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A novel algorithm to allocate customers and retailers in a closed-loop supply chain under probabilistic demand

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Abstract

In this paper, a closed-loop supply chain (CLSC) is modeled to obtain the best location of retailers and allocate them to other utilities. The structure of CLSC includes production centers, retailers' centers, probabilistic customers, collection, and disposal centers. In this research, two strategies are considered to find the best location for retailers by focusing on 1- the type of expected movement 2- expected coverage (distance and time) for minimizing the costs and maximizing the profit by considering the probabilistic customer and uncertainly demand. First of all, the expected distances between customers and retailers are calculated per movement method. These values are compared with the Maximum expected coverage distance of retailers, which is displayed in algorithm 1 heuristically, and the minimum value is picked. Also, to allocate customers to retailers, considering the customer's movement methods and comparing it with Maximum expected coverage time, which is presented in Algorithm 2 heuristically, the minimum value is chosen to this end, a bi-objective nonlinear programming model is proposed. This model concurrently compares Strategies 1 and 2 to select the best competitor. Based on the chosen strategy, the best allocation is determined by employing two heuristic algorithms, and the locations of the best retailers are determined. As the proposed model is NP-hard, a meta-heuristics (non-dominated sorting genetic) algorithm is employed for the solution process. Afterward, the effectiveness of the proposed model is validated and confirmed, and the obtained results are analyzed. For this purpose, a numerical example is given and solved through the optimization software**.**

Keywords: Closed-loop supply chain; Expected coverage distance; Expected coverage time; NSGA-II

Introduction

Supply chain networks are classified into two general categories of (1) traditional supply chains as a forward or an open-loop chain and (2) integrated chains that are composed of components such as raw materials, manufacturing facilities, distribution centers, and customers all connected by the flow of material in a forward chain and by information flow in a reverse chain [1]. In recent decades, interest in reverse logistics (RL) and closed-loop supply chains (CLSCs) has grown significantly due to increasing environmental problems. Regular forward supply chains include the forward flow of products through suppliers, plants, and distributors to customers. RL consists of the reverse flow of products starting from end customers and involves collecting, inspecting, repairing, disassembly, disposal, recycling, and remanufacturing of products [2], [3]. Even during and after a critical circumstance (like pandemic COVID-19), it is essential to activate the opened- and closed-loop system through an efficient and resilient supply chain network [4]. In this regard, many large companies such as Xerox, Canon, Kodak, Dell, and Acer have put efforts into green operations. For instance, Dell has recently changed their business approach to zero-waste manufacturing and renewable-energy usage [5].

Nowadays, supply chain (SC) systems have become more complex and dynamic with wide geographical coverage. Hence, SC is exposed to a broad range of uncertainties, some of which may cause disruptions in the SC [6]. While composing a supply chain network (SCN), it should be noticed that SC breakdowns are unplanned and erratic events that upset the normal flow of goods and supplies in the chain. Consequently, corporations in the SC are prone to commercial and operational risks [7] as disruption in infrastructures has relatively significant impacts on the operation and performance of the SC [8]. Accordingly, the strategy should be chosen to be aware of these contradictions and the necessity of choosing between competitive criteria such as speed, efficiency, quality, cost, and satisfaction, or mixed models [9].

Allocating retailers in a closed-loop supply chain structure include production centers, retailers centers, probabilistic customers, collection, and disposal centers. Comparing the type of movement and possible coverage and use of innovative algorithms in the allocation is one of the topics that is less addressed in recent articles [4],[6],[8],[10],[11].[12],[13], [14],[15],[16],[17].

LITERATURE REVIEW

In this section, some of the recent studies in the context of the location-routing-inventory problem, closed-loop supply chain, and their synthesis are presented to show the necessity of this research. In this paper, the related literature is reviewed according to two research domains of CLSC by considering certain and uncertain conditions. In [18], the CLSCN equilibrium is investigated based on three practical factors: two complementary types of suppliers, the manufacturer's risk-averse character, and the suppliers' capacity constraints. In [19], a CLSC model comprising an original manufacturer, third-party remanufacturer, and retailer was investigated. In the model proposed, the remanufacturer can only recycle and remanufacture the patented products with patent licensing from the original manufacturer, where newly manufactured and remanufactured products are sold together in the same market at different prices. The demand for the two types of products is sensitive to their retail prices. [20] Studied, the implications of the government's tariffs on optimal pricing decisions in a dual-channel SC with one manufacturer and one retailer by taking into account the retailer services examined.

In [21], a multi-objective Mixed Integer Programming Model was developed for a CLSCN design problem. In addition to the overall costs, the model optimized overall carbon emissions and the responsiveness of the network. In this study, an improved genetic algorithm based on the framework of NSGA II is developed to solve the problem and obtain Pareto-optimal solutions. Also, in [22] a multi-period, multi-product, multi-echelon, and multi-customer CLSCN was Designed and optimized for a mobile phone network considering different types of product returns. The researchers in [23] studied a two-echelon SC with one manufacturer and one retailer, based on which three reverse channels were formatted. The main reason behind developing the model was to investigate how the wholesale price, the retail price, the collection rate, and the total channel profits were affected by the choice of the reverse channel structure. In [24], a two-period closed-loop green supply chain (CLGSC) model with a single manufacturer and a single retailer was developed to investigate the impact of green innovation, marketing effort, and collection rate of used products on the SC decisions. In [25], a multi-objective optimization model was designed for the integrated product family. Next, CLSC was formulated based on a cooperative game model to minimize the manufacturer's total cost and maximize suppliers' total payoffs. In [26], a dual-channel CLSC structure was proposed in which a manufacturer sells to a retailer and a direct online channel. The components of this model behave in a Stackelberg game. Here, the manufacturer is the channel and owns the traditional retail channel and an Internet-based direct channel, and the retailer is a channel follower and sells products in the traditional retail channel. [27] Proposed a bi-level game-theoretic model proposed to investigate the effects of governmental financial intervention on green supply chain. This problem is formulated as a bi-level program for a green supply chain that produces various products with different environmental pollution levels. in [28], the authors introduced a CLSC model with dual competitive sales channels. They considered three reverse channel structures in the CLSC Manufacturer collecting (Model M), retailer collecting (Model R), and third-party collecting (Model C) structures. In addition, they showed that a simple price contract consisting of the wholesale price, direct channel price, and transfer price of the used product (in Model R and Model C), with a complementary profit-sharing mechanism, can effectively coordinate dual-channel CLSCs under different recycling channel structures. In [29], a dynamic equilibrium model of oligopolistic CLSCN was developed to account for the seasonality of demand. In this model, demands and returns are uncertain and time-dependent. Also, the dynamic Cournot-Nash equilibrium of the oligopolistic network is constructed by evolutionary variation inequality and projected dynamical systems. [30] Proposed to design a tri-objective multi-echelon multiproduct multi-period supply chain model, which incorporates product development and new product production and their effects on supply chain configuration.

The authors of [31] developed a new mathematical formulation for a multi-objective stochastic CLSCN designed considering social impacts. Based on the proposed model, three novel hybrid meta-heuristics were applied to a strategically CLSC. These researchers considered economic and social aspects simultaneously using the suppositions. In [32], a deterministic mixed-integer optimization model and robust counterparts were developed to cope with the uncertainty of recycled products and customer demand in the fashion industry. These researchers showed that a robust counterpart with a budget of uncertainty is equivalent to a robust counterpart with a box uncertainty under special conditions. In [33], the closed-loop supply chain network design was considered for hazardous products (HP-CLSCND), including both forward SC and reverse SC. Overall, the showed that the uncertainty inherent in CLSCN will significantly affect the overall performance of the CLSCN design. The model focuses on the HP-CLSCND problem with uncertain demands and returns. A two-stage stochastic programming model (scenario-based) is proposed for solving this model in which risk restriction constraint and reward-penalty mechanisms are simultaneously considered. The objective of this modeling is to make ordering decisions to minimize the total system cost. They introduced a two-stage tri-level optimization model with a rolling horizon to address the uncertain demand and lead time regardless of their underlying distributions. In [34], a two-stage mixed-integer problem (MIP) model was proposed that combines the conditional value at risk and the structure of a CLSCN. In this model, end-customer areas are divided into two parts, namely, the primary market and the secondary market. This approach can prevent a distinct impact on economic performance via changes in quality. In [35], a mixed-integer nonlinear programming (MILP) model was proposed to integrate pricing with facility location and inventory control decisions in a CLSCN in the information and communications technology (ICT) industry. This model maximizes the total profit obtained by selling the new ICT products or collecting the used ICT products. In addition, in [36], a new nature-inspired algorithm (Whale Optimal station Algorithm (WOA)) and a popular algorithm (Particle Swarm Optimization (PSO)) were employed to solve this problem. In addition to this development, Genetic Algorithm (GA) and Simulated Annealing (SA) are the well-known meta-heuristics widely used in the literature. Finally, they used some evaluation metrics to assess the Pareto optimal fronts of algorithms' quality and conduct a comparative study. In [37], a comprehensive CLSC was developed and a network was designed that considers both the cost objective and the service efficiency objective of the warehouses/ hybrid facilities. The proposed MILP model assists decision-making in the location of facilities and distribution planning in a CLSCN under the two objectives. In another study [38], a two-stage stochastic programming model was presented for the design of a green CLSC. This model presents an upper bound of emission capability that helps governments and industries to control greenhouse gas emissions were considered. The researchers in [10] investigated a CLSC with different grades extracted from a melting process in a reverse flow based on demand planning to handle the uncertainty of the model. Modeling emphasizes high profitability due to uncertainty in demand. They investigated various issues in this field and used a robust and modified GA for optimization. In [11], a new fuzzy-stochastic multi-objective mathematical model was proposed for sustainable CLSCN design. The model aims at balancing the trade-off between cost-effectiveness and environmental performance under different types of uncertainty. The environmental performance of the CLSCN design is measured by carbon emission. Furthermore, the network flexibility is modeled and incorporated in the decision-making so that customer demands can be fulfilled by different means.

Mathematical Model

- **Network structure:** The structure of the CLSCN (Fig. 1) includes production centers, retail centers, probabilistic customers, collection centers, and disposing centers.

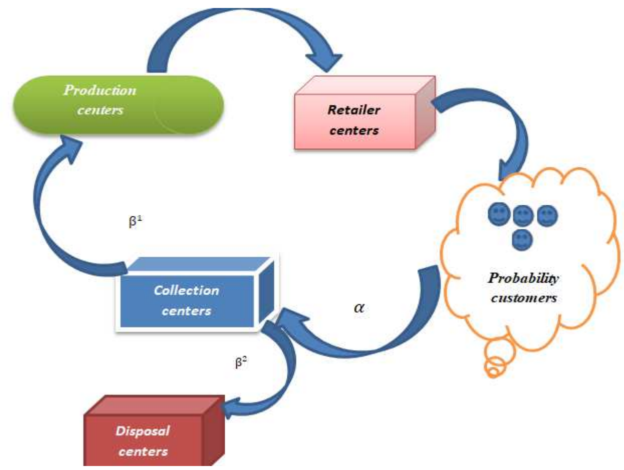

FIG. 1
SCHEMATIC DIAGRAM OF THE MODELED CLSC

In Fig. 1, the customers are chosen probabilistically with lower and upper time bounds and have minimum and maximum coverage radii to reach retailers. Also, retailers have lower and upper bounds such that they have minimum and maximum coverage ranges to provide customer service.

- **Model Assumption**:
- The single-period model is considered;
- Insufficiency is allowed;
- Transportation costs are fixed over a period;
- All customers must receive their services;
- Every customer can visit more than one retail center to receive services;
- The customers' and retailers' motion type is either Rectangular, Euclidean, Euclidean Square, or Chebyshev;
- The motion happens on a page;
- Based on the relocation distance, the transportation time is constant and invariable;
- All chain parameters and variables are definite, excluding the customer place, customer coverage time, retailer coverage distance, and customers demand;

- No cost is considered for keeping the goods;
- The retail coverage distance for random customers is not constant at each stage;
- The customer coverage time of retailers is not constant at every stage;
- The quantity of goods in retailers is always constant.

The behavior between customers and retailers in this model is taken from SNAP Company in Iran.it shown in the form of an industrial model (FIG.1)

Sets

In this section, all the indexes used in modeling the problem are presented.

$p = 1,2, \dots P$	Index of collection centers that have the potential to produce
$j = 1,2, \dots J$	Index of collection of retail centers that have the selling potential
$i = 1,2, \dots I$	Index of collection of probabilistic customers
$cc = 1,2, \dots CC$	Index of collection centers that have the potential to collect
$dis = 1,2, \dots DIS$	Index of collection of disposal centers that have the elimination potential
$r = 1,2, \dots R$	Index of the produced goods

e, e'= The set of all echelons $(e, e' \in \{p, j, i, cc, dis\})$

k, k'= The set of facilities in the echelon $(k_e, k'_e \in \{1, \dots K_e\})$

- **Model parameters:** In this section, all the parameters associated with the problem are introduced.

TABLE1
PARAMETERS USED IN MODEL

cap_e	Capacity of facility $e \in \{p, j, i, cc, dis\}$
cap_{k_e}	Capacity of facility $k_e \| e \in \{p, j, cc, dis\}$
τ_{k_e}	The amount of facility $k_e \| e \in \{p, j, cc, dis\}$
v_r	The amount of the type r production
Fc_{k_e}	Fixed cost of the facility $k_e \| e \in \{p\}$
FC_j	Fixed cost of the *jth* potential retailer center
FC'_{k_e}	Fixed cost of the facility $k_e \| e \in\in \{cc, dis\}$
L_j	Standard radius of the service distance for the *jth* retailer
T_i	Standard radius of time of service receiving for the *ith* probabilistic costumer
$l_{j,i,r}$	Distance covered by the *jth* retailer transfer to provide service for the *ith* probabilistic customer due to send the type r product
$t_{i,j,r}$	Time covered by the *ith* probabilistic customer spend to gain service for the *jth* retailer due to receive the type r product
e_j	Upper and lower limit values from the standard distance radius
θ_i	Upper and lower limit values from the standard time radius
μ_{i1}	The average horizontal coordinates of the *ith* probabilistic customer
μ_{i2}	The average vertical coordinates of the *ith* probabilistic customer
σ_{i1}^2	The variance of horizontal coordinates of the *ith* probabilistic customer
σ_{i2}^2	The variance of vertical coordinates of the *ith* probabilistic costumer
d_{j1}	Spatial horizontal coordinates of the *jth* retailer
d_{j2}	Spatial vertical coordinates of the *jth* retailer
$\pi_{j,i,r}$	The shortage costs of the type *rth* product from the *jth* potential retailer center to the *ith* probabilistic customer
$\pi'_{i,j,r}$	The shortage costs of the type *rth* product from the *ith* probabilistic customer to the *jth* potential retailer center
α	The percentage of the costumers returned goods. (α ≤ 1)
β^1, β^2	The percentage of the products that can be revived in the collection center and eliminated in the disposal center ($\beta^1 + \beta^2 = \alpha$)
$Cost_{k_e k'_{e'},r}$	The relocation cost of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{p, j\})$
$Cost_{j,i,r}$	The relocation cost of the type *r* product from the *jth* potential retailer center to the *ith* probabilistic customer
$Cost'_{i,j,r}$	The relocation cost of the type *r* product from the *ith* probabilistic customer to the *jth* potential retailer center
$Cost'_{k_e k'_{e'},r}$	The relocation cost of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$dic_{k_e k'_{e'},r}$	The transferring distance of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{p, j\})$
$dic_{j,i,r}$	The transferring distance of the type *r* product from the *jth* potential retailer center to the *ith* probabilistic customer
$dic'_{i,j,r}$	The transferring distance of the type *r* product from the *ith* the probabilistic customer to the *jth* potential retailer center
$dic'_{k_e k'_{e'},r}$	The transferring distance of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$x_{k_e k'_{e'},r}$	The amount of the type *r* product send from the facility center k_e to facility center $k'_{e'}$ $(e, e' \in \{p, j\})$
$x_{j,i,r}$	The amount of probabilistic demand of the type *r* product send from *jth* potential retailer center to the *ith* probabilistic customer
$x'_{i,j,r}$	The amount of probabilistic demand of the type *r* product received the *ith* probabilistic customer from, the *jth* potential retailer center
$x'_{k_e k'_{e'},r}$	The amount of the type *r* product send from the facility center k_e to facility center $k'_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$dicR$ $dicED$ $dicEDS$ $dicCH$	Rectangular motion Euclidean motion Euclidean Square motion Chebyshev motion
COV^1 COV^2	Expected distance coverage Expected time coverage

Decision variable: In this paper, only one decision variable (0 and 1) is to designate customers to retailers or vice versa.

$$Q_1 = \begin{cases} 1 & \textit{if the retailer is allocated to the costumer} \\ 0 & \textit{otherwise} \end{cases}$$

- **MODEL FORMULATION:** The mathematical model of this chain is in two stages. In the first step, the mathematical model is formulated among probabilistic customers and retailers, and in the second stage, the entire problem is formulated.

Step 1: Calculations are quite effective among retailers and probabilistic customers. Therefore, it is assumed that the type of movement by retailers to customers or vice versa and calculating the distance coverage radius of the retailers and the time coverage radius of customers and comparing them with each other according to the heuristic algorithms. Accordingly, their minimum value is chosen and considered as the output of this section.

$$f = min\Big(\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min(\ f_{1(j,i)}, f_{3(j.i)})\} \times Q_1 + \{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,jr} \times Cost'_{i,j,r} \times min(\ f_{2(i,j)}, f_{4(i,j)})\} \times (1 - Q_1)\Big) \quad (1)$$

S.T

$$\theta_i < T_i \qquad \forall i \in I \quad (2)$$

$$\text{e}_j < L_j \qquad \forall j \in J \quad (3)$$

$$Q_1 + (1 - Q_1) = 1 \quad (4)$$

Equation (1) presents the objective function of the first step of the process among retailers and probabilistic customers within a parenthesis. This process has two parts, which are distinguished by "{}". The first part of the selection calculates the minimum coverage of the expected distance and the expected motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) of retailers' grants services to customers. In addition to movement and coverage types, this section presents the amount of probabilistic customer demand. The second part of the selection includes the minimum calculation of expected time coverage and expected motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) of customers regarding the services received from retailers. Lastly, by comparing the chosen minimum costs, the lowest value is selected as the output. The output of this part explains whether the retailers convey the goods or the customers come to receive them. Constraint (2) gives the maximum amount of time coverage radius of the customer. Constraint (3) gives the maximum motion radius of retailers. Finally, Constraint (4) displays choosing the decision variable.

Step 2: In this step, according to Fig. 1 of the modeling process, the input and output ports of the desired chain are calculated. The first objective function shows the value of the first target function (displacement cost), and the second objective function shows the profits from the sale of goods. For more information, see the related Equation $f_{1(j,i)}$, $f_{2(i,j)}, f_{3(j.i)}, f_{4(i,j)}$E($S$) and E ($S'$) in **APPENDIX**.

$$TOTAL\ OBJECT1 = min\Big(\Big\{\sum_{k_e \in \{p\}} Fc_{k_e} + \sum_{k_e \in \{p,j\}} \sum_r x_{k_e k'_{e'},r} \times Cost_{k_e k'_{e'},r} \times dic_{k_e k'_{e'},r}\Big\} + min\{f\} + \Big\{\sum_{k_e \in \{cc,dis\}} FC'_{k_e} + \sum_{k_e \in \{i,cc,dis,p\}} \sum_r x'_{k_e k'_{e'},r} \times Cost'_{k_e k'_{e'},r} \times dic'_{k_e k'_{e'},r}\Big\}\Big) \quad (5)$$

Equation (5) represents the objective function of the process, which consists of 3 parts, each separated by a "{}" from the others. Part 1 includes the sum of the fixed and the variable costs of the transfer of goods, which are shipped from the production centers to the retailer centers. Part 2 is calculated using Equation (1). Finally, Part 3 includes the entire fixed and variable cost of products returned by customers to the collection centers. Afterward, fixed and variable costs of the transfer of goods from collection centers to repair and disposing centers are calculated. Finally, calculated fixed and variable costs of the transfer of goods from the repair center to distribution and warehouse and disposing centers are computed.

$$TOTAL\ OBJECT2 = max(SA_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\} \times Q_1) + (BU'_{i,j,r} \times min\{E(x'_{i,j,r}), cap_j \ \pi'_{i,j,r} \times max[(E(x'_{i,j,r}) - cap_j), 0] \times 1 - Q_1) \quad (6)$$

Equation (6) presents the Profit function of the probabilistic customer by uncertain demand.

S.T

$$\sum_{k_e | e \in \{p,j,cc,dis\}} \tau_{k_e,r} \le cap_{e \in \{p,j,cc,dis\}} \qquad \forall r \quad (7)$$

$$\sum_r \tau_{k_e,r} \le cap_{k_e} \quad \forall k_e | e \in \{p, j, cc, dis\} \quad (8)$$

$$\sum_{k_e | e \in \{p,,j,cc,dis\}} \tau_{k_e,r} \le v_r \qquad \forall r \quad (9)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,p\})} x'_{k_e k'_{e'},r} + \sum_{k_e,k'_{e'} | e,e' \in \{p,j\}} x_{k_e k'_{e'},r} \le \sum_{k_e | e \in \{p\}} \tau_{k_e,r} \qquad \forall r \quad (10)$$

$$\sum_r x_{k_e k'_{e'},r} \le \sum_r \tau_{k_e,r} \quad (11)$$

$$\forall x_{k_e k'_{e'}} \in (e, e' \in \{p, j\})$$

$$\forall \tau_{k_e} | e \in \{j\}$$

$$\sum_{x_{k_e k'_{e'}} \in (e,e' \in \{p,j\}),r} x_{k_e k'_{e'},r} = \left(\sum_{i=1}^I \sum_{j=1}^J E(x_{j,i,r})\right) \times Q_1 + \left(\sum_{j=1}^J \sum_{i=1}^I E(x'_{i,j,r})\right) \times (1 - Q_1) \qquad \forall r \quad (12)$$

$$\left(\sum_i^I \sum_j^J \alpha_{j,i,r} \times \sum_{i=1}^I \sum_{j=1}^J E(x_{j,i,r})\right) \times Q_1 + \left(\sum_j^J \sum_i^I \alpha_{i,j,r} \times \sum_{j=1}^J \sum_{i=1}^I E(x'_{i,j,r})\right) \times (1 - Q_1) = \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (13)$$

$$\sum_i^I \sum_j^J \alpha_{j,i,r} \times Q_1 + \sum_j^J \sum_i^I \alpha_{i,j,r} \times (1 - Q_1) \leq 1 \quad \forall r \quad (14)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} \leq \sum_{k_e | e \in \{cc\}} \tau_{k_e,r} \quad \forall r \quad (15)$$

$$\sum_{cc=1}^{CC} \sum_{i=1}^I \beta^1_{i,cc,r} \times \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,p\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (16)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,p\})} x'_{k_e k'_{e'},r} \leq \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (17)$$

$$\sum_{cc}^{CC} \sum_i^I \beta^2_{i,cc,r} \times \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,dis\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (18)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,dis\})} x'_{k_e k'_{e'},r} \leq \sum_{k_e | e \in \{dis\}} \tau_{k_e,r} \quad \forall r \quad (19)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,dis\})} x'_{k_e k'_{e'},r} \leq \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (20)$$

$$\sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,p\})} x'_{k_e k'_{e'},r} + \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{cc,dis\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r} \in (e,e' \in \{i,cc\})} x'_{k_e k'_{e'},r} \quad \forall r \quad (21)$$

Constraint (7) gives the maximum capacity of all facility centers Constraint (8) gives the maximum capacity Constraint (9) declares the maximum amount of production r in each facility center. Constraint (10) shows the maximum capacity of goods joining the production centers. Constraint (11) shows the maximum entry capacity of each retailer center. Constraint (12) indicates the balance of entry and exit of goods in retail centers. According to Algorithms 1 and 2, this restriction determines whether the retailers send the goods to the customers or the customers refer to the retailers to receive the goods. Constraint (13), depending on the decision variable, shows the return percentage of goods from customers. Constraint (14) gives the total percentage of goods returned by customers a maximum of 1. Constraint (15) gives the maximum capacity of goods joining the collection centers by all customers. Constraint (16) shows the percentage of goods shipped from the collection center to the production centers. Constraint (17) states that the maximum number of collection center goods equals the number of returned goods. Constraint (18) gives the percentage of goods sent from the total collection center to the disposal centers. Constraint (19) shows that the maximum number of goods sent from the collection centers to the disposal centers is equal to the maximum capacity of the disposal centers. Constraint (20) asserts that the maximum number of destroyed goods is equal to the number of returned goods. Finally, Constraint (21) presents the balance of goods entering and leaving in all collection centers.

SOLUTION APPROACH

In addition to decreasing costs by choosing the best place for retailers, the proposed model lowers the amount of carbon dioxide in retailers and customers by allowing the best place for retailers. In this model, while retailers can be selected to send services, customers can also refer to receive services. Due to the probabilistic nature of customers, first, the expected distances between customers and retailers are calculated per movement methods performed (Rectangular, Euclidean, Euclidean Square, and Chebyshev). These values are compared with the MECD of retailers, which is displayed in Algorithm 1 heuristically, and the minimum value is picked. Also, the minimum value is chosen to allocate customers to retailers considering the customer's movement methods and comparing it with MECT, which is presented in Algorithm 2 heuristically. In the end, choosing the minimum cost from the two mentioned methods determines the allocation and the way to provide the service.

- **Algorithm 1**: Assigning retailers to customers

Step 1: Initialization

-Generate the average longitudinal and transverse to the number of probabilistic customers

$\boldsymbol{\mu}_{11}, \boldsymbol{\mu}_{21}, \ldots, \boldsymbol{\mu}_{I1}.$

$\boldsymbol{\mu}_{12}, \boldsymbol{\mu}_{22}, \ldots, \boldsymbol{\mu}_{I2}.$

Step 2: Computing expected distance and cost

- Compute expected distance based on the type of sending the goods from retailers to probabilistic customers

-Calculate the cost of sending goods from retailers to potential customers using the second step

Step 3: Computing distance coverage radius

- Computes the maximum distance based on the second step

$maximum\ distance = max\left(E\left[(dicR_{j,i}), (dicED_{j,i}), (dicEDS_{j,i}), (dicCH_{j,i})\right]\right)$

$l_{j,i,r} = rand([1, maximum\ distance])_{J \times I \times R}$

Step 4: Computing maximum and the minimum distance coverage radii

-For all retailers to provide services, calculate the minimum and the maximum distance coverage radius separately;

Min and Max = $[L_j - e_j \quad L_j + e_j]$

Step 5: assign retailer

If:

$(l_{j,i,r}) \leq [L_j - e_j]$ *Then, calculate* $\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])\}$

And assign the retailer to the $max(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])$

If:

$L_j - \text{e}_j < (l_{j,i,r}) \leq L_j$, *then, calculate* $\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])\}$
And assign the retailer to the $max(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])$

If:

$L_j < (l_{j,i,r}) \leq L_j + \text{e}_j$, *then, calculate* $\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])\}$
And assign the retailer to the $max(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])$

If:

$(l_{j,i,r}) > L_j + \text{e}_j$, *then calculate* $\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])\}$
And assign the retailer S to the $max(E[dic_{j,i,r}], E[COV^1(l_{j,i,r})])$

- **Algorithm 2**: *Assigning customers to retailers*

Step 1: Initialization
Generate the average longitudinal and transverse to the number of probabilistic customers
$\mu_{11}, \mu_{21}, \dots, \mu_{I1}$.
$\mu_{12}, \mu_{22}, \dots, \mu_{I2}$.

Step 2: Computing expected distance and cost
Compute expected distance based on the type of sending the goods from retailers to probabilistic customers (Rectangular, Euclidean, Euclidean Square, and Chebyshev),
Calculate the cost of sending goods from retailers to potential customers using the second step

Step 3: Computing time coverage radius
Using the second step, compute the maximum distance.
$MaxTime = max(E[(dicR_{i,j}), (dicED_{i,j}), (dicEDS_{i,j}), (dicCH_{i,j})])$
$t_{i,j,r} = rand([1, \ MaxTime])_{I\times J\times R}$

Step 4: Computing maximum and the minimum time coverage radius
For all retailer service providers, calculate the minimum and the maximum time coverage radius separately;
Min and max=$[T_i - \theta_i \quad T_i + \theta_i]$

Step 5: assign customer

if :

$(t_{i,j,r}) \leq [T_i - \theta_i]$ *then calculate* $\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min([dic_{i,j,r}], E[COV^2(t_{i,j,r})])\}$
And assign the customer to the max $([dic_{i,j,r}], E[COV^2(t_{i,j,r})])$

if :

$T_i - \theta_i < (t_{i,j,r}) \leq T_i$
then calculate $\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min([dic_{i,j,r}], E[COV^2(t_{i,j,r})])\}$
And assign the customer to the $max([dic_{i,j,r}], E[COV^2(t_{i,j,r})])$

If:

$T_i < (t_{i,j,r}) \leq T_i + \theta_i$, *then calculate* $\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min([dic_{i,j,r}], E[COV^2(t_{i,j,r})])\}$
And assign the customer to the max $([dic_{i,j,r}], E[COV^2(t_{i,j,r})])$

If:

$(t_{i,j,r}) > T_i + \theta_i$ *then calculate* $\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min([dic_{i,j,r}], E[COV^2(t_{i,j,r})])\}$
And assign the customer to the max $([dic_{i,j,r}], E[COV^2(t_{i,j,r})])$

The minimum cost is chosen by comparing the outputs of Algorithm 1 and Algorithm 2.

Non-Sorting-Genetic Algorithm II: General problem solving: The use of meta-heuristic algorithms will have highly profitable results in solving complex difficult problems. Therefore, the use of the NSGA-II in solving unrestricted multi-objective problems is expanding swiftly [39]. In this paper, due to the double-objective nature and multiplicity of constraints, this algorithm is employed to solve the general model of the SC. Fig. 2 presents the flowchart of this algorithm [40].

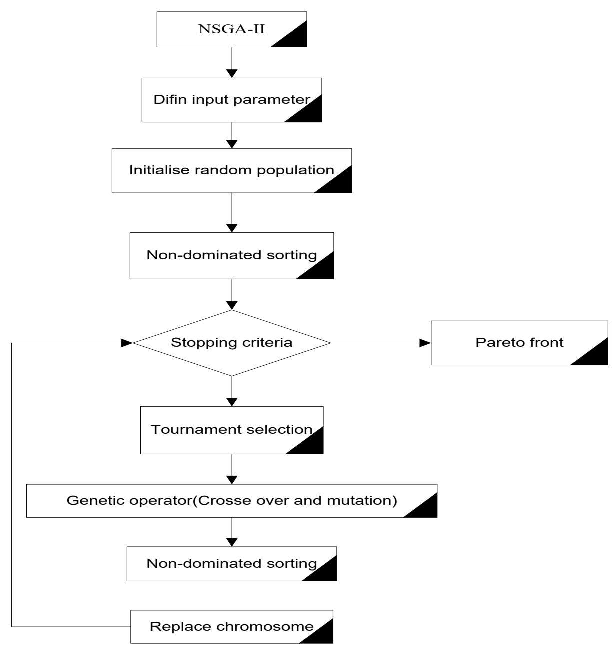

FIG. 2
FLOW CHART OF THE NSGA-II

Proposed chromosome: The structure of the chromosome is defined to be consisting of several components, in which the variable related to location and survivors are considered as one part. The variables in this part are defined as is shows figure 3.

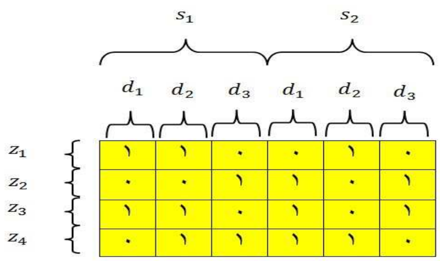

FIG.3
PROPSED CHROMOSOME

Figure 4 shows the intersection of the proposed chromosome. As can be seen, a single point intersection has been used. In this form, a point is randomly selected and the corresponding genes are moved

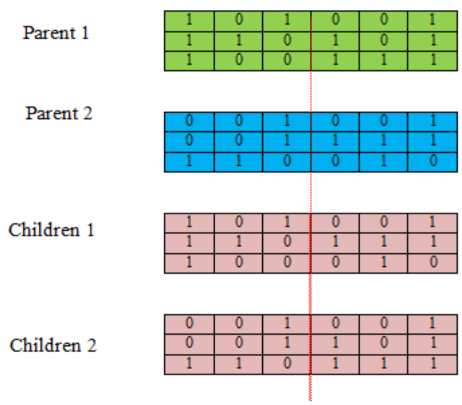

FIG.4
PROPSED CHROMOSOME

Mutation operation: Figure 5 shows the mutation operator. For this purpose, a row is selected as desired and the selected row is reversed.

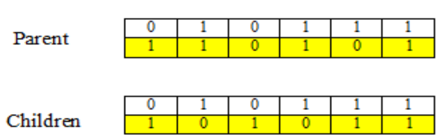

FIG.5
MUTATION OPERATION

To estimate the algorithm parameters, the result of 100 experiments designed for this problem shows that in each experiment, the algorithm parameters change and the results change accordingly. Figure 6 shows the results of the Taguchi approach for estimating parameters.

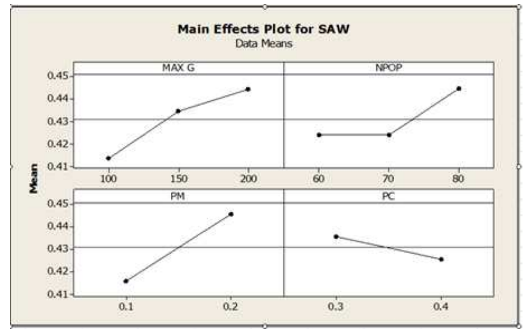

FIG.6
TAGUCHI PARAMETER ADJUSTMENT

In addition, a Taguchi method is used to set the parameters of these algorithms to enhance their performance. Table 2 shows proposed values for NSGA II algorithm parameters

TABLE 2
PROPOSED VALUES FOR NSGA II ALGORITHM PARAMETERS

mutation	Cross over	Pop number	Max number
0.2	0.4	80	200

Numerical example

Fig. 1 presents a numerical example for the CLSC model to understand the problem model. This problem is solved using MATLAB R2018b coding.

Computational results

The CLSC, as a multi-objective issue, is among the most important branches of SC problems. One of the primary measures in such issues is to decrease the service distance of retailers or lessen the time for customers to reach the service centers. In this study, which is a special case of CLSC issues, the problem is addressed by presenting heuristic allocation algorithms and focusing on retailers with known coordinates and the level of their coverage distance to send services. Moreover, customers have probabilistic coordinates and the coverage time of visiting the retail centers. In this model, the optimal allocation is done for the first time by simultaneously comparing the distances and expected coverage of retailers and time and expected coverage of probabilistic customers. Also, the best places of retailers are discovered using the NSGA-II algorithm. Afterward, the distance coverage radius between retailers and the time coverage radius of the customers is calculated considering the amount of standard radius, upper and lower bounds of each of the retailers and customers. To prevent further dissemination in solving this example, we held the potential location search range for probabilistic customers within [4000, 7000] and the optimal search location for retail centers within [3000, 7000] spans. Thus, the optimal coordinates of retailers are calculated in the same span.

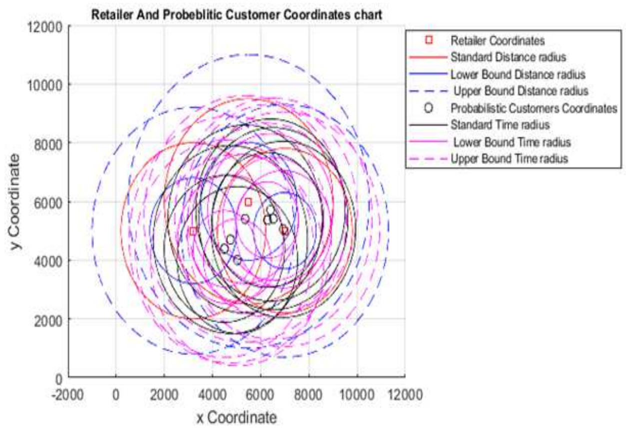

FIG.7
INITIAL COORDINATES OF RETAILERS AND PROBLEMATIC CUSTOMERS AND STANDARD LOWER AND UPPER COVERAGE RADIUS

Fig. 8 shows the Random points, Algorithm first point, and the Best point of the total cost and profit simultaneously until reaching the optimal point.

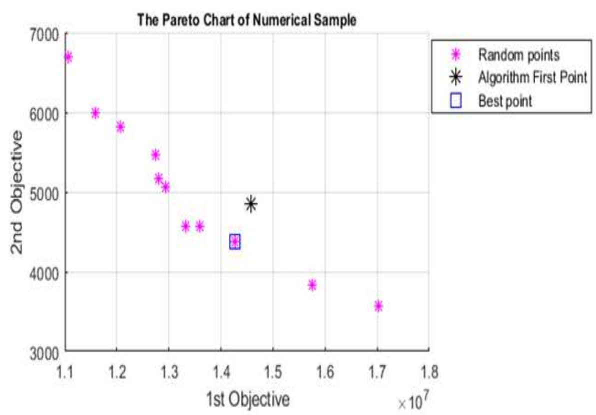

FIG.8
THE PARETO FRONT CHART OF NUMERICAL SAMPLE

The results of problem-solving before and after solving with the NSGA-II algorithm are presented in Table 3. The results show that customers should turn to retailers for services.

TABLE 3
TOTAL COST AND PROFIT FUNCTION

	Results check	**Cost and profit calculated by allocating the proposed algorithm before NSGA-II**		**Cost and profit calculated by allocating the proposed algorithm after NSGA-II**	
		Cost function	Profit function	Cost function	Profit function
1	The total cost of the system, if retailers provide services to probabilistic customers	14806589	5082	14385401	4449
2	The total cost of the system, if probabilistic customers refer to retailers for service	14572321	4913	14260093	4311
Conclusion		Customers refer to retailers for services		Customers refer to retailers for services	

Fig. 9 presents the initial coordinates of retailers and probabilistic customers. Also, based on the results of Table 3, it presents the calculated optimal coordinates that retailers can offer to these customers.

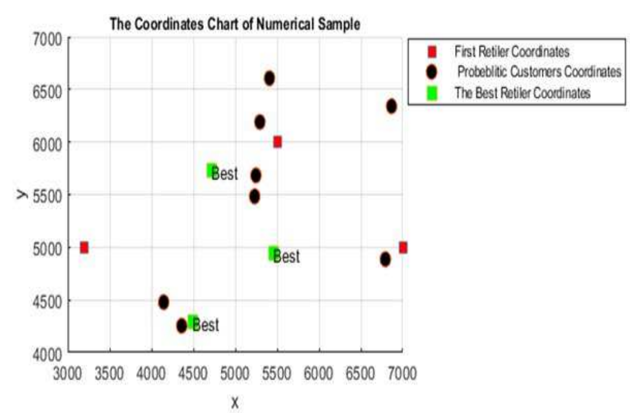

FIG. 9
OPTIMAL COORDINATES OF RETAILERS

Table 4 provides Retailer coordinates before and after solving by NSGA-II.

TABLE 4
RETAILER COORDINATES BEFORE AND AFTER SOLVING BY NSGA-II

Number Retailer	Coordinates of the retailers before solving with NSGA-II	Coordinates of the retailers after solving with NSGA-II
1	[3200, 5000]	[4489.6, 4288.2]
2	[5500, 6000]	[5457.6, 4939.4]
3	[7000, 5000]	[4715.9, 3572.7]

TABLE5
PROBABILISTIC CUSTOMERS ASSIGNED TO RETAILERS

		Before solving by Nsga2			After solving by Nsga2		
		j=1	j=2	j=3	j=1	j=2	j=3
i=1	r=1	✓				✓	
	r=2		✓			✓	
i=2	r=1			✓			✓
	r=2		✓				✓
i=3	r=1		✓			✓	
	r=2	✓				✓	
i=4	r=1		✓				✓
	r=2	✓					✓
i=5	r=1		✓			✓	
	r=2	✓				✓	
i=6	r=1	✓					✓
	r=2			✓			✓
i=7	r=1			✓		✓	
	r=2			✓		✓	
i=8	r=1		✓		✓		
	r=2	✓			✓		

Regarding the results of this problem before and after solving NSGA-II and the allocation of retailers to customers, Table 5 shows customers should refer to retailers for receiving desired goods.

Table 6 presents the customers' decision regarding the type of motion or the use of coverage before and after the solution by the algorithm NSGA-II.

TABLE6
DECIDING ON THE TYPE OF MOVEMENT OR COVERAGE

		Before solving by Nsga2			After solving by Nsga2		
		j=1	j=2	j=3	j=1	j=2	j=3
i=1	*r=1*	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
i=2	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
i=3	*r=1*	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
i=4	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**dicCH**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**dicCH**
i=5	*r=1*	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
i=6	*r=1*	COV^2	COV^2	COV^2	COV^2	dicCH	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicCH	**COV^2**
i=7	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
i=8	*r=1*	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**

Conclusion

In this paper, a closed-loop supply chain (CLSC) is modeled to obtain the best location of retailers and allocate them to other utilities. The structure of CLSC includes production centers, retailers' centers, probabilistic customers, collection, and disposal centers. In the first step, considering the probabilistic place of customers and the fixed location of retailers, the expected distance is calculated at the first step according to the type of movement (Rectangular, Euclidean, Ecclesiastical Square, And Chebyshev). Next, based on the coverage radius, the distance between retailers and the time of probabilistic customers is calculated probabilistically by integral calculations. Additionally, customers were allocated to retailers or vice versa by presenting Algorithms 1 and 2. In the second step, which is the general solution to the problem, the NSGA-II algorithm is applied. The results of applying the model to the studied example indicate that concerning costs and profit. Based on these results, customers are recommended to refer to retailers to receive services. Moreover, the type of motion was hinted at considering the calculated expected coverage time. Furthermore, new coordinates are calculated for retailers such that to provide the lowest cost for customers and enable the optimal allocation of retailers to customers. Finally, routing, relocating time, probabilistic inventory in warehouse can be possible themes for future research using various scenarios in a time window.

APPENDIX:

$$f_{1(j,i)} = \min\left(E\left[dic_{j,i}\right]\right)$$
$$= min\left(E\left[\left(dicR_{j,i}\right),\left(dicED_{j,i}\right),\left(dicEDS_{j,i}\right),\left(dicCH_{j,i}\right)\right]\right)$$
$$= min\left[\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right)\right),\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}}\right)\right)\right),\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(d_{j1}-\mu_{i1}\right)^2+\sigma_{i1}^2+\left(d_{j2}-\mu_{i2}\right)^2+\sigma_{i2}^2\right)\right),\left(max\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1}-\mu_{i1}\right|,\left|d_{j2}-\mu_{i2}\right|\right)\right)\right]$$

$$f_{2(i,j)} = \min\left(E\left[dic_{i,j}\right]\right)$$
$$= min\left(E\left[\left(dicR_{i,j}\right),\left(dicED_{i,j}\right),\left(dicEDS_{i,j}\right),\left(dicCH_{i,j}\right)\right]\right)$$
$$= min\left[\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left|\mu_{i1}-d_{j1}\right|+\left|\mu_{i2}-d_{j2}\right|\right)\right),\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}}\right)\right)\right),\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\mu_{i1}-d_{j1}\right)^2+\sigma_{i1}^2+\left(\mu_{i2}-d_{j2}\right)^2+\sigma_{i2}^2\right)\right),\left(max\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left|\mu_{i1}-d_{j1}\right|,\left|\mu_{i2}-d_{j2}\right|\right)\right)\right]$$

Lemma1: The following equations are always confirmed.

$$\left[dicR_{j,i}\right] = \sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right)$$

$$\left[dicR_{i,j}\right] = \sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left|\mu_{i1}-d_{j1}\right|+\left|\mu_{i2}-d_{j2}\right|\right)$$

Proof: a_{i1} *and* a_{i2} are independent of each other. As a result, we have the independence of customers of this model.

$$E\left[dic_{R_{j,i}}\right] = E\left[l\left(d_j,a_i\right)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\left(d_j,a_i\right)f_i\left(a_i\right)la_{i1}la_{i2}$$
$$\int_{-\infty}^{+\infty}\left(\left|d_{j1}-a_{i1}\right|f_i\left(a_i\right)la_{i1}la_{i2}+\int_{-\infty}^{+\infty}\left|d_{j2}-a_{i2}\right|f_i\left(a_i\right)la_{i1}la_{i2}\right) = \int_{-\infty}^{+\infty}\left(\left(\left|d_{j1}-a_{i1}\right|f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right)+\int_{-\infty}^{+\infty}\left(\left(\left|d_{j2}-a_{i2}\right|f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right) =$$
$$\left[\left|\left(\left[\int_{-\infty}^{+\infty}\left(d_{j1}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]-\left[\left(a_{j1}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]\right)\right|\right]+\left[\left|\left(\left[\int_{-\infty}^{+\infty}\left(d_{j2}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]-\left[\left(a_{j2}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]\right)\right|\right]$$
$$=\left[\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right]$$

Lemma 2: The following equations are always confirmed [41].

$$\boldsymbol{E\left[dicED_{j,i}\right] = \sum_{j=1}^{J}\sum_{i=1}^{I}\left[\left(\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}}\right)\right]}$$

$$\boldsymbol{E\left[dicED_{i,j}\right] = \sum_{i=1}^{J}\sum_{j=1}^{I}\left[\left(\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}}\right)\right]}$$

Lemma 3: The following equations are always confirmed**.**

$$E\left[dicEDS_{j,i}\right]=\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(d_{j1}-\mu_{i1}\right)^2+\sigma_{i1}^2+\left(d_{j2}-\mu_{i2}\right)^2+\sigma_{i2}^2\right)$$

$$E\left[dicEDS_{i,j}\right]=\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\mu_{i1}-d_{j1}\right)^2+\sigma_{i1}^2+\left(\mu_{i2}-d_{j2}\right)^2+\sigma_{i2}^2\right)$$

Proof:

$$E\left[l\left(d_j,a_i\right)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\left(d_j,a_i\right)f_i\left(a_i\right)la_{i1}la_{i2}$$

The Parameters a_{i1}, a_{i2} are independent of each other. As a result, the customers of this model are independent.

$$\boldsymbol{f_j = \left(a_{i1}\times a_{i2}\right) = f_{i1}\left(a_{i1}\right)\times f_{i2}\left(a_{i2}\right)}$$

$$E\left[l\left(d_j,a_i\right)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(d_{j1}-a_{i1}\right)^2+\left(d_{j2}-a_{i2}\right)^2\right]f_i\left(a_{i1}\right)\times f_i\left(a_{i2}\right)la_{i1}la_{i2}$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(d_{j1}-a_{i1}\right)^2 f_i\left(a_{i1}\right)\times f_i\left(a_{i2}\right)la_{i1}la_{i2}+\left(d_{j2}-a_{i2}\right)^2 f_i\left(a_{i1}\right)\times f_i\left(a_{i2}\right)la_{i1}la_{i2}\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left({d_{j1}}^2+{a_{i1}}^2-2d_{j1}a_{i1}\right)\times f_i\left(a_{i1}\right)\times f_i\left(a_{i2}\right)la_{i1}la_{i2}\quad+\left({d_{j2}}^2+{a_{i2}}^2-2d_{j2}a_{i2}\right)\times f_i\left(\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} d_{j1}^2\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}+\int_{-\infty}^{+\infty} a_{i1}^2\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}-\int_{-\infty}^{+\infty}\left(2d_{j1}a_{i1}\right)\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}+\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} d_{j2}^2\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}+\int_{-\infty}^{+\infty} a_{i2}^2\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}-\int_{-\infty}^{+\infty} 2d_{j2}a_{i2}\times f_i\left(a_{i1}\right)la_{i1}\times f_i\left(a_{i2}\right)la_{i2}\right]a_{i1}\right)\times f_i\left(a_{i2}\right)la_{i1}la_{i2}\right] = \left[d_{j1}^2+\left(\sigma_{i1}^2+\mu_{i1}^2\right)-2d_{j1}\mu_{i1}\right]+\left[d_{j2}^2+\left(\sigma_{i2}^2+\mu_{i2}^2\right)-2d_{j2}\mu_{i2}\right]$$

On the other hand, the following statements are always valid:

$$\int_{-\infty}^{+\infty} a_{ik}f_{ik}\left(a_{ik}\right)la_{ik} = \mu_{ik}$$

$$\int_{-\infty}^{+\infty} {a_{ik}}^2 f_{ik}\left(a_{ik}\right)la_{ik} = \sigma_{ik}^2+\mu_{ik}^2$$

$$\int_{-\infty}^{+\infty} f_{ik}\left(a_{ik}\right)la_{ik} = 1$$

Lemma 4: The following equations are always confirmed [42].

$f_{3(j,i,r)}$=*Max* $E\left(COV^1\left(l_{j,i,r}\right)\right)$ =

$$
\begin{cases}
1 & l_{jir} \le L_j - \mathrm{e}_j \\
\left(\frac{1+\left(\frac{L_j - l_{jir}}{\mathrm{e}_j}\right)}{2}\right) + 2\left(\frac{L_j - l_{jir}}{\mathrm{e}_j}\right) \times ln2 - \frac{1}{2}\left(\left(\left(\frac{L_j - l_{jir}}{\mathrm{e}_j} + 1\right)^2 \times ln\left(1 + \left(\frac{L_j - l_{jir}}{\mathrm{e}_j}\right)\right)\right) - \left(\left(\frac{L_j - l_{jir}}{\mathrm{e}_j}\right)^2 \times ln\left(\frac{L_j - l_{jir}}{\mathrm{e}_j}\right)\right)\right) & L_j - \mathrm{e}_j < l_{jir} \le L_j \\
\left(\frac{1-\left(\frac{l_{jir} - L_j}{\mathrm{e}_j}\right)}{2}\right) - 2\left(\frac{l_{jir} - L_j}{\mathrm{e}_j}\right) \times ln2 + \frac{1}{2}\left(\left(\left(\frac{l_{jir} - L_j}{\mathrm{e}_j} + 1\right)^2 \times ln\left(1 + \left(\frac{l_{jir} - L_j}{\mathrm{e}_j}\right)\right)\right) - \left(\left(\frac{l_{jir} - L_j}{\mathrm{e}_j}\right)^2 \times ln\left(\frac{l_{jir} - L_j}{\mathrm{e}_j}\right)\right)\right) & L_j < l_{ijr} \le L_j + \mathrm{e}_j \\
0 & l_{ijr} > L_j + \mathrm{e}_j
\end{cases}
$$

Lemma 5: The following equations are always confirmed [42].

$f_{4(i,j,r)} = Max\ E\left(COV^2(t_{i,j,r})\right) =$

$$
\begin{cases}
1 & t_{ijr} \le T_i - \theta_i \\
\left(\frac{1+\left(\frac{T_i - t_{ijr}}{\theta_i}\right)}{2}\right) + 2\left(\frac{T_i - t_{ijr}}{\theta_i}\right) \times ln2 - \frac{1}{2}\left(\left(\left(\frac{T_i - t_{ijr}}{\theta_i} + 1\right)^2 \times ln\left(1 + \left(\frac{T_i - t_{ijr}}{\theta_i}\right)\right)\right) - \left(\left(\frac{T_i - t_{ijr}}{\theta_i}\right)^2 \times ln\left(\frac{T_i - t_{ijr}}{\theta_i}\right)\right)\right) & T_i - \theta_i < t_{ijr} \le T_i \\
\left(\frac{1-\left(\frac{t_{ijr} - T_i}{\theta_i}\right)}{2}\right) - 2\left(\frac{t_{ijr} - T_i}{\theta_i}\right) \times ln2 + \frac{1}{2}\left(\left(\left(\frac{t_{ijr} - T_i}{\theta_i} + 1\right)^2 \times ln\left(1 + \left(\frac{t_{ijr} - T_i}{\theta_i}\right)\right)\right) - \left(\left(\frac{t_{ijr} - T_i}{\theta_i}\right)^2 \times ln\left(\frac{t_{ijr} - T_i}{\theta_i}\right)\right)\right) & T_i < t_{ijr} \le T_i + \theta_i \\
0 & t_{ijr} > T_i + \theta_i
\end{cases}
$$

Lemma 6: The following equations are always confirmed.

$$E\left[dicCH_{j,i}\right] = max\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1} - \mu_{i1}\right|, \left|d_{j2} - \mu_{i2}\right|\right)$$

$$E\left[dicCH_{i,j}\right] = max\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left|\mu_{i1} - d_{j1}\right|, \left|\mu_{i2} - d_{j2}\right|\right)$$

Proof:

$$E\left[l(\boldsymbol{d}_j, \boldsymbol{a}_i)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(MAX\left(\left|d_{j1} - a_{i1}\right|, \left|d_{j2} - a_{i2}\right|\right)f_i(a_i)la_{i1}la_{i2}\right) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} MAX\ \left(\left(\left|d_{j1} - a_{i1}\right|f_i(a_i)la_{i1}, \left|d_{j2} - a_{i2}\right|\right)f_i(a_i)la_{i2}\right) = MAX\left(\left|\int_{-\infty}^{+\infty} d_{j1}f_i(a_i)la_{i1}\right| - \left|\int_{-\infty}^{+\infty} a_{i1}f_i(a_i)la_{i1}\right|\right), \left(\left|\int_{-\infty}^{+\infty} d_{j2}f_i(a_i)la_{i2}\right| - \left|\int_{-\infty}^{+\infty} a_{i2}f_i(a_i)la_{i2}\right|\right) = MAX\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1} - \mu_{i1}\right|, \left|d_{j2} - \mu_{i2}\right|\right)$$

Lemma 7: The following equations are always confirmed [41].

$$\boldsymbol{E}\left[\boldsymbol{dicED}_{j,i}\right] = \sum_{j=1}^{J}\sum_{i=1}^{I}\left[\left(\sqrt{(\boldsymbol{d}_{j1} - \boldsymbol{\mu}_{i1})^2 + (\boldsymbol{d}_{j2} - \boldsymbol{\mu}_{i2})^2}\right) + \frac{1}{2}\left(\frac{\boldsymbol{\sigma}_{i1}^2 + \boldsymbol{\sigma}_{i2}^2}{\sqrt{(\boldsymbol{d}_{j1} - \boldsymbol{\mu}_{i1})^2 + (\boldsymbol{d}_{j2} - \boldsymbol{\mu}_{i2})^2}}\right)\right]$$

$$\boldsymbol{E}\left[\boldsymbol{dicED}_{i,j}\right] = \sum_{i=1}^{J}\sum_{j=1}^{I}\left[\left(\sqrt{(\boldsymbol{\mu}_{i1} - \boldsymbol{d}_{j1})^2 + (\boldsymbol{\mu}_{i2} - \boldsymbol{d}_{j2})^2}\right) + \frac{1}{2}\left(\frac{\boldsymbol{\sigma}_{i1}^2 + \boldsymbol{\sigma}_{i2}^2}{\sqrt{(\boldsymbol{\mu}_{i1} - \boldsymbol{d}_{j1})^2 + (\boldsymbol{\mu}_{i2} - \boldsymbol{d}_{j2})^2}}\right)\right]$$

DECLARATIONS:

THE AUTHORS DECLARE THAT THEY HAVE NO CONFLICT OF INTEREST TO THE PUBLICATION OF THIS ARTICLE.

Lemma 8: The following equations are always confirmed.

$$\boldsymbol{E(S) = Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\}}$$

$$\boldsymbol{E(S') = Cost'_{i,j,r} \times min\{E(x'_{i,j,r}), cap_j\} - \pi_{i,j,r} \times max[(E(x'_{i,j,r}) - cap_j), 0]}$$

PROOF:

$$\boldsymbol{E(S) = Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times \mathbf{max}\,\{(E(x_{j,i,r}) - cap_j), 0\}}$$

$$\boldsymbol{= Cost_{j,i,r} \times \left[\int_0^{cap_j} x_{j,i,r} \times f_x(x_{j,i,r})dx + \int_0^{\infty} cap_j \times f_x(x_{j,i,r})dx\right] - \pi_{j,i,r} \times \left[\int_{cap_j}^{\infty}(x_{j,i,r} - cap_j) \times f(x_{j,i,r})dx\right]}$$

$$\boldsymbol{= Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\}}$$

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