Instability of Hydromagnetic Hybrid Flow Fe₂O₃-Fe₃O₄ /H₂O of Thermocapillary Layers of Shear-Thinning Nanofluids

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Instability of Hydromagnetic Hybrid Flow $Fe_2O_3-Fe_3O_4/H_2O$ of Thermocapillary Layers of Shear-Thinning Nanofluids

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ABSTRACT: Nanofluids have inordinate uses in industries due to their high rate of heat transformation. The enhancement of heat transfer rates, an advanced type of nanofluids called “Hybrid nanofluid” is discovered. Hybrid nanofluids are created by the suspension of two or more type of nano sized atoms in base fluid such as water, oil etc. In current work, the instability of Hydromagnetic Poiseuille flow of thermocapillary layers of liquid of shear thinning hybrid nanofluid of $Fe_2O_3-Fe_3O_4$ in water based fluid is investigated. The transverse magnetic field is imposed on the flow. The solution of obtained ODEs for current problem is found using a numerical method called “cheybeshev collocation method”. The eigenvalue problem from generalized Orr-sommerfeld equations is solved using the algorithm of “QZ” abbreviated as (Qualitat and Zuverlassigkeit). The instability of water based hybrid nanofluids for variation of different parameters are analyzed and discussed through graphs. It has been found that Reynolds number and prandtl number has destabilizing effect on the flow while wave number and magnetic parameter has stabilizing impact on the fluid flow.

Keywords: Hybrid nanofluid, Heat transfer, Hydromagnetic, Poiseuille flow, Thermocapillary layers

INTRODUCTION

Hydromagnetic also known as Magneto hydro dynamics (MHD) or Electromagnetic effects is the review of the behavior of conductive fluids in a magnetic field. The magnetic field convinces an electric current in the flowing conductive fluid. The electric current flowing through the conductive fluid generates a force on the fluid and affects the magnetic field. A famous physicist Hannes Alfven [1] belongs to Sweden was first introduced the concept of MHD flow. The stability analysis of the hydromagnetic fluid flow has one of the most important research fields. Many research papers have been published on the instability of MHD flow in recent years. Z. Hussain

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[2] worked on the instability of 3D electrical conducting of MHD Couette flow. It is concluded from his research that there is stabilizing effect on Hartmann flow due to the magnetic field with exact magnitude. The stability of MHD flow of Hartmann layers between corresponding plates by imposed transverse magnetic field is investigated by Z. Yang and Z. Hussain [3]. Many other researches [4-6] have been done on the MHD instabilities.

Thermocapillary flow is the motion of fluid due to the stress of surface by induced temperature gradient. In past decades, the researchers of nanaofluid instability have performed a huge amount of work on thermocapillary flow. Schatz [7] and Davis [8] considered the thermocapillary instabilities theoretically and using experiments. In some previous studies, the impact of viscous elasticity on the instability of thermocapillary layers of liquid has been considered [9-14], which indicates that instability is significantly affected by elasticity. The thermocapillary flows used in many applications experimentally, for example in coating of film [15], lithography [16-17] and many more.

Nanofluids are fluid systems containing colloidal dispersions of nanoparticles in base fluid such as any low viscosity liquid. The small sized solid particles suspending in an energy transfer fluid can increase its thermal conductivity and provide an advanced and effective way to significantly enhance its heat transfer features. Nanofluids have produced in many ways such as the most common methods are spreading methods by using an ultrasonic investigation, vaporization methods, gas scattering methods, and chemical precipitation techniques. Nanofluids have many applications in different industries such as in automotive industry it is used for enhancing the lubricants and liquids coolant in the systems of cooling-air radiator. In both applications the effective heat transfers properties of several kinds of nanofluids are used. The enhancement of heat transmission basically depends on the thermal conductivities of nanoparticles, the volume concentration of particles and the mass flow rate. The enhancement of heat transformation totally depends on thermal conductivity for fixed volume concentration and the rate of mass flow. The thermal conductivity may be improved by formulating hybrid nanoparticles. Hybrid nanofluids are formed by the composition of two or more type of nano-meter sized particles in base fluid such as water, oil etc. The inspiration for preparing hybrid nanofluids is to attain more enhancement of heat transformation by increasing the thermal conductivity of these nanofluids.

A huge amount of work has been done by researchers on the instability of hybrid nanofluids in recent years. D. Dhinesh Kumar [18] and N. A. C. Sidik [19] systematically reviewed the preparation, description, properties and instability of hybrid nanofluid. M. S. Kumar [20] worked on the thermal conductivity and rheological properties of Cu-Zn hybrid nanoparticles with different base fluids. He used photo capturing and DLS methods for analyzing the stability of hybrid nanofluids. The results of his studies shows that hybrid nanofluids shown stability for 72 h and nanoparticles starts accumulation and all the particles settled down at 168 h. The thermal conductivity of CNT-SiO₂ based water hybrid nanofluid was investigated experimentally by A. S. Dalkilic [21]. He concluded from his research that the thermal conductivity of hybrid nanofluids increase with volume fraction and temperature. A. I. Ramadhan [22] investigated the stability analysis of Al₂O₃-TiO₂-SiO₂ tri-hybrid nanofluid in base fluid mixture of water plus ethylene glycol. From his studies it is concluded that tri-hybrid nanoparticles effectively prepared and attained stability. The qualitative studies on the stability of Al₂O₃-SiO₂ based water hybrid nanofluid was carried out by H. Ismail [23]. The enhanced stability, optical properties and high solar heat transformation competency of SiC-MWCNTs hybrid nanofluids for DASC applications was studied by Xiaoke Li [24]. The influence of various surfactants and ultrasonication time on the instability and thermal physical characteristics of hybrid nanoparticles of graphene nanoplatelets and TiO₂ based water and ethylene glycol was analyzed by H. W. Xian [25]. It is shown from his article results that hybrid nanofluids has greater thermal conductivity than other simple nanofluids at all ranges of temperature and concentration and both nanofluids shows the Newtonian behavior from which shear stresses increases linearly by increasing shear rate.

In this article, the instability of Hydromagnetic or MHD Poiseuille flow of thermocapillary layers of liquid of shear thinning Hybrid nanofluid of Fe₂O₃-Fe₃O₄ in water based fluid is examined. The transverse magnetic field is executed on the flow. The solution of obtained ODEs for current problem is found using a numerical method called “cheybeshev collocation method”. The eigenvalue problem from generalized Orr-sommerfeld equations is solved by using the QZ algorithm to obtaining the eigenvalues. The instability analysis of water based hybrid nanofluids for variation of different parameters are explored and deliberated through graphs.

FORMULATION OF PROBLEM

In this research work, the unsteady Magneto-hydro-dynamic Poiseuille flow of water based hybrid nanofluid of ferro oxide plus Iron oxide is considered under the impact of transverse magnetic field. The fluid is incompressible, electrically conductive and viscous. We choose the thermocapillary liquid model which was proposed by Davis and Smith [26]. In this model the layer of fluid lies above an infinitely inflexible plane and exerts a temperature gradient on its open surface as shown in Figure 1. The convection is managing by the thermal capillary forces while the tension on the surface is big sufficient to preserve the surface smooth. The flow of the fluid is moving in x-axis between two plates with ' $2H$ ' distance apart, while the direction of velocity gradient is normal to it. Both plates are taken to be electrically nonconductive with temperature T . The constant magnetic field ' B ' is executed in the perpendicular direction of plates.

FIGURE 1: Geometrical configuration of the problem

In this problem we consider the Carreau fluid model from Lodge's molecular network theory [27] to define the shear-thinning nanofluid behavior. The constitutive equation is

$$\tau = \hat{\mu}(\dot{\gamma})\tilde{\gamma} \quad (1)$$

Where $\tilde{\gamma} = \nabla V + (\nabla V)^T$ represented the strain-rate tensor, τ is the stress tensor, $V(u, v, 0)$ is the velocity vector and $\hat{\mu}$ represents the viscosity of the fluid that can be defined as,

$$\hat{\mu} = \hat{\mu}_{\infty} + (\hat{\mu}_0 - \hat{\mu}_{\infty}) \left(1 + (\hat{\lambda} \dot{\gamma})^2\right)^{(n-1)/2} \quad (2)$$

Here $\hat{\mu}_0$ and $\hat{\mu}_{\infty}$ are the viscosities at zero shear rate (characteristic viscosity) and infinite shear rate respectively ($\hat{\mu}_0 \geq \hat{\mu}_{\infty}$). $\hat{\lambda}$ represents the material time constant and the index n shows the degree of shear thinning ($n \leq 1$). For $n=1$ the Carreau fluid becomes Newtonian fluid. It can also be seen that with the decreasing of n or increasing of $\hat{\lambda}$, degree of shear-thinning always increases.

Using the non-dimensional parameters, the viscosity becomes,

$$\mu = \mu_{\infty} + (1 - \mu_{\infty}) \left(1 + (\lambda \dot{\gamma})^2\right)^{(n-1)/2}, \text{ Where } \mu_{\infty} = \hat{\mu}_{\infty} / \hat{\mu}_0, \lambda = \hat{\lambda} U / L \quad (3)$$

In this problem, $\mu_{\infty} = 0.01$ is used. The dimensionless governing equations for above problem are,

Equation of continuity:- $\nabla \cdot V = 0,$ (4)

Momentum equation:- $R \left[\frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla P + \nabla \cdot \tau + Bo \cdot \theta \hat{j} + N(J \times B)$ (5)

Energy equation:- $\frac{\partial \theta}{\partial t} + (V \cdot \nabla) \theta = \frac{1}{Ma} \nabla^2 \theta$ (6)

Ohm's law and charge equation can be defined as,

$$J = (V \times B) - \nabla \phi \quad (7)$$

$$\nabla \cdot J = 0 \quad (8)$$

Here $J=(J_x, J_y, J_z)$, P , B , θ , ϕ , are the current density, pressure, total magnetic field, temperature and electric potential respectively.

Boundary conditions for above equations are defined in the following way,

$$u = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = 0, \frac{\partial T}{\partial y} = 0 \quad \text{at } x = \pm H \quad (9)$$

The dimensionless parameters from governing equations are as follows:

$R = \rho UL / \hat{\mu}_0$ is Reynolds number, where ρ is the density of the fluid, L is the depth of the layer, and U represents the characteristic velocity which is define as $U = b\gamma L / \hat{\mu}_0$. The interaction parameter N can be defined as $N = B_0^2 L \sigma / \rho U$ is the proportion of electro-magnetic to inertia forces. N is also the relationship of Hartmann number and Reynolds number i.e, $N = Ha^2 / R$, where $Ha = B_0 L \sqrt{\sigma / \rho \nu}$. Here, B_0 is the imposed magnetic field, $\nu = \mu / \rho$ represents kinematic viscosity, and σ represents the electrical conductivity.

$Ma = b\gamma L^2 / \hat{\mu}_0 \alpha$ is known as Marangoni number, where α is the thermal diffusivity. Ma can also be defined as the relationship of Re and Pr ($Pr = \nu / \alpha$ the relation of momentum diffusivity to thermal diffusivity) i.e, $Ma = R \cdot Pr$. Prandtl number can also be written as $Pr = \hat{\mu}_0 C_p / K$, where C_p & K are the specific heat and thermal conductivity respectively.

$Bo = \rho g a L^2 / \gamma$ represents the dynamic Bond number, from which we can measured the gravity effect. Here, a is thermal expansion coefficient and g represents the gravitational acceleration. The density of fluid related with the temperature is defined as $\rho = \rho_0 \left[1 - a(\tilde{T} - \tilde{T}_0) \right]$, where $\tilde{T}$ is the fluid temperature and $\tilde{T}_0$ is reference temperature.

The relations between the base fluid and Hybrid nanofluids for different thermal physical properties are follows:

The dynamic viscosity μ_{hnf} is defined as,

$$\hat{\mu}_{0,hnf} / \hat{\mu}_{0,f} = 1 / (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} \quad (10)$$

The density ρ_{hnf} is defined as,

$$\rho_{hmf} / \rho_f = (1 - \phi_2) \left[(1 - \phi_1) + (\phi_1 \rho_{1s} / \rho_f) \right] + (\phi_2 \rho_{2s} / \rho_f) \quad (11)$$

Electrical conductivity σ_{hmf} is defined as,

$$\sigma_{hmf} / \sigma_{bf} = \left[\frac{\sigma_{2s} + 2\sigma_{bf} - 2\phi_2 (\sigma_{bf} - \sigma_{2s})}{\sigma_{2s} + 2\sigma_{bf} + \phi_2 (\sigma_{bf} - \sigma_{2s})} \right], \text{ Where } \sigma_{bf} / \sigma_f = \left[\frac{\sigma_{1s} + 2\sigma_f - 2\phi_1 (\sigma_f - \sigma_{1s})}{\sigma_{1s} + 2\sigma_f + \phi_1 (\sigma_f - \sigma_{1s})} \right] \quad (12)$$

The specific heat capacity is defined as,

$$\frac{(\rho C_p)_{hmf}}{(\rho C_p)_f} = (1 - \phi_2) \left[(1 - \phi_1) + \frac{\phi_1 (\rho C_p)_{1s}}{(\rho C_p)_f} \right] + \frac{\phi_2 (\rho C_p)_{2s}}{(\rho C_p)_f} \quad (13)$$

The thermal conductivity is of the form,

$$\frac{K_{hmf}}{K_{bf}} = \frac{K_{2s} + 2K_{bf} - 2\phi_2 (K_{bf} - K_{2s})}{K_{2s} + 2K_{bf} + \phi_2 (K_{bf} - K_{2s})} \quad \text{Where} \quad \frac{K_{bf}}{K_f} = \frac{K_{1s} + 2K_f - 2\phi_1 (K_f - K_{1s})}{K_{1s} + 2K_f + \phi_1 (K_f - K_{1s})} \quad (14)$$

The subscripts $hmf, f, 1s$ and $2s$ represents the Hybrid nanofluids, basic fluid, solid particles of ferro oxide and solid particles of Iron oxide respectively. Using these above relations the dimensionless parameters used in governing equations, Reynolds number, interaction parameter, Bond number and Marangoni number can be written as,

$$R_{hmf} = M \cdot R \quad (15)$$

$$\text{Where } M = \left[(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} \right] \left[(1 - \phi_2) \left[(1 - \phi_1) + (\phi_1 \rho_{1s} / \rho_f) \right] + (\phi_2 \rho_{2s} / \rho_f) \right]$$

$$N_{hmf} = N \cdot H \quad (16)$$

$$\text{Where } H = \frac{\left[\sigma_{bf} / \sigma_f \right] \left[\left[\sigma_{2s} + 2\sigma_{bf} - 2\phi_2 (\sigma_{bf} - \sigma_{2s}) \right] / \left[\sigma_{2s} + 2\sigma_{bf} + \phi_2 (\sigma_{bf} - \sigma_{2s}) \right] \right]}{\left[(1 - \phi_2) \left[(1 - \phi_1) + (\phi_1 \rho_{1s} / \rho_f) \right] + (\phi_2 \rho_{2s} / \rho_f) \right]}$$

$$Bo_{hmf} = Bo \cdot I \quad (17)$$

Where
$$I = \left[(1 - \phi_2) \left[(1 - \phi_1) + \left(\phi_1 \rho_{1s} / \rho_f \right) \right] + \left(\phi_2 \rho_{2s} / \rho_f \right) \right]$$

$$Ma_{mf} = R \cdot \text{Pr} \cdot K \quad (18)$$

Where

$$K = \left[(1 - \phi_2) \left[(1 - \phi_1) + \frac{\phi_1 (\rho C_p)_{1s}}{(\rho C_p)_f} \right] + \frac{\phi_2 (\rho C_p)_{2s}}{(\rho C_p)_f} \right] / \left[\frac{K_{bf}}{K_f} \right] \left[\frac{K_{2s} + 2K_{bf} - 2\phi_2 (K_{bf} - K_{2s})}{K_{2s} + 2K_{bf} + \phi_2 (K_{bf} - K_{2s})} \right]$$

Putting equations (15-18) in momentum and energy equations we get,

From eq. (5)
$$A_1 \left[\frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla P + \nabla \cdot \tau + A_3 \cdot \theta \hat{j} + A_2 (J \times B) \quad (19)$$

From eq. (6)
$$\frac{\partial \theta}{\partial t} + (V \cdot \nabla) \theta = \frac{1}{A_4} \nabla^2 \theta \quad (20)$$

Where $M \cdot R = A_1$, $N \cdot H = A_2$, $Bo \cdot I = A_3$, and $R \cdot \text{Pr} \cdot K = A_4$

LINEAR INSTABILITY ANALYSIS

In this problem the viscosity depends on the strain-rate tensor, so from the strain-rate $\tilde{\gamma}$ perturbation we can get the perturbation of viscosity μ' . Thus, it can be inferred from equation (1) that the perturbation on stress tensor contains two portions, one is due to strain-rate $\tilde{\gamma}$ perturbation and other is caused by perturbation of viscosity μ' , i.e.,

$$\tau = \mu \tilde{\gamma} + \mu' \tilde{\gamma}_0 \quad (21)$$

Where $\tilde{\gamma}_0$ is the strain-rate tensor of simple flow. In our problem we can be defined the basic flow as

$$\tilde{\gamma}_0 = \begin{pmatrix} 0 & \dot{\gamma}_0 \\ \dot{\gamma}_0 & 0 \end{pmatrix}, \dot{\gamma}_0 = D\bar{U}, D = \frac{d}{dy} \quad (22)$$

In addition of infinite perturbation in the form of normal modes to the basic flow can be defined as,

$$(V, \theta, P, \tau) = (\bar{V}, \bar{\theta}, \bar{P}, \bar{\tau}) + \delta \quad (23)$$

Where
$$\delta = (\hat{u}, \hat{v}, \hat{\theta}, \hat{P}, \hat{\tau}) e^{[\omega t + ikx]} \quad (24)$$

Where $\omega = \omega_r + i\omega_i$ is the complex phase velocity and k is real wave number.

From equations (23) and (24), we get

$$\tilde{\gamma} = \begin{pmatrix} 2ik\hat{u} & D\hat{u} + ik\hat{v} \\ D\hat{u} + ik\hat{v} & 2D\hat{v} \end{pmatrix} \quad \text{and} \quad \tilde{\gamma}_0 = \begin{pmatrix} 0 & D\hat{u} + ik\hat{v} \\ D\hat{u} + ik\hat{v} & 0 \end{pmatrix}$$

Putting all above expressions of $\tilde{\gamma}$, μ' , and $\tilde{\gamma}_0$ in eq (21), we get the linearized relation between stress and strain-rate perturbation as,

$$\hat{\tau} = \mu \begin{pmatrix} 2ik\hat{u} & D\hat{u} + ik\hat{v} \\ D\hat{u} + ik\hat{v} & 2D\hat{v} \end{pmatrix} + \frac{d\mu}{d\dot{\gamma}_0} \dot{\gamma}_0 \begin{pmatrix} 0 & D\hat{u} + ik\hat{v} \\ D\hat{u} + ik\hat{v} & 0 \end{pmatrix} \quad (25)$$

Where
$$\frac{d\mu}{d\dot{\gamma}_0} \dot{\gamma}_0 = (1 - \mu_\infty)(n-1) \left(1 + (\lambda \dot{\gamma}_0)^2\right)^{(n-3)/2} (\lambda \dot{\gamma}_0)^2 \quad (26)$$

Here, above equation deduced from equation (3). From this equation we can see that when $n=1$, the second part of the right side of equation (25) disappears, so that Carreau fluid converts into Newtonian fluid. For $n < 1$, the viscosity corresponds to $\dot{\gamma}_{12}$ is

$$\mu_t = \mu + \frac{d\mu}{d\dot{\gamma}_0} \dot{\gamma}_0 \quad (27)$$

The viscosity for the other components of $\tilde{\gamma}$ is μ . μ_t is the tangent viscosity (tangent line slope for the curves of shear rate vs. shear stress). From equation (26) we can also be seen that for shear thinning fluid $n < 1$, $\mu_t \leq \mu$.

Now substituting equations (23) and (24) in the equations (4), (19), and (20), the linearized perturbation equations are follows,

From (4),
$$ik\hat{u} + D\hat{v} = 0 \quad (28)$$

From (19), In x-direction,

$$A_1 [\omega\hat{u} + \bar{U}ik\hat{u} + \hat{v}D\bar{U}] = -ik\hat{P} + \tau'_x - A_2\hat{u} \quad (29)$$

In y-direction,

$$A_1 [\omega\hat{v} + \bar{U}ik\hat{v}] = -D\hat{P} + \tau'_y + A_3 \cdot \hat{\theta} \quad (30)$$

As

$$\nabla \hat{\tau} = \begin{pmatrix} ik\hat{\tau}_{11} + D\hat{\tau}_{12} & ik\hat{\tau}_{12} + D\hat{\tau}_{22} \end{pmatrix}$$

From equation (29), we get

$$A_1 [\omega\hat{u} + \bar{U}ik\hat{u} + \hat{v}D\bar{U}] = -ik\hat{P} + (ik\hat{\tau}_{11} + D\hat{\tau}_{12}) - A_2\hat{u} \quad (31)$$

From equation (30), we get

$$A_1 [\omega\hat{v} + \bar{U}ik\hat{v}] = -D\hat{P} + (ik\hat{\tau}_{12} + D\hat{\tau}_{22}) + A_3 \cdot \hat{\theta} \quad (32)$$

Where $D = \frac{d}{dy}$

Putting value of $\hat{u}$ from equation (28) in equation (31) and multiplying by (ik) we get,

$$A_1 [-\omega D\hat{v} - \bar{U}ikD\hat{v} + \hat{v}ikD\bar{U}] = k^2\hat{P} + ik(ik\hat{\tau}_{11} + D\hat{\tau}_{12}) + A_2D\hat{v} \quad (33)$$

Now in order to eliminate pressure, multiplying equation (32) by (k^2) and equation (33) by (D) and adding we get

$$\begin{aligned} & A_1\bar{U}ik^3\hat{v} + A_1ikD^2\bar{U}\hat{v} - A_1\bar{U}ikD^2\hat{v} - ik^3\hat{\tau}_{12} - k^2D\hat{\tau}_{22} + k^2D\hat{\tau}_{11} \\ & - ikD^2\hat{\tau}_{12} - k^2 \cdot A_3 \cdot \hat{\theta} - A_2D^2\hat{v} = A_1\omega(D^2 - k^2)\hat{v} \end{aligned} \quad (34)$$

Here, from equation (25), we have

$$\hat{\tau}_{11} = -2\mu D\hat{v} \quad (35)$$

$$\hat{\tau}_{12} = \mu_t \left(-D^2\hat{v} - k^2\hat{v} \right) \quad (36)$$

$$\hat{\tau}_{22} = 2\mu D\hat{v} \quad (37)$$

From temperature equation (20), we get

$$A_4 \left[-\frac{\partial \bar{\theta}}{\partial x} D\hat{v} - \bar{U}k^2\hat{\theta} + ik \frac{\partial \bar{\theta}}{\partial y} \hat{v} \right] + ik^3\hat{\theta} - ikD^2\hat{\theta} = -\omega ik A_4 \hat{\theta} \quad (38)$$

Where characteristic magnitude of velocity and temperature for Poiseuille flow are defined as

$$\bar{U}(y) = [\cosh(Ha) - \cosh(Ha \cdot y)] / [\cosh(Ha) - 1] \quad (39)$$

$$\bar{\theta}(y) = 0.5(1 - y) \quad (40)$$

The boundary conditions for perturbation flow are now becomes

$$\hat{u} = 0, \quad D\hat{u} = D\hat{\theta} = 0 \quad \text{at} \quad x = \pm H \quad (41)$$

Equations (34) and (38) are now becomes in the form of ODEs for $\hat{v}$ & $\hat{\theta}$. The linear instability of these ODEs can be measured as two point BVP. We solved this problem in the form of eigenvalue problem by using chebyshev collocation method.

NUMERICAL METHOD

For each positive integer n , the n th degree Chebyshev series of type first $T_n(x)$ can be written as,

$$T_n(\cos \theta) = \cos n\theta, \quad (42)$$

Let the Chebyshev series expansion of eigenfunction $\hat{v}(y)$ and $\hat{\theta}(y)$ for current problem are,

$$u_j = \sum_{n=0}^{N_l} a_n T_{n_j}, \quad u_j' = \sum_{n=0}^{N_l} a_n T_{n_j}', \quad u_j'' = \sum_{n=0}^{N_l} a_n T_{n_j}'', \quad u_j^{(4)} = \sum_{n=0}^{N_l} a_n T_{n_j}^{(4)}$$

$$\theta_j = \sum_{n=0}^{N_1} b_n T_{n_j}, \theta_j' = \sum_{n=0}^{N_1} b_n T_{n_j}', \theta_j'' = \sum_{n=0}^{N_1} b_n T_{n_j}'' \quad (j=1,2,\dots,N_1) \quad (43)$$

Where a_n and b_n are coefficients of the series.

Using these above representation, equations (34) and (38) can be written as,

$$\begin{aligned} & \sum_{n=0}^{N_1} \left[\left[\mu_i k T^{(4)} + \left\{ -A_1 \bar{U} i k + 2\mu_i k^3 - 4\mu k^2 - A_2 \right\} T'' + \left\{ A_1 \bar{U} i k^3 + A_1 i k D^2 \bar{U} + \mu_i k^5 \right\} T \right] a_n - \left[A_3 k^2 T \right] b_n \right] \\ & = \omega \sum_{n=0}^{N_1} \left[A_1 T'' - A_1 k^2 T \right] a_n \end{aligned} \quad (44)$$

From (38), we have

$$\sum_{n=0}^{N_1} \left[\left[A_4 \frac{\partial \bar{\theta}}{\partial x} T' - A_4 i k \frac{\partial \bar{\theta}}{\partial y} T \right] a_n + \left[i k T'' + A_4 \bar{U} k^2 T - i k^3 T \right] b_n \right] = \omega \sum_{n=0}^{N_1} \left[A_4 i k T \right] b_n \quad (45)$$

Equations (44) and (45) are the general form of Orr–Sommerfeld equations for this problem. In the matrix form,

$$\begin{aligned} AX &= \omega BX \\ AY &= \omega BY \end{aligned} \quad (46)$$

Where

$$X = (a_0, a_1, \dots, a_n), Y = (b_0, b_1, \dots, b_n).$$

RESULTS AND DISCUSSION

In this section, the outcomes of numerical calculations of eigenvalue problems using the numerical method of “chebyshev collocation method” will be discussed. Table 1 lists the thermophysical properties of water and hybrid nanoparticles of ferro oxide and iron oxide used in this work. The research is carried out through graphs of neutral and growth rate curves. The values of material time constant (λ), the strain-rate tensor ($\tilde{\gamma}$) and degree of shear thinning (n), used in recent problem are 2, 0.01 and 0.3 respectively. The value for Bond number (Bo) is fixed taken as 3 in current problem and other parameters are varied.

In Figure 2, the flow velocity curves used in current work for different magnetic parameter (Ha) values are drawn. The impact of magnetic parameter on the simple velocity is displayed in this figure. The figure shows the velocity profile of the Poiseuille flow between two horizontal parallel plates of Ha from 1 to 10. From the figure it is shown that, the flow is convinced by the pressure gradient classified by a two-dimensional parabolic velocity distribution that is symmetrical to the straight middle plane. In order to increase the value of Ha the velocity distribution converges from laminar flow to turbulent flow.

Table 1: Numerical values for Thermo-physical properties of Nanoparticles and water

	Electric conductivity $\sigma(\Omega \cdot m)^{-1}$	Specific heat capacity $C_p(J \cdot kg^{-1}/K)$	Density $\rho(kg \cdot m^{-3})$	Thermal conductivity $K(Wm^{-1}/K)$
H₂O	0.05	4179	997	0.613
Fe₂O₃	25000	670	5180	9.7
Fe₃O₄	740000	670	5200	6

FIGURE 2: Curves for velocity distribution for third problem (Poiseuille Flow).

Impact of wave number on growth rate

The growth rate curves ω vs. Ha for wave number are presented in figure 3(a). The values of wave number are taken from $k=0.5$ to 1 with interval of 0.1 . Reynolds number and Prandtl number are taken as 5000 and 6.4 respectively. The values for volume fraction for ferro oxide $\phi_1=0.1$ and Iron oxide $\phi_2=0.04$ are used. From the curves it is analyzed that by increasing the wave number, the growth rate stabilizes rapidly and the curves are closer to each other. There is one global maximum point for each curve. In figure 3(b), the graph for growth rate ω vs. Pr is presented for variation of wave number k . By increasing the values of k from 0.5 with interval of 0.1 , it is seen that the curves of growth rate are going from unstable to stabilizing region. For $k=0.6$ and onward steps are generated for each curve from starting point and after some interval the curves becomes linear. Figure 3(c) represents the graph for growth rate curves ω vs. Reynolds number for different wave number. The wave numbers are taken from $k=0.4$ to 0.9 with 0.1 interval. Here also it is perceived from the figure that with increasing of wave number the growth rate destabilizing to stabilizing region rapidly. The configuration for each curve is similar. For higher values of Reynolds number the curves becomes linear. It is concluded that wave number have the stabilizing effect on the growth rate.

FIGURE 3: Graphs for growth rate ω vs. (a). Ha (b). Pr and (c). R for variation of wave number k

Impact of magnetic parameter on growth rate

The frequencies for growth rate ω vs. k for variation of magnetic parameter Ha are plotted in figure 4(a). The values for Ha are taken from 0.1 to 4.1 with unit interval. It is analyzed from the curves of the figure that curves starting from same point destabilizing first and from maximum point they becomes stabilizes. For increasing the magnetic parameter the growth rate stabilizes rapidly and from $Ha > 3$ stabilizing slowly. For each value of Ha curves have one global maximum. Curves pattern are similar for each case. In figure 4(b), growth rate curves ω vs. Pr for different values of magnetic parameter Ha from 0.1 to 10.1 with interval of 2 are presented. By increasing the magnetic parameter the growth rate first stabilizing fast and from $Ha > 4$ stabilizing effect

becomes slow. The curves contain steps which become small to larger with increasing of Prandtl number, and after some interval of Pr the curves becomes linear. Figure 4(c) presented the curves for growth rate ω vs. R for variation of Ha from 0.1 to 8.1 with interval of 2. From the figure it is analyzed that the growth rate lie in the stabilizing region, and for increasing the Ha , the growth rate stabilizes first and for $Ha > 4$, it stabilizes slowly. The magnetic parameter also plays an important role on the stabilizing. It is concluded that by increasing magnetic parameter from $Ha > 4$, stabilizing effect becomes slow.

FIGURE 4: Graphs for growth rate ω vs. (a). k (b). Pr and (c). R for variation of magnetic parameter Ha

Impact of Reynolds number on growth rate

The curves for growth rate ω vs. Ha for variation of Reynolds number are shown in figure 5(a). Reynolds number varies from 5000 to 35000 with the gap of 5000. It is seen that when the value of Reynolds number increases the growth rate destabilizes slowly. The growth rate curves become linear after some interval of Ha . For $R=10000$, 15000 and 20000 curves become linear at same line horizontally and similarly for $R=25000$, 30000 and 35000. The growth rate curves are closer to each other by increasing the Reynolds number. In figure 5(b), the graph for growth rate ω vs. k for Reynolds number from 5000 to 25000 is plotted. This graph displays that as the values of R increases the growth rate curves destabilizes very slowly. The curves starting from same point on vertical axis stabilizes first and with increasing the wave number they reach on peak point and after that they are destabilized. The maximum point occurs at $k=0.22$ and each curve has a unique maximum point on vertical axis. Figure 5(c) shows the graph for growth rate ω vs. Pr for variation of Reynolds number. The values of magnetic parameter and wave number are taken to be fixed as 2.1 and 0.8 respectively. Here also it is analyzed that by increasing the value of R the growth rate is going to unstable region slowly. The step wise curves become closer to each other for higher values of Reynolds number. It is concluded that Reynolds number have good destabilizing impact on the growth rate.

FIGURE 5: Graphs for growth rate ω vs. (a). Ha (b). k and (c). Pr for variation of Reynolds number R

Impact of Prandtl number on growth rate

The frequencies of growth rate ω vs. Ha for variation of Prandtl number are shown in figure 6(a). Prandtl number varies from 4.4 to 14.4 with an interval of two. The curves of this graph tell that there is a destabilizing effect on the growth rate with increasing the Pr . The distance between the curves becomes less with a small rate. The curves become linear after some value of magnetic parameter. In figure 6(b), the curves for the growth rate with respect to the wave number for variation of Pr are presented. The growth rate destabilizes slowly by increasing the Prandtl number. All the curves' maximum point lies in the unstable region. Each curve has a maximum point from which the growth rate starts destabilizing. Figure 6(c) represents the growth rate curves of ω - R for variation of Prandtl number. From the figure it is investigated that with increasing of Pr the growth rate destabilizes slowly. The curves for each value of Pr have almost the same pattern. For higher values of Reynolds number, the curves developed into linear in horizontal axis. It is concluded from all the discussion that Prandtl number has an important role on the destabilizing of growth rate.

FIGURE 6: Graphs for growth rate ω vs. (a). Ha (b). k and (c). R for variation of Prandtl number Pr

Impact of ϕ_I on growth rate

The influence of volume fraction for ferro oxide on growth rate ω vs. Ha is displayed in figure 7(a). The values of ϕ_I are taken from 0.1 to 0.8 with 0.2 intervals. From the curves of this figure it is shown that the growth rate curves lies in the stabilizing region at starting value of $\phi_I=0.1$ and more stabilizing for increasing the volume fraction. When $\phi_I>0.5$ it is observed that growth rate starts destabilizing more rapidly. The curves pattern also change becomes linear to banding curves. In figure 7(b), the curves for growth rate ω vs. k for different values of ϕ_I are displayed. It is perceived from the figure that with increasing the values of volume fraction the growth rate first stabilizing and after some interval for $\phi_I>0.3$, growth rate is going to the unstable region very fast. The curves pattern is almost same for different values of ϕ_I , and each curve has a global maximum point. After some interval of wave number on x -axis the curves becomes linear. Figure 7(c) represents the growth rate curves ω vs. Pr for variation of ϕ_I from 0.1 to 0.8 with interval of 0.2. It is analyzed that growth rate first stabilizing and then destabilizing rapidly. All the curves moves diagonally from stabilizing to destabilizing region. By increasing the volume fraction the curves pattern also changes from stepwise curves to linear. The figure 7(d) displays the curves of growth rate ω vs. Reynolds number for variation of ϕ_I . In this figure it is also investigated that till $\phi_I<0.3$ the growth rate slowly stabilized, and for $\phi_I>0.3$ it goes to the unstable region more rapidly. The curves starting from same point first moves in y -direction and then bands to x -direction becomes linear with increasing of Reynolds number. From the influence of ϕ_I on growth rate it is concluded that volume fraction of ferro oxide has more destabilizing influence on the growth rate of the flow.

FIGURE 7: Graphs for growth rate ω vs. (a). Ha (b). k (c). Pr and (d). R for variation of ϕ_1

Impact of ϕ_2 on growth rate

The impact of volume fraction of Iron oxide on the growth rate is displayed in below figures. In figure 8(a) with the function of magnetic parameter growth rate curves are plotted. ϕ_2 represents the volume fraction for Iron oxide. The values for ϕ_2 are taken from 0.04 to 0.44 with interval of 0.1. The curves in this figure show the stabilizing and destabilizing effect on the growth rate. It is evaluated that growth rate first stabilizing and from $\phi_2 > 0.14$, it goes to destabilizing region rapidly. The curves become linear from different points on magnetic parameter Ha. As the volume fraction increases the linearity point on x -axis also increases. From starting values of ϕ_2 the curves totally lies in the stable region and by increasing the values of volume fraction they goes to unstable region. The figure 8(b) shows the growth rate with respect to wave number for variation of volume fraction. The curves starting from unstable region first destabilizing and after the maximum points they become stabilized. From the figure it is analyzed that as the values of volume fraction rises the growth rate stabilizing slowly and for $\phi_2 > 0.14$, destabilizing effect occurs rapidly. The unique maximum point exists for each curve. In figure 8(c), for variation of volume fraction, the growth rate curves as a function of prandtl number is displayed. Here also it is investigated that with increasing of ϕ_2 the growth rate goes stabilizing to unstable region rapidly. The distance between the growth rate curves increases with increasing the volume fraction. Figure 8(d) indicates the graph for growth rate ω vs. R for variation of volume fraction ϕ_2 . The curves are banding types which bands from vertical direction to horizontal direction with increasing of Reynolds number and then linearized in x -axis. The growth rate rapidly destabilizes with increasing of ϕ_2 . It is concluded that volume fraction of Iron oxide has same destabilizing effect as the volume fraction of ferro oxide by increasing. Thus volume fraction plays an important role on the destabilizing of growth rate.

FIGURE 8: Graphs for growth rate ω vs. (a). Ha (b). k (c). Pr and (d). R for variation of ϕ_2

Impact of wave number on neutral stability curves

The curves for stability analysis of magnetic parameter (Ha) with respect to Prandtl number (Pr) for variation of wave number (k) plotted in figure 9(a). Wave number varies from 0.5 to 0.9 with 0.1 interval. The values for other parameters are used in current problem are $Ha=2.1$, $R=5000$, $\phi_1=0.1$ and $\phi_2=0.04$. The above area of these curves represents the unstable and below area represents the stable area. It is observed from the figure that with increasing of wave number the stability of the flow also increases rapidly. In figure 9(b), the neutral stability curves of R - Ha for variation of different wave number are plotted. The curves upper part shows the unstable region and lower part shows the stable region. From this graph the curves represent that stabilizing region decreases by increasing the wave number i.e., the flow is going to be destabilized. For higher Reynolds number the stability curves become linear horizontally on fixed value of magnetic parameter. Each curve has one global maximum corresponding to different values of wave number. Figure 9(c) represents the instability curves of R vs. Pr for $k=0.5-0.9$ with 0.1 interval. Stability occurs below from these curves and upper area represents unstable region. Stabilizing area of the flow increases rapidly for greater values of wave number. It is also detected that by increasing the Prandtl number, the Reynolds number approaches to zero. It is determined that stabilizing effect occurs for higher values of wave number.

FIGURE 9: Curves for neutral stability (a). Ha - Pr (b). R - Ha and (c). R - Pr for variation of k

Impact of magnetic parameter on neutral stability curves

The effect of magnetic parameter on the stability of hybrid flow is presented in below figures. In figure 10(a), stability curves of Pr with respect to k for increasing the magnetic parameter values from $Ha=0.1-4.1$ with unit interval are plotted. The curves show the increasing of wave number with the increasing of Prandtl number. The above area from these curves indicates the stable and below area indicates the unstable region. It is examined that the flow stabilizes rapidly by increasing the magnetic parameter. The instability curves of Reynolds number vs. wave number for increasing the magnetic parameter are shown in figure 10(b). The upper part of these banding curves represents the stabilizing region and lower part shows the destabilizing region. The curves starting from origin moves up and after some interval of wave number they bands to x -direction and becomes linearized for higher Reynolds number. Instability of the flow decreases by increasing magnetic parameter. Figure 10(c) indicates the neutral curves for stability of flow of R vs. Pr for variation of different Ha . These curves show the stability from below part and instability from above part. As the magnetic parameter increases the stability of the flow also increases rapidly. It is concluded that magnetic parameter has stabilizing influence on the flow.

FIGURE 10: Curves for neutral stability (a). Pr - k (b). R - k and (c). R - Pr for variation of Ha

Impact of Reynolds number on neutral stability curves

The neutral stability curves Ha vs. k for variation of Reynolds number are presented in figure 11(a). Reynolds number is taken from 5000-25000 with interval of 5000. The upper part designates the stabilizing region and lower part designates destabilizing region. The stability of the flow decreases steadily by increasing the Reynolds number. For higher Reynolds number the distance between the curves approaches to zero. For increasing the value of Ha the wave number decreases. In figure 11(b), the graph for neutral stability Ha vs. Pr for increasing the values of Reynolds number from 5000 to 45000 with interval of 10000 is shown. The below area from these stability curves represents the stabilizing region and above area from the curves shows unstable

region. It is perceived that unstable region steadily increases with increasing of Reynolds number and stability curves become very closer to each other for higher Reynolds number. In this case the curves show that by increasing the magnetic parameter, Prandtl number also increased. Figure 11(c) indicates the stability curves of Prandtl number with respect to wave number for different values of R . The upper part shows the stable and lower part indicates the unstable region. From the figure it is analyzed that by increasing the Reynolds number the influence on the flow is destabilizing very slowly. From the discussion it is determined that Reynolds number has an important impact on the destabilizing of the flow.

FIGURE 11: Curves for neutral stability (a). $Ha-k$ (b). $Ha-Pr$ and (c). $Pr-k$ for variation of R

Impact of Prandtl number on neutral stability curves

The impact of Prandtl number on the stability of the flow is analyzed through graphs. The graph for instability curves of $Ha-k$ for variation of different values of Pr is presented in figure 12(a). Prandtl number varies from $Pr=4.4$ to 12.4 with interval of 2. Stable region occurs above from the curves and unstable area occurs below from these curves. This figure tells that there is destabilizing impact on the stability of the flow by increasing the Prandtl number i.e, the flow steadily destabilized with increasing of Pr . The distance between the curves become smaller for higher Prandtl number. In figure 12(b), the curves for stability of the flow of R vs. Ha for different Prandtl number from $Pr=6.4-14.4$ are plotted. Above part of these curves is stabilizing and below part is unstable region. It is investigated from the figure that stabilizing area decrease with increasing the Prandtl number. The distance between the curves decreases with small rate of change. There exists one global maximum point for each curve. Figure 12(c) indicates the neutral stability curves of R vs. k for different values of Pr . Stability occurs above and instability occurs below from these curves. The pattern for all the curves is almost same. It is seen from this graph that similar to above cases the flow is going to be destabilizing with increasing the Prandtl number. It is concluded that like the Reynolds number, Prandtl number also has destabilizing impact on the stability of the hybrid flow.

FIGURE 12: Curves for neutral stability (a). Ha - k (b). R - Ha and (c). R - k for variation of Pr

Impact of ϕ_1 on neutral stability curves

The influence of volume fraction on the stability of the flow is deliberated through graphs. ϕ_1 represents the volume fraction for ferro oxide. The curves of $Ha-k$ for increasing ϕ_1 are presented in figure 13(a). In this figure the upper area indicates stable and lower area indicates the unstable regions. The value of ϕ_1 varies from 0.1 to 0.7 with 0.1 intervals. It is seen from the figure that flow of the hybrid fluid first stabilizes steadily and for $\phi_1 > 0.4$, flow goes to be rapidly destabilized. In figure 13(b), the curves for stability of the flow of $Ha-Pr$ for variation of volume fraction are plotted. In these curves lower part represents the stable and upper part represents the unstable region. $\phi_1 = 0.4$ is the neutral point from which the flow changes from stabilizes to destabilizing. It is investigated that flow of the hybrid fluid first stabilizing and then destabilizes at same rate by increasing the volume fraction. Figure 13(c) displayed the stability graph of Pr with respect to k for variation of different volume fraction values. Stability occurs above from these curves and below part shows the destabilizing region. Here also it is perceived that till the value of $\phi_1 = 0.4$, the flow steadily stabilizes and then starts to destabilize very fast for higher values of volume fraction. The instability curves of R vs. Ha for increasing the volume fraction presented in figure 13(d). The value of ϕ_1 varies from 0.01 to 0.13 with interval of 0.03. It is analyzed that flow is stabilized by increasing the volume fraction of ferro oxide. Figure 13(e) shows the R vs. k stability curves for variation of ϕ_1 . From $\phi_1 = 0.1$ to 0.4, there is stabilizing effect on the flow of hybrid fluid and from $\phi_1 > 0.4$, the fluid flow again destabilizing rapidly. In figure 13(f), R vs. Pr instability curves are plotted for variation of volume fraction of ferro oxide ϕ_1 . In this graph the banding curves represents the neutral points from which the flow stabilizes or destabilizes. It is realized that as the prandtl number goes to their greater values, Reynolds number on x -axis approaches to zero. The below part of these curves indicates the stabilizing region and upper area shows the destabilizing region. From the figure it is investigated that the fluid flow first stabilizes and for $\phi_1 > 0.4$, flow starts destabilizes again more rapidly. From all the graph discussion for variation of volume fraction, it is concluded that volume fraction has both stabilizing and destabilizing consequences on the fluid flow.

FIGURE 13: Curves for neutral stability (a). $Ha-k$ (b). $Ha-Pr$ (c). $Pr-k$ (d) $R-Ha$ (e) $R-k$ and (f) $R-Pr$ for variation of ϕ_1

Impact of ϕ_2 on neutral stability curves

The neutral instability curves of magnetic parameter as a function of wave number for increasing the volume fraction of Iron oxide ϕ_2 are displayed in figure 14(a). The volume fraction values varies from $\phi_2=0.04-0.44$ with 0.1 intervals. The smooth curves in this figure show the neutral points for stability. The upper area shows the stability and below part shows the unstable region. It is observed that the flow of recent hybrid fluid starts to stabilize and then destabilizes rapidly after some interval. $\phi_2=0.14$ is the neutral value of volume fraction from which the flow starts destabilized. Figure 14(b) represents the curves of $Ha-Pr$ for variation of ϕ_2 . These curves show the increase in Prandtl number by increasing the magnetic parameter. Stability of the flow occurs below from these curves and upper part indicates the instability area. The flow first stabilize and from $\phi_2>0.14$, it starts to destabilizing again more rapidly. All these smooth curves move in the same direction. In figure 14(c), the graph of Pr vs. k for increasing the volume fraction values are presented. In this case also it is analyzed that the fluid flow goes stabilizes to destabilizing region with the increase of ϕ_2 . The curves for neutral stability of $R-Ha$ for varying the volume fraction values are shown in figure 14(d). The above part indicates the stable region and below part of these curves represents the unstable region. The curves start from different points on x -axis i.e, for different values of R . It is investigated that by increasing the values of ϕ_2 the flow first stabilizing and then destabilizes very fast. Figure 14(e) indicates the R vs. k stability curves for variation of ϕ_2 from 0.04 to 0.44 with interval of 0.1 . These curves show that as the volume fraction of Iron oxide increases, the flow of the hybrid fluid stabilizes slowly and for $\phi_2>0.14$ the flow starting destabilizes with high speed. All the curves starting from the origin moves vertically and after some interval of wave number they become linearized in x -direction. The curves for linear instability of R vs. Pr for variation of volume fraction ϕ_2 are displayed in figure 14(f). In this case it is also analyzed that there is more destabilizing effect for $\phi_2>0.14$ on the fluid flow. From all the discussions of variation of volume fraction it is concluded that the influence of volume fraction of Iron oxide ϕ_2 on the stability of the flow is same as ϕ_1 . Both volume fractions have first stabilizing and then rapidly destabilizing impact on the stability of the flow.

FIGURE 14: Curves for neutral stability (a). $Ha-k$ (b). $Ha-Pr$ (c). $Pr-k$ (d) $R-Ha$ (e) $R-k$ and (f) $R-Pr$ for variation of ϕ_2

CONCLUSIONS

We have analyzed the instability of Hydromagnetic Poiseuille flow of thermocapillary layers of liquid of shear thinning hybrid nanofluids of $Fe_2O_3-Fe_3O_4$ in water based fluid. The transverse magnetic field imposed on the flow. The solution of obtained ODEs for recent problem is found using a numerical method called “cheybeshev collocation method”. The eigenvalue problem from generalized Orr-sommerfeld equations is solved using the algorithm of QZ. The instability of water based hybrid nanofluids for variation of different parameters are analyzed and discussed through graphs. All the above discussions are concluded as, a suitable method is used for comparison of instabilities of hybrid nanofluid based water flow. It is investigated that by increasing the values of different parameters, wave number and magnetic parameter (Ha) has the stabilizing effect on the fluid flow. Reynolds number and prandtl number affected destabilizes to the hybrid nanofluid flow. The volume fraction has both the stabilizing and destabilizing impact on the fluid flow.

Nomenclature:

V	Velocity vectors of flow
C_p	Heat capacity
$(C_p)_{hnf}$	Heat capacity of Hybrid nanofluid at fixed pressure
J	Current density
B	Total magnetic field
B_0	Forced magnetic field
K_{hnf}	Thermal conductivity of Hybrid nanofluid
P	Pressure
U	Characteristic velocity
L	Depth of the layer
b	Temperature gradient on the surface
Pr	Prandtl number
Ha	Hartmann number

R	Reynolds number
Bo	Bond number
Ma	Marangoni number
α	Thermal expansion coefficient
$\tilde{T}$	Temperature of fluid
$\tilde{T}_0$	Reference temperature
g	Gravitational acceleration
t	Time
k	Wave number

Greek symbols:

τ	Stress tensor
$\tilde{\gamma}$	Strain-rate tensor
$\dot{\gamma}$	Strain-rate tensor magnitude
$\hat{\mu}$	Dynamic viscosity
$\hat{\mu}_0$	Characteristic viscosity (at zero shear rate)
$\hat{\lambda}$	Material time constant
n	Degree of shear thinning
$\hat{\mu}_\infty$	Viscosity at infinite shear rate
μ_{hnf}	Hybrid nanofluid dynamic viscosity
ρ_{hnf}	Hybrid nanofluid density
θ	Dimensionless temperature of the fluid
φ	Electric potential
ϕ_1	Volume fraction of ferro oxide
ϕ_2	Volume fraction of Iron oxide
σ	Electric conductivity

σ_{hmf}	Electric conductivity of Hybrid nanofluid
ν	Kinematic viscosity
α	Thermal diffusivity
ω	Complex phase velocity

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Figures

Figure 1

Geometrical configuration of the problem

Figure 2

Curves for velocity distribution for third problem (Poiseuille Flow).

Figure 3

Graphs for growth rate ω vs. (a). Ha (b). Pr and (c). R for variation of wave number k

Figure 4

Graphs for growth rate ω vs. (a). k (b). Pr and (c). R for variation of magnetic parameter Ha

Figure 5

Graphs for growth rate ω vs. (a). Ha (b). k and (c). Pr for variation of Reynolds number R

Figure 6

Graphs for growth rate ω vs. (a). Ha (b). k and (c). R for variation of Prandtl number Pr

Figure 7

Graphs for growth rate ω vs. (a). Ha (b). k (c). Pr and (d). R for variation of ϕ_1

Figure 8

Graphs for growth rate ω vs. (a). Ha (b). k (c). Pr and (d). R for variation of ϕ_2

Figure 9

Curves for neutral stability (a). Ha - Pr (b). R - Ha and (c). R - Pr for variation of k

Figure 10

Curves for neutral stability (a). Pr - k (b). R - k and (c). R - Pr for variation of Ha

Figure 11

Curves for neutral stability (a). Ha - k (b). Ha - Pr and (c). Pr - k for variation of R

Figure 12

Curves for neutral stability (a). Ha - k (b). R - Ha and (c). R - k for variation of Pr

Figure 13

Curves for neutral stability (a). $Ha-k$ (b). $Ha-Pr$ (c). $Pr-k$ (d) $R-Ha$ (e) $R-k$ and (f) RPr for variation of ϕ_1

Figure 14

Curves for neutral stability (a). Ha - k (b). Ha - Pr (c). Pr - k (d) R - Ha (e) R - k and (f) RPr for variation of ϕ_2