An Extended Approach to Value Chain Analysis

Klemen Knez (Klemen.Knez@fdv.uni-lj.si)
University of Ljubljana: Univerza v Ljubljani https://orcid.org/0000-0003-4772-7074
Andreja Jaklič
University of Ljubljana: Univerza v Ljubljani
Metka Stare
University of Ljubljana: Univerza v Ljubljani

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An Extended Approach to Value Chain Analysis

Klemen Knez[a*], Andreja Jaklič[a], Metka Stare[a]

Abstract

In the article, we propose a comprehensive methodology of value chain analysis in the international input-output framework, introducing a new measure of value chain participation and an extended typology of value chains, including a new measure of fragmentation of domestic value chain production. This allows for the simultaneous analysis of both global and domestic production fragmentation, complex patterns of their evolution and their impact on economic development. The main contribution of the proposed methodology is conceptual: it allows the measurement of all value chain paths that pass through each country-sector from production to final consumption, whether the path includes downstream linkages, upstream linkages or a combination of both at the same time. The empirical application of this methodology shows the importance of including domestic fragmentation in value chain analyses: Both global and domestic fragmentation levels of production show a significant positive correlation with economic growth. This implies that the effects of global production fragmentation need to be analysed together with the changing structure of domestic production fragmentation to obtain a complete picture, which could provide important information for policy-making and industrial policy.

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Introduction

In recent decades, the increasing complexity of the division of labour has manifested itself in the fact that a growing share of production takes place within value chains, both domestically and globally. Theoretical and empirical approaches to the analysis of value chains have advanced rapidly, but are currently very eclectic and heterogeneous. The first definitions of commodity chains[1] can be traced back to the world-systems[2] theory: "What we mean by such chains is the following: take an ultimate consumable item and trace back the set of inputs that culminated in this item - the prior transformations, the raw materials, the transportation mechanisms, the labor input into each of the material processes, the food inputs into the labor. This linked set of processes we call a commodity chain. (Hopkins & Wallerstein, 1977)". In the 1990s, the research programme of global commodity chains was first systematically outlined by Gereffi's seminal contribution (1994), in which he defined three interlocking dimensions of research: Input-output dimension, spatial dimension and the question of commodity chain governance[3]. This research period

[a]Centre of International Relations, University of Ljubljana, Ljubljana, Slovenia

[*]Corresponding author, mail: Klemen.Knez@fdv.uni-lj.si

[1]Global commodity chain framework is a predecessor of global value chain framework.

[2]Embracing a historical and macroeconomic approach to the analysis of the global division of labour, world-systems approach examines the unequal patterns of exchange along global commodity chains as well as different structural patterns of international integration of core, periphery and semi-periphery (Arrighi & Drangel, 1986).

[3]Governance was conceived either as consumer-driven (apparel sector) or producer-driven (auto-motive sector).

was characterised by a reorientation from a historical and macroeconomic perspective towards a special focus on industrial chains, with numerous case studies on value chains. The global value chain framework emerged at the beginning of the new century with the explicit aim of unifying the previous research conducted under heterogeneous terminology (Gereffi *et al.*, 2001). On the one hand, the global value chain approach increased the focus on the enterprise level and merged with literature from international business and management[4], while also drawing from new institutional transaction cost approach[5]. On the other hand, the creation of international input-output tables[6] led to a revival of the aggregated macroeconomic approach to global value chains, albeit with a different focus compared to the world-systems approach.[7]

In this article, we present a novel conceptualization of value chains and a methodology for measuring participation rates in the international input-output framework. Compared to the most widely used measurement of value chain participation introduced by Wang et al. (2017), we make two fundamental conceptual improvements. First, our methodology creates a single and consistent measurement of value chain participation at country-sector level, as opposed to two (upstream and downstream) participation rates, which is a feature of Wang's methodology. We thus create a single consistent variable at the country-sector level that measures the overall level of participation in value chains, thus enabling us to empirically test many research theses that were previously either limited to the aggregate level or had to be articulated separately in terms of measuring the impact of upstream and downstream integration. Second, our methodology allows for extensions of the typology of value chains that are not possible with Wang's approach to decomposition of production activities. We introduce a novel measure of the domestic value chain participation rate to measure the share of production that represents the extent of domestic production fragmentation. Since we distinguish between domestic production, which is fragmented, and domestic production, which is not fragmented (it consists of producing a direct value for consumption without cooperation between firms), our concept of the domestic value chain is a completely new and different concept compared to Wang's domestic component, which does not distinguish between the two and combines both categories into one concept. In Wang's methodology, this undifferentiated element of decomposition is simply referred to as the domestic component and should not be confused with our separate concepts of domestic value chain and no value chain production activities - both represent a production that directly and indirectly involves only domestic firms, while the domestic value chain represents a domestic production that is fragmented (includes domestic cooperation between firms), and no value chain represents a domestic production that is not. While the Wang's share of the domestic component is only a simple residual - a negation of the share of global production fragmentation and the global Ricardian trade share that does not provide information on the nature of the domestic economy, our novel methodology allows us to measure the extent of fragmentation of domestic production in addition to the usual study of international production fragmentation.

In order to derive a single measure of value chain participation on country-sector level (compared to existing upstream and downstream measures), we cannot rely on the existing approach, which uses a value added export matrix describing the value flows between any two country-sectors in the economy.

[4]Porter's (1985) concept of the intra-firm value chain is often used to discuss the specialisation of enterprises, and core competencies and business literature on multinational enterprises overlap with the global value chain framework.

[5]Which was used to extend the producer-driven and consumer-driven governance typology of commodity chain research to a more general typology of value chain linkages, from transactions on the completely free market to a strict hierarchy (Gereffi *et al.*, 2005).

[6]In international economics, the use of input-output methodology gained importance as researchers of various international incentives integrated nationally based input-output tables into harmonised global input-output tables. Most prominent are the World Input-Output Database (Timmer *et al.*, 2015), the OECD Trade in Value Added and the EORA (Lenzen *et al.*, 2013).

[7]While all heterogeneous approaches to value chains focus on a development issue, the late GVC approach is taken up by international institutions to highlight gains from liberalisation and industrial upgrading, while world-systems approach critically examines unequal rewards along the value chain and different structural integration patterns that may cause perpetuation of unequal development (Gereffi, 2018; Taglioni & Winkler, 2016; Mirodout *et al.* 2013).

Instead, we need to observe the asymmetric value chain stemming both downstream and upstream from each particular country-sector concerned *simultaneously*. In other words, we need to observe all paths of value creation that pass through a given country-sector, as well as all paths of its value realisation. Only by looking at each value chain as an asymmetric tree of upstream and downstream linkages can we establish a unique measure of global and domestic integration at country-sector level. Thus, we provide methodological tools that allow us to explore the complex interrelationship of global and domestic value chains and their evolution over time. We believe that this will help us to understand the diverse patterns of structural integration of different countries or sectors and the different effects of such patterns on economic development. Although this is primarily a methodological contribution, we will use very basic empirical data to try to show the possible link between the level of global and domestic production fragmentation and overall economic growth.

The article is structured as follows: In section 1 we review the existing value chain indicators and address their shortcomings. In section 2 we present the new conceptualisation of the value chain in the international I-O framework and define our object of disaggregation. In section 3 we present a new value chain typology and derive participation shares. In section 4 we present basic empirical results of the new methodology, show which insights into economic structures can be gained on the basis of new value chain measures and which links exist between value chain integration patterns and overall economic growth. Finally, we discuss the contributions of the paper and possibilities for further research.

1 Literature review

Most recent macroeconomic analyses of the global value chains are based on the use of international input-output methodology. As international I-O data are essentially an integrated standard accounting data set harmonised at sectoral level, there is a lack of any information on the typology of value chain governance. Therefore, the international I-O database cannot be the only source for the study of production networks, which theoretically differ from purely open trade transactions by including at least a certain level of hierarchy, and which investigate the local embedding of production linkages (Henderson et al., 2002; Hess & Coe, 2006). However, the general framework of global value chains functions without such distinctions and makes the international I-O data set one of its most important sources of information. The primary benefit of applying the I-O methodology in global value chain analysis is to obtain aggregated information about the structure of value chains, as opposed to isolated firm-specific case studies that can provide a more detailed understanding of different aspects of a particular value chain. Thus, of the 3 dimensions of commodity chain research according to Gereffi (1994), both the I-O aspect and the spatial dimension can be carried out within the international I-O approach, while the governance aspect can't. On the one hand, the international I-O data allow for an analysis of trade in terms of value added, which is hidden behind classical trade statistics, expressed in gross values, and more accurately (by eliminating double counting) reflects the actual trade relations (Johnson & Noguera, 2012). On the other hand, it contains in-depth structural information on the sectoral interdependence of the economy at both national and global level, thus offering the opportunity to empirically capture and measure value chains. Various aggregated and sectoral global value chain indicators, indices and measures have been proposed, which have been derived in the international I-O framework. GVC indicators can be roughly divided into measures of length[8] and participation rates, which we will discuss briefly.

Early I-O measures of the GVC structure were simple upstream and downstream indicators that corresponded to the measure of distance to final demand (upstream) and the Leontief measure of backward linkage (downstream) and were often referred to as the length of a value chain (Ahmad *et al.*, 2017). Fally

[8] Relative position indices can easily be derived from length measures as simple ratios.

(2011) and Antras *et al.* (2012) defined the downstream indicator to "reflect how many plants (stages) are involved in production one after the other" up to the point observed and the upstream indicator to "measure how many plants this product will pass through (e.g. by assembly with other products) before it reaches final demand (Fally, 2011, 10)". Fally (2011) defined them as a number of vertical stages weighted by the value added of each stage, with the distance between each stage set to 1.[9] Since then, the average vertical distance has been the basic measure of the length of the value chain in the international I-O framework. Miller and Temurshoev (2015) further specified the existing measures by presenting upstream and downstream indicators in a matrix formulation using Ghosh's forward and Leontief's backward coefficient matrices (Ghosh, 1958; Leontief, 1936). These upstream and downstream measures are simple measures of the upstream and downstream length of value chains measured by the average vertical distance. Within this framework, further improvements were introduced by Muradov (2016), who focused on the separation of the domestic from the global production component when calculating the length of value chains.

The conceptualisation of GVC participation measures is largely based on the work of Johnson and Noguera (2012), who produced a value-added export matrix that captures information on value flows in the economy between any two points (country-sectors) in the economy. This provides the basis for the disaggregation of value at the country-sector level, depending on whether the value was produced domestically for domestic consumption or whether it involved cross-border transactions for either final or productive consumption (Koopman *et al*, 2014; Wang *et al*, 2017). Since the value-added export matrix informs us about the source and destination of value added and covers all possible paths between any two country-sectors in the economy, there are two indicators of the share of GVC participation - the upstream and downstream share. The conception of the upstream participation share of participation starts from the value added of the individual industries (country-sectors), disaggregating all possible paths leading to the realisation of their value, while the conception of the downstream share of participation starts with final consumption, disaggregating all possible paths of the downstream production linkages. Within this framework, disaggregation is defined on the domestic part, the "Ricardian trade" in finished goods, the simple GVC and the complex GVC, which is currently the most widely used accounting framework for GVC participation, which has so far been used by the best known research on GVC carried out jointly by the WTO, the WB group, the OECD, IDE -JETRO, RCGVC-UIBE and the China Development Research Foundation (GVC Development Reports).

We address two main shortcomings of the existing indicators. The first is the lack of a single uniform criterion for participation rates at the country-sector level. While the existing indicators for downstream and upstream participation rates are consistent at the aggregate level, they are different and provide two different types of information at the country-sector level. This is relevant for some types of analysis that deal with the relationship between upstream and downstream participation in GVCs, but there are a variety of situations where a common measure of GVC participation, defined uniformly at country-sector level, is required either as the main object of the analysis or as a supplementary or control variable. It is also obvious that a simple solution, such as using the average of existing upstream and downstream indicators, cannot be justified in theory.[10]

[9] Using a method similar to that used to calculate the average propagation length required for the analysis of the dynamic response to shocks, defined by Dietzenbacher and Romero (2007).

[10] If, for example for a given country-sector, its share in the upstream global value chain is high (close to 100%) and its share in the downstream global value chain is relatively low (close to 0%), then the average share in the value chain would be around 50%, which is misleading because the value chain as a whole is almost entirely global (using the criterion that the value crosses borders at least once). As far as value chain paths are concerned, despite the low share of the downstream global value chain paths, the same paths in high share (close to 100%) continue in the upstream global value chain, so that production as a whole has a very high global share (close to 100%), while the use of the average of the upstream and downstream indicator does not correspond to the definition of the global value chain.

The second, less technical aspect is the lack of a measure of domestic value chain fragmentation in existing indicators. The existing disaggregation of participation shares into the "domestic component" and the GVC participation rates (and the Ricardian trade share) consists of a simple duality that should by construction sum to 1, with the definition of the domestic component amounting to the simple absence of compliance with the GVC criteria. This is the reason why, in the existing disaggregation, the share of the domestic component in regressions (due to collinearity) is never used and is never even examined as a theoretical concept. It is simply a residual, a share that does not interest researchers, since all the information they disaggregate is included in their GVC participation rates. Within the existing approaches, the researchers focus exclusively on the international dimension of production fragmentation, neglecting the potential of the international I-O methodology, which allows an analysis of domestic production fragmentation. Our approach is novel in this area, as it proposes a concept of the domestic fragmentation that can be measured on its own and according to its own definition and is not collinear with the sum of the GVC participation rate. This multiplies the research opportunities offered by the value chain methodology based on the international input-output structure, as it allows for a general analysis of both domestic and global production fragmentation and their interdependence, as well as the possible mutual effects of their development.

The proposed methodological approach is fundamentally different from that of Johnson and Noguera (2012) and Wang et al. (2017). The approach of Johnson and Noguera (2012) defined the value added export matrix, the elements of which represent all value transfers between any two country-sectors of the economy along any possible value chain path. This was used by Wang et al. (2017) to define different value chain participation rates (simple global value chain, complex global value chain, Ricardian trade). He defined 2 value chain participation rates: The downstream value chain participation rate is defined by decomposing all value chain paths leading from primary production to the respective country-sector, while the separate upstream value chain participation rate is defined by decomposing all upstream value chain paths leading from the respective country-sector to final consumption. Existing value chain participation rates are thus two-sided indicators, while there does not yet exist a consistent uniform measure of value chain participation at country-sector level. In contrast to the most widespread existing approaches, our starting point for decomposition is not the value-added export matrix, which describes *all the value chain paths between two specific country-sectors* in the economy. Instead, the starting point of our decomposition is a set of presented value chain tree matrices (τ_i) that describe all value chain paths, *from any* country-sector of primary origin *to any* country-sector of production for final consumption that *passes through* (include a production stage of) *a single* particular country-sector. This formulation is the first attempt to capture the information of the asymmetric value chain tree, which is a specific characteristic of each individual country-sector (Figure 1). The proposed value chain tree matrices are unique in that they allow us to simultaneously capture the structure of the downstream and upstream value chain paths and define value chain participation rates as a single measure for each country-sector. The main point of the proposed methodology is to enable a disaggregation of value chains based solely on the structure of value chain paths - taking into account whether these paths include only domestic production fragmentation, international production fragmentation or no production fragmentation at all. This allows us to introduce a concept of domestic value chain fragmentation that is impossible to create within the existing framework of 2 bipartite participation indices.

Using this methodology, we show that the increase in global fragmentation of production in recent decades was the general trend in most countries (with backlash in recent years), but different institutional arrangements as well as different structural economic positions led to different types of global economic integration, which had diverse effects on domestic fragmentation. Using our methodology, we will empirically demonstrate that in many countries with high growth and increasing levels of global integration, domestic value chain fragmentation also increased. There are, however, cases where domestic fragmentation stagnated or even declined while the global value chain fragmentation increased. Different types of integration in global value chains are the result of a variety of structural and institutional developments.[11] On the one hand, the simultaneous increase in domestic and global fragmentation can only be a consequence of the growing complexity and division of labour. On the other hand, the simultaneous increase in global fragmentation and the drastic decline in domestic integration may be a consequence of the fracturing of domestic vertically integrated companies, parts of which integrate into global value chains as subsidiaries, or a consequence of the gradual replacement of domestic suppliers by globally traded inputs, which may increase following a foreign takeover or privatisation. Diverse possibilities amount to different structural positions of each production unit within global production as a whole, and different structural positions may imply different levels of dependence, which can be a factor of economic performance, especially in times of crisis (Horvath & Grabowski, 1999).

2 The Value Chain Tree

2.1 Conceptualisation

We understand the value chain as a series of stages in the production of a product or service for the end user, where each stage adds value and the total value of the end product is the sum of the value added at each stage. For the value chain to exist, there must be at least two separate production stages. The existing GVC framework is analytically and empirically based on the idea that value is created in the production process and added to the value already present in the used intermediate goods. The old value (value of intermediaries) is only transferred to the new product, while the newly created value is added linearly to the transferred value. The same idea is also behind the elimination of double counting in standard gross trade statistics and exploration of the hidden underlying trade in value-added, which provides insight into international structure of trade (Johnson & Noguera, 2012). We use the same basic assumptions for more general value chain analysis.

We examine the structure of the economy from the perspective of a small unit[12] (companies belonging to a particular country and sector) and capture its structural position within domestic and international production by measuring the degree of integration in domestic or global value chains. Each production unit is located within the production structure with a number of transactions linking enterprises. On the one hand, the conditions of production are linked to the inputs produced by other enterprises in downstream linkages and, on the other hand, the final consumption of its product may only be reached after a series of upstream linkages in which its output is used as an input by other enterprises.

Thus, if one concentrates on a specific unit (country-sector) and aims to capture the upstream and downstream value chain linkages *simultaneously*, the value chain can be viewed as a tree, in contrast to the snake or spider analogy (see Figure 1).[13] In the general case, the product is partly consumed immediately after production but also partly sent to further stages of production and from each of these upstream

[11]For example, the concept of integrated periphery was introduced to describe a specific type of integration in the case of the Slovak and Czech car industry, which is characterised by the proximity of consumer markets, cheaper labour force, the absence of positive spillover effects and the absence of domestic linkages (Oldřich & Vladan, 2019; Pavlínek, 2018).

[12]In our derivation, which is consistent with most existing international I-O data, the country-sector is the smallest object of analysis. When we refer to our methodology and derive it, the reference to the country-sector is a reference to the smallest object of analysis given by the level of detail of the I-O data set. If the I-O data sets were built on a more detailed structure at enterprise level structure (greatly increasing dimension), the proposed methodology and measures would work in the same way, with the value chain still structured around the smallest possible unit - in this case the enterprise. Despite the starting point of analysis of value chain structure being the smallest units of analysis, the approach offers many different aggregation possibilities to capture the changing economic structure of production as a whole.

[13]Vertical and horizontal fragmentation of production is often represented by metaphors of snakes (sequential value transfers from one company to further stages in linear sequence) and spiders (simultaneous value transfers from different companies to the same company) (Baldwin & Venables, 2013).

stages it is further decomposed in the same way (*etc. ad infinitum*), spreading out like twigs and leaves until it ends completely in final consumption. Similarly, the primary value-creating activity can be represented by the structure of the roots, whereby value is only partially created in each stage, since it requires pre-existing intermediates, which in turn are further decomposed in the same way *ad infinitum*.

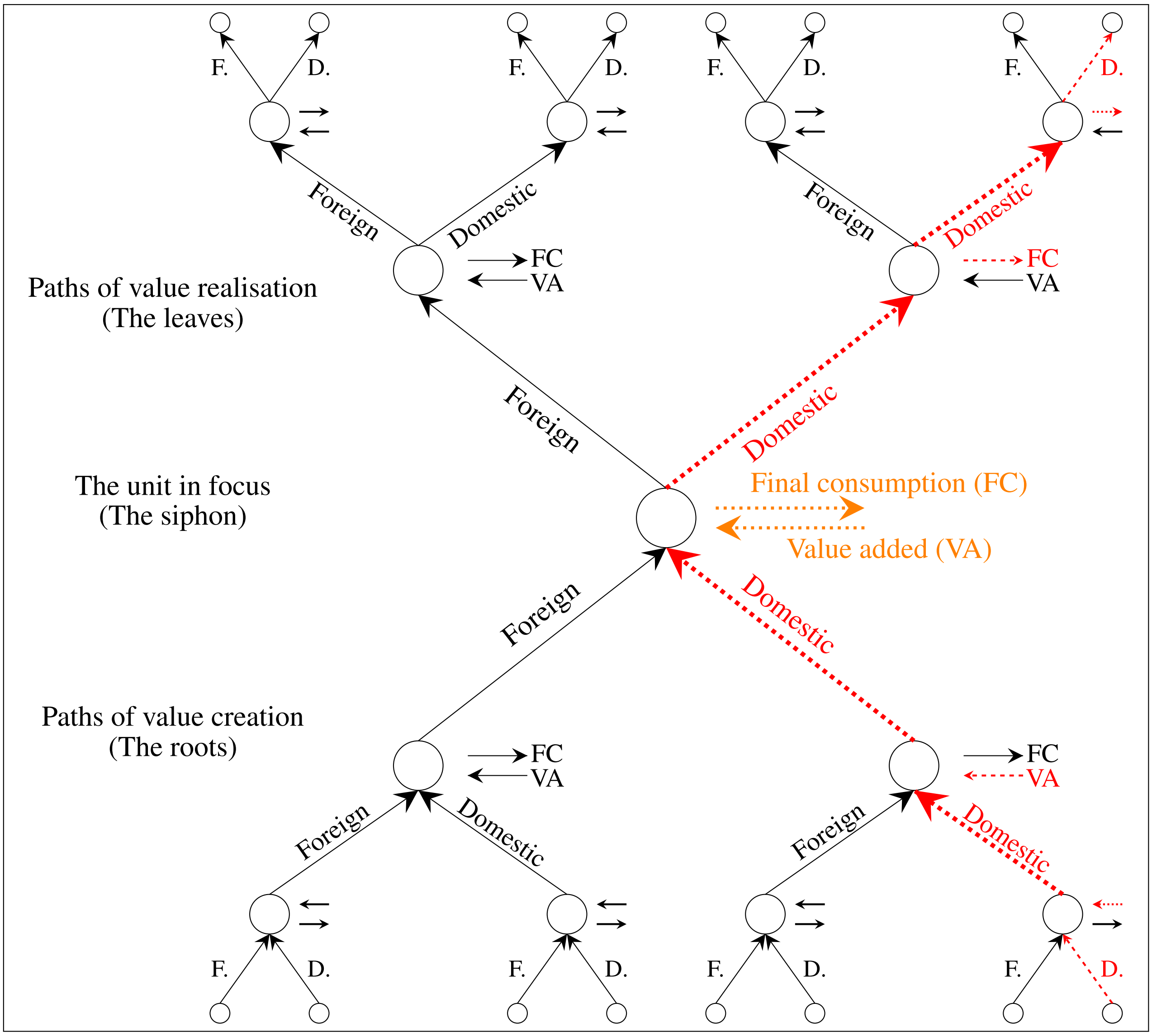


Figure 1: Value chain tree. Source: own conceptualisation and design.

To conceptualise and measure the value chain structure of each specific smallest unit of analysis (country-sector), we introduce the concept of a value chain path. As shown in Figure 1, the value can be decomposed with respect to two dimensions: Origin (where the value was created primarily) and final stage of production (where the final product for consumption is finished). The matrix of value added exports, which is already well known and frequently used, describes all direct and indirect value added paths between any two units (country-sectors) in the networked production system (Johnson & Noguera, 2012). However, the indicators constructed with this conceptualisation are essentially one-sided. The measure of downstream GVC participation conceptualises all possible downstream paths to the focused country-sector, and *vice versa* the upstream measure conceptualises all possible upstream paths. Our goal to derive a single participation measure at the country-sector level requires the derivation of an object that can track the value passing through a specific country-sector in focus along all possible paths from origin to end use. In this way, we decompose the value that is part of the production process of a given country-sector along all its paths, which include not only downstream paths leading to the focused country-sector and upstream paths leading from it to the final consumption, but also, and above all, paths that combine upstream and downstream linkages and pass through the focused country-sector.

In general, any value share can originate in any country-sector, and the same value share can also reach final consumption as a product of any country-sector. What we do differently from Johnson and Noguera is to add the third dimension[14] - the midpoint - the siphon through which the value from any origin to any final stage flows (Figure 1). The value chain tree of each country-sector is defined as the structure of the value chain paths, where this country-sector is the siphon through which the value chain paths pass. We show that each unit of analysis (country-sector) has a unique value chain structure that represents its structural position in the economy. Its output can be decomposed along every possible path within its value chain tree - i.e. along every value chain path that has its primary origin in any country-sector, passes through downstream linkages to the production stage of the country-sector that defines the value chain tree (the siphon), and ends in final consumption through upstream linkages as the final product of any country-sector.

Understanding the structure of value chains by empirically measuring all such paths of each country-sector (smallest unit of analysis) is already an end in itself and can contribute to the further understanding of the economy and its changing structure in terms of global integration, its specific regional and sectoral forms and the complex interactions between domestic and global production fragmentation.

2.2 Derivation

Object of disaggregation is country-sector's total output. Each country-sector's total output is disaggregated along both downstream and upstream linkages that are unique to its specific value chain structure. Downstream disaggregation represents all possible value chain paths from origin of production and upstream disaggregation all possible paths to satisfy final demand, both with respect to the unique value chain tree of each country-sector. In this way, we disaggregate the same object - the total output of each country-sector - *simultaneously* along its downstream and upstream paths.

In contrast to the existing approaches, which are based on the matrix of value added exports covering all value added flows between any two country-sectors in the economy, we propose a new object - a set of matrices that describe the value chain structure of each country-sector separately, covering all value chain paths from each primary origin to each final stage via the output of a single specific country-sector (Figure 1). In this conceptualization, each country-sector has a corresponding value chain tree described by the value chain tree matrix - while the value chain structure of the economy as a whole is described by the set of such matrices.

We derive our disaggregation within the static international Leontief demand driven model[15]. The usual pairs of indices characterizing the country and sector of origin (s,i) and the final destination (d,j) are replaced by a single index for each country-sector for a more transparent notation. Since we no longer work in the $n \times n$ dimensional space, but in the $n \times n \times n$ dimensional space, we would need 3 pairs of indices, 1 pair for the country-sector of origin, 1 pair for the final stage and also 1 pair for the country-sector which is the siphon through which all possible value chain paths exist and characterize its specific value chain structure. Instead, we work with only 3 indices, one for the country-sector of origin (k), one for the final stage country-sector (j) and one to characterize the country-sector value chain tree - the country-sector representing the siphon through which the value chain paths pass (i).[16]

[14] Formal addition of further n dimensions to the usual $n \times n$ dimensions.

[15] The definitions of the notation are in appendix A

[16] The simplification consists only of the notation. We keep all the complexity of the block-matrix structure of the international I-O data and remove only the large number of indices, which would make the equations much more difficult to read.

$$x = C\mathbf{1} + F\vec{1} \tag{2.1}$$

$$x = Ax + f \tag{2.2}$$

We start with the upstream part, by using standard Leontief's derivation:

$$x = (I - A)^{-1} f \tag{2.3}$$

$$x = (I - A)^{-1} \hat{f}\mathbf{1} \tag{2.4}$$

$$\hat{x}^{-1}(I - A)^{-1} \hat{f}\mathbf{1} = \mathbf{1} \tag{2.5}$$

Definition 2.1. Upstream output decomposition W
$W = \hat{x}^{-1}(I - A)^{-1}\hat{f}$

The matrix W represents the upstream output decomposition along all upstream value chain paths. Its element w_{ij} represents the share of the total output of the country-sector i that reaches final consumption as the end product of the country-sector j, along all possible upstream production fragmentation paths in the economy. The i-th row of W represents the disaggregation of the total output of the i-th country-sector into output shares according to its final production stages that account for all direct and indirect paths of upstream value transfers leading to the full realisation of total output (by being used directly or indirectly by other country-sectors as intermediate productive consumption). Each i-th row of W can thus be characterised as a discrete probability distribution. On the one hand, the upstream output shares of each country-sector i add up consistently to 1: $\sum_{j=1}^{n} w_{ij} = 1 \; \forall i$. On the other hand, there is a clear economic interpretation of the probability distribution: w_{ij} represents the probability that a randomly selected part of the total output of the i-th country-sector will eventually be consumed as the final product of the country-sector j, along any upstream value chain path.

For the downstream part we begin with identity:

$$x^T = \mathbf{1}^T \hat{x} \tag{2.6}$$

$$x^T = \mathbf{1}^T (I - A)(I - A)^{-1} \hat{x} \tag{2.7}$$

$$x^T = v_C^T (I - A)^{-1} \hat{x} \tag{2.8}$$

$$x^T = \mathbf{1}^T \hat{v_C} (I - A)^{-1} \hat{x} \tag{2.9}$$

$$\mathbf{1}^T = \mathbf{1}^T \hat{v_C} (I - A)^{-1} \tag{2.10}$$

Definition 2.2. Downstream output decomposition Z
$Z = \hat{v_C}(I - A)^{-1}$

The matrix Z represents the downstream output decomposition along all downstream value chain paths. Its element z_{ki} represents the share of the total output of the country-sector i that is primarily created in the country-sector k, along any possible downstream production fragmentation path in the economy. The i-th column of Z represents the disaggregation of the total output of the i-th country-sector into output shares, which represent all direct and indirect paths of downstream value transfer from each country-sector that has contributed to the production of its output (through direct or indirect production of intermediate productive consumption used by i). Each i-th column of Z can thus be characterized as a discrete probability distribution. On the one hand, the downstream output shares of the individual country-sectors i add up consistently to 1: $\sum_{k=1}^{n} z_{ki} = 1 \; \forall i$. On the other hand, there is a clear economic interpretation of the probability distribution: z_{ki} represents the probability that a randomly selected part of the total output of the i-th country-sector was produced by the country-sector k, along any downstream value chain path.

The two matrices presented, W and Z, can appear as the two sides of the same coin - similar to the forward and backward decomposition, which has been largely exhausted in the international input-output literature. However, if we focus on a single country-sector (i), the i-th column of Z and the i-th row of W represent two probability distributions that take into account the transfers in the value chain, which result in two completely different and independent types of information. The i-th column of Z contains the information on the downstream structure of the value chain of the respective i-th country-sector and the i-th row of W contains the information on the upstream structure of the value chain of the respective i-th country-sector. For a given i-th country-sector, the two probability distributions are asymmetrical. Most importantly, both probability distributions relate to the same object of investigation - the total output of the country-sector i. Using the total output of each country-sector seems to be the only way to derive a disaggregation of the same object on its upstream and downstream value chain. While the upstream part can be derived consistently as a disaggregation of the value added shares, this is impossible for the downstream part, since only the total output makes up the entire downstream chain as opposed to the value added. In other words - the total output is an object that has both an upstream and a downstream path, while the value added has only one upstream path. Using the total output share as the basis for disaggregating on the individual country-sector level is therefore a legitimate choice. This is the main reason why we derived the W matrix in terms of shares of total output (2.4, 2.5) and not, as usual, in terms of shares of value added - to make it perfectly clear that both upstream and downstream disaggregation have the same object - the total output of i, which includes both the value added of the country-sector i and the total value added of the other country-sectors (j) downstream. The same object (total output) is then distributed along the upstream value chain paths (as determined by the i-th column of W) until it reaches final consumption along an upstream value chain path.

All input-output analyses assume the homogeneity of the smallest classification object (in our case country-sector). The level of detail of the data corresponds to the level of detail of the sector (and country) classification and within a country-sector there is no further information and rather strict homogeneity assumptions apply. We use the assumption of homogeneity of production of each country-sector to combine the two probability distributions.

w_{ij} represents the share of the total output of the i-th country-sector, which was primarily produced by the country-sector j. Due to the homogeneity of the total output of the i-th country-sector, the z_{ki} represents not only the probability that a random part of the total output of the i-th country-sector reaches the final consumption as product of k, but also the probability that a random part of **any share** of the output of the i-th country-sector reaches the final consumption as product of k. Since w_{ij} is a share of the i-th country-sector's total output, its upstream decomposition is clearly and uniquely defined by the i-th column of Z.

The product $w_{ij}z_{ki}$ thus simply represents the probability that a certain part of the total output of the i-th country-sector is primarily produced in j and reaches final consumption as the product of k along any value chain path (upstream, downstream or a combination thereof) passing through i. In other words, it represents a share of the total output of i that was produced by j and reached final consumption as a product of k. A simple multiplication of probabilities requires that the two events - a random portion of the total output of i produced by j and a random portion of the total output of i completed for consumption by k - are statistically independent. First, if certain parts of the total output of a particular country-sector were to behave differently from certain other parts of the same output, this would violate the homogeneity assumption, which is the basic assumption of the input-output structure and methodology. Second, at the level of economic theory it is relatively easy to argue about the statistical independence of the structure of upstream and downstream value chains: Nothing about the downstream structure of production in the i-th country-sector implies anything about its upstream structure and *vice versa*. Both are calculated independently and provide completely different information: the downstream decomposition gives the information about the inputs produced by other country-sectors that are used directly or indirectly in the production process of the i-th country-sector, and the upstream decomposition gives the information about how the product of the i-th country-sector is consumed either directly or as part of the final product of other country-sectors.

Two separate vectors which disaggregate the value chain paths of the downstream (i-th row of Z) and upstream value chain (i-th column of W), thus span an entire matrix of total output shares that capture the value chain tree structure of the i-th country-sector. We combine them with the direct product that defines the matrix of the value chain tree for each country-sector (i) by multiplying each element of $Z\vec{e_i}$ (the i-th column of Z) by each element of $\vec{e_i}^T W$ (the i-th row of W).

Definition 2.3. Value chain tree matrix
$\tau_i = Z\vec{e_i} \otimes \vec{e_i}^T W$; $\tau_i \in \mathbb{R}^{n\times n}$, where $\vec{e_i} \in \mathbb{R}^n$ represents standard orthonormal basis of $\mathbb{R}^n$

This defines each element of the value chain tree matrix $t_{ijk} \in \tau_i$ as $t_{ijk} = w_{ij} z_{ki}$. Each element of the value chain tree matrix τ_i thus represents a share of the total output of the country-sector i, which is primarily produced in the country-sector j and consumed as an end product of the country-sector k, along any upstream and downstream value chain path.

The main point of our derivation is not the expressed final value distribution of the total output of each country-sector along any of its upstream and downstream value chain paths, but the expression of the total output distribution (of the respective country-sector) along any value chain path, be it a downstream value chain path, an upstream value chain path or any combination of both paths at the same time.

$$\tau_i = \hat{v_C}(I - A)^{-1}\vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1}(I - A)^{-1}\hat{f} \tag{2.11}$$

The structure of the value chain tree matrices allows us to focus our disaggregation on the composition of the value chain paths covered by the two global Lenotief inverses in the equation, the first representing all upstream parts of the value chain paths and the second representing all downstream parts of the value chain paths.

A single value chain path is determined by a series of concrete transactions between companies: It is a unique path from primary value creation (value created in production, not transferred from intermediate products) to value realisation (final consumption, not productive consumption of intermediate products), which passes through the production stage of the i-th country-sector. The total output of i is not only disaggregated along all possible paths leading from any country-sector of origin via the country-sector i to any country-sector of final stage production (as determined by τ_i), but it is also disaggregated in much finer detail, along all the unique value chain paths that pass through i. That a concrete value chain path is only a part of the value chain tree matrix can be easily recognised if both inverses in τ_i are replaced by an infinite series ($(I - A)^{-1} = I + A + A^2 + \dots$). Such disaggregation then results in an infinite number of value chain paths, and the total output of the i-th country-sector is distributed over all these paths.

A concrete value chain path share of the total output of i is determined by the Leontief technical coefficients $a_{ij} \in A$. For example, take a concrete value chain path consisting of the value primarily produced in the country-sector CS_1[17], then used as an intermediate in CS_2, which in turn is used as an intermediate in i (the country-sector whose value chain is broken down), and then sent as an intermediate to CS_3, which is then sent as an intermediate to CS_4, where it is finished and sold for consumption. This value chain path has origin (CS_1), midpoint (i) and final destination of production (CS_4), as well as the concrete path, which has a length of 5 (5 country-sectors contribute to production from origin to final

[17] CS_k represents an index for different country-sectors. $a_{CS_1 CS_2}$ thus represents a single Leontief technical coefficient indicating that the value produced by CS_2 requires a $a_{CS_1 CS_2}$ share of CS_1 input.

consumption). The share of the total output of the i-th country-sector that can be attributed to this specific path is:

$$v_{CS_1}a_{CS_1CS_2}a_{CS_2i}x_i^{-1}a_{iCS_3}a_{CS_3CS_4}f_{CS_4} \tag{2.12}$$

A specific unique value chain path of the i-th country-sector's value chain tree, that has origin in j and final stage in k can be written as:

$$\Pi_{p=1}^{d}v_{CS_0}a_{CS_{p-1}CS_p}x_i^{-1}\Pi_{p=d}^{u-1}a_{CS_pCS_{p+1}}f_{CS_u} \tag{2.13}$$

Such a path has a downstream length of d and an upstream length of $u-1-d$ and the path is determined by a unique set of transaction between firms from the origin to the final stage (from origin $j = CS_0$, to CS_1, to CS_2, ..., to $i = CS_d$, and further to CS_{d+1}, CS_{d+2}, ..., to $k = CS_u$). Leontief technical coefficients $a_{CS_{p-1}CS_p}$ determine each transaction between firms. The summation along the total output shares of i attributed to all such unique value chain paths, taking into account all permutations of possible transaction sequences and also all possible lengths (all possible length combinations of downstream and upstream lengths) as well as all possible origins and final stage destinations results in a unit:

$$\mathbf{1}^T\tau_i\mathbf{1} = \mathbf{1}^T\hat{v_C}(I-A)^{-1}\vec{e_i} \otimes \vec{e_i}^T\hat{x}^{-1}(I-A)^{-1}\hat{f}\mathbf{1}^T = 1 \tag{2.14}$$

Our conceptualization allows us to define decomposition criteria that can be applied to each value chain path of the value chain tree of the i-th country-sector. Based on this property, we will decompose the value chain structure of each country-sector separately in the following section.

3 The Value Chain Typology

3.1 Definitions

The framework of the international I-O analysis allows the separate analysis of final transactions to consumers and transactions between companies. Based on this characteristic, we propose a typology of value chains based solely on the structure of linkages between enterprises, while adding a further decomposition with regard to different possible transactions to reach the final consumption *post festum*.[18] Each matrix τ_i expressed by equation 2.13, represents the desegmentation of the total product of the country-sector i along different downstream and upstream paths. When we refer to the value chain, we refer to the specific share of value (share of output) that corresponds to a particular value chain path. Path[19] of each value share generally includes any combination of domestic and cross-border transactions between firms, which can take place both downstream and upstream in relation to the respective country-sector. Our criteria for the value chain typology thus refer to each specific value share corresponding to a single path within a value chain tree specific to each country-sector.

Definition 3.1. Domestic value chain
Domestic value chain (DVC) is a value that involves *at least 1 transaction between domestic firms* and involves *only domestic transactions* between firms along its path.

[18] For example, Wang's disaggregation into simple and complex GVCs uses the number of cross-border transactions, regardless of whether the value crossed border for production or whether it is only an export to end users. Such a criterion mixes two conceptually different transactions, resulting in unnecessary calculation complexity and the impossibility of further conceptual disaggregation. Existing definitions of the typology of value chains, like all such definitions, are constructed in a relatively arbitrary way. More important than strict adherence to the prevailing definitions is the clarity of the proposed revision and the presentation of the conceptual relationship of the new concepts to the old ones. With our proposal, we are making a more detailed decomposition, which will allow researchers to construct an indicator that is better suited to their research questions. Since the revised typology is based on a more detailed decomposition compared to the currently prevailing typology, researchers can (by simply adding components of the revised decomposition) also replicate objects that correspond to existing studies.

[19] Here we examine the path of production fragmentation, while the path to final consumption, which represents an additional transaction, are analysed in section 3.5.

Definition 3.2. Global value chain
Global value chain (GVC) is a value that involves *at least 1 cross-border transaction* between firms along its path. We further distinguish two types of global value chains: simple and complex.
Definition 3.2.1 Simple global value chain
Simple global value chain (SGVC) is a value that involves *exactly 1 cross-border transaction* between firms anywhere along its path.
Definition 3.2.2 Complex global value chain
Complex global value chain (CGVC) is a value that involves *more than 1 cross-border transactions* between firms along its path.

Definition 3.3. No value chain
No value chain (NVC) is a value that *does not involve any transactions* between firms and has no path within production.

A few brief comments on our definitions and their interpretation. No material product or service belongs to a single classification of value chain, and no enterprise can be considered part of a single type of value chain. The output of each enterprise belongs to a variety of value chain paths. In general, one part of the output comprises many cross-border transactions, another part only domestic transactions, and yet another part their relatively complex interrelationship. Each product (or country-sector in our case) can be assigned different shares of the value chain paths. These shares are objects that provide information on the structure of the economy. For example, there is virtually no enterprise that could be classified exclusively as part of no value chain, but some enterprises that provide services (e.g. domestic services) have a relatively high share of output that has no value chain path, especially in services, where salaries account for almost all of the enterprise's expenditure and where their product directly satisfies final demand. On the one hand, enterprises that specialise in intermediate goods are always part of a value chain, whether domestic or global. On the other hand, even modern industries, such as food processing and pharmaceuticals, also have a certain (usually small) share of value added that is not part of any value chain (no value chain share), corresponding to the share of domestic value added in these industries that is also directly consumed (part of output that has no value chain path). The value chain shares and their changes are the object that provide information on the structure of the economy, whether at sector or country level. As the economy develops, the division of labour also increases, which corresponds to an increasing fragmentation of production, in particular international production fragmentation and a decrease in shares where there is limited or no value chain fragmentation. Compared to the existing typology of value chains, this revised typology allows for an analysis of the relationship between global and domestic fragmentation, which could be particularly relevant for the policies of developing countries.

3.2 The Decomposition of Paths

Value chain typology is established on the basis of criteria along the entire value chain. For this reason, we disaggregate the value chain tree matrices τ_i in terms of criteria for different types of value chain paths. Our decomposition consists of the decomposition of two Leontief inverses, which can be interpreted as a decomposition of the downstream part and upstream part of each value chain path, as defined by equation 2.11: $\tau_i = \hat{v_C}(I-A)^{-1}\vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1}(I-A)^{-1}\hat{f}$. The decomposition is constructed on the basis of the criteria of the number of cross-border and domestic transactions between firms that are consistent with the revised value chain typology.

First, we investigate the decomposition of only a single Leontief inverse (which is interpreted symmetrically with respect to our criteria in the upstream and downstream value chain) and only then we analyze the decomposition of all value chain paths characterized by the two Leontief inverses. The international I-O data have a specific block matrix structure in which block diagonal elements represent domestic transactions between firms and block off-diagonal elements represent international transactions between firms

(A_D denotes domestic - block diagonal - and A_{CB} cross-border - block-off diagonal - part of A), which allows us to decompose the Leontief inverse in the following way:

$$\begin{aligned}(I-A)^{-1} &= (I-A_D)^{-1} + (I-A)^{-1} - (I-A_D)^{-1} = \\ &= I + A_D + A_D^2 + A_D^3 + \dots + (I-A)^{-1} - (I-A_D)^{-1} = \\ &= I + A_D(I + A_D + A_D^2 + \dots) + (I-A)^{-1} - (I-A_D)^{-1} = \\ &= I + A_D(I-A_D)^{-1} + (I-A)^{-1} - (I-A_D)^{-1} = \\ &= \underbrace{I}_{1.)} + \underbrace{A_D(I-A_D)^{-1}}_{2.)} + \underbrace{(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}}_{3.)} + \\ &\quad + \underbrace{(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}}_{4.)}\end{aligned} \tag{3.1}$$

1.) I obviously represents the part of the output that contains *no transactions* between firms - no value chain linkages. In the upstream part, it represents the share of total output that directly satisfies final demand (i.e. no upstream value chain), and in the downstream part it represents the direct value added of the country-sector whose production is being decomposed (i.e. no downstream value chain).

2.) $A_D(I-A_D)^{-1} = A_D + A_D^2 + A_D^3 + \dots$ represents the part of output that contains *at least 1* transaction between domestic firms and contains *only domestic transactions* between firms.

3.) $(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}$ represents the part of the output that contains *at least 1 transaction* between firms and contains *exactly one cross-border transaction* between firms somewhere along its value chain path. This can be demonstrated by paraphrasing the part as all possible combinations of a single cross-border transaction among any possible set of domestic transactions between firms that occur before or after the single cross-border transaction:

$$\begin{gathered}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1} = \\ = A_{CB} + A_{CB}A_D + A_{CB}A_D^2 + \dots + \\ + A_DA_{CB} + A_DA_{CB}A_D + A_DA_{CB}A_D^2 + \dots + \\ + A_D^2A_{CB} + A_D^2A_{CB}A_D + A_D^2A_{CB}A_D^2 + \dots + \\ \vdots\end{gathered}$$

4.) $(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}$ represents the part of the output that contains *at least two or more transactions* between firms, of which *at least two are cross-border transactions*. This follows logically from the fact that part 1), 2) and 3) cover the total output that contain less than two cross-border transactions, and that the full Leontief inverse covers the total output.

3.3 Value chain tree matrix decomposition

We proceed by disaggregation of all value chain paths as they are structured in value chain tree matrices. Using the decomposition of the Leontief inverse that we disaggregated in the previous subsection and inserting it into equation 2.11, we obtain 16 components (4×4 product) for each matrix τ_i.[20] This disaggregation along both the upstream and downstream paths is the basis for deriving value chain shares that correspond to our typology. We decompose each τ_i matrix describing all possible value chain paths of the output of the i-th country-sector into a matrix consisting of domestic value chain paths only, a matrix containing all possible global value chain paths (as well as simple and complex global value chain paths separately), and a matrix consisting only of the value that has no value chain path.

[20] Details on disaggregation are given in Appendix B.

Definition 3.4. Domestic value chain tree τ_i^{DVC}

$$\begin{aligned}\tau_i^{DVC} =& \hat{v_C} A_D (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \hat{f} + \hat{v_C} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} A_D (I - A_D)^{-1} \hat{f} + \\ &+ \hat{v_C} A_D (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} A_D (I - A_D)^{-1} \hat{f}\end{aligned}$$

The domestic value chain tree represents all value chain paths of output of each country-sector, which according to definition 3.1 are part of the domestic value chains. In Figure 1 the domestic value chain paths are marked in red. Domestic value chain paths are defined as all paths that contain at least one red-coloured linkage (representing transactions between domestic enterprises) and include only red-coloured linkages and orange paths (representing the value creation or realisation in the respective country-sector in focus). The first part ($\hat{v_C} A_D (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \hat{f}$) covers the downstream domestic value added (downstream domestic path), which ends as the i-th country-sector final stage (no upstream path), the second part ($\hat{v_C} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} A_D (I - A_D)^{-1} \hat{f}$) covers the value added of the i-th country-sector (no downstream path) that is transferred via the upstream domestic value chain (upstream domestic path), and thirdly ($\hat{v_C} A_D (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} A_D (I - A_D)^{-1} \hat{f}$) comprises the downstream domestic value added that is used as an intermediate product in production of i and then used as intermediary further in the upstream domestic value chain until it reaches final demand (both downstream and upstream domestic path). All three cases meet the definition of the domestic value chain.

Definition 3.5. Global value chain tree τ_i^{GVC}

$$\begin{aligned}\tau_i^{GVC} =& \hat{v_C} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \hat{f} + \\ &+ \hat{v_C} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} \hat{f} + \\ &+ \hat{v_C} \big[(I - A)^{-1} - (I - A_D)^{-1}) \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \hat{f}\end{aligned}$$

The global value chain tree represents all paths of output of the individual country-sector, which are part of global value chains according to definition 3.2. In Figure 1, the global value chain paths are represented by all paths containing at least one black-coloured linkage (representing cross-border transactions between enterprises). Global value chain paths can contain any number of red (domestic) and orange (no value chain) linkages, as long as there is at least one black (cross-border) linkage along their path. The first element ($\hat{v_C} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \hat{f}$) covers the downstream domestic and no value chain paths, which have global upstream linkages (simple or complex), second element ($\hat{v_C} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} \hat{f}$) covers downstream global linkages (simple or complex), which have a upstream domestic or no value chain path and a third element ($\hat{v_C} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} \big[(I - A)^{-1} - (I - A_D)^{-1}\big] \hat{f}$) covers the value that has global paths both upstream and downstream. All these cases correspond to our definition of the global value chain.

Definition 3.5.1 Simple global value chain tree τ_i^{SGVC}

$$\begin{aligned}\tau_i^{SGVC} =& \hat{v_C} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} A_{CB} (I - A_D)^{-1} \hat{f} + \\ &+ \hat{v_C} (I - A_D)^{-1} A_{CB} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} \hat{f}\end{aligned}$$

The simple global value chain tree represents all paths of output of each country-sector that are part of simple global value chains as defined by 3.2.1 The first element ($\hat{v_C} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} A_{CB} (I - A_D)^{-1} \hat{f}$) covers a downstream domestic and no value chain path that has a simple global upstream linkages and a second element ($\hat{v_C} (I - A_D)^{-1} A_{CB} (I - A_D)^{-1} \vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1} (I - A_D)^{-1} \hat{f}$) covers downstream simple global linkages that have an upstream domestic or no value chain path. These are the only cases that fit our definition of simple global value chain. A value chain path covering both downstream and upstream simple global linkages already has more than 1 cross-border transaction and is therefore part of the complex global value chain.

Definition 3.5.2 Complex global value chain tree τ_i^{CGVC}

$$\begin{aligned}\tau_i^{CGVC} =& \hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1}-(I-A_D)^{-1}-(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\hat{f}+\\ &+\hat{v_C}\big[(I-A)^{-1}-(I-A_D)^{-1}-(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f}+\\ &+\hat{v_C}\big[(I-A)^{-1}-(I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1}-(I-A_D)^{-1}\big]\hat{f}\end{aligned}$$

The complex global value chain tree represents all paths of output of the individual country-sectors that are part of complex global value chains as defined in 3.2.2 The first element ($\hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1}-(I-A_D)^{-1}-(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\hat{f}$) covers the downstream domestic and no value chain path, having complex global upstream linkages, second element ($\hat{v_C}\big[(I-A)^{-1}-(I-A_D)^{-1}-(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f}$) comprises downstream complex global linkages, which have an upstream domestic or no value chain path and a third element ($\hat{v_C}\big[(I-A)^{-1}-(I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1}-(I-A_D)^{-1}\big]\hat{f}$) represents the combinations of the global downstream and upstream paths (simple-simple, simple-complex, complex-simple, complex-complex). All these elements correspond to our definition of the complex global value chain, because in all cases the value crosses borders for production at least twice.

Definition 3.6. No value chain tree τ_i^{NVC}

$$\tau_i^{NVC} = \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f}$$

No value chain tree represents the part of the output of each country-sector that is not part of a value chain according to definition 3.3. In Figure 1, no value chain path is represented by the orange colour only (any other linkage represents a value chain path). Only the share of value added that is produced in the respective country-sector in focus (no downstream stages) and also completed for final consumption (no upstream stages) in the same production phase fulfils this criterion. Since the I-O method distinguishes between the product used as an intermediate product within the same sector[21] and the product manufactured for final consumption, the use of this definition as no value chain does not depend on the level of detail of I-O data disaggregation. The cyclical effect of the production of intermediate goods within the same country-sector is already included in the domestic value chain tree and after taking into account all defined value chain paths (domestic, simple and complex global value chain paths), a value share remains without a value chain path and with a simple representation as a value added of the country-sector that is also directly consumed. This represents a value that has no path in terms of transactions that represent a fragmentation of production.

This concludes the value chain tree decomposition, which can be written as:

$$\tau_i = \tau_i^{DVC} + \tau_i^{GVC} + \tau_i^{NVC} \tag{3.2}$$

$$\tau_i^{GVC} = \tau_i^{SGVC} + \tau_i^{CGVC} \tag{3.3}$$

3.4 The Value Chain Participation Rates

In section 2 we have shown that a set of value chain tree matrices τ_i represent all possible value chain paths of the output of each country sector and that the summation along all shares of total output assigned to

[21]This is determined by the pure diagonal elements of the Leontief technical matrix A. Each a_{ii} represents the portion of the total product of the i-th country-sector that requires the use of the intermediate product of the same country-sector in the production process, thus covering the cyclical transactions within a sector. These cyclical transactions are of course included in the decomposition of the domestic value chain and not no value chain, since cyclical transactions represent a domestic value chain fragmentation.

all such unique value chain paths yields a unity for each value chain tree (equation 2.14). In other words, we have presented a unique disaggregation of the output of each country-sector along all its value chain paths. In the same way, the summation along the two disaggregating dimensions of our decomposed set of matrices (global, domestic and no value chain tree matrices) captures the total share of the total output of each country-sector i that meets the criteria by which the value chain paths were decomposed by including either only domestic value chain paths, only global value chain paths, or only values that have no value chain paths at all. In other words, the summation of the disaggregated value chain matrices along any origin and end stage represents the share of output of each country-sector that has either a domestic, a global or no value chain.

Definition 3.7. Domestic value chain share $DVCs$

$$DVCs \in \mathbb{R}^n;\ DVCs_i = \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{DVC};\ DVCs = \begin{bmatrix} \mathbf{1}^T \mathcal{T}_1^{DVC} \mathbf{1} \\ \mathbf{1}^T \mathcal{T}_2^{DVC} \mathbf{1} \\ \vdots \\ \mathbf{1}^T \mathcal{T}_n^{DVC} \mathbf{1} \end{bmatrix}$$

Domestic value chain share represents a share of each country-sector's output that has domestic value chain path.

Definition 3.8. Global value chain share $GVCs$

$$GVCs \in \mathbb{R}^n;\ GVCs_i = \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{GVC};\ GVCs = \begin{bmatrix} \mathbf{1}^T \mathcal{T}_1^{GVC} \mathbf{1} \\ \mathbf{1}^T \mathcal{T}_2^{GVC} \mathbf{1} \\ \vdots \\ \mathbf{1}^T \mathcal{T}_n^{GVC} \mathbf{1} \end{bmatrix}$$

Global value chain share represents a share of each country-sector's output that has global value chain path.

Definition 3.8.1 Simple global value chain share $SGVCs$

$$SGVCs \in \mathbb{R}^n;\ SGVCs_i = \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{SGVC};\ SGVCs = \begin{bmatrix} \mathbf{1}^T \mathcal{T}_1^{SGVC} \mathbf{1} \\ \mathbf{1}^T \mathcal{T}_2^{SGVC} \mathbf{1} \\ \vdots \\ \mathbf{1}^T \mathcal{T}_n^{SGVC} \mathbf{1} \end{bmatrix}$$

Simple global value chain share represents a share of each country-sector's output that has simple global value chain path.

Definition 3.8.2 Complex global value chain share $CGVCs$

$$CGVCs \in \mathbb{R}^n;\ CGVCs_i = \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{CGVC};\ CGVCs = \begin{bmatrix} \mathbf{1}^T \mathcal{T}_1^{CGVC} \mathbf{1} \\ \mathbf{1}^T \mathcal{T}_2^{CGVC} \mathbf{1} \\ \vdots \\ \mathbf{1}^T \mathcal{T}_n^{CGVC} \mathbf{1} \end{bmatrix}$$

Complex global value chain share represents a share of each country-sector's output that has complex global value chain path.

Definition 3.9. No value chain share $NVCs$

$$NVCs \in \mathbb{R}^n;\ NVCs_i = \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{NVC};\ NVCs = \begin{bmatrix} \mathbf{1}^T \mathcal{T}_1^{NVC} \mathbf{1} \\ \mathbf{1}^T \mathcal{T}_2^{NVC} \mathbf{1} \\ \vdots \\ \mathbf{1}^T \mathcal{T}_n^{NVC} \mathbf{1} \end{bmatrix}$$

No value chain share represents a share of each country-sector's output that has no value chain path.

$$DVCs_i + GVCs_i + NVCs_i = \sum_{j=1}^{n}\sum_{k=1}^{n}\left(t_{ijk}^{DVC} + t_{ijk}^{GVC} + t_{ijk}^{NVC}\right) = \sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk} = 1 \tag{3.4}$$

$$SGVCs_i + CGVCs_i = \sum_{j=1}^{n}\sum_{k=1}^{n}\left(t_{ijk}^{SGVC} + t_{ijk}^{CGVC}\right) = \sum_{j=1}^{n}\sum_{k=1}^{n} t_{ijk}^{GVC} = GVCs_i \tag{3.5}$$

With this we conclude our disaggregation of each country-sectors total output, with respect to its specific value chain integration based on interlinkages between firms. We can summarize our decomposition in the simple vector form:

$$DVCs + GVCs + NVCs = \mathbf{1} \tag{3.6}$$

$$GVCs = SGVCs + CGVCs \tag{3.7}$$

3.5 Decomposition of transaction to final consumer

Since all value chain paths within production are covered and decomposed, we still have one last transaction to the consumer to complete the value chain path from production to consumption. We can decompose the final transaction to the consumer, with the criterion whether it is a transaction to domestic consumers or a cross-border transaction (export of the final product for consumption). The reference to domestic consumption refers to the country-sector in which the last stage of production took place and not to the country-sector whose value chain we analyse. Each country-sector has a unique value chain and a specific structure of value chain paths. The completion of each value chain path by a transaction to the consumer can be achieved by an additional cross-border transaction of export of the final product or consumption in the country where the product was finalised. Such an additional decomposition of the value chain paths allows a more detailed analysis of the value chains.

The I-O data include the information on the transaction to final consumers within matrix F, which can be decomposed on its cross-border and domestic flows to final consumers ($F = F_{CB} + F_D$), due to its block vector structure. We construct a matrix of all cross-border final consumption flows and matrix of all domestic consumption flows:

$$\hat{f} = \hat{f}_D + \hat{f}_{CB} \tag{3.8}$$

Every value chain path within production can thus be further decomposed with additional criterion for transaction to final consumers. Each set of disaggregated value chain matrices, defined by the equation 3.2 and 3.3, can be separated on two matrices, one covering all the production paths that end in domestic final consumption (no export - τ_i^{NE}) and the other covering all the production value chain paths that end with exporting for final consumption (τ_i^{E}).

$$\tau_i = \hat{v_C}(I-A)^{-1}\vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1}(I-A)^{-1}\hat{f}_D + \hat{v_C}(I-A)^{-1}\vec{e_i} \otimes \vec{e_i}^T \hat{x}^{-1}(I-A)^{-1}\hat{f}_{CB} = \tau_i^{NE} + \tau_i^{E} \tag{3.9}$$

Due to simple additive properties of operation all decomposed value chain tree matrices are similarly decomposed on ones with exporting or with no exporting as final transaction.

$$\tau_i = \tau_i^{NE} + \tau_i^{E} = \tau_i^{GVC-NE} + \tau_i^{DVC-NE} + \tau_i^{NVC-NE} + \tau_i^{GVC-E} + \tau_i^{DVC-E} + \tau_i^{NVC-E} \tag{3.10}$$

$$\tau_i^{GVC} = \tau_i^{GVC-NE} + \tau_i^{GVC-E} = \tau_i^{SGVC-NE} + \tau_i^{CGVC-NE} + \tau_i^{SGVC-E} + \tau_i^{CGVC-E} \tag{3.11}$$

The value shares that are part of each value chain path are thus further decomposed, as explained in subsection 3.4. The final decomposition of output is thus a decomposition along each value chain, as defined by criteria that simultaneously take into account transactions related to production fragmentation (different value chains) and the final transaction to the consumer. A share of value that has either a domestic, global or no value chain has as its final transaction to the consumer either export or no export transaction, which provides a detailed decomposition of the participation shares that can be used to construct different composite indices suitable for different research questions.

$$DVCs^{NE} + GVCs^{NE} + NVCs^{NE} + DVCs^{E} + GVCs^{E} + NVCs^{E} = \mathbf{1} \tag{3.12}$$

$$GVCs = SGVCs^{NE} + CGVCs^{NE} + SGVCs^{E} + CGVCs^{E} \tag{3.13}$$

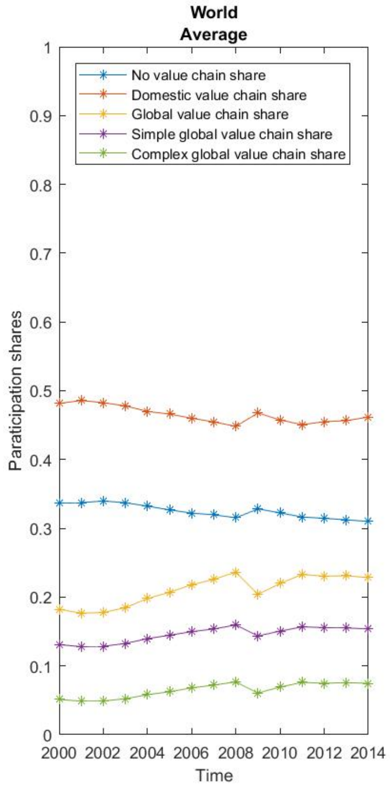
4 Empirical results

The proposed measures broaden the scope for empirical application and static analysis of international production and trade. The contribution of our approach consists of the simultaneous insight into domestic and global value chains, which allows to study their interaction and structural changes in the economies. All elements of the new typology may vary over time, from country to country and from sector to sector and are relevant research topics. Derived participation shares are also simple fragmentation measures, and each smallest unit of analysis (country-sector) is represented by a single measure (scalar share) that covers the extent of overall value chain fragmentation, as opposed to separate downstream and upstream indicators.

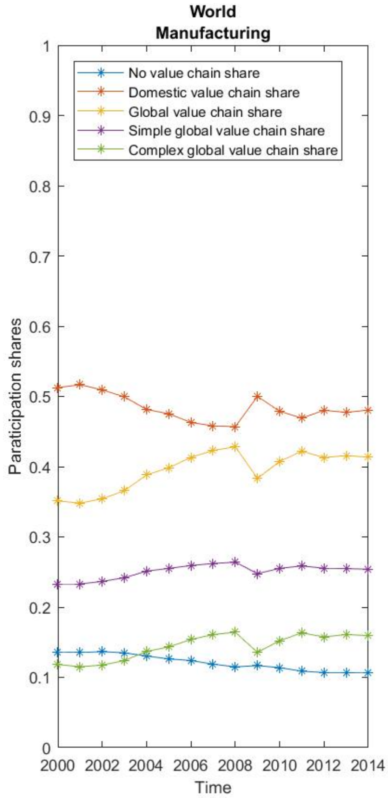
Due to the limitations of the paper and its primarily methodological focus, we present only some very basic empirical results. First, we show the global averages of value chain participation rates based on the WIOD 2016 data and the global average participation rates for the manufacturing and service sectors separately (Figures 2, 3 and 4). Using our methodological approach, we observe that the global average GVC share of world output is consistently above 20%, reached almost 24% at the peak before the global recession, and then stagnated slightly below this level until 2014 (Figure 2). This suggests that the most recent estimates of GVC's share of between 10% and 15% (Dollar, 2017, p. 2; Li *et al.*, 2019, p. 12) may be undervalued. As expected, the manufacturing sector is globally integrated to an above-average extent, with the share in the global value chain rising from 35% to over 40% before the crisis and then stagnating around this level after a brief recovery. The share of the complex global value chain shows the highest relative growth, while the average increase in global value chain integration is greater than the decline in domestic value chain integration. Interestingly, the decline in global integration in times of crisis had almost no impact on the part of economy without value chain fragmentation, while domestic fragmentation increased almost in proportion to the decline in global integration. Thus, the crisis did not lead to a general decline in production fragmentation, but only to a decrease in its global character. For services, on the other hand, less than 15% of total output has a global value chain path, although services show some increase in global integration, mainly due to decreasing domestic integration (which could be attributed to the globalisation of business services), while the part of economy without a value chain appears relatively stable. For this reason, vulnerability to external financial shocks was much less pronounced in services during the crisis.

As the data for the world average hide large differences between countries, we also show the value chain participation shares of manufacturing for China, the US and the average of the economically most integrated new EU members - 3 Baltic and 4 Visegrad countries (Figures 5, 6 and 7), which show structural differences and diverse patterns of development in global and domestic integration. China has on average a high share of domestic production integration (around 65%) and is one of the few economies where the share of domestic integration has increased by almost 10 percentage points over the period 2004 -2014. In the United States, the picture is reversed, while the already lower average share of domestic integration is steadily decreasing. A completely different pattern is evident in the Baltic and Visegrad European countries, which became EU members in the new millennium. On average, the integration in the global value chains in the manufacturing industries of these countries increased from already high 53% to 69% during the observed period. At the same time there was a huge relative decline in the already below-average share of domestic fragmentation from 32% to 18%. Interestingly, almost all the growth in the global value chain share in the Central-Eastern EU countries was due to the increase in complex global value chain linkages, while simple global value chain linkages remain relatively stable.

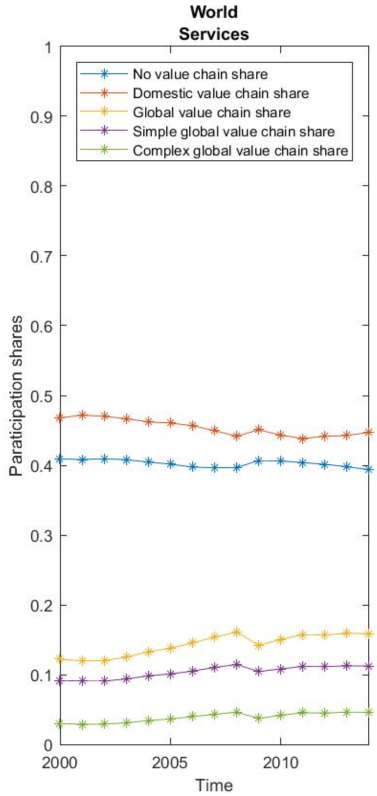
Finally, we use the fact that we have created a uniform participation rates by performing a simple OLS regression to test the relationship between the level of domestic and global fragmentation and economic growth measured by GDP. Since the use of a panel data approach is not appropriate in this context (the



(a) *Figure 2: World average participation rates.*



(b) *Figure 3: World average of manufacturing.*

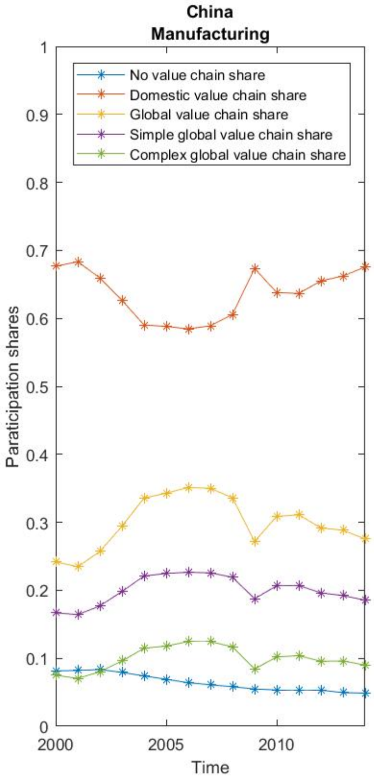


(c) *Figure 4: World average of services.*

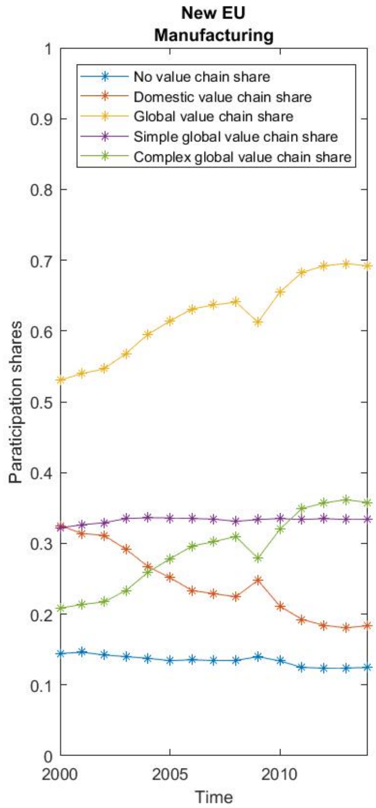
Source: WIOD, 2016; own calculations.

explanation of short-term productivity fluctuations can hardly be explained by the economic structure expressed in value chain shares), we use a cross-sectional approach to test the long-term effects of different levels of domestic or global fragmentation on economic growth. Our observations relate to the 43 countries included in the WIOD 2016 data, and the variables are their average annual growth, the average DVC and GVC shares, with the average logarithm of GDP as a control for convergence, the average logarithm of the annual population as a control for the size of the country, and the EU control dummy for potential EU specifics.

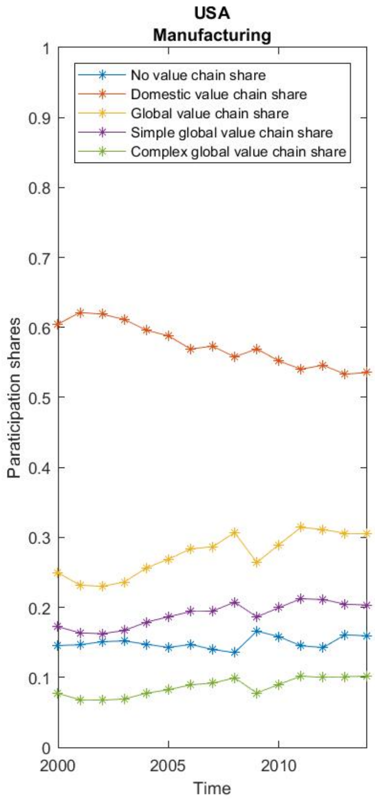
The regression results are shown in Table 1. The logarithm of GDP is a significant variable and is negatively related to growth. The result simply reflects the fact that a higher GDP implies a lower potential for higher growth rates. Taking this into account, both the DVC share and the GVC share are highly significant variables that have a positive effect on growth rates. It is therefore both domestic and global integration that can have a significant impact on economic growth. The same result applies after the introduction of additional controls on country size and EU specifics. Due to the primarily methodological orientation of the article, we refrain from a detailed interpretation of the regression results. However, it should be noted that it is difficult to separate cause and effect in the application of econometric analyses - the country in recession for external reasons could experience a decline in global and domestic production fragmentation due to those same external reasons. In any case, there is a correlation between economic growth and the degree of production fragmentation, whether it is domestic or global. A country that experiences an overall decrease in production fragmentation (domestic fragmentation declines faster then global increases), regardless of an increase in global production integration, could experience a negative



(a) *Figure 5: China manufacturing participation rates.*



(b) *Figure 6: New EU countries manufacturing.*



(c) *Figure 7: USA manufacturing participation rates.*

Source: WIOD, 2016; own calculations.

impact on economic growth compared to similarly developed countries, in line with our findings.[22] An increase only in participation in the global value chains is therefore not necessarily enhancing the growth due to various forms of integration[23] with different effects on domestic integration, which is also an important factor in determining economic growth. Further studies are needed to examine the relationship between domestic and global fragmentation and diverse patterns of structural integration.

[22] The Greek and Italian economies, which experienced the longest recession in the EU over the period, experienced precisely this pattern (general reduction in production fragmentation, primarily reduction in domestic production fragmentation and increased integration of the global value chains).

[23] A variety of institutional and structural economic positions leads to a diverse effects of global integration at country level.

Table 1: Regression results.

	(1) Yearly growth	(2) Yearly growth	(3) Yearly growth
logGDP	-0.013***	-0.013***	-0.013***
	(0.00)	(0.00)	(0.00)
DVC share	0.169***	0.181***	0.183***
	(0.03)	(0.04)	(0.04)
GVC share	0.163***	0.160***	0.162***
	(0.03)	(0.03)	(0.03)
logPOP		-0.001	-0.001
		(0.00)	(0.00)
EU			-0.003
			(0.00)
Constant	0.036	0.049	0.055
	(0.03)	(0.03)	(0.04)
R^2	0.819	0.821	0.824
F	57.126	42.314	33.806

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: WIOD, 2016; WB; own calculations.

5 Conclusion

We propose a new methodology for measuring the participation shares of different types of value chains in the international input-output framework. We address the lack of a consistent unitary measure of value chain integration at the country-sector level by proposing a new concept of the value chain tree for each country-sector, covering all value chain paths from value creation (downstream linkages) through a single country-sector and to final consumption (upstream linkages) simultaneously. By capturing the structure of all value chains in a series of value chain tree matrices, we add a new mathematical object that serves as a basis for deriving the proposed new indicator of value chain participation, which we contribute to the collection of existing indicators.

This methodology allows us to introduce an extended typology of value chains by distinguishing and disaggregating all production activity into the following types: no value chain, domestic value chain and global value chain - further differentiated into simple and complex global value chains. The most important new conceptual subdivision in the extended typology relates to the subdivision of the existing 'domestic component' into no value chain and domestic value chain. This subdivision, which is only possible with the proposed methodology, provides a better representation of domestic production interdependencies and allows comparative analyses of the simultaneous development of domestic and foreign production interdependencies, thus enabling aggregated analyses of domestic and global production fragmentation and its interrelated development as influenced by outsourcing or offshoring. A major change introduced by a new typology is also its fundamental production-related character: all distinctions between different types of value chains are made only with regard to (potential) production fragmentation, with separated examination of the transaction to the final consumer - which may or may not be cross-border. This affirms concept of value chain as related primarily to the fragmentation of production, while the *post festum* differentiation is also derived on the basis of the last transaction to the final consumer.

The proposed methodology and typology of value chains provides researchers with new opportunities to carry out future research at different levels of disaggregation, be it comparative geographical analysis (e.g. comparing the evolution of value chain measures between two countries or between groups of countries) or observing the evolution of value chains in different sectoral disaggregations. The preliminary illustration of the new methodology, which attempts to link both domestic and global production fragmentation with long-term growth rates, shows the positive correlation between both global and domestic production fragmentation with economic growth. This result could indicate that it is the general complexity of the division of labour, reflected in the general fragmentation of production, that is primarily correlated with growth, irrespective of its global or domestic nature. Thus, the proposed measure and the new typology of value chains, in particular the novel conceptualization of domestic value chain fragmentation, could bring to light important information that was hidden in the existing typology, which conceptualised the domestic component only as a negation of the global value chain and thus did not allow research with explicit research questions related to domestic integration. The complex development of globalisation in recent decades and the recent shifts towards localization and regionalization of economic integration caused by political, economic and external factors make the new approach increasingly relevant. The proposed measure, particularly in conjunction with data from other sources, could further deepen the theoretical and empirical investigation.

In conclusion, it can be said that our new methodological approach and the new extended typology of value chains associated with it provide fertile ground for deeper insights into different types of value chains as well as a broader set of tools useful for various extensions of research.

6 Declarations

6.1 Availability of data and materials

The datasets analysed during the current study are available at http://www.wiod.org.

6.2 Competing interests

The authors declare that they have no competing interests.

6.3 Funding

The research was conducted with funding from Slovenian Research Agency (ARRS), record no. 52075.

6.4 Authors' contributions

KK contributed the methodological derivations and empirical results, all three authors contributed to the literature review, discussion of results and extensive proofreading.

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Appendix A - notations

$n_S \in \mathbb{N}$ number of sectors.
$n_C \in \mathbb{N}$ number of countries.
$n \in \mathbb{N}$; $n = n_S * n_C$ number of country-sectors.

$\mathbf{1} \in \mathbb{R}^n$ vector of ones.
$\vec{1} \in \mathbb{R}^{n_C}$ vector of ones.
$\vec{e_i} \in \mathbb{R}^n$; $e_{ij} = \delta_{ij}$ standard orthonormal basis of $\mathbb{R}^n$.
$I \in \mathbb{R}^{n \times n}$ identity matrix.

$x \in \mathbb{R}^n$ total output vector.
$\hat{x} \in \mathbb{R}^{n \times n}$; $\hat{x} = diag(x)$ total output matrix.
$C \in \mathbb{R}^{n \times n}$ intermediate consumption matrix.
$F \in \mathbb{R}^{n \times n_C}$ final consumption matrix on country level.[24]
$f \in \mathbb{R}^n$; $f = F\vec{1}$ total final consumption vector.
$\hat{f} \in \mathbb{R}^{n \times n}$; $\hat{f} = diag(f)$ total final consumption matrix.

$A \in \mathbb{R}^{n \times n}$; $A = C\hat{x}^{-1}$ Leontief technical coefficient matrix.
$G \in \mathbb{R}^{n \times n}$; $G = \hat{x}^{-1}C$ Ghosh technical coefficient matrix.

$v \in \mathbb{R}^n$; $v^T = x^T - \mathbf{1}^T C = \mathbf{1}(\hat{x} - A\hat{x}) = \mathbf{1}^T(I - A)\hat{x}$ vector of total value added.
$\hat{v} \in \mathbb{R}^{n \times n}$; $\hat{v} = diag(v)$ total value added matrix.
$v_C \in \mathbb{R}^n$; $v_C^T = v^T\hat{x}^{-1} = \mathbf{1}^T(I - A)$ vector of value added coefficients - value added share in total output.
$\hat{v}_C \in \mathbb{R}^{n \times n}$; $\hat{v}_C = diag(v_C)$ value added coefficients matrix.

C, A and G have block matrix structure $\mathbb{R}^{(n_S \times n_S) \times (n_C \times n_C)}$, while F has a block vector structure $\mathbb{R}^{n_S \times (n_C \times n_C)}$. Diagonal block elements with respect to countries represent domestic intermediate transfers and domestic consumption and off diagonal block elements represent transactions that crossborder either for intermediate use or final consumption.

$C = C_{CB} + C_D$
$A = A_{CB} + A_D$
$G = G_{CB} + G_D$
$F = F_{CB} + F_D$
$f_{CB} \in \mathbb{R}^n$; $f_{CB} = F_{CB}\vec{1}$ total final consumption by exporting.
$f_D \in \mathbb{R}^n$; $f_D = F_D\vec{1}$ total final consumption by domestic transactions.
$\hat{f}_{CB} \in \mathbb{R}^{n \times n}$; $\hat{f}_{CB} = diag(f_{CB})$ total final consumption by exporting matrix.
$\hat{f}_D \in \mathbb{R}^{n \times n}$; $\hat{f}_D = diag(f_D)$ total final consumption by domestic transactions matrix.

[24] In international I-O framework F is usually disaggregated on country level as well as in additional dimension of final consumption (household, government and non-profit consumption, fixed capital formation and changes in inventories), which is in our derivation irrelevant and left out. Disaggregation by countries is relevant to enable separation of domestic final consumption and export.

Appendix B - τ_i decomposition

$$\tau_i = \hat{v_C}(I-A)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A)^{-1}\hat{f} =$$
$$= \hat{v_C}\Big[I + A_D(I-A_D)^{-1} + (I-A_D)^{-1}A_{CB}(I-A_D)^{-1} +$$
$$+(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\Big]\vec{e_i}\otimes$$
$$\otimes\vec{e_i}^T\hat{x}^{-1}\Big[I + A_D(I-A_D)^{-1} + (I-A_D)^{-1}A_{CB}(I-A_D)^{-1} +$$
$$+(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\Big]\hat{f} =$$
$$= \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} + \hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} + \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} + \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\hat{f} + \hat{v_C}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} +$$
$$+\hat{v_C}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\hat{f} +$$
$$+\hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\hat{f} +$$
$$+\hat{v_C}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\vec{e_i}\otimes$$
$$\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big)\hat{f} =$$
$$= \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} + \hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\hat{f} + \hat{v_C}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}A_D(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}A_D(I-A_D)^{-1}\hat{f} + \hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\hat{f} =$$
$$\tau_i^{NVC} + \tau_i^{DVC} + \tau_i^{GVC} = \tau_i$$

$$\tau_i^{GVC} = \hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big((I-A)^{-1} - (I-A_D)^{-1}\big)\hat{f} =$$
$$\hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}(I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}(I-A_D)^{-1}\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\hat{f} +$$
$$+\hat{v_C}\big[(I-A)^{-1} - (I-A_D)^{-1} - (I-A_D)^{-1}A_{CB}(I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}(I-A_D)^{-1}\hat{f} +$$
$$+\hat{v_C}\big[(I-A)^{-1} - (I-A_D)^{-1}\big]\vec{e_i}\otimes\vec{e_i}^T\hat{x}^{-1}\big[(I-A)^{-1} - (I-A_D)^{-1}\big]\hat{f} =$$
$$= \tau_i^{SGVC} + \tau_i^{CGVC} = \tau_i^{GVC}$$

Figures

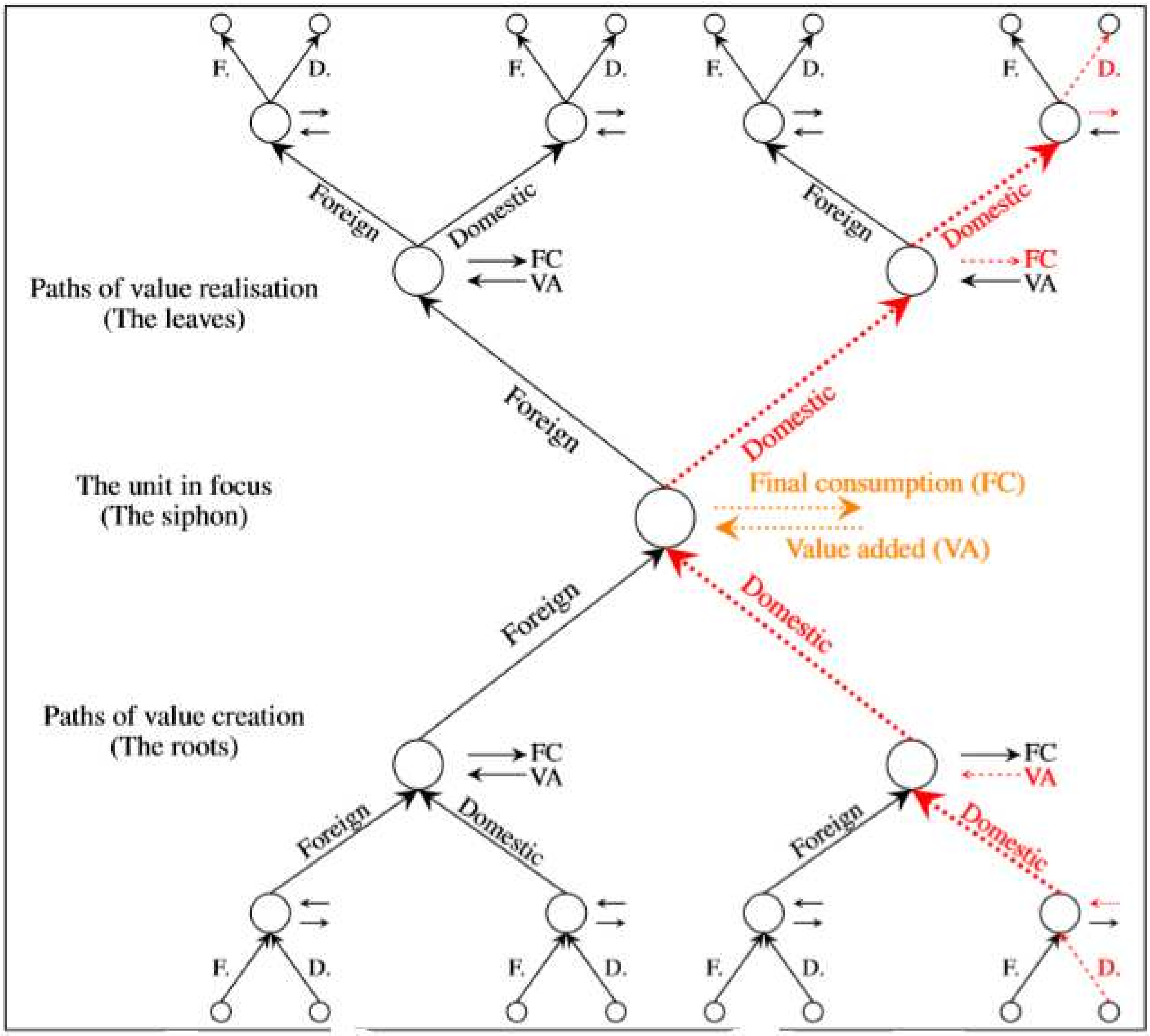

Figure 1

Value chain tree. Source: own conceptualisation and design.

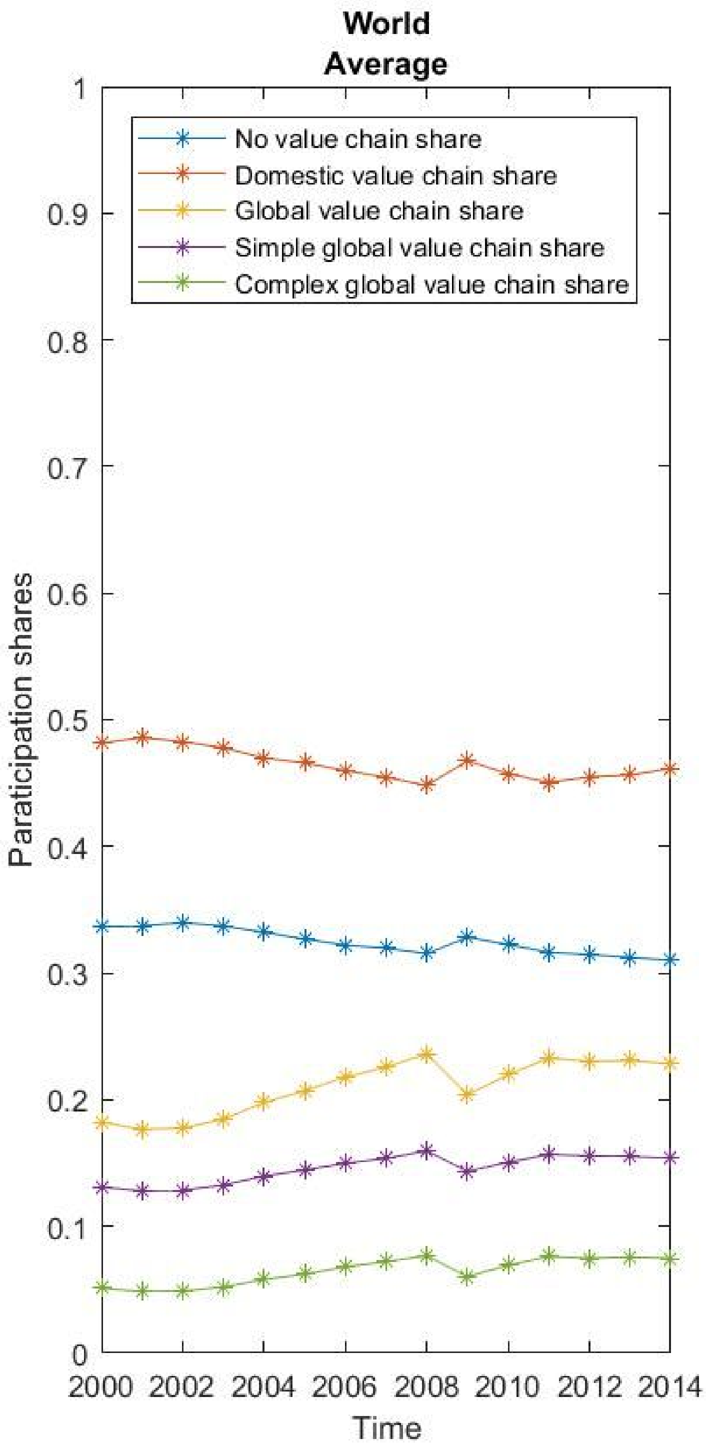

Figure 2

World average participation rates.

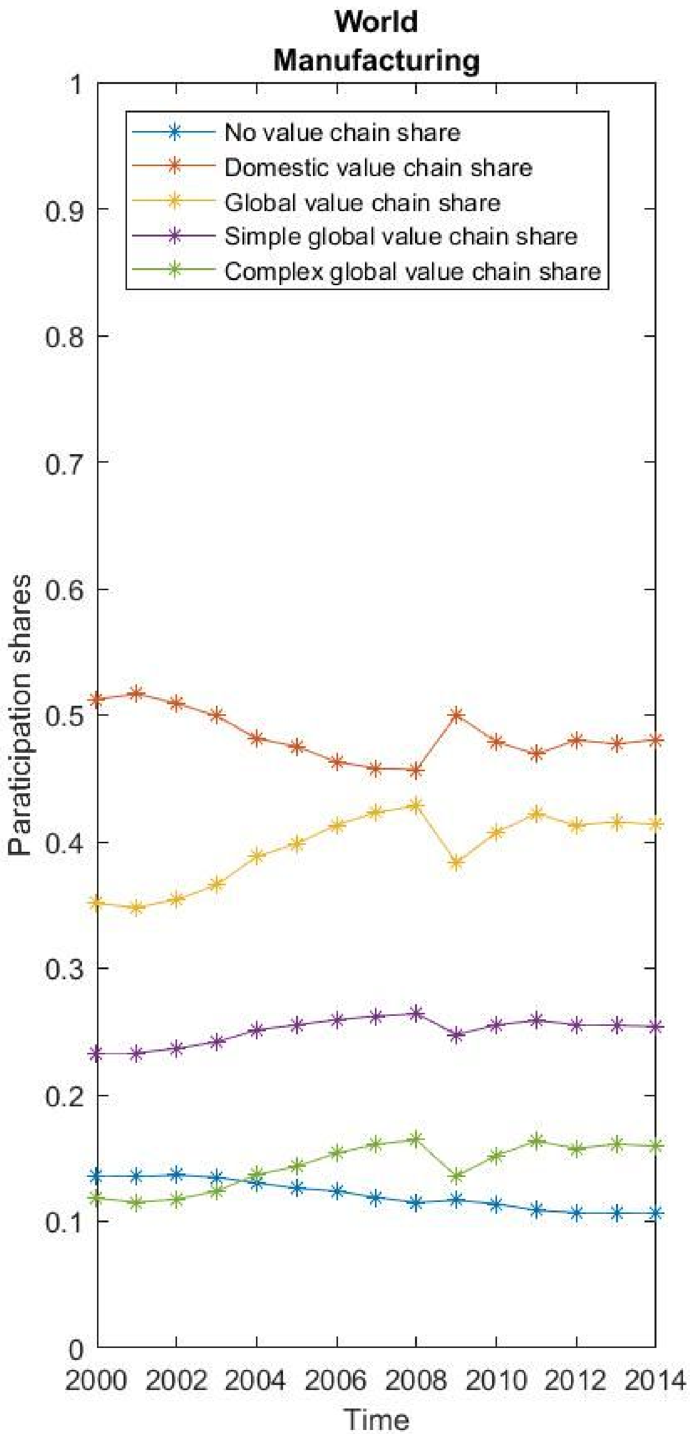

Figure 3

World average of manufacturing

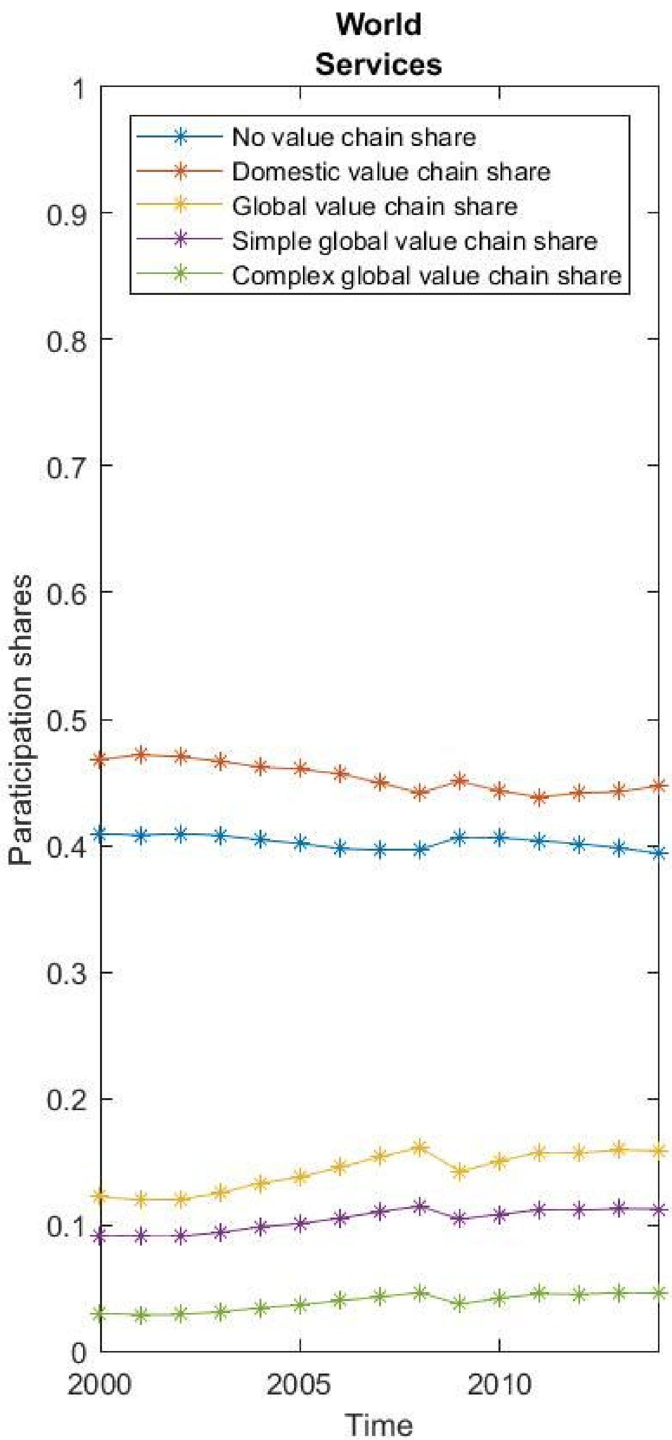

Figure 4

World average of services

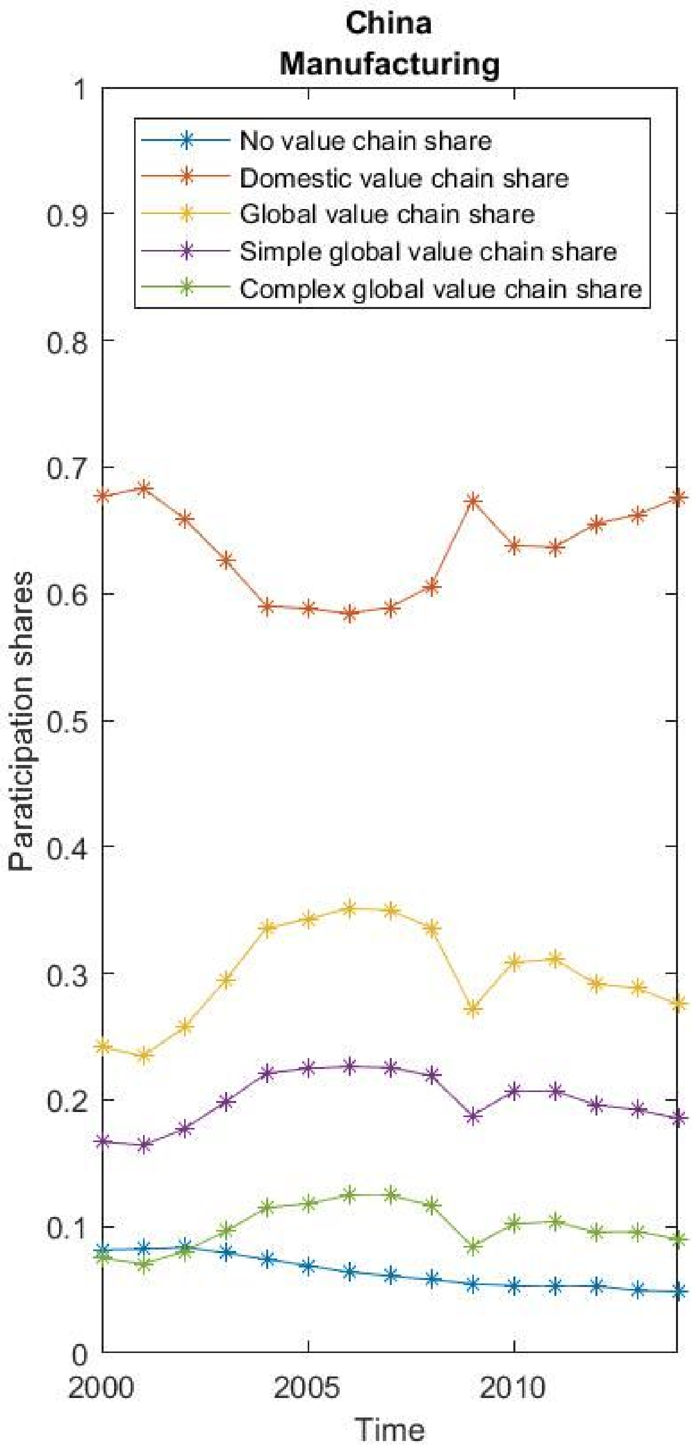

Figure 5

China manufacturing participation rates

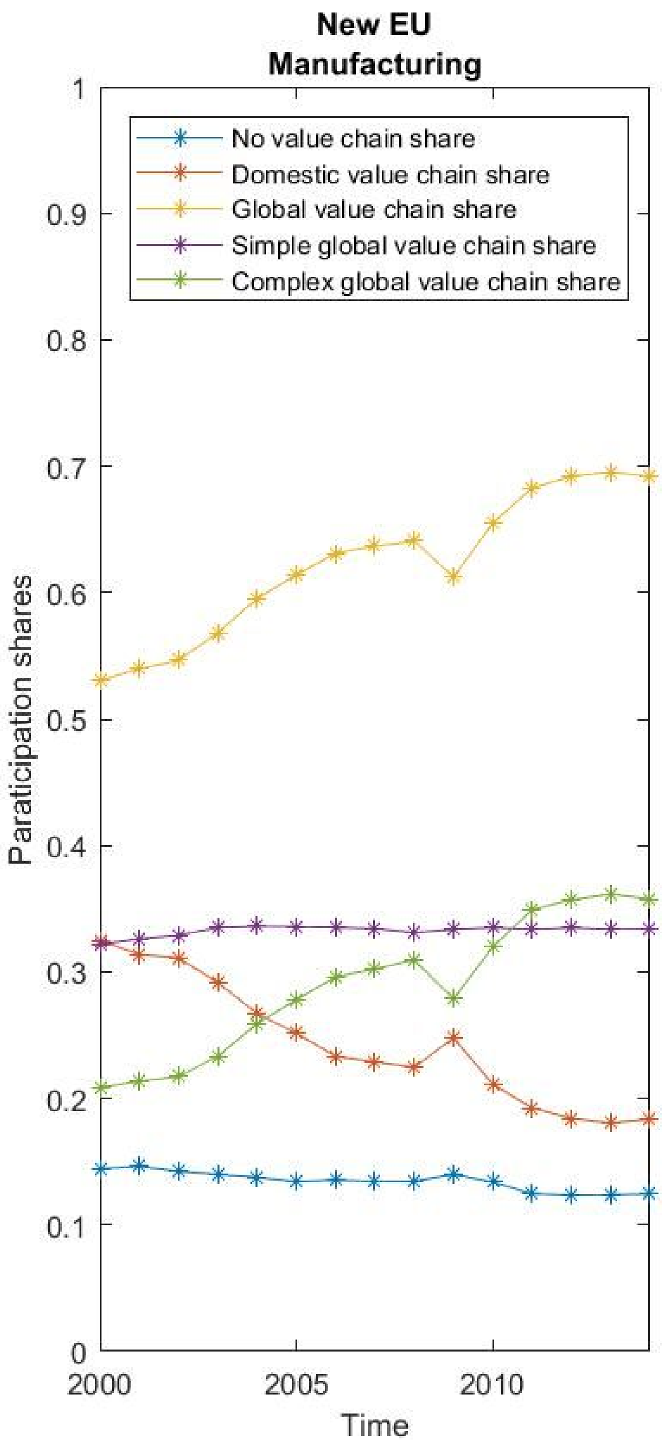

Figure 6

New EU countries manufacturing

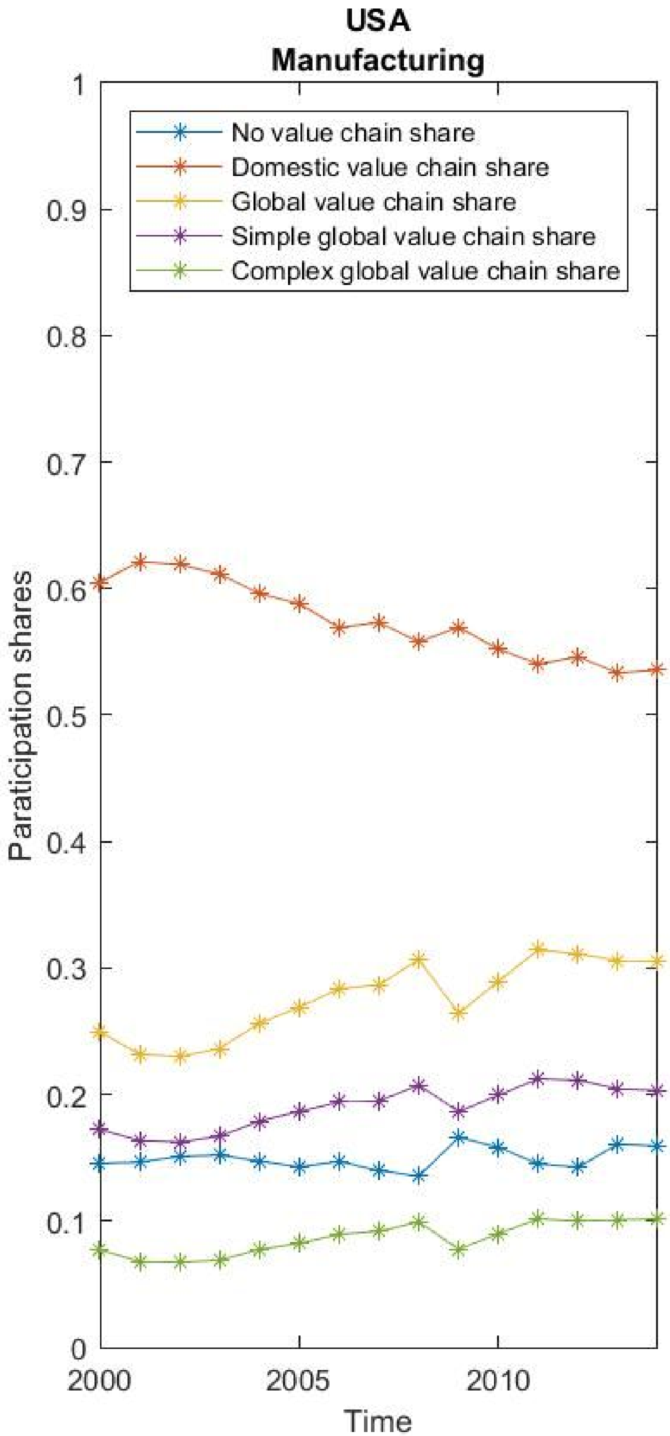

Figure 7

USA manufacturing participation rates