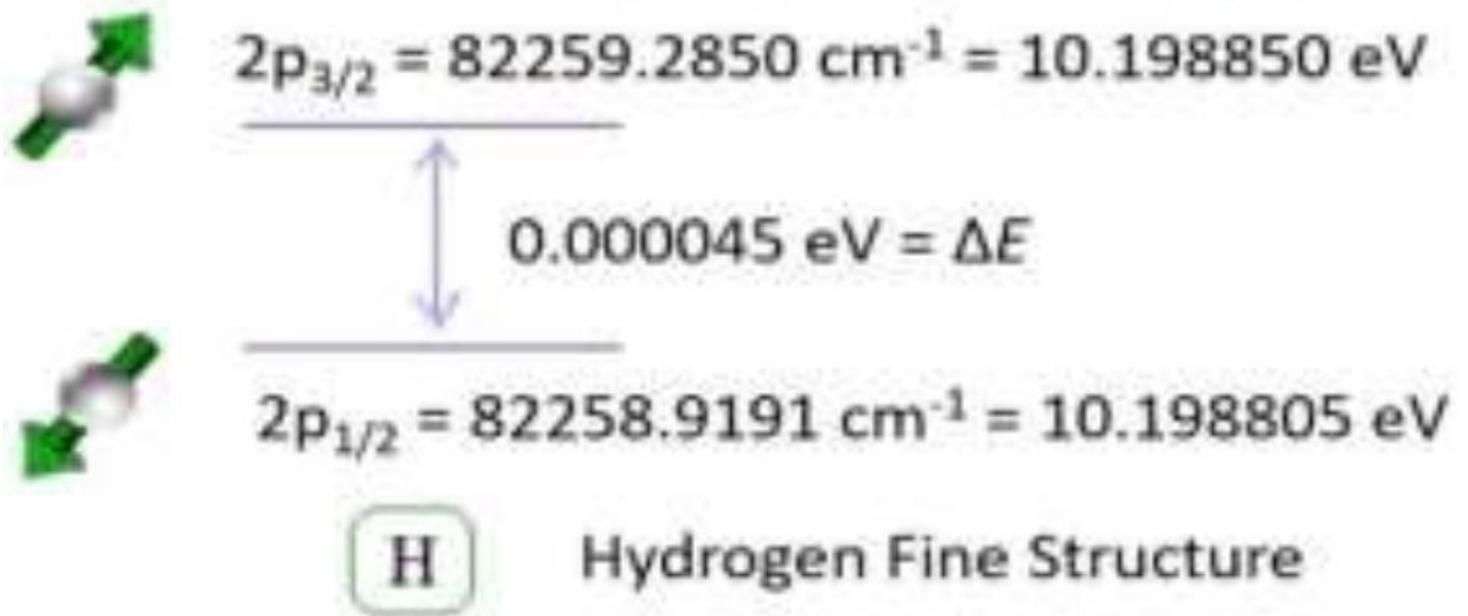


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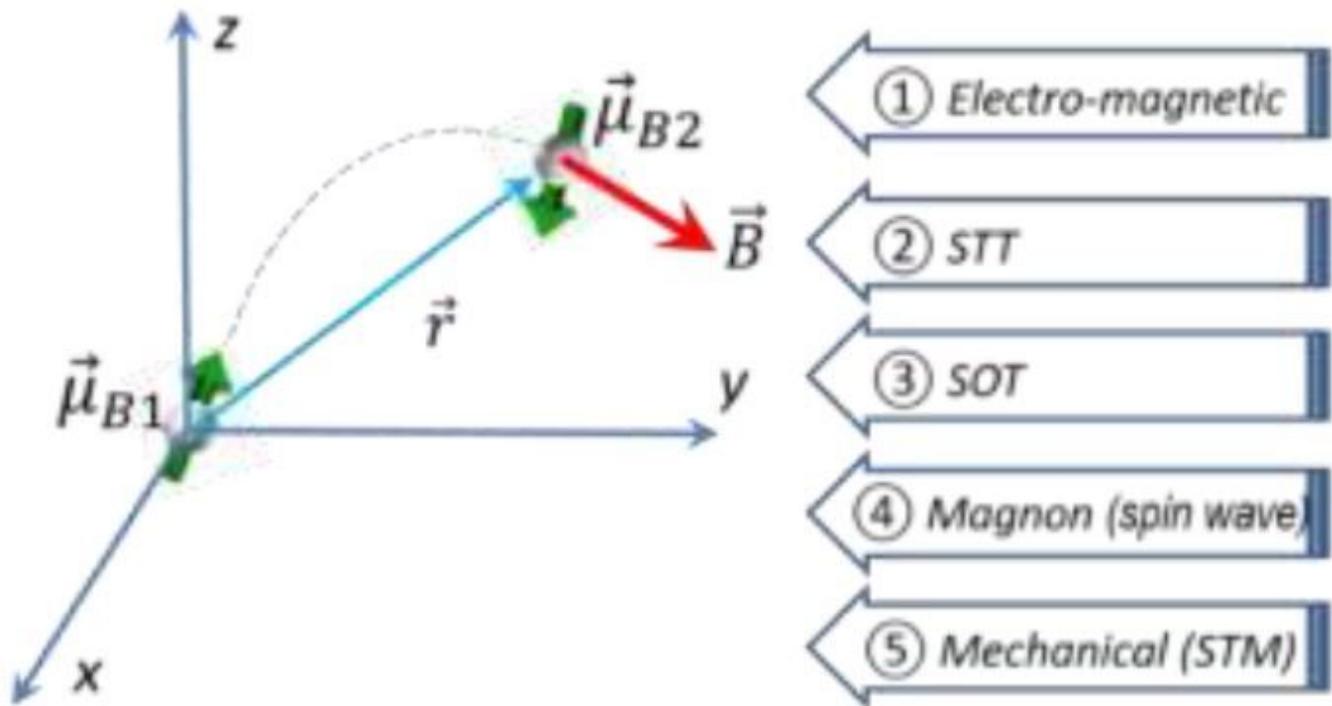


Figure 5

One Bohr magneton applies a magnetic field B on another magneton. Note that the magnetic field experienced by one magneton may not be the same as the field experienced by the other considering their possible orientations. In this work, we have covered five different energy-inputting means to manipulate the spin-spin interactions (see main text for details).

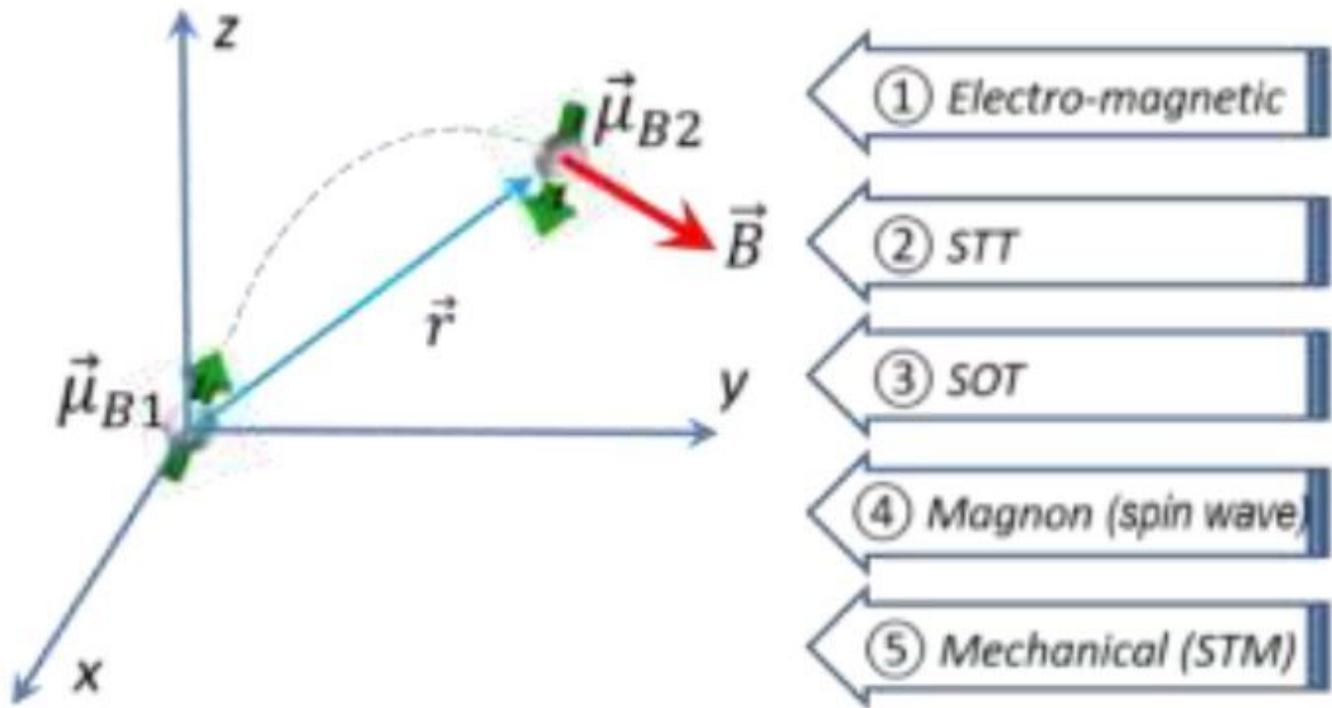
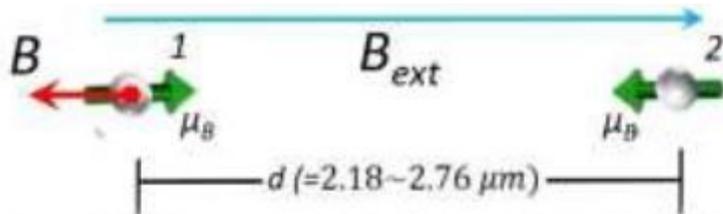
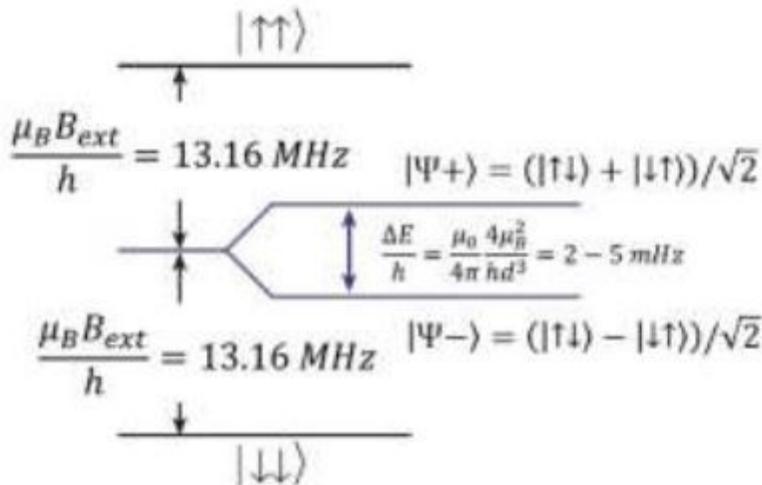


Figure 5

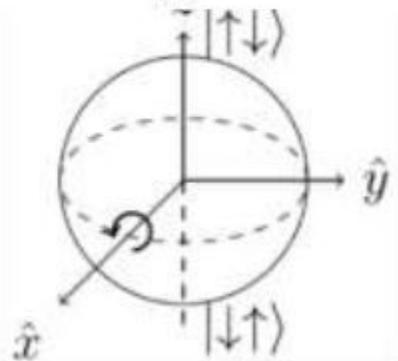
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(a) The (spin-1/2 valence) electron 2 of one trapped $^{88}\text{Sr}^+$ ion applies a magnetic field B on the (valence) electron 1 of the other co-trapped $^{88}\text{Sr}^+$ ion ;



(b) Two spins' energy diagram with magnetic interaction eigenvectors $|\Psi_{\pm}\rangle = \frac{(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)}{\sqrt{2}}$;

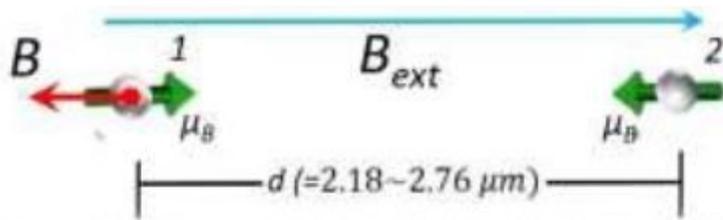


(c) Geometric Bloch representation of the decoherence-free subspace.

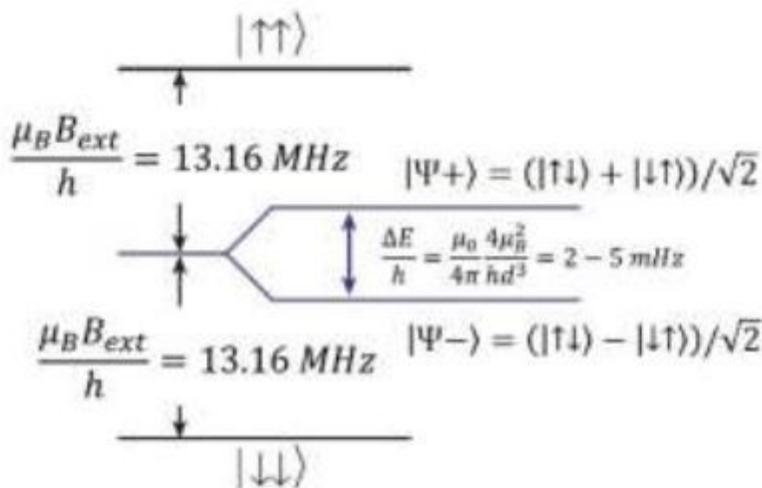
Figure 6

In 2014, the weak, millihertz-scale magnetic interaction between the two ground-state spin-1/2 valence electrons of two $^{88}\text{Sr}^+$ ions against relatively strong magnetic noise across a separation ($d = 2.18 \sim 2.76 \mu\text{m}$) was observed and measured [13]. It was found that the two ions can be tightly confined and laser-cooled to their mechanical ground state [13]. Note that an external magnetic field $B_{\text{ext}} (\neq 0)$ was applied along the line (\hat{x}) connecting the two ions during the measurements. In a quantum system, such a magnetic field promotes the quantum mixing of the 'up' and 'down' spin orientations. One can initialize the system to $|\uparrow\uparrow\rangle$ along that direction and measure the state projections on the $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ basis

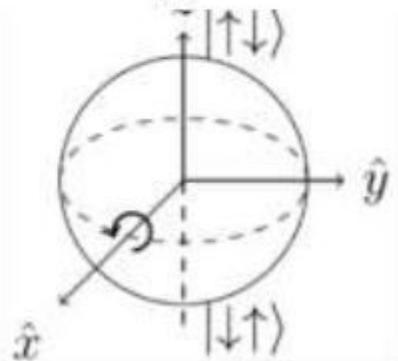
along \hat{x} and rotations of the system state around the \hat{x} axis (the system dynamics are nearly ideal (\hat{y} \hat{z} rotation)). The energy splitting $\Delta E = \hbar \omega = \hbar \mu_B / d$, where \hbar is the Planck constant, f is the frequency, c is the speed of light in vacuum and λ is the wavelength.



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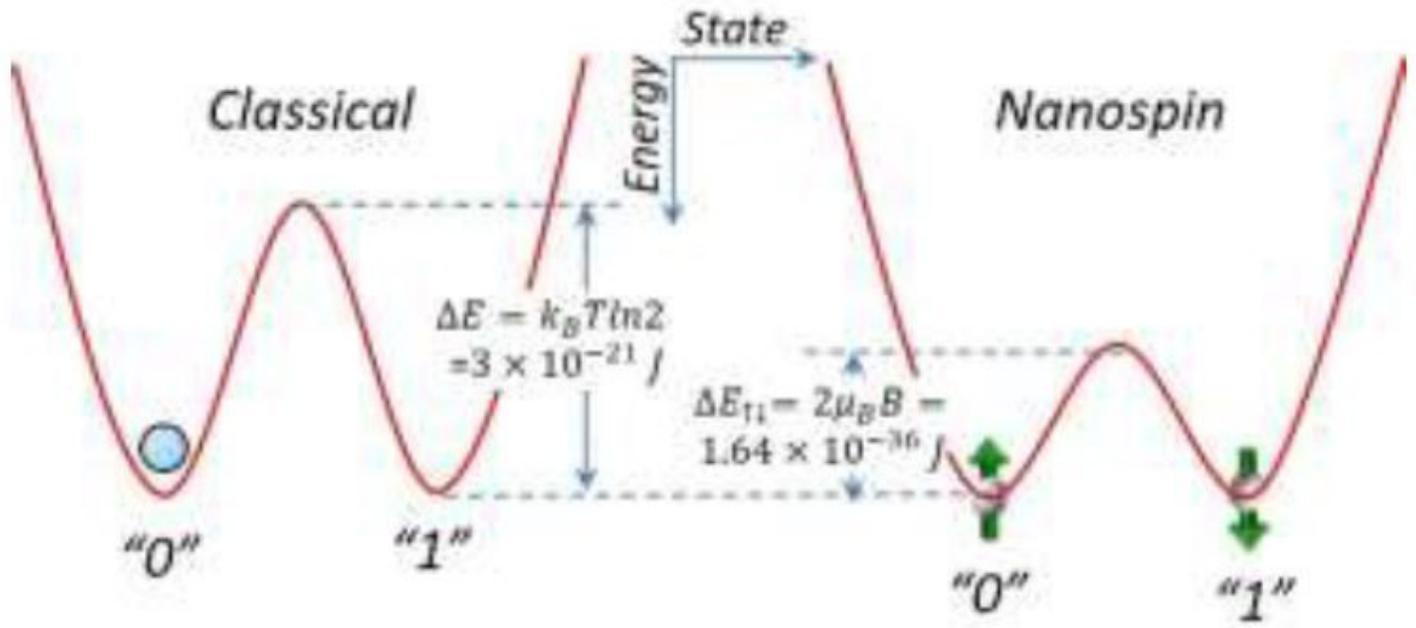


Figure 7

Schematic comparison between the (classical) energy bound and our new energy bound for nanospin. To erase a classical bit of information, the barrier in the bistable potential well separating the two bistable states can be lowered (as illustrated in Fig. 1), and a possible thermal fluctuation allows the system to fluctuate between the two quasidegenerate states (Note that Landauer's principle is valid even if the gap is zero.). As mentioned in the main text, quantum tunneling between quasidegenerate ± 1 states can be promoted, which speeds up spin reversal.

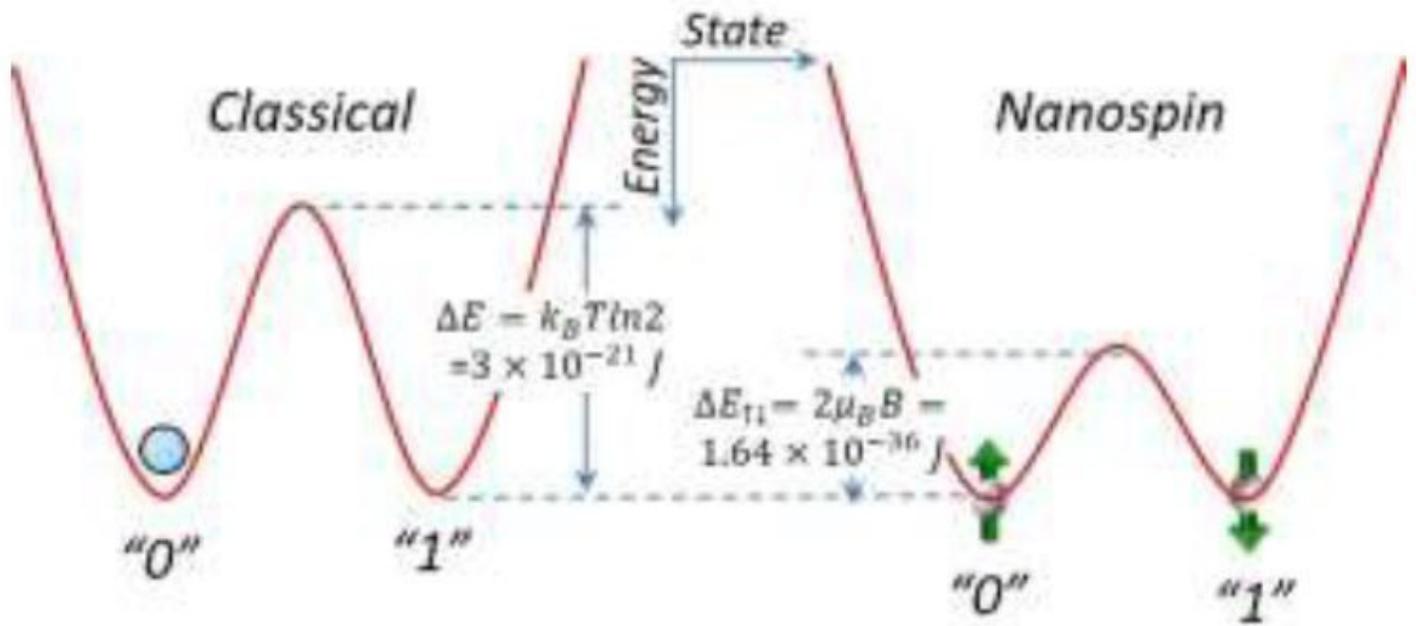


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