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An Image-based Gradation Calculation Method Considering Crushed Stone Morphology

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An image-based gradation calculation method considering crushed stone morphology is proposed in this paper. Eight kinds of crushed stones are prepared to figure out the effect of the particle shape on the gradation. All particles are pictured and the photos are processed to obtain particle shape indices by using the image-based method. A minimum Feret diameter analysis method is used, which can reflect the influence of the particle shape. Then the paper proposes a developed overall regularity to describe the morphology of particles, which is calculated by the weight of three shape indices: aspect ratio, convexity and sphericity. The developed overall regularity is applied on gradation curve equations to provide an image-based gradation calculation method. And the provided method is validated to well fit the actual case. Furthermore, the method is also applied on two examples of gradation designation and the result proves to be feasible and practicable.

Crushed stone is small pieces of the broken rock due to natural reasons or man-made destruction, which has irregular size, shape and texture. Materials with different particle size distribution are often used to form coarse aggregate¹⁻³, backfill material⁴⁻⁷ and rockfill⁸⁻¹⁰ in high fill, subgrade and mining engineering. There are various parameters describing soil materials, among which the particle size and gradation are commonly used. Gradation is known to be defined as the distribution of aggregate particles of different sizes, which can be determined by sieve analysis test. However, the pore of the sieves are generally composed by a lattice square according to the method of British Standards Institute¹¹, which sometimes makes particles of quite different characteristics have the same size¹². This brings about difficulties to design the gradation for different kinds of materials in engineering.

Particle geometry is a result of the genesis of the parent rock, stages of transportation and depositional history, and chemical and biological effects¹³. For particulate materials including crushed stones, the characteristics of the particle shape govern their complex mechanical behavior at multi-scales, about which the research interest has been growing in recent years^{14,15}. Since Wadell¹⁶ first proposed the shape parameter: sphericity, the parameter evaluating the particle shape has been developed and the number of the parameter has been added. Traditional methods are to make use of charts such as the charts of Krumbein and Sloss¹⁷ and Cho et al.¹⁸. The roundness and sphericity of a given particle are assessed by visual comparison with the reference shapes in these charts. Lee et al.¹⁵ not only redefined sphericity but also selected

41 elongation, slenderness, and convexity to describe the irregularity of sand grains.
42 Berrezueta et al.¹⁹ evaluated the representativity of four independent 2D particle
43 parameters which included fractal dimension, Wadell's roundness, a perimeter-area
44 normalized ratio and sphericity. Liu et al.²⁰ proposed an empirical small-strain shear
45 modulus model including a Fourier-based particle shape factor modified by gradation
46 through a series of shear wave velocity tests.

47 Recently, with the application of the computer image processing technology, the
48 accuracy of the particle shape measurements has been improved. An image-based
49 method is known to be a relatively convenient and accurate method, which can obtain
50 particle shape data by using image processing software²⁰⁻³². Basically, particles are
51 usually pictured finely or roughly and the photos are processed in specific software
52 such as ImageJ^{20,29,31} and OnDemand3D³² to obtain the particle shape indices. Some
53 researchers used professional instruments to measure the particle shape of sand and
54 reviews the particle shape effect on mechanical behavior such as QICPIC^{21,22,27},
55 Sedimaging²⁴ and CX2³⁰ systems. Moreover, some computational algorithm was
56 proposed to be integrated into existing optical soil characterization systems^{25,26}. The
57 software or apparatus used by these researchers are mainly focus on soil or sand
58 materials. Different from these small particles, the shape measurement of crushed stone
59 is easier and the requirement of apparatus is lower.

60 The gradation of materials is usually described by non-uniformity coefficient (C_u)
61 and curvature coefficient (C_c), which is determined by effective particle size (D_{10}),
62 mean particle size (D_{50}) and controlling particle size (D_{60}). However, the method
63 cannot fully express the characteristics of the gradation. A gradation curve equation
64 method is proposed by researchers to overcome the shortcomings. Since Fuller and
65 Thompson³³ and Talbot and Richart³⁴ proposed the equations of maximum density
66 gradation curves and fractal gradation curves, respectively, the studies on gradation
67 curve equations have been developed. Zhu et al.³⁵ and Guo et al.³⁶ proposed a gradation
68 equation mainly focused on soil and coarse-grained soil. Then Wu et al.³⁷ improved the
69 method to reduce the required parameters. Zhu et al.³⁸ provided a gradation design
70 method for rockfill materials based on fractal theory. Sun et al.³⁹ studied the particle
71 generation procedure in DEM and proposed a gradation algorithm based on the particle
72 number. The gradation curve equations are used to design the required gradation and
73 has been gradually applied on the engineering.

74 To figure out the effect of particle shape when designing the gradation of aggregate,
75 in this paper, eight kinds of crushed stones are prepared. The stones are divided into
76 two groups which are small particle group and large particle group. The gradations for
77 each group are processed to be the same by sieving. Meanwhile, the image processing
78 software are used to obtain shape indices, including the minimum Feret diameter, aspect
79 ratio, convexity and sphericity. The minimum Feret diameter of particles which can
80 reflect the particle shape effect are measured to form gradations. To quantitatively
81 analyze this particle shape effect, a developed overall regularity is proposed, which is
82 calculated by the weight of the aspect ratio, convexity and sphericity. The developed

overall regularity is applied on the gradation curve equations to provide an image-based gradation calculation method. The new proposed gradation calculation method is validated to be well fitted and the method is applied to two examples of gradation designation. The objective and methodology are shown in Table 1.

Opinions	Methodology
The sieve analysis method sometimes makes particles of quite different characteristics have the same size. A developed measure method should be proposed to reflect the particle shape effect.	Eight kinds of crushed stones with different particle shape are sieved to be the same gradation. Additionally, the minimum Feret diameter (D^{Fmin}) of particles are measured to form gradations. The gradations formed by sieve analysis and D^{Fmin} analysis are compared to reflect the particle shape effect on gradation curves.
The particle shape of crushed stones can be quantified to be an index, which can evaluate whether a particle is rounded or angular.	The image processing software are used to obtain shape indices, including the aspect ratio, convexity and sphericity. Take the weight of these indices, a developed overall regularity (OR) is proposed to quantitatively analyze the particle shape effect.
The current gradation curve equation can be developed to consider the particle shape effect.	A developed continuous gradation equation is proposed. The key parameters in the equation are related to the developed overall regularity (OR). Moreover, to reflect the particle shape diameter, the traditional diameter of the sieved particle (D) is replaced by the minimum Feret diameter (D^{Fmin}).

Table 1. The opinions and the methodology.

Significance of the proposed method

In this paper, a gradation calculation method considering the shape of crushed stone based on image analysis is proposed.

The proposed method is relatively accurate comparing with the traditional gradation calculation method. Generally, particles are sieved to obtain sizes to form the gradation. However, the sieve hole of the sizing screen is round or square, which may mislead the designation in the engineering because particles with different particle shape can have the same gradation by sieving. To overcome the weakness of the current method, based on the image-based method, a more accurate gradation calculation method is proposed.

The proposed method is also practical and effective in the engineering. Comparing with sand, clay and other small size particles, the crushed stone is relatively large in size. It's enough to picture the crushed stone using normal camera instead of using high-precision equipment. By using image processing software, the particle shape indices are easy to be obtained. These make the proposed method applicable on the engineering.

Results

Particle material and gradation. Eight kinds of materials with different particle shapes are involved in the experiment as Fig. 1. Table 2 shows the summary of the geological classification and characteristics, particle shape classification of the materials.

Fig. 1. Eight kinds of materials used in the experiment.

Stone	Geological classification and characteristics	Particle shape
White pebbles	Nearly pure silica stone with smoother surface	Sub-rounded
Black stone	Carbonate stone	Angular
Basalt	Porous stone about 40% void ratio, 45% silica and 30% Fe_2O_3 , CaO	Angular
Perlite	Fragile vitreous stone composed of 70% silica and 12% alumina	Sub-rounded
Medical stone	Aluminosilicate stone with dense composition	Sub-angular
Guangxi stone	Carbonate stone with quartz grain	Angular
Sepenggiante	Hard Marble stone with the grain of tree	Sub-rounded
Limestone	Carbonate stone with roughness surface, CaCO_3 over 90%	Sub-angular

Table 2. Summary of the geological classification and characteristics and particle shape classification.

According to their sizes, the eight kinds of crushed stones are divided into two categories: one includes white pebbles, black stone, basalt, perlite, medical stone with the larger size; the other includes Guangxi stone, sepenggiante, limestone with the smaller size. The particle size distribution curves of the eight crushed stone are controlled by sieving to be exactly the same, giving the mean particle diameter $D_{50}^{sieve} = 15.5 \text{ mm}$ and uniformity coefficient $C_u^{sieve} = 1.34$ for smaller particles as the black line in Fig. 2a; $D_{50}^{sieve} = 20.5 \text{ mm}$ and uniformity coefficient $C_u^{sieve} = 1.15$ for larger particles as the black line in Fig. 2b.

Fig. 2. Particle size cumulative distribution for sieve analysis and D^{Fmin} analysis.

Besides the conventional sieving results, an imaging-based method is also used to obtain particle size and gradation. Over 2000 particles are pictured to obtain the particle indices, and the number of each kind of stones is the same for a qualitative comparison. Several indices measured by this method can describe the size of the particle, along which the minimum Feret diameter (D^{Fmin}) is used in this paper, and D^{Fmin} means the smallest distance between two tangents on opposite sides of the particle, as shown in Fig. 3a.

Particle shape measurements. Three shape indices are used in this paper and have been widely used in recent years^{21,22,27,28}, namely the aspect ratio (AR), the convexity (C) and the sphericity (S), as shown in Fig. 3: AR is defined as the ratio between D^{Fmin} and D^{Fmax} ; C is defined by the ratio between the area of the particle (A) and the area of the convex hull of the particle ($A + B$); S is defined as the ratio between the perimeter of a circle with the same area as the projected area of the particle and its real perimeter.

Fig. 3. Definition of shape and size indices.

Fig. 4 presents the cumulative distribution of shape measurements of eight crushed stones. Different from the results of the glass beads by Yang and Luo²⁷, the range of the

convexity (C) is normally from 0.9 to 1 for crushed stone, but the range of the aspect ratio (AR) and the sphericity (S) are similar. For a given material, it is reasonable to choose the shape value corresponding to the 50% cumulative distribution as a representative value²⁷. In this method, shape data of the eight crushed stones is provided in Table 3.

Fig.4. Cumulative distribution for (a) aspect ratio, (b) convexity and (c) sphericity.

Material	Aspect ratio (AR_{50})	Convexity (C_{50})	Sphericity (S_{50})	OR of Yang and Luo	OR of present study
White pebbles	0.708	0.977	0.842	0.842	0.648
Black stone	0.683	0.971	0.792	0.815	0.562
Basalt	0.686	0.949	0.681	0.772	0.367
Perlite	0.701	0.988	0.869	0.853	0.720
Medical stone	0.700	0.958	0.813	0.824	0.523
Guangxi stone	0.659	0.967	0.740	0.789	0.486
Sepenggiante	0.681	0.968	0.788	0.812	0.543
Limestone	0.696	0.976	0.781	0.818	0.588

Table 3. The 50% cumulative distribution data of the eight materials.

As shown in Fig. 5, there is a relatively fixed range for each shape indices of aspect ratio, convexity and sphericity, but not distributes in the range of 0~1. This means to reflect the real weight of these shape indices, the range needs to be processed. Yang and Luo²⁷ provided a calculation method to describe the morphology of particles, referred to as overall regularity (OR), and can be calculated as the average of the value of three shape indices $(AR_{50} + S_{50} + C_{50})/3$. This means the weights of the three particle shape indices are equal. However, as for crushed stone in this paper, the value of OR will reflect more characteristics of S and C in this method, because there are narrower range and larger value for S and C than AR . To better reflect the weight of each index, the calculation of the overall regularity (OR) is modified as the following equation:

$$OR = \left(\frac{AR_{50} - AR_{min}}{AR_{max} - AR_{min}} + \frac{C_{50} - C_{min}}{C_{max} - C_{min}} + \frac{S_{50} - S_{min}}{S_{max} - S_{min}} \right) / 3 \quad (1)$$

where the subscript “ max ” and “ min ” means the upper and lower bound of the 95% confidence interval, respectively, as shown in Fig. 5. The figure shows the counting number distribution for the three shape indices. It is obvious that the contribution of each particle shape index is concentrated in a certain range. Eq. (1) makes the weights of the three indices the same through the normalization method. Thus, OR with Eq. (1) can describe the particle shape in a collective manner.

Fig. 5. Counting number distribution for (a) aspect ratio, (b) convexity and (c) sphericity.

Gradation equations. Traditional calculation of particle gradation needs to draw an initial gradation curve first, and interpolate to get the value of effective particle size (D_{10}) and controlling particle size (D_{60}). Then calculate uniformity coefficient (C_u) and curvature coefficient (C_c) to check whether the result meets the engineering requirements, and adjusting the initial curve continuously until it finally meets. Comparing the complicated process shown above, the calculation process can be simplified by using the gradation equation method.

The continuous gradation equation proposed by Zhu et al.³⁵ and Guo et al.⁴¹ is used in this paper:

$$P = \frac{1}{(1-b)(D_{max}/D)^{a+b}} \times 100\% \quad (2)$$

where D is any particle size within the gradation range; P is the percentage of particles smaller than D ; D_{max} is the maximum particle size; a and b are dimensionless parameters need to be determined. For minimum Feret diameter (D^{Fmin}) analysis, Eq. (2) can be expressed as following:

$$P = \frac{1}{(1-b^{Fmin})(D_{max}^{Fmin}/D^{Fmin})^{a^{Fmin}+b^{Fmin}}} \times 100\% \quad (3)$$

where the superscript “ $Fmin$ ” means the parameter is obtained by minimum Feret diameter analysis, and the superscript “ $sieve$ ” used in the following part means the parameter is obtained by sieve analysis.

Zhu et al.³⁵ indicated that the gradation curve shape was like ‘S’ for a higher value of b . Because the shapes of the gradation curves in Fig. 2 are all like ‘S’, $b = b^{Fmin} = 0.95$ is considered to be reasonable. Fitting D^{Fmin} curves with Eq. (3), the values of a^{Fmin} for each kind of particles are as shown in Table 4. And fitting sieve curves with Eq. (2), the values of a^{sieve} for sieve analysis of small and large particles are calculated to be 10.37 and 15.28, respectively. The ratios between a^{Fmin} and a^{sieve} are also shown in Table 4. Fig. 6 shows the relationship between overall regularity (OR) and parameter a^{Fmin}/a^{sieve} . A linear equation is provided as following:

$$a^{Fmin}/a^{sieve} = 0.4OR + 0.5 \quad (4)$$

Gradation curves	a^{Fmin}	a^{Fmin}/a^{sieve}	D_{max}^{Fmin} (mm)	$D_{max}^{Fmin}/D_{max}^{sieve}$
White pebbles	7.735	0.746	22.282	1.114
Black stone	7.589	0.732	24.015	1.201
Basalt	6.912	0.667	26.169	1.308
Perlite	8.438	0.814	21.330	1.066
Medical stone	7.268	0.701	24.071	1.203
Guangxi stone	10.52	0.688	29.970	1.199
Sepenggiante	10.94	0.716	29.541	1.182
Limestone	11.24	0.736	29.071	1.163

Table 4. The values of a^{Fmin} and D_{max}^{Fmin} for each kind of gradation curves.

It is obvious that a certain range of OR can be found for particles with different particle shapes in Fig. 6. In the engineering, particle shape indices are needed to calculate OR , sometimes making the work relatively complex and inconvenient. Thus, the paper provides a approach to obtain the value of OR as shown in Fig. 6: nearly spherical particles are classified as ‘rounded’ and $OR = 0.8\sim 0.9$; elliptical or well-shaped particles are classified as ‘sub-rounded’ and $OR = 0.6\sim 0.8$; particles with some edges and corners are classified as ‘sub-angular’ and $OR = 0.4\sim 0.6$; crushed and irregular particles are classified as ‘angular’ and $OR = 0.3\sim 0.4$.

Fig. 6. Curve fitting for overall regularity (OR) versus a^{Fmin}/a^{sieve} .

The value of D_{max}^{Fmin} is also related to OR , as shown in Fig. 7. The figure shows that more angular particles have larger D_{max}^{Fmin} comparing to rounded particles. It is noted that the D_{max}^{Fmin} size of most rounded particles is exactly the same as the sieve pore size, referred to $D_{max}^{Fmin} = D_{max}^{sieve}$. Then an asymptotic relationship is more suitable between $D_{max}^{Fmin}/D_{max}^{sieve}$ and OR . The equation can be expressed as following:

$$D_{max}^{Fmin}/D_{max}^{sieve} = e^{-3OR} + 1 \quad (5)$$

Fig. 7. Curve fitting for overall regularity (OR) versus $D_{max}^{Fmin} / D_{max}^{sieve}$.

Then combining Eq. (3) with Eq. (4) and Eq. (5), the following equation can be provided to calculate the gradation considering the effect of particle morphology:

$$P = \frac{1}{(1-b^{Fmin})[(e^{-3OR}+1)D_{max}^{sieve}/D^{Fmin}]^{(0.4OR+0.5)a^{sieve}} + b^{Fmin}} \times 100\% \quad (6)$$

The validation of the method. The image-based apparatus, known as QICPIC, have been widely used in recently years to characterize soil particle shape^{14,21,22,27}. The detailed description of how the apparatus works can be found in Sympatec⁴². Three kinds of particles with different shape are used to figure out the influence of the particle shape on gradation by Yang and Luo²⁷: glass beads (well-rounded), Fujian sand (sub-angular) and crushed glass beads (angular). As shown in Fig. 8, the particles are sieved as the same gradation with $C_u = 1.20$ and $D_{50} = 512.5 \mu m$, together with gradations by D^{Fmin} analysis with apparatus QICPIC. Using the method of Eq. (2), for $b = 0.95$, the gradation curve of sieve analysis is fitted and the result proves to be $a^{sieve} = 9.450$. The ranges of the aspect ratio (AR), the convexity (C) and the sphericity (S) are $0.6 \sim 0.1$, $0.9 \sim 0.1$ and $0.6 \sim 0.1$, respectively. The 50% cumulative distribution of the shape indices provided by Yang and Luo²⁷ and the values of the calculated OR with Eq. (1) are shown in Table 5. The maximum sieve diameter (D_{max}^{sieve}) is $700 \mu m$, then the D^{Fmin} gradation curves can be drawn with Eq. (6) in Fig. 8, which are well fitted with the results provided by the test of Yang and Luo²⁷.

Material kinds	Aspect ratio (AR_{50})	Convexity (C_{50})	Sphericity (S_{50})	overall regularity (OR)
Glass beads	0.974	0.974	0.944	0.845
Fujian sand	0.745	0.956	0.891	0.550
Crushed glass beads	0.653	0.929	0.822	0.324

Table 5. Shape indices and calculated OR .

It is noted that Eqs. (2) and (3) focuses on the continuous gradation with ‘S’ shape when b approaches 1. For a smaller value of b , the gradation curve shape proves to be hyperbolic, and Eqs. (2) and (3) are still applicable. However, further researches is needed when the curve shape is not continuous, and some researchers have been studied the related contents^{40,43}.

Fig. 8. Validation for three different particles.

Application of the method. Usually, there are two cases to calculate the particle gradations according to the particle parameters: ① Several particle parameters (e.g. D_{50} , C_u , C_c) have been determined and a specific gradation needs to be designed; ② A particle parameter is fixed and other parameters are changed in a certain interval, and a series of gradations need to be designed. The non-uniformity coefficient (C_u) and the curvature coefficient (C_c) are defined by the following equation:

$$C_u = D_{60}/D_{10} \quad (7)$$

$$C_c = D_{30}^2 / (D_{10}D_{60}) \quad (8)$$

where D_{10} , D_{30} and D_{60} are the particle sizes of 10%, 30% and 60% by weight, respectively. The calculation method by using the new image-based gradation calculation method is shown in detail in the following sub-sections.

An example of three different kinds of particle shapes of sands is used to design gradation curves by Ren et al.⁴⁴. Calcareous sand, standard sand and glass beads are used and can be classified as ‘angular’, ‘sub-angular’ and ‘rounded’. The parameters $C_u = 8$ and 10 and $C_c = 1.5$ are needed and $D_{max}^{sieve} = 8$ mm. Based on Eq. (2), for $C_u = 8$ and $C_c = 1.5$, the following equations can be provided as:

$$\begin{cases} C_u = \frac{D_{60}}{D_{10}} = \left[\frac{6(1-0.1b)}{1-0.6b} \right]^{1/a^{sieve}} = 8 \\ C_c = \frac{D_{30}^2}{D_{10}D_{60}} = \left[\frac{3(1-0.1b)(1-0.6b)}{2(1-0.3b)^2} \right]^{1/a^{sieve}} = 1.5 \end{cases} \quad (9)$$

The result of Eq. (9) is $a^{sieve} = 0.241$ and $b = 0.925$. Then using the same method, $a^{sieve} = 0.237$ and $b = 0.870$ for the case of $C_u = 10$ and $C_c = 1.5$.

Taking the median value of the gradation range of ‘angular’, ‘sub-angular’ and ‘rounded’, OR is assumed to be 0.35, 0.5 and 0.85, respectively. By the method of Eq. (6), the calculation results are shown in Fig. 9. Then using methods to measure D^{Fmin} for these particles to form the drawn D^{Fmin} gradation curve. At that time, these particles are composed of the required gradation considering the effect of particle shape. This process is the same as Fig. 12.

Fig. 9. Calculation of a case of the specific gradation.

An example of coarse-grained soil to study the permeability by Peng⁴⁵. The $D_{max}^{sieve} = 20$ mm for the aggregate of the coarse-grained. The percentage of particles smaller than 10 mm needs to be 55%~75%. Then the non-uniformity coefficient $C_u = 2.2$ and a series of changing C_c is required. Based on Eq. (2), the following equations can be provided as:

$$\begin{cases} 55\% < \frac{1}{(1-b)\left(\frac{D_{max}^{sieve}}{10}\right)^{a^{sieve}} + b} < 75\% \\ C_u = \frac{D_{60}}{D_{10}} = \left[\frac{6(1-0.1b)}{1-0.6b}\right]^{1/a^{sieve}} = 2.2 \end{cases} \quad (10)$$

Fig. 10. Calculation of a case of a series of gradations.

The sieve analysis result of Eq. (10) is plotted on the plane of $b - a^{sieve}$, as shown in Fig. 10a. Because the used particles can be classified as ‘sub-regular’, $OR = 0.5$ is assumed in this paper. Then calculating D_{max}^{Fmin} to be 24.46 mm with Eq. (5) and replacing a^{sieve} with $[a^{Fmin}/(0.4OR + 0.5)]$, the D^{Fmin} analysis result of Eq. (10) is plotted on the plane of $b - a^{Fmin}$, as shown in Fig. 10b. Four points are chosen for each analysis of $b = 0.9, 0.92, 0.94$ and 0.96 as the red circle in the figure, then required C_c is calculated in Eq. (6) to be 1.054, 1.049, 1.044, 1.040 and 0.738, 0.734, 0.731, 0.728 for sieve analysis and D^{Fmin} analysis, respectively.

Discussion

The pictures of the eight kinds of crushed stone particles are processed to obtain the particle shape indices. As shown in Fig. 2, the gradation determined by the minimum Feret diameter analysis proves to be different from which is determined by sieve analysis. The gradation curve provided by D^{Fmin} as the size index has been found to be close to but different from that determined by sieve analysis²¹, and the influence of the particle shape can be reflected with the D^{Fmin} analysis²⁷. Comparing with the sieve gradation, the minimum Feret diameter gradation can reflect the effect of the particle morphology. The difference between the two gradation curves is relatively small for rounded particles. Conversely, it is large for angular particles. Additionally, Table 6 shows more details about the size distribution curves.

Material	D_{10}^{Fmin} (mm) ^a	D_{50}^{Fmin} (mm) ^b	D_{60}^{Fmin} (mm) ^c	C_u^{Fmind}	C_u^{Sieve}
White pebbles	11.5	14.9	15.6	1.35	1.34
Black stone	12.2	16.0	16.8	1.37	1.34
Basalt	12.9	16.8	17.7	1.37	1.34
Perlite	11.4	15.0	15.6	1.38	1.34
Medical stone	12.1	15.8	16.3	1.35	1.34
Guangxi stone	18.1	21.6	22.4	1.24	1.15
Sepenggiant	18.3	21.4	22.1	1.21	1.15
Limestone	18.7	22.1	22.8	1.22	1.15

Table 6. The shape data of the eight materials for the minimum Feret (D^{Fmin}) analysis. ^a D_{10} is the particle size of the 10% size distribution curve. ^b D_{50} is the mean particle size. ^c D_{60} is the particle size of the 60% size distribution curve. ^d C_u is the uniformity coefficient defined by D_{60}/D_{10} .

Fig. 11 shows the overall regularity (OR) calculation results of Eq. (1) and the method of Yang and Luo²⁷. The difference is obvious for the rather different kinds of particles by using the method of the present study, comparing with the method of Yang and Luo²⁷. The calculation results of Yang and Luo²⁷ for the used particles all concentrate at 0.8. By using the present method, the range of OR are from 0.3 to 0.8. For example, $OR = 0.367$ for basalt which is irregular and porous, and $OR = 0.720$ for perlite which is sub-rounded and smooth. This proves that for different kinds of

particles, the proposed developed method can distinguish the morphology of them more obviously.

Fig. 11. The results of the method of Yang and Luo²⁷ and the present study.

Fig. 12 shows the process of obtain gradations considering the effect of the particle shape. In the engineering, some parameters are required to design or calculate the gradation: uniformity coefficient (C_u), curvature coefficient (C_c), effective particle size (D_{10}), mean particle size D_{50} , controlling particle size D_{60} or the maximum particle size D_{max} . These parameters are enough to determine a certain gradation curve or a serials of gradation curves. Then with Eq. (2), a^{sieve} and b^{sieve} can be obtained. The overall regularity (OR) can be assumed or be calculated more precisely with Eq. (1). Because D_{max}^{sieve} is usually provided first, the gradation curve of D^{Fmin} analysis can be drawn in $D^{Fmin} - P$ plane with Eq. (6). The above steps can be accomplished in paper. Then using methods to measure D^{Fmin} for these particles to form the drawn D^{Fmin} gradation curve. At that time, these particles are composed of the required gradation considering the effect of particle shape.

The key part of the proposed method is to obtain D^{Fmin} of the particles. However, different from the sieve analysis, measuring the minimum Feret diameter D^{Fmin} is relatively more difficult. If there are few particles, a calipers can be used to simply measure D^{Fmin} . At present, there is no convenient way to obtain D^{Fmin} for a lot of particles. It's time-consuming to measure D^{Fmin} one by one by using calipers. The image-based method is a relatively practicable approach. Some apparatus are used to measure particle shape parameters in laboratory tests¹⁴. Moreover, after the particles are pictured by a clear enough camera, the image processing software and numerical calculation software can also be used to obtain the shape parameters. Further studies and more apparatus need to be discussed and developed in the future.

Some problems should also be further studied. The image-based method in this paper is focused on 2D, which is a relatively effective method. The 3D methods have been developed in these years⁴⁷, which is a potential method in the future.

Fig. 12. Process of obtain gradations considering the effect of the particle shape.

Methods

Sieve experiment. The eight kinds of crushed stone used in this study have been sieved. The circular sieve aperture diameter is 10 mm to 30 mm with 1 mm interval. To obtain representative and repeatable data for each material, about 5 kg for each kind of crushed stone with hundreds of particles are used to establish cumulative distribution curves. The sieve mesh and crushed stones of different sizes sieved is shown as Fig. 13.

Fig. 13. Sieve experiment. (a) sieve mesh; (b) sieved stone.

Morphological method. The morphology of the crushed stone can be roughly divided into four kinds: ‘rounded’, ‘sub-rounded’, ‘sub-angular’ and ‘angular’. However, to quantify the morphology of the particles, many particle shape indices are introduced such as ‘aspect ratio’, ‘convexity’, ‘sphericity’, ‘elongation’, ‘slenderness’ and so on. This paper adopts ‘aspect ratio’ (AR), ‘convexity’ (C) and ‘sphericity’ (S) to evaluate

the morphology of a particle as Fig. 3, in which the ‘aspect ratio’ determines whether the shape is close to the normal shape, ‘convexity’ measures the compactness of a particle and ‘sphericity’ quantifies the degree of similarity between a particle and a sphere. Then an index OR which is calculated by the weight of the AR , C and S is used to conveniently describe the overall regularity of a particle.

Fig. 14. The image processing procedure to obtain the data of shape indices, which includes (a) taking picture to get real image of the particle, then (b) converting the real image into the binary image, then (c) measuring the shape indices in software, finally (d) obtaining the data of the shape indices.

Image-based method. All particles in the experiment are pictured by camera. Because the particle sizes are around 20 mm, a camera with normal configuration (80 mega pixels) is adequate to capture the particle shape characteristics¹⁹. Then an image processing software Image-Pro Plus 6.0⁵¹ is used to convert the real image to binary image to obtain shape indices. The experiment procedure is shown as Fig. 14.

Gradation curve model. The form of continuous gradation equation is adopted as Eq. (2). To establish the relationship between gradation equation and the particle shape effect, the overall regularity (OR) is introduced to modify the parameter (a^{sieve}) and the particle size (D^{sieve}). The relationship between the parameter a^{sieve}/a^{Fmin} in Eq. (2) and the overall regularity (OR) is found to be linear as shown in Fig. 6. Additionally, an asymptotic relationship between D^{Fmin}/D^{sieve} and the overall regularity (OR) is proposed as Eq. (5). Finally, the continuous gradation equation considering particle shape effect can be obtained as Eq. (6).

Data availability

The data that support the findings of this study are not publicly available due to privacy or ethical restrictions.

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Author contributions

Zexiong Shi and Liang Deng conducted sieve experiments, divided eight kinds of crushed stone into unified gradation, photographed the photos of Fig. 13 and Fig. 14a and sampled all the crushed stone sieved. Xin Ding processed all the crushed stone data through image and data processing software including Figs. 14b, c and d, and wrote part of the paper. By analyzing and processing the data, Li Pang introduced the gradation curve formula model to obtain the gradation curve model considering the influence of particle shape, and wrote the rest of the paper. Chong Jiang summarized the whole paper, gave guiding opinions, and corrected the mistakes in the process of writing the paper.

Competing interests

The authors declare no competing interests.