The Λ CDM Model

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The Λ CDM Model

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Abstract – A model for the reduction in Universal density over time, the “ Λ CDM” model, is derived using gas thermodynamics with $z = 1089$ as the starting point. In the Λ CDM model, the Universe is pushing itself apart with internal gas pressure. A simple three-term Hubble expression H_G is derived and found to be independent, or zero-order, in temperature and molecular weight of the gas. Isoentropic expansion of the gas at any z yields an entropic energy term which is modified to include energetic electrons, derived in turn from high-energy photons. These electrons are proposed as the source of the “dark energy” term found in the Λ CDM model.

The presently preferred description of Universal expansion is the flat-universe Λ CDM model. Its empirical accuracy is accepted as high. The Λ CDM model treats Universal expansion as a function of the sum of the density parameters Ω , three of which (Ω_{rad} , Ω_b , and Ω_c) have comoving mass densities ρ_{rad} , ρ_b , and ρ_c . The fourth parameter Ω_Λ , termed “dark energy”, has a density ρ_Λ which is not comoving, but rather is the same for any volume of space at any time. This model is given in simple form by (1):

$$H^2(a) = H_0^2 [\Omega_{rad} a^{-4} + \Omega_b a^{-3} + \Omega_c a^{-3} + \Omega_\Lambda] \quad (1)$$

Where $H(a)$ is the Hubble parameter at scale factor $a = 1/(1+z)$, z is the cosmic redshift $(\lambda_e - \lambda_0)/\lambda_e$ of an emitted photon of known λ_0 , and H_0 is the present-day Hubble constant, 67.70 Km/sec/Mpc,¹ from the Planck 2018 survey.² The Ω values add up to one and relate their density to the present-day *critical density* $\rho_{crit} = 3H_0^2/8\pi G = 8.6075 \times 10^{-27}$ Kg/m³, where G is Newton’s constant, 6.67408×10^{-11} m³/(Kg-sec²):

$$1 = \sum \Omega_x = \sum \frac{\rho_x}{\rho_{crit}} \quad (2)$$

The critical density ρ_{crit} is the total mass-energy density which, within the Λ CDM model, gives an exact balance to the energy loss from gravity over time.³ By convention, the Ω parameters are expressed with the value $h = H/100$. Table 6 of ref. 2 gives the baryon $\Omega_b h^2 = 0.022447$, so at $z = 0$ $\Omega_b = (10^4 \times 0.022447)/H_0^2 = 0.04898$. Similarly for cold dark matter (CDM), $\Omega_c h^2 = 0.11923$ and $\Omega_c = 0.26014$ right now. For Ω_{rad} at $z = 0$, we treat neutrino energy as relativistic and

¹ This is equal to 2.194×10^{-18} sec⁻¹, and H_0 in sec⁻¹ is used throughout, except for Ω determinations.

² Planck Collaboration, *Astronomy & Astrophysics* 641 (2020) A1. DOI: 10.1051/0004-6361/201833880.

³ In this paper we will be working whenever possible in units of mass M and mass density $\rho = \epsilon/c^2$.

combine its energy with CMB photons.⁴ Ref. 2 gives the redshift of matter-radiation equality at $z = 3387\text{-}3402$, which corresponds at $z = 0$ to $\Omega_{rad} \approx 9.1 \times 10^{-5}$. For Ω_A , Ref. 2 gives $\Omega_A = 0.6894$. We get $\Omega_A = 1 - (0.04898 + 0.26014 + 9 \times 10^{-5}) = 0.6908$; we will use our value. An important variable in the Λ CDM model is the baryon mass density at $z = 0$. Its value is $\Omega_b \rho_{crit} = 4.216 \times 10^{-28} \text{ Kg/m}^3$ by our calculation, close to Ryden and Pogge's value.⁵

The sources of the Λ CDM model are the Friedmann and fluid equations, and the equation of state. We compare, in advance, the differences between the Λ CDM and GCDM models. The reader is referred to some more recent textbooks for a summary of the Λ CDM model.⁶ We focus not on the equations themselves but their assumptions and conclusions, especially the fluid equation, where the differences between the models are most clear. Not all of these differences will be subject to quantitative analysis in the following discussion, but rather serve as postulates of the GCDM model for further quantitative treatment, analysis, and disproof.

The first difference is that the GCDM model considers only the internal kinetic energy $U_i = \frac{1}{2} M \bar{v}_i^2$ of gaseous baryons as contributing to expansion, whereas both the fluid and state equations view the baryon rest mass $U_b = Mc^2$ as a component of internal energy, and therefore the overwhelmingly dominant constituent: $U_b - U_i \approx U_b$.

The second difference is that in the fluid equation, the internal pressure P of the gas is attractive and retards expansion, whereas in the GCDM model, P is repulsive and the source of expansion.

The third difference is that the Λ CDM model makes no separate provision for *entropic energy* $T\Delta S$. The GCDM model includes $T\Delta S$.

The fourth difference is that in the fluid equation, U_b of a comoving volume is constant and there is no heat input Q . In the GCDM model, U_i is fed by Q from high-energy stellar photons and active galactic nuclei.

The fifth difference is that the dark energy component of the Λ CDM model is of unknown origin and nonconservative. In the GCDM model, this is caused by entropic energy gain $T\Delta S$ from photoionization, and is conservative.

We construct the GCDM model using Newtonian laws, and include relativistic energy (mass) and entropic effects later on. First, we select a time: $z = 1089$, just after recombination. This is the earliest at which monatomic gas thermodynamic laws can be reasonably applied. Baryonic

⁴ CMB = cosmic microwave background.

⁵ in Ryden, B. R., Pogge, R.W.; *Interstellar and Intergalactic Medium* (2021), Cambridge University Press, ISBN 978-1-108-74877-3, page 198. They report $4.21 \times 10^{-28} \text{ Kg/m}^3$.

⁶ a) in Ryden, B. R., *Introduction to Cosmology, second edition* (2017), Cambridge University Press, ISBN 978-1-107-15483-4, pages 58-60.

b) in Liddle, A., *An Introduction to Modern Cosmology, third edition* (2015), John Wiley and Sons, ISBN 978-1-118-50214-3, pages 26-27.

matter was almost all neutral gas and acoustic oscillation was minimal so the Universe had constant density. We use the convention that baryons were then present as a mixture of 75% monatomic hydrogen (H_1) : 25% helium (He) by weight, or 92.256 mole % H_1 : 7.744 mole % He. This gives a mean molecular weight $\mathcal{K} = 1.2399 \times 10^{-3}$ Kg/mol.⁷ The baryon density ρ_{1089} was $(\rho_0/a^3) = (4.216 \times 10^{-28})(1090)^3 = 5.46 \times 10^{-19}$ Kg/m³, or 2.6×10^8 atoms per cubic meter. The background radiation had just decoupled and the baryon temperature at $z = 1089$ will be set to the proposed value, 2971 K (CMB = 2.7255 K)($1/a = 1090$).

Absent high-energy photons, a gaseous Universe is adiabatic insofar as U_i is concerned, and the Universe is a “closed” system, so if isoentropically expanding, U_i does all the work: $-\Delta U_i = W$. In a classic setting, there are two kinds of adiabatic gas expansion: reversible and free. Reversible expansion is isoentropic by definition. When a gas expands reversibly, U_i decreases, the gas performs work, and the temperature and pressure drop. When a gas expands freely, U_i does not decrease and only the pressure drops. The temperature stays the same:

$$\partial V = \partial V_S + \partial V_T \quad (3)$$

Both happen cosmically. In *cosmic isoentropic expansion* (∂V_S), not all of the internal kinetic energy lost ($-\Delta U_i$) performs work against gravity. The excess, *entropic energy*, may give additional volume increase. This *cosmic entropic expansion* differs from classic free expansion in that entropic energy is lost to gravity. As with classic free expansion, the internal kinetic energy of the gas, and so its temperature, remains unchanged (∂V_T).

We start with isoentropic expansion. Consider a finite sphere around a single atom of H_1 , of radius r about Earth size (6.3781×10^6 m), at 2971 K, which at ρ_{1089} has baryon mass $M = 593$ Kg. This sphere is still in thermal equilibrium, a major but necessary departure from reality. Nonequilibrium thermodynamics must be set aside so that the underlying transfer of conserved energy is more clearly described. The sphere’s gravitational potential energy (U) is:⁸

$$U = \frac{-3GM^2}{5r} \quad (4)$$

The Λ CDM model contains cold dark matter (CDM), $\Omega_c/(\Omega_b + \Omega_c) \approx 84\%$ of all cold mass in the Universe, and doesn’t act as a gas. Its only influence is gravitational. This is also needed in the GCDM model, and included as a *mass multiplier* $\eta = (\Omega_b + \Omega_c)/\Omega_b = 6.3111$:

$$U = \frac{-3GM^2}{5r} = \frac{-3G(\eta M)^2}{5r} = \frac{-3G(6.3111M)^2}{5r} \quad (5)$$

⁷ Atomic weights: $H_1 = 0.00100797$ kg/m³; He = 0.0040260 kg/m³. Heavier elements and diatomic hydrogen are treated as negligible. Hydrogen is in its isotopic distribution.

⁸ The terms U_i , U_2 , U_r , and U_s refer to gravitational potential energy. The term U_i is used to denote the internal kinetic energy of a gas or plasma, and U_b , the rest mass of the baryons, is used briefly in an energy context.

The multiplier η is adjusted to include photon and neutrino mass later on (eq. 29). The ideal gas law is:

$$PV = nRT = \frac{MRT}{\mathcal{K}} \quad (6)$$

Where R is the universal gas constant ($=8.31446 \text{ J-mole}^{-1}\text{K}^{-1}$). The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3 \quad (7)$$

When (6) and (7) are combined we get the *internal pressure* (P_I):

$$P_I = \frac{nRT}{V} = \frac{\rho RT}{\mathcal{K}} = \frac{MRT}{\mathcal{K}V} = \frac{3MRT}{4\mathcal{K}\pi r_1^3} \quad (8)$$

Entering our values for M , T , and r , we obtain $P_I = 1.09 \times 10^{-11} \text{ Pa}$. We will also suppose that the sphere isn't getting any bigger over time. It is but for now we'll say it isn't. We increase the sphere's radius by $\sqrt[3]{1.01}$, giving a volume increase of one percent.⁹ Work is performed against gravity:

$$U_r = U_1 - U_2 = \frac{-3GM'^2}{5} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (9)$$

Where U_1 and U_2 are the gravitational potential energies at radii r_1 and r_2 respectively. Entering the values for M , r_1 and r_2 we find that $U_r = 2.916 \times 10^{-13} \text{ J}$. The internal kinetic energy loss ($-\Delta U_i = W = E$) is, however, much greater than U_r .¹⁰

$$-\Delta U_i = W = E = \left(\frac{3}{2} \right) P_I V_1 \left(\left(\frac{V_2}{V_1} \right)^{\frac{2}{3}} - 1 \right) \quad (10)$$

Where W has the classic meaning of work performed by the gas, P_I is the internal pressure before expansion, and V_1 and V_2 are the before and after volumes of the sphere respectively. V and P can be calculated from (7) and (8). Entering these into (10) gives $E = 1.17 \times 10^8 \text{ J}$. This is 10^{20} times as much energy released as absorbed. The excess (E_k) is now outward, *radial kinetic energy*:

$$E_k = E + U_r = E + (U_1 - U_2) = \left(\frac{3}{2} \right) P_I V_1 \left(\left(\frac{V_2}{V_1} \right)^{\frac{2}{3}} - 1 \right) - \frac{3GM'^2}{5} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (11)$$

The radial kinetic energy E_k is exactly the entropic energy. They are the same thing:

$$E_k = \Delta(TS) \approx T\Delta S + S\Delta T \approx T\Delta S \quad (12)$$

⁹ We define the *increment* as either $r_2 - r_1 = \Delta r_i$ or as a dimensionless $\frac{r_2 - r_1}{r_1} = \frac{\Delta r_i}{r}$ depending on context.

¹⁰ This and the other thermodynamic expressions are found in many textbooks and, *eg*, Wikipedia.

At $r = 6 \times 10^6$ m, gravity loss is negligible and $E_k = E \approx 10^8$ J. The internal pressure drops to a new value, P_2 :

$$P_2 = P_1 \left(\frac{V_2}{V_1} \right)^{-\frac{5}{3}} \quad (13)$$

Eq. (13) gives $P_2 = 1.06 \times 10^{-11}$ Pa. Dividing E_k by V_2 gives the increase in *entropic pressure* (ΔP_S):

$$\Delta P_S = \frac{E_{k2} - E_{k1}}{V_2} \quad (14)$$

Our sphere was static to start so $E_{k1} = 0$. Our expanded sphere has $\Delta P_S = 1.07 \times 10^{-13}$ Pa, or 1% of P_2 . It's important to emphasize that P_S does not add to P_2 , but is instead a vector quantity which results in radial increase only. If we ignore or “freeze” U_i , each atom can then be seen as moving in a straight line away from the center, like a bunch of tiny rockets blasting away from their despoiled planet, or a bomb going off.¹¹ Entropic pressure already existed in the sphere since the Universe has been expanding all along.

The temperature drop is given as:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{2/5} \quad (15)$$

The temperature drops from 2971 to 2951 K or 0.7%. This can be compared to the CMB-derived temperature: $(2.7255)(1090/\sqrt[3]{1.01}) = 2961$ K, or 0.3% for $r_2 = \sqrt[3]{1.01}r_1$. The gas is cooling slightly faster than the photons. Eq. (15) is also used to verify the calculations. The internal kinetic energy U_i of a monatomic ideal gas is given by:

$$U_i = \frac{3}{2} \frac{MRT}{\mathcal{K}} = \frac{3}{2} \frac{\rho RTV}{\mathcal{K}} \quad (16)$$

The new internal energy $U_{i'}$ can be found with the T_2 value from (15). The residual error $U_i - (U_{i'} + E)$ is exactly zero to the limit of the spreadsheet. We can also get temperature drops over large z changes with eq. (16). Starting at $z = 1089$ and $T = 2971$ K, we set the increment to 10, basically one large step, and get $z = 108.9$. This gives $T_{108.9} = 24.6$ K, and $U_{i,108.9} = 0.0083 U_{i,1089}$. Before star formation around $z = 100$, the Universe was colder than liquid nitrogen, having exhausted 99+% of its internal kinetic energy.¹²

The linear rate of expansion, or *increment radial velocity* ($v_{s'}$) of the sphere is:

¹¹). Some say that “all bombs are entropic” in the sense that there is an instant rise in U_i within the bomb, followed by a large entropic increase when the bomb casing ruptures. This analogy may help the reader to better understand eq. (12).

¹² Calculations at this large of an increment are unreliable, but ΔU_i remains accurate regardless of increment size.

$$v_{s'} = \sqrt{\frac{2E_k}{M}} \quad (17)$$

Direct use of (17) from (11) ignores the fact that the sphere is already expanding, so it's inaccurate and quite low. It's also increment-dependent; larger $\frac{\Delta r_i}{r}$ gives larger $v_{s'}$ values. We can get correct values (v_s) at an instant in time ∂t with (11). We set $\frac{\Delta r_i}{r} = 10^{-9}$ and increase r independently. First we define the *gravity ratio* (X):

$$X = \frac{U_2 - U_1}{E} \quad (18)$$

With our above T , ρ , and $\Delta r_i/r$ held constant, we increase r stepwise. The mass rises and $v_{s'}$ falls until the *adiabatic radius*, or *endpoint* (r_e) is reached, where $X = 1$ and $\frac{\partial E_k}{\partial r} = 0$. This *adiabatic sphere* conserves energy around the central atom; its surface is the *adiabatic surface*. Below the *cutoff radius* ($r_c = 0.003r_e$), gravity can be neglected and (11) simplifies to (10). These *small spheres* all have the same $v_{s'}$, regardless of size, and a constant $\frac{E_k}{M}$ value.¹³

$$\frac{E_k}{M} = \frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V}{\partial M} \frac{\partial E}{\partial V} = \left(\frac{RT}{\mathcal{K}P} \right) \frac{\partial E}{\partial V} = \frac{RT}{\mathcal{K}} \left(\frac{\partial E}{P \partial V} \right) = \frac{RT}{\mathcal{K}} \quad (19)$$

This gives the *initial radial velocity* (v_i):

$$v_i = \sqrt{\frac{2E}{M}} = \sqrt{\frac{2RT}{\mathcal{K}}} \quad (20)$$

We can compare this with the PV change (E) upon incremental increase and see if it's consistent. We increment a small sphere, giving T_2 , and examine the residual error (21):

$$\frac{\frac{1}{2}M \left[\left(v_{i(T_1)} \right)^2 - \left(v_{i(T_2)} \right)^2 \right] - E}{E} \quad (21)$$

By use of (21) we find that in fact, v_i is better expressed by rearranging (16):

$$\frac{U_i}{M} = \frac{3}{2} \frac{RT}{\mathcal{K}} \quad (22)$$

which gives:

$$v_i = \sqrt{\frac{2U_i}{M}} = \sqrt{\frac{3RT}{\mathcal{K}}} = \sqrt{\frac{3(8.3145)(2971)}{(1.23988)}} = 7731 \text{ m/sec} \quad (23)$$

¹³ For isentropic adiabatic expansion, $\partial E = P \partial V$.

By use of (23) instead of (20), the residual error (21) is minimal (2×10^{-8}). Again using (23), the residual error of v_i for a small sphere is:

$$\frac{(v_{i(T_2)} + v_{s'}) - v_{i(T_1)}}{v_{i(T_1)}} = 4.5 \times 10^{-5} \quad (24)$$

Which is about as good as we are going to get with an increment this large.¹⁴ Now that we have a proper value of v_i , we can suggest an expression for v_s :

$$v_s = \frac{v_{s'}}{v_{s'_0}} v_i \quad (25)$$

Where $v_{s'_0}$ is the constant value of $v_{s'}$ at $r < r_c$. Eq. (25) gives a zero value at the endpoint, and gives v_i at low r . The endpoint is found by convergence of r around $X = 1$, which at $\rho_{(z=1089)} = 5.46 \times 10^{-19} \text{ Kg/m}^3$ and $T = 2971 \text{ K}$, gives $r_e = 1.28 \times 10^{17} \text{ meters}$ (4.1 parsecs).

The choice of r_c is best seen graphically. Figure 1 shows a semilog map of v_s/v_{s0} vs. r/r_e from $10^{-5} r_e$ to $0.1 r_e$.¹⁵ Below $r_c/r_e = 0.003$, v_s is constant to 4 ppm and drops at higher r/r_e as gravity takes its toll. The radial velocity (v) of the adiabatic sphere is the sum:

$$v = (v_i) \left(\frac{r_c}{r_e} \right) + \sum_{r_c}^{r_e} \left(\frac{r}{r_e} v_s \right) \rightarrow (v_i) \left(\frac{r_c}{r_e} \right) + \int_{r_c}^{r_e} \left(\frac{v_s}{r_e} \partial r \right) \quad (26)$$

At our chosen T and ρ , for all $r < r_c$, $v = 23.2 \text{ m/sec}$. That leaves the remaining 99.7% of v to be found. Integration of (11) is problematic so we resort to a map (Figure 2) whose cumulative value at $r = r_e$ is 6103 m/sec. Adding 23.2 to this gives 6126 m/sec, or $0.79245 \pm 0.00005 v_i$. If r_c/r_e is kept constant, the proportion of v_i not lost to gravity, or *curvature* K , shows little change with either ρ , T , or $\mathcal{K}$; K is constant to the 4th decimal place.¹⁶

In the special case of atoms separated by $2r_e$, their adiabatic spheres are joined at a tangent point and they are moving apart at $2v$. More generally, for any two atoms separated by a distance r , their recession rate v_r is:

$$v_r = K \frac{r}{r_e} v_i \quad (27)$$

Rearrangement of (27) gives the fundamental equation of this paper:

¹⁴ Spreadsheet errors begin to creep in at $\Delta r_i/r < 10^{-9}$. At $\Delta r_i/r = 10^{-8}$, the residual v_i error is 1.4×10^{-4} .

¹⁵ The temperature and density were found to be irrelevant. Also, both r_c and r_e are independent of $\Delta r_i/r$ over a wide range; $\Delta r_i/r = 10^{-8}$ was used for Fig. 1.

¹⁶ Initial H_G values reported herein were derived starting with eq. (20) using 998-999 steps of linearly increasing r/r_e , beginning at r_c and ending at $r/r_e = 0.999$ or 1. The integrals were calculated with the plotting program, Dplot, giving third-order correlation > 0.9999 in all cases. Replacement of the integral constant with $v_i(r_c/r_e)$ gave the reported $K = v/v_i$, 0.79245. Using 499 data points gave the same result.

$$H_G = \frac{Kv_i}{r_e} \quad (28)$$

Where $H_G = v_r/r$ is the comoving Hubble constant of the GCDM model. At $z = 1089$ eq. (28) gives $H_G = 4.79 \times 10^{-14}/\text{sec}$, or $21816H_{A0}$. This is 0.950 or 95% of the $H_{A,1089}$ value found from (1). If we set $K = 1$, we get $H_G = 6.04 \times 10^{-14}/\text{sec}$, or $27531H_{A0}$. This is 120% of $H_{A,1089}$. Use of (28) at varying T from 100 to 4000K at $z = 1089$ gives the same result to five decimal places every time. More extensive input change reveals no temperature dependence. The model is also zero-order in $\mathcal{K}$. A Universe made of xenon atoms (0.131 Kg/mole) at the same density returns 100% ($H_{G,1089} = 21816 H_{A,0}$) of our primordial mix. The mass density ρ is the only remaining thermodynamic variable in the model, and it's a function of z . This relation between H_G and z is strictly monotonic and exclusive of other variables.

We now include relativistic mass. In general relativity, energy and mass are equivalent. Relativistic energy from photons and neutrinos¹⁷ makes a contribution to gravity through its equivalent mass density ρ_{rad} , and is incorporated into the M' term of (5) through the mass multiplier η :

$$\eta(a) = \left[\frac{\Omega_{rad}a^{-4} + \Omega_b a^{-3} + \Omega_c a^{-3}}{\Omega_b a^{-3}} \right] \quad (29)$$

which at $z = 1089$ gives $\eta = 8.3363$, about a 1/3 increase. Consequently r_e shrinks to 9.69×10^{16} m with a resulting increase in H_G : $6.32 \times 10^{-14} \text{ sec}^{-1}$. This is 125% of the H_A value found from (1).

We now examine the GCDM model at $z = 1$ to 10, where non-Planck astronomers actually look. This gives substantial negative deviance from the Λ CDM model at $z = 0$ to 2. We suppose this is due to dark energy. A more useful comparison of the two models is thus performed by removing Ω_Λ from (1):

$$H'^2(a) = H_0^2 [\Omega_{rad}a^{-4} + \Omega_b a^{-3} + \Omega_c a^{-3}] \quad (30)$$

Values of H_G/H_A and H_G/H'_A vs. z are shown in Figure 3 for $z = 10$ to 0.¹⁸ While H_G/H_A drops off sharply at low z , H_G/H'_A remains constant. This linear relation suggests that the two models are simply connected by the slope. We suppose that the slope deviance ($1.09 - 1 = 0.09$) of the line (H_G/H'_A vs. z) may be from a missing entropic term. We've seen that isentropic treatment of an expanding sphere of monatomic gas gives entropic energy E_k inside the sphere. This entropic energy may give hitherto unaccounted radial increase. We add an entropic increment $\Delta r_S = r_3 - r_2$:

¹⁷ As before, this considers neutrino energy to be entirely relativistic for all z .

¹⁸ These numbers were calculated at $T = 2971\text{K}$; any T may be used and gives the same result.

$$\Delta r_{(i+S)} = \Delta r_i + \Delta r_S \quad (31)$$

Both increments occur at the same time ∂t and so the same speed, but additional distance is traveled with Δr_S . This is found by iteration. At $r < r_c$, the gravity terms are negligible, so we will say the increments are equal: $\Delta r_{(i+S)} = 2\Delta r_i$.¹⁹ When $r > r_c$, gravity kicks in and Δr_S shrinks,²⁰ analogous to an ascending rocket that's run out of fuel. The entropic gravity loss U_{S_0} is:

$$U_{S_0} = -\frac{3GM'^2}{5} \left(\frac{1}{\Delta r_{S_0}} \right) \quad (32)$$

Where Δr_{S_0} is the starting value for iteration of the free increment. We recalculate E_k :

$$E_{k_1} = E + U_r + U_{S_0} = E_{k_0} - \frac{3GM'^2}{5} \left(\frac{1}{\Delta r_{S_0}} \right) \quad (33)$$

Where the initial values are $E_{k_0} = E_k$ and E_{k_1} is the kinetic energy gain adjusted for entropic gravity loss. We resize Δr_S :

$$\Delta r_{S(n+1)} = \Delta r_{S(n)} \sqrt{\frac{E_{k(n+1)}}{E_{k(n)}}} \quad (34)$$

The new Δr_S is entered back into eqs. (32) - (34) to get new values U_{S_n} and $\frac{E_{k(n+1)}}{E_{k(n)}}$, repeating until $\Delta r_{S(n+1)} = \Delta r_{S(n)}$.²¹ The obtained $\frac{\Delta r_S}{\Delta r_i}$ is used for the next step, and the steps carried out as for the isentropic model, to the same endpoint. The resulting *entropic curvature* K_S , also shown in Fig. 2, is only minimally less and has a similar invariance with ρ , T , and $\mathcal{K}$: $K_S = 0.9935 K$. Entropic expansion doesn't seem to be very important in the overall scheme of things so we continue to use K and not K_S .²²

Our slope deviance issue, H_G/H'_Λ vs. z ($= 1.09$), wasn't solved by an entropic term, so we look elsewhere: the critical density ρ_{crit} . In the Λ CDM model, ρ_{crit} is found from H_0 . With GCDM, " ρ_{crit} " is not dependent on H_0 so we can choose a best fit value: $\rho' = 0.84\rho_{crit}$. This adjustment, also shown in Figure 3, gives a result within 0.05% of the Λ CDM H'_Λ values from $z = 0$ to 5. The gravitationally unbound mass fraction in the Universe is presently estimated at about 0.85 and its volume fraction at around 0.9, giving an unbound density of $0.94\rho_{crit}$. Our

¹⁹ The entropic increment Δr_{S_0} can be made larger but the end result is the same.

²⁰ Since both r_1 and r_2 are independent variables, the isentropic increment Δr_i is unchanged.

²¹ These iterations were performed sequentially and not recursively. Ten is enough for convergence to six decimal places.

²² There may be some dependence of K_S on the number of data points. More steps may give $K_S \rightarrow K$, but this hasn't been thoroughly investigated.

finding suggests that the volume fraction of unbound matter is closer to 1 than present estimates, or the mass fraction is lower than 0.85, perhaps as low as 0.76. We are tempted to conclude from Figure 3 that the partitioning of Universal mass into gravitationally bound and unbound domains happened very early on, but such conjecture is best left to the reader.

None of the above discussion comes any closer to explaining the phenomenon of dark energy. The Λ CDM model gives us the means to do so from known and conserved sources, rather than from the proposed (and nonconservative) “vacuum energy field” of the Λ CDM model. The Λ CDM model has three terms: v_i , K , and r_e . If we want to increase H_G , we need to increase v_i or K , decrease r_e , or some combination. This boils down to finding a source of entropic energy that does not derive from the internal kinetic energy U_i of the adiabatic sphere. We use the expression:

$$E_S \approx T \Delta S_\beta \quad (35)$$

Where E_S is the added entropic energy and ΔS_β is the change in entropy from a source β . We modify (23) by adding E_S :

$$v_i = \sqrt{\frac{2(U_i + E_S)}{M}} = \sqrt{\frac{2U_i}{M} + \frac{2E_S}{M}} = \sqrt{\frac{2U_i}{M} + \frac{2T\Delta S_\beta}{M}} = \sqrt{\frac{3RT}{\mathcal{K}} + \frac{2T\Delta S_\beta}{M}} = \sqrt{T \left(\frac{3R}{\mathcal{K}} + \frac{2\Delta S_\beta}{M} \right)} \quad (36)$$

As yet we have no *a priori* means of evaluating ΔS_β and therefore E_S , but we can find values for $(U_i + E_S)$ which give a best fit with H_A . Since $\rho' = 0.84\rho_{crit}$ gives an excellent fit to H' over the range $z = 0$ to 5, we will use $\rho'(z)$. We use the same temperature, 4000K, for all calculations. The curvature K is kept at 0.79245; r_e is left unchanged.²³ The best fit value is found by varying E_S/U_i to convergence around $H_G/H_A = 1$. A plot of best-fit E_S/U_i values vs. $(z+1)$ is shown in Figure 4. At $z = 0$, $E_S/U_i = 2.24$, and drops steadily to 0.01 at $z = 5$. Whatever the source, E_S is clearly the most important contributor to H_G presently, and played little role in earlier times.

We now propose that high-energy photons are the source of E_S . Star formation is believed to have commenced around $z = 100$, maybe earlier; new old stars are being found at this is written. Some give off light with photon energy sufficient to ionize H_1 , yielding a proton and a free electron. This photon energy is partitioned into potential energy gain nE_i , electron energy nE_β , and added proton kinetic energy nE_{H^+} :

$$nE_\lambda = \frac{nhc}{\lambda} = nE_i + nE_\beta + nE_{H^+} \quad (37)$$

Where n is the number of impact photons, E_λ is the mean photon energy, h is Planck's constant = 2×10^{-25} J-m, λ is a suitable averaged wavelength of the impact photon, c is the speed of light = 3×10^8 m/sec, E_i is the work function of the impacted particle, E_β is the mean energy of the emitted

²³ K actually varies at the third decimal place, increasing with E_S , but this error is minor enough that we can set K as a constant. The endpoint r_e is very nearly independent of E_S , constant to about 1 ppm.

electron, and E_{H^+} is the mean kinetic energy added to the proton. For H_1 the work function E_i is its ionization potential (13.6 eV or 1.6×10^{-19} J). A plasma is created, which at this density has thermodynamic behavior much like any other monatomic gas.²⁴ We focus on the latter two terms of (37), E_β and E_{H^+} , and return to our sphere upon which the GCDM model is based. The direction of motion of the protons and electrons after photon impact is random. In order for them to make any meaningful contribution to (26) through (36) they have to be moving faster than v , or they will never cross the adiabatic surface of the sphere, and their movement simply adds to U_i . We calculate v at $T = 4000\text{K}$ and $\mathcal{K} = 0.00124 \text{ Kg/m}^3$, which from (23) gives $v_i = 8971 \text{ m/sec}$ and $v = Kv_i = 7109 \text{ m/sec}$. For an impact photon of $hc/\lambda = 14 \text{ eV}$, 13.6 eV is absorbed and the remaining 0.4 eV is partitioned into proton (E_{H^+}) and electron (E_e) kinetic energies; the electron carries away 99.945% of the energy.²⁵ The particle speeds v'_e and v'_{H^+} are, at $hc/\lambda = 14 \text{ eV}$, respectively:

$$v'_e = \sqrt{\frac{2E_e}{m_e}} = \sqrt{\frac{2(0.4\text{eV})(0.99945)(1.6022 \times 10^{-19}\text{J/eV})}{9.1094 \times 10^{-31}\text{Kg}}} = 370,000 \text{ m/sec} \quad (38)$$

which is plenty, and:

$$v'_{H^+} = \sqrt{\frac{2E_{H^+}}{m_{H^+}}} = \sqrt{\frac{2(0.4\text{eV})(0.00055)(1.6022 \times 10^{-19}\text{J/eV})}{1.673 \times 10^{-27}\text{Kg}}} = 205 \text{ m/sec} \quad (39)$$

which isn't enough to escape the sphere. It appears that the principal source of E_s is photoionized electrons. These do not necessarily have to come from hydrogen. There's plenty of, *e.g.*, helium around, albeit requiring a much higher hc/λ , 24.6 eV.

This added entropic energy, which corresponds to dark energy in the ΛCDM model, is a function of the impact photon flux across the surface of the adiabatic sphere. A complex subject, it's well beyond the scope of our discussion.²⁶ Briefly, the high-energy photon content of the Universe is estimated from the Wein tail of the Boltzmann distribution function at the surface of short-lived type O giant stars,²⁷ and their rate of formation.²⁸ One thing we can conclude from Fig. 4 is that the amount of high-energy photons is up in recent times, and their sources, such as type O stars and the like, are present in increasing number.

The author declares no competing interest.

²⁴ Actually, plasma at low density is much more kinematically responsive than the corresponding neutral gas. Protons, for example, are charged and repel according to an inverse-square law. This causes path deflection at distances many times that of the van der Waals radius of the neutral atom.

²⁵ Momentum is conserved.

²⁶ For some details of photon production, the reader is referred to chapters 4 and 9 of ref. 5.

²⁷ In the Harvard system (OBFGKM), "O" is the hottest.

²⁸ There is also a contribution from active galactic nuclei, subdwarf O stars, and probably other sources as well.

Figure 1.pdf

Figure 2.pdf

Figure 3.pdf

Figure 4.pdf

Figures

Figure 1.
Small sphere cutoff, $r_c = 0.003 r_e$

Figure 1

Small sphere cutoff, $r_c = 0.003 r_e$

Figure 2.
Normalized sphere radial velocity
vs. normalized sphere radius

Figure 2

Normalized sphere radial velocity vs. normalized sphere radius

Figure 3.
 H_G/H_Λ and H_G/H'_Λ vs. z
 at density $\rho(z) = 0.84\rho_{\text{crit}}$ and $\rho(z) = \rho_{\text{crit}}$

Figure 3

H_G/H_Λ and H_G/H'_Λ vs. z at density $\rho(z) = 0.84\rho_{\text{crit}}$ and $\rho(z) = \rho_{\text{crit}}$

Figure 4.
Best-fit values of E_S/U_I for $H_G = H_\Lambda$

Figure 4

Best-fit values of E_S/U_I for $H_G = H_\Lambda$

Supplementary Files

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