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Physical Sciences - Article

Keywords:

Posted Date: January 14th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1258188/v1>

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Faulting of rocks as a critical energy process

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ABSTRACT

Faulting of rocks is a dominant earth process that governs small-scale fracturing, formation of tectonic plate boundaries, and earthquakes occurrence¹⁻⁴. Since the 18th century, the mechanical settings for rock faulting were commonly analyzed with the Coulomb criterion⁵ that offers empirical, useful tools for scientific and engineering applications^{1,6-12}. Here we revisit the processes of rock faulting by an alternative approach that incorporates elastic energy, strain-state, and three-dimensional deformation; these mechanical fundamentals are missing in Coulomb criterion. We propose that a stressed rock-body fails as two conditions are met: (1) The elastic energy generated by the loading system equals or exceeds a critical energy intensity that is required for the faulting process; (2) The internal strain of the stressed rock-body due to slip and dilation along the developing faults equals the strain-state created by the loading system to maintain physical continuity^{13,14}. Our simulations reveal that meeting these energy and strain conditions requires an orthorhombic, polymodal fault geometry that is similar to natural and experimental fault systems¹⁵⁻²⁰. The application of our formulation to hundreds of rock-mechanics experiments^{11,21-28} provides a new, comprehensive benchmark for rock-faulting.

INTRODUCTION

Since Charles-Augustin de Coulomb addressed the issue of soil stability in 1773, his failure criterion has become the prevailing analytical tool of soil and rock mechanics^{1,5,6,8}. The rationale of Coulomb criterion is straightforward: a body of soil or rock fails by shear fracturing when the stresses acting on a given fracture reach its strength:

$$|\tau| = C + \mu\sigma_n \quad (1)$$

where τ and σ_n are the acting shear and normal stresses, C is the fracture cohesion, and μ , indicates the fracture strength dependency on the normal stress, which is known as the coefficient of 'internal friction'²⁹. This simple and elegant expression was applied to countless rock failure experiments and adopted for the interpretation of fault systems in the field^{4,6}. But the Coulomb criterion cannot explain the observed orthorhombic fault patterns in experiments and field (Fig. 1)¹⁵⁻²⁰, apparently due to omitting strain and energy considerations that are paramount components of mechanical processes.

In 1913, von Mises proposed that ductile materials yield once the deviatoric stresses reach a critical level^{30,31}. This criterion, which is related to the elastic strain energy³², was successfully applied to 3-D experimental yielding of ductile materials, mostly metals. However, this criterion ignores the effects of the mean-stress that has strong influence on yielding and shear of brittle solids like rocks^{1,9,33}.

Later, Alan Arnold Griffith developed a theory of tensile fracturing by analyzing the stress distribution around flaws in a brittle solid³⁴. He showed that the propagation of a critically stressed flaw is controlled by the balance between the elastic energy release and the energy dissipation by the new surface-area of the growing fracture. With this energy-controlled theory, Griffith established the field of fracture mechanics^{35,36}. A few attempts to adopt this theory to shear faulting of rocks under compression were partially successful^{25,37,38}. Following Griffith's energy-based mechanism, we envision that rock faulting is constrained by critical energy level, and develop here a new approach that incorporates strain-state and elastic energy limit. The applications of this concept to a range of rock-failure experiments provide an energy-based rock failure criterion, and naturally reveal the orthorhombic structure of in-situ observed faults.

HYPOTHESES

We consider an intact, homogeneous, linear elastic, brittle rock, at a quasi-static compressional loading system. The bulk elastic strain energy of the tested rock sample, U , is a product of the applied three-dimensional stresses and the elastic moduli of the rock sample. For simplicity, the analysis refers to the principal stress coordinate system, in which σ_i , and ε_i ($i = 1-3$), are the principal stresses and the associated strains, respectively.

It is suggested that the rock failure by shear along one or more faults (Fig. 1) is controlled by the conditions formulated by these hypotheses:

- A. Critical energy: Faulting (i.e., shear fracturing) occurs when $U \geq U_C$, where U is the elastic energy density generated by the loading system, and U_C is the critical energy density necessary for dissipation by the faulting processes. We employ the total elastic energy that incorporates both the mean-stress and the distortion-energy because the failure of brittle rocks strongly depends on the mean-stress and normal stress^{1,4,9}.
- B. Strain compatibility: The internal strain-state is compatible with the external, elastic strain-state, ε_i that is generated by the external loading stresses^{13,39}, σ_i . The internal strain is accommodated by slip along the developing faults and by dilation normal to them.

CRITICAL-ENERGY-OF-FAULTING-FAILURE (CEFF)

General elasticity theory indicates that loading of a linear, elastic solid by stress-state, σ_i , can be translated to elastic strain-state, ε_i :

$$\varepsilon_1 = [\sigma_1 - \nu(\sigma_2 + \sigma_3)]/E, \quad (2)$$

where ν and E are the Poisson's ratio and Young modulus, and the other two principal strains (ε_2 and ε_3) are determined by the 2nd and 3rd permutations⁸. The elastic energy per unit volume, namely the elastic-energy-density, is defined as one half the product of the stress and the associated strain⁸, $U = \frac{1}{2} \sigma \varepsilon$. Under three-dimensional deformation, U is effectively presented by the stress invariants³²:

$$U = 0.5 [I_1^2 - 2(1 + \nu) I_2]/E, \quad (3)$$

where $I_1 = (\sigma_1 + \sigma_2 + \sigma_3)$ and $I_2 = (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)$ are the first and second invariants in the principal stress system (Supplementary). The elastic energy, U , depends on the three principal stresses, and we found that it is both simple and unique to relate U_C , the critical elastic energy density at failure, to the maximum stress, σ_1 (Methods, Extended Data Fig. 1).

The critical energy hypothesis implies that the elastic energy density at yielding stage, U_C , defines a failure envelope that delineates the transition from intact to faulted states. This relationship is demonstrated for the Shirahama sandstone with 37 failure experiments¹¹ (Extended Data Table 1, Fig. 2A). The elastic energy density, which is calculated from the experimental stress data and the elastic moduli, displays power relationships, $U_C = a (\sigma_1)^b$, where a and b are constants of $7.36 \cdot 10^{-5}$ and 1.87, respectively. The power law relationship is further demonstrated for 13 published datasets of experimental rock faults (Methods, Extended Data Table 1, Fig. 2B). Most of the experiments were conducted under the general, polyaxial loading, $\sigma_1 > \sigma_2 > \sigma_3$, also called ‘true-triaxial’^{23,40}; the rest of the experiments are of triaxial loading, $\sigma_1 > \sigma_2 = \sigma_3$, and uniaxial loading, $\sigma_1 > \sigma_2 = \sigma_3 = 0$ (Extended Data Table 1). The systematic, unique relations of the critical energy to σ_1 (Methods, Figs. 2A, B), demonstrates that U_C can be considered as a robust, energy-based faulting criterion. We also noted a clear separation between the eight sets of brittle, strong rocks and the five sets of ductile, relatively weak faults. This separation reflects the effect of the relatively smaller Young moduli of the ductile, weak rock resulting in larger elastic energy for a given σ_1 . We abbreviate the Critical-Energy-of-Faulting-Failure as *CEFF*, and later discuss its physical source.

INTERNAL STRAIN

The external loading of a rock-body generates an elastic strain-state (Fig. 2C, Equation 2), and hypothesis B implies that at failure this strain is internally accommodated by slip along faults (Fig. 2D). Further, the internal strain by faults’ slip should equal the externally applied strain while the energy dissipation along the faults is constrained by the available critical energy (hypothesis A). A general, 3-D strain-state (Fig. 2C) has orthorhombic symmetry (three mutually perpendicular principal strain axes). To accommodate such 3-D strain by fault slip requires four faults in orthorhombic (=polymodal) symmetry^{14,41}. This requirement is analogue to the five slip systems in metal plastic deformation¹³. Notably, due to the symmetry of the principal stress axes, an orthorhombic fault system can be reproduced by mirror and rotation actions on a single preferred fault³⁹. Similar rotation and mirror actions are commonly utilized to define the conjugate set of Coulomb criterion, and other failure criteria, e.g., von-Mises, or Tresca⁴².

We search for a fault system that satisfy the above two hypotheses. The search is based on simulations of a single preferred fault that is defined by its normal angles $(\theta_1, \theta_2, \theta_3)$ with respect to the three-dimensional axes (X_1, X_2, X_3) of the principal stress state $(\sigma_1, \sigma_2, \sigma_3)$, Fig. 3A (Methods). At failure, the rock-body is internally deformed by fault slip along the D_S axis and dilation D_D normal to it. The faulting energy dissipation is the work done by D_S and D_D under the corresponding stresses applied on the preferred fault. The preferred fault geometry is derived here

by a multi-parameter minimization of the differences between the external elastic deformation (strain, and energy), and the internal strain and energy dissipation by the fault (Methods). Our dedicated Matlab code, OrthoFault, and a few experimental datasets, are available in GitHub (<https://github.com/nadavwetzler/OrthoFault.git>).

The predicted faults orientations under polyaxial loading ($\sigma_1 > \sigma_2 > \sigma_3$) are demonstrated for the Shirahama sandstone that includes 37 failure experiments (Extended Data Table 1, Fig. 3B). The calculated preferred faults are shown by red great circles with black dots for the slip axes. Next, we detail the results of one selected experiment with strain-state of maximum vertical shortening and horizontal extensions (Fig. 3C, D). The orthorhombic faults of this experiment are reproduced by rotation and mirror operations on the preferred fault about the principal planes, and are displayed by blue circles (Fig. 3B) and 3-D view (Fig. 3C). We further calculated the preferred fault orientations in 14 sets with 344 individual experiments (Extended Data Table 1). These calculations indicate that the slip axis of a given fault is not necessarily dip-parallel, strike-parallel, or parallel to a principal stress axis (Fig. 3B, C, Extended Data Fig. 2, Fig. S4). These features contrast the classical Coulomb criterion that predicts a single conjugate set (two pink great circles in Fig. 3B) with fault planes that parallel the σ_2 axis^{6,8}.

ENERGY DISSIPATION DURING FAULTING

We now examine the physical significance of the proposed critical energy density, $CEFF$, (Fig. 2A, B) by following the rationale of Griffith's theory³⁴. According to this theory, tiny flaws within a brittle solid may locally amplify the stresses beyond the local strength, however, a flaw would grow unstably only when the released elastic energy exceeds the dissipated energy by the growing fracture. The dissipated energy is defined as the 'fracture energy', G , and it quantifies the energy required to form new surface-area during growth of a tensile fracture^{35,36}. Similarly, the fracture energy of a shear fault quantifies the energy dissipated by the processes of fragmentation, micro-fracturing, and heating during faulting of an intact rock⁴³. The fracture energy of faulting was measured by the integration of the post-failure stage of experimental stress-strain curve⁴⁴ (Fig. S1) revealing $G = 0.3\text{-}10 \cdot 10^4 \text{ Jm}^{-2}$ for experimental faulting^{28,44,45}.

In a given experiment, the dissipated-energy-density, W_D , can be evaluated by

$$W_D = G \cdot A / V \quad (4)$$

where A is the fault area, and V is the sample volume. The dissipated-energy-density, W_D , is calculated here for a series of nine triaxial tests on Stripa granite samples^{28,46} (Methods). The range of measured fracture energy during faulting in these experiments is $0.3\text{-}8.12 \cdot 10^4 \text{ Jm}^{-2}$ (Extended Data Table 2) similar to typical experimental values^{44,45}. We calculated W_D for these experiments by using the observed fault area A (Insert Fig. 4A, Methods, Supplementary), the reported G , and the sample volume, V . The W_D and U_C energy parameters, which are independently calculated, show similar, correlated trends when plotted relatively to σ_1 (Fig. 4A). We argue that this correlation between U_C and W_D indicates the fundamental significance of hypothesis A: The critical energy generated by the loading system, $CEFF$, facilitates the failure by providing the energy to be dissipated by the faulting.

The last statement raises a central question: what happens if the local stresses in an experiment exceed the local strength while the total energy is below the critical energy? This question cannot be addressed by Coulomb's criterion, but Griffith's approach may offer an explanation. It is likely that an incipient fault would grow if the local stresses exceed the local strength, however, if $U < U_c$ this failure will remain local without crossing the sample. Indeed, local yielding events before rock failure are very common, and experimentally manifested by countless microfracturing, distributed damage and micro-faults as monitored by acoustic emission⁴⁷ and microscopic analyses³. We show here one example of local faulting during a triaxial experiment on Mount Scott granite (Extended Data Table 1). Fig. 4B displays the stress-strain relationship and the elastic energy density, U , of this experiment. Both curves show two faulting events recognized by distinct stress and energy drops. The first event, with small stress drop, indicates partial, local faulting within the sample, but during this first event, $U < U_c$. We argue that additional damage and energy are needed to rupture the whole sample by a crosscutting fault.

APPLICATION OF FAULTING SIMULATIONS

The OrthoFault code calculations of a given experimental series reveal a linear relationship between the normal stress, σ_n , on the preferred fault, and the shear stress, τ_s , along its slip axis (Methods, Extended Data Table 1, Fig. 4C),

$$\tau_s = A + B * \sigma_n, \quad (5)$$

where A and B are constants. This function defines a new strength criterion that is based on strain compatibility and energy constrain. We also noted distribution similarity between shear strength, τ_s , the energy dissipation, $CEFF$, and the stress/strain invariants (Methods, Extended Data Fig. 4) which indicate that the new strength criterion characterizes the energy dissipation during faulting.

The new strength envelope has similar form to Coulomb criterion (equation 1) and Byerlee law⁹, allowing useful comparisons. First, the B parameter of (5) is functionally equivalent to μ , the mysterious 'internal friction' parameter in (1)^{6,7,29}. We thus argue that fault dissipative energy, which is analyzed here (Fig. 4A), is the physical basis of 'internal friction'. Second, our analyses of 13 experimental sets show that Coulomb envelope is systematically higher than our strength envelope (column 5, Extended Data Table 1 and Fig. 2). For example, the Shirahama sandstone set (red line, Fig. 4C) predicts shear strength that is 26-46% lower than the Coulomb strength for these experiments (black line, Fig. 4C) (the later was calculated directly from the experimental σ_1 and σ_3 ⁸). Thus, the CEFF model predicts more conservative rock strength than Coulomb criterion, and this result could bear major implications to rock-engineering designs.

We finally examine the orientations of the preferred faults relatively to experimental observations. For this comparison, we use the results of the Berea sandstone set²² that is one of the very few sets reporting fault patterns under polyaxial loading (Fig. S4). In this set, the fault systems developed in intact cubic samples that were loaded under polyaxial strain state with principal axes perpendicular to the cubes faces (Fig. S4). The eight faults, which were mapped on the exterior of three samples, trend diagonally to the principal axes. This observation is consistent with

orthorhombic fault patterns in nature and other experiments (Fig. 1). The eight faults and the predicted preferred faults for the same experimental conditions (by OrthoFault) share similar orientation as displayed in a combined stereonet and rose diagram (Figs. 4D, S4D). This similarity further supports the concept of orthorhombic faulting under polyaxial loading and the ability to predict their orientations by using OrthoFault.

We analyze here rock faulting processes by integrating the mechanical fundamentals of energy, stress, and strain. The analysis application to multiple experimental series revealed a systematic dependence of rock failure state on the critical intensity of the elastic energy. This energy is essential for the faulting process, and it is partly or fully dissipated by slip and dilation along the developing faults. Our analysis provides the theoretical basis for faulting, and predicts the observations of orthorhombic fault patterns in the field and experiments. Lastly, the newly derived strength envelope offers a conservative strength prediction that can be applied to a range of rock-engineering situations.

ACKNOWLEDGMENTS

We benefitted from discussions and comments by Vladimir Lyakhovsky, Einat Aharonov, Yossi H Hatzor, Brett Carpenter, Zonghu Liao, and Amotz Agnon. We thank Alexey Ryabov for providing the Matlab code for the subplot lettering AddLetters2Plots (<https://www.mathworks.com/matlabcentral/fileexchange/95478>), MATLAB Central File Exchange. Retrieved July 9, 2021. The research was partially supported through the project “Rock Mechanics of Reservoir Rock” provided by China University of Petroleum-Beijing (ZR), and the Israeli Science Foundation #363/20 (NW).

Fig. 1. Examples of orthorhombic (i.e., polymodal) fault patterns developed experimentally under three-dimensional, polyaxial (i.e., true-triaxial) loading and naturally in the field. A, B. Experimental faulting is demonstrated of two cubic samples of Candoro limestone (A) and Westerly granite (B) under 3D strain loading²². C. Simplified map of normal fault system in Chimney Rock area, central Utah (redrawn after Fig. 9 in Krantz¹⁶). The field mapping identified four sets of normal faults that are shown by their trend and their inclination direction (down-dip displacement) as marked by the small arrows. Each set is marked by a different color. Note that the two dominant trends and the four inclination directions together form an orthorhombic pattern as displayed schematically by the insert 3-D block and in a stereographic projection.

Fig. 2. The two hypotheses of the critical energy model. A. The ‘critical energy’, U_c , calculated for the 37 failure experiments (black dots) of polyaxial loading ($\sigma_1 > \sigma_2 > \sigma_3$) of the Shirahama sandstone¹¹ as a function of the maximum normal stress, σ_1 , with fitted power relationship, $U_c = a (\sigma_1)^b$, with a and b as constants $7.36 \cdot 10^{-5}$ and 1.87. B. The calculated experimental relationship of U_c and the corresponding σ_1 for 13 experimental datasets, all of which display similar power relationships (Extended Data Table 1). Note the clear separation between the eight sets of brittle, strong rocks and the five sets of ductile, weak faults. C. Schematic display of the stress and strain states during the loading of a rock block. The externally applied polyaxial stresses ($\sigma_1 > \sigma_2 > \sigma_3$) deform the initial sample (dotted prism) into the pre-failure strained sample with strains $\epsilon_1 \neq \epsilon_2 \neq \epsilon_3$, that depend on the elastic properties and stresses magnitudes (equation 2) (gray prism). D. Faulting at failure form an orthorhombic pattern with multiple planes (dashed lines), sub-parallel to the principal orientations, deforming the rock-body by the same strain magnitudes of the external loads (3-D green arrows in C) (hypothesis B).

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Fig. 3. Determination of the expected fault patterns according to the requirements of hypotheses A and B: The internal strain by fault-slip and fault-dilation equals the external elastic strain, and that the dissipation energy during faulting is constrained by the available elastic energy. A. The geometry of a preferred fault (gray triangular) in the principal stresses coordinate system. The fault is defined by the angles θ_1 , θ_2 and θ_3 of its normal to the principal stress axes X_1 , X_2 , X_3 . The slip along the fault is defined by corresponding three components, D_1 , D_2 , and D_3 , and the fault normal dilation by D_d . B. Stereonet plot of the preferred faults (red great circles) with the related slip axis (black dot) for the Shirahama sandstone experiments¹¹. These fault systems were calculated by the OrthoFault code (Methods). An orthorhombic fault set (four faults in dashed blue circles) is displayed for one selected preferred fault (blue great circle) with the corresponding slip direction (green arrows); the orthorhombic set is produced by rotation and mirror of the calculated preferred one (text). The conjugate fault set predicted by Coulomb criterion is shown for comparison (pink great circles) C. The calculated orthorhombic fault set (blue circles in B) is shown in 3-D block view with corresponding strain-state displayed by three pairs of arrows in the center, and the corresponding strain Mohr diagram (D).

Fig. 4. Investigation of the critical energy model of rock-faulting, *CEFF*. A. The elastic energy density at failure, *CEFF*, in triaxial experiments of Stripa granite as function of σ_I (red dots), and the dissipated-energy-density, W_D , due to faulting (equation4) in these experiments (blue squares) (Methods, Extended Data Table 2). Calculations are detailed in Methods. The inset shows a cross-cutting fault in the center of the sample (yellow ellipse), and the largest possible fault (dashed red ellipse); see Fig. S2 and Supplementary. B. Stress-strain relations (blue) and contemporaneous elastic energy (red) in a triaxial experiment of Mount Scott granite²⁴. Note two faulting events (text). C. The strength envelope as a shear-stress vs normal-stress line (red) that was calculated by OrthoFault code for the Shirahama sandstone experiments¹¹ (red dots). The Coulomb strength envelope for these experiments (black line) is shown for comparison (text). Note that the present predicted shear strength that is 26-46% lower than the Coulomb strength. D. Observed faults in three samples of Berea sandstone under polyaxial strain loading (Fig. S5), and calculated faults by OrthoFault code. Insert is a block-diagram of the experimental faults in sample RD-42. Fault orientations are displayed on a combined stereonet plot and rose diagram (NW quarter). The stereonet shows poles to eight observed faults in three experiments (red dots), and poles to preferred faults predicted by OrthoFault for these three experiments (purple dots). The rose diagram shows the dip-direction of the same data (experimental in blue sectors, and preferred faults in red sectors).

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Elastic strain energy density, U , and maximum stress, σ_1 .

The critical elastic energy density at failure, $CEFF$, is a function of the three-dimensional loading stresses. We present $CEFF$ as function of σ_1 alone (Figs. 2A, B, 4A) as if the other two principal stresses may be ignored. This choice provides unique relationship of critical energy and the stress-state in addition of allowing simple and intuitive presentation. For the demonstration, we consider the experimental stage of rock failure (hypothesis A) in the series of Shirahama sandstone under three-dimensional loading. These experiments display power relations between $CEFF$ and σ_1 (Fig. 2A), and we examine here the uniqueness of these relationships.

Extended Data Fig. 1 displays, in addition to $CEFF$, the strain energy density, U_i , for a range of hypothetical triaxial experiments under constant maximum principal stress of $\sigma_1 = 200$ MPa, and confining pressures that ranges from $C_{Pi} = 0$ (uniaxial) to $C_{Pi} = 120$ MPa. The elastic energy density, U_i , corresponding to each of the hypothetical tests is calculated by using the $\sigma_1 = 200$ MPa, and C_{Pi} values, increasing from right-to-left in the secondary coordinate system. The plot (Extended Data Fig. 1) displays the C_{Pi} (green dot) and the associated U_i (blue dots with horizontal line). The lowest energy of $U_i = 0.93 \text{ MJm}^{-3}$ is associated with $C_{Pi} = 120$ MPa, and it is clearly within the intact rock state. At this state, the energy density level is insufficient to promote failure. Decreasing C_{Pi} , under the constant σ_1 , increases U_i until under $C_{Pi} = 35$ MPa the energy density reaches $U_i = U_C = 1.35 \text{ MJm}^{-3}$, and both green and blue dots are on the failure curve. It is predicted that under these conditions of $\sigma_1 = 200$, $\sigma_2 = \sigma_3 = 35$ MPa, a sample of Shirahama sandstone will fail by faulting. For lower confining pressures, the elastic energy is above the critical value of the failure envelope indicating that this rock cannot support this stress state.

Obviously, there are many other paths from the intact-state to the faulted-state, but it is clear that all these paths must cross the U_C curve at a single value of σ_1 . Thus, the critical elastic energy density, $CEFF$, is uniquely defined by the σ_1 value, and this uniqueness allows using the $CEFF$ - σ_1 power relations in the main text (Figs. 2A, B, 4A, S1).

The OrthoFault Code for $CEFF$ analysis

The present critical energy model of rock faulting is based on two hypotheses and the assumption that these hypotheses can be realized by slip and dilation along preferred faults. The preferred faults are determined by numerical search with a dedicated Matlab program. The input file for a given dataset includes a table of the three principal stresses at experimental failure, and the Young modulus and Poisson's ratio parameters of the tested rock (Extended Data Table 1). The calculations are conducted in the following steps:

- I. Calculate and plot the general input data: principal stresses, the principal strains ε_1 , ε_2 and ε_3 (equation 2) at failure, the stress invariants, I_1 and I_2 , and the elastic energy density during failure, U_C (equation 3) (Figs. 2A, B, Extended Data Fig. 2).
- II. Guessing an initial preferred fault orientation with slip intensity, D_S and dilation, D_D . This fault is defined by directional cosines of its normal (N_1 , N_2 , N_3), and the slip axis is defined by its direction cosines (S_1 , S_2 , S_3) in the principal stress coordinates. One can show that N_1 and N_2 are independent, and the other four directional cosines can be determined from N_1 and N_2 and the strain-ratio $\varepsilon_2/\varepsilon_1$ ³⁹.
- III. Calculation for the initial preferred fault:

- a. The normal stress across the initial fault, σ_n , and the shear stress, τ_s , in the slip direction based on the known stress-state ($\sigma_1, \sigma_2, \sigma_3$) (input), and the initial fault parameters (N_i, S_i) were defined in step II.
 - b. The internal strain-state due to the slip, D_S , and dilation, D_D , along the initial fault.
 - c. The dissipated work, W , by the fault slip under τ_s for distance D_S , and work against σ_n by dilation D_D .
- IV. Steps II and III are repeated in a grid scanning the full range of N_1, N_2 (0-1), in search for the minimum D_S , and D_D , for the best-fit preferred fault. We used the Levenberg-Marquardt nonlinear least-squares method provided by MATLAB. The best-fit fault has these features: (1) the slip and dilation generate an internal three-dimensional strain-state that equals the elastic strain-state generated by the principal stresses; (2) the dissipated work by the slip and dilation is smaller than the critical energy, $CEFF$; and (3) the slip intensity, D_S , is minimized.
- V. The calculations of step IV produced one fault, but due to the symmetry of the stress system, four faults in orthorhombic symmetry (Figs. 1, 3C) have the same stress and strain of the preferred fault, and have equal likelihood to form. This situation is equivalent to the development of a conjugate set of faults under 2D strain loading (Reches, 1983a, b).
- VI. Steps I-V are for one experiment with known stress-state, and calculated strain-state (Fig. 3C) and preferred faults (Fig. 3D). The calculations are then repeated to all experiments in the dataset to produce plots of: (1) the preferred faults with slip axes (Fig. 3B); and (2) a Mohr diagram with τ_s vs σ_n of the preferred faults. This plot represents the strength of the analyzed rock according to the present theory.

Fracture energy and fault slip during failure

We explored the relationships between the critical elastic density, $CEFF$ (hypothesis A, text), and the energy dissipation during experimental faulting^{28,44}. In rock failure experiments, the dissipated work during faulting is evaluated by integrating the area under the stress-displacement curve after the peak stress (Fig. S1). This measured mechanical work is considered the shear fracture energy^{28,44}. We examine here the results of Hakami⁴⁶ and Hakami and Stephansson²⁸ who presented all the data needed for dissipation calculations: sample dimensions, fault angle, shear fracture energy, G , and fault slip during failure, Δu (Extended Data Table 2). The triaxial experiments were conducted on Stripa granite while using intact samples and samples with well healed fracture (column 2, Extended Data Table 2). The determined fracture energy in these experiments ranges $G = 0.36\text{-}8.12 \cdot 10^4 \text{ Jm}^{-2}$ (column 5, Extended Data Table 2) which is similar to the values reported by Wong (1982) of $G = 0.3\text{-}7.3 \cdot 10^4 \text{ Jm}^{-2}$ for experimental measurements (there Table 2).

Hakami and Stephansson²⁸ used cylindrical samples of 21.8 mm diameter and twice the height. In the experiments the confining pressure was first increased to an initial selected level (column 3A, Extended Data Table 2), and then the confining chamber was locked allowing the pressure to rise with the increasing axial load (initial and final confining pressures in column 3, Extended Data Table 2). The images of the faulted samples (Figs. 5.18-5.23 in Hakami⁴⁶) indicate that the faults cut across the central part of the samples (Fig. S3A-E). Thus, the area of the faults is in the range of 0.5-0.75 of the largest possible fault (Fig. S3F). We calculated the fault area (column 9) by using the fault angle (column 4, Extended Data Table 2), and sample dimensions; the 0.5-0.75 range was used to estimate the area error. The total dissipated energy during each experiment (column 10A, Extended Data Table 2) was calculated by using the

fracture energy, G (column 5, Extended Data Table 2), and the estimated fault area. Then the dissipated energy per the volume is calculated by using equation 4 (text),

$$W_D = G \cdot A/V$$

where V is the sample volume. The faulting dissipated energies are listed as total energy during failure (column 10A, Extended Data Table 2), and as energy density (column 10B, Extended Data Table 2). Fig. 4A (text) displays the elastic energy density, $CEFF$, (column 8, Extended Data Table 2), and the dissipated energy density, W_D , (column 10B, Extended Data Table 2) due to the formation of the sample-crossing fault. The significance of the good agreement between the two sets of values, which were independently calculated, is discussed in the main text.

Hakami and Stephansson²⁸ also reported the faults' slip during failure, Δu (their Table 1, column 6A in Extended Data Table 2). This central measurement can be compared to our calculated slip along the preferred faults. Our calculations are based on hypothesis B (fault strain equals the elastic strain), and the preferred fault satisfies this strain requirement with the best-fit minimum slip, D_{ST} (Methods). The slip magnitude that we calculated for the nine experiments indicate $D_{ST} = 1.78$ -6.78 mm. However, as our strain and energy calculations are in SI units, the results are implicitly related to slip within a $1m^3$ rock-body. Thus, we scaled these slip values to the dimensions of a given sample by using the relations,

$$\text{Expected fault slip } (\Delta u_{\text{expected}}) = \text{calculated slip } (D_{ST}) \cdot (\text{sample height}) / 1m$$

The scaling to the 0.0436 m sample height of Stripa experiments returned values of $\Delta u_{\text{expected}} = 0.08$ -0.30 mm (column 6B, Extended Data Table, 2). The average measured slip is crudely twice the calculated slip (column 6, Extended Data Table 2), and both the measured and calculated slip display irregular scatter with a weak increasing trend with σ_1 (Fig. S3). We claim that this agreement, within a factor of two, between the measured and predicted slip values supports the validity of our approach. More refined agreement may be achieved in dedicated experimental work.

CEFF strength criterion

It is shown in the main text that the OrthoFault calculations allow to define a new strength criterion which is based on strain compatibility and energy constrain. This criterion has the linear relationship (Extended Data Table 1, Fig. 4C),

$$\tau_s = A + B \cdot \sigma_n, \quad (5)$$

where σ_n is the normal stress, τ_s is the shear stress on the preferred fault along its slip axis, and A and B are constants. We examined here the relationships between the shear strength, τ_s , of the strength criterion, the energy dissipation during failure, $CEFF$, and the stress/strain invariants by plotting these parameters for the experimental series of Dunham dolomite (Extended Data Fig. 3). The clear similar dependence of both shear strength and energy dissipation on the stress/strain invariants support the notion that the physical control of the strength function (equation 5) is the energy dissipation by faulting during failure. Further, this result offers an explanation for the elusive term of 'internal friction' of Coulomb criterion (μ in equation 1). This term has the form of friction, however, common 'friction' between two solid blocks cannot exist before a fault is formed (Anderson). Handin⁷ stated: "In cohesive material, however, internal friction is a fictitious quantity that cannot be measured directly." Our analysis, e.g., Extended Data Fig. 3, indicates that 'internal friction' (μ in equation 1 and B in equation 5) actually represents the intensity of the energy dissipation during faulting (Fig. Extended Data 3), and it is related to 'friction' only by formula similarity.

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Extended Data Fig. 1. The relationship between the total elastic strain energy and the magnitude of principal stress is shown for the 37 experiments of the Shirahama sandstone (red dots).

Shirahama-sandstone

523 Extended Data Fig. 2. The graphic output of OrthoFault (Methods) simulations of the

Shirahama sandstone experimental dataset¹¹. Additional examples are displayed in Figs. S6-8. A. Stereonet plot of the preferred faults (red great circles) with the related slip axis (black dot). The orthorhombic fault set (blue circles) is displayed with the corresponding slip direction (light blue dots) is produced by rotation and mirror of the calculated preferred one (text). The conjugate fault set of Coulomb criterion is shown by two black circles. B. The '*critical energy*', U_C , as a function of the maximum normal stress, σ_I , with fitted power relationship, $U_C = a (\sigma_I)^b$, and the calculated dissipated energy by fault slip and dilation. C. The calculated strength envelope as a shear-stress vs normal-stress (red line) for the 37 experiments (red dots). The Coulomb strength envelope for these experiments (black line) was calculated by using σ_I vs σ_3 relations shown in I. D. Mohr plot of the loading principal stresses at failure. E. The orthorhombic fault set (four dashed blue circles) of for one selected preferred fault (blue great circle) with the corresponding slip direction (pink arrows). F. The calculated orthorhombic fault set (dashed blue in E) is shown in 3-D block view with corresponding strain-state displayed by three pairs of arrows in the center, and the corresponding strain and stress Mohr diagrams (G).

542 Extended Data Fig. 3. OrthoFault calculated relations of faulting failure in the experimental series
 543 of Dunham dolomite. The first invariants of stress and strain are linearly related to each other,
 544 and for this rock $A=41.3$, and $B=0.66$. The power fit functions are: Shear stress = $7.55 \cdot 10^5 \cdot I_1^{1.17}$,
 545 $r^2 = 0.75$ and Dissipation energy = $3.55 \cdot 10^5 \cdot I_1^{1.98}$, $r^2 = 0.79$

Extended Data Table 1. We utilized 14 published experimental series for the application and testing of the present model. The data include eight series with polyaxial (true-triaxial) loading, three series of triaxial experiments, and two series of extension triaxial tests. The datasets include eight cases of brittle, relatively strong rocks (sets: 1, 3, 4, 6-9, 12, 14), and five of more ductile, relatively weaker rocks (sets: 2, 5, 10, 11, 13). The elastic constants of Young modulus and Poisson's ratio (column 3) are from the experimental sources and from other published sources.

	1	2			3	4	5		6
Set	Rock type with dataset source in reference list	Tests by number of loading axes			Elastic moduli Young (GPa), Poisson's	a and b in $U_C = a[\sigma_I]^b$, r^2	A, B (MPa) on Mohr plot (eq. 5)		Comments
		Uni-	Tri-	Poly-			Preferred fault	Coulomb criterion	
1	KTB amphibolite ¹¹	3	4	35	95, 0.26	$8.5 \cdot 10^{-6}$, 1.91; 0.99	0.81, 35	0.98, 88	Only 3D tests used
2	Yuubari shale ¹¹			26	15, 0.2	$3.7 \cdot 10^{-5}$, 1.97; 0.97	0.54, 16	0.54, 35	
3	Dunham dolomite ¹¹			52	111, 0.22	$6.7 \cdot 10^{-5}$, 1.97; 0.99	0.62, 49	0.66, 118	
4	Mt Scott granite ²⁴	3	9		78.4, 0.22	$8.3 \cdot 10^{-6}$, 1.95; 0.99	1.14, 55	1.16, 62	
5	Shirahama sandstone ¹¹			37	11.5, 0.38	$7.3 \cdot 10^{-5}$, 1.85; 0.99	0.55, 12	0.82, 21	
6	Solenhofen limestone ¹¹			29	72.5, 0.34	$2.9 \cdot 10^{-5}$, 1.77; 0.99	0.63, 44	0.58, 107	
7	Westerly granite ²³			45	70, 0.2	$8.9 \cdot 10^{-6}$, 1.96; 0.99	0.93, 38	1.24, 62	
8	Westerly granite ²¹		6		70, 0.2	$9.3 \cdot 10^{-6}$, 1.95; 0.99	1.23, 47	1.29, 47	Compression loading
9	Westerly granite ²¹		7		70, 0.2	$1.3 \cdot 10^{-5}$, 1.97; 0.99	0.41, 131	1.81, 49	Extension loading
10	Maha Sarakham salt ²⁶			35	22.2, 0.37	$4.1 \cdot 10^{-5}$, 1.83; 0.96	0.73, 6.7	0.61, 15.4	
11	Berea sandstone ²²			12	26.4, 0.12	$6.0 \cdot 10^{-6}$, 2.45; 0.90	0.50, 11	1.05, 17	
12	Tautona quartzite ²⁷	3	17		76.8, 0.21	$9.3 \cdot 10^{-6}$, 1.92; 0.99	0.56, 32	0.38, 77	Only triaxial tests used
13	Carrara marble ²⁵		12		40, 0.18	$5.6 \cdot 10^{-5}$, 1.78; 0.99	0.81, 14	1.23, 22	Extension loading
14	Stripa granite ²⁸		9		84, 0.22	$4.9 \cdot 10^{-6}$, 2.02; 0.99	0.93, -18.7	0.41, 78.6	Includes fracture energy

Extended Data Table 2. Experimental and calculated parameters of the triaxial failure experiments on Stripa granite samples. Experimental data from Hakami⁴⁶ and Hakami and Stephansson²⁸, expected parameters were calculated in the present study.

1	2	3A, 3B		4	5	6A, 6B		7	8	9	10A, 10B	
Sample#	Sample type ¹	Confinment (MPa)		Fault angle (°)	G , 10 ⁴ J/m ²	Fault slip Δu , (mm)		σ_f at failure (MPa)	Elastic energy ² MJ/m ³	Crossing fault area ³ (10 ⁻³ m ²)	Dissipated energy on crossing fault	
		Initial	Final			Measured	calculated				Total (J) ⁴	Density ⁵ (MJ/m ³)
SG15	Intact	20	63	20	1.2	0.27	0.25	573	1.99	0.782	5.96	0.37
SG14	Intact	20	81	23	0.76	0.22	0.20	527	1.63	0.794	3.85	0.24
SG6	Single	20	38	34	0.36	0.16	0.08	213	0.26	0.811	2.03	0.12
SG11	Intact	50	108	30	1.77	0.40	0.20	505	1.46	0.900	9.54	0.59
SG4	Single	50	77	17	0.53	0.24	0.14	239	0.32	0.862	2.59	0.16
SG12	Intact	50	79	20	2.86	0.55	0.14	340	0.65	0.781	14.20	0.87
SG5	Single	80	136	42	3.94	0.45	0.30	303	0.52	0.794	24.74	1.52
SG13	Intact	80	129	28	8.12	0.63	0.25	624	2.22	1.005	42.91	2.64
SG10	Intact	80	125	30	7.73	0.67	0.25	620	2.20	0.845	41.64	2.56
Mean values					3.08	0.40	0.20		1.25			1.01

1. Intact sample or sample with healed single fracture
2. Calculated for Young Modulus $E=75$ GPa and Poisson's ratio $\nu=0.25$
3. Area of likely single fault in the sample (see Fig. S2).
4. Total dissipated energy = $G \cdot$ fault area
5. Dissipated energy density is the total dissipated energy per volume of the sample

Faulting of rocks as a critical energy process

SUPPLEMENTARY INFORMATION

This document covers topics with additional information to the main text. The references and figures of the main text are cited by the original numbering, and the figures and tables of this document are marked by 'S'. To maintain coherent presentation in this Supplementary document, it includes some repetitions of text and figures of the main manuscript.

THE PRESENTED TOPICS ARE:

Text S1. Elastic energy formulations

Text S2: Fracture energy and fault slip during failure

Text S3: Comparing experimentally observed faults with predicted preferred faults

Text S4: Strength dependence on sample size

Text S5: Examples of OrthoFault code calculations for faulting analysis

TEXT S1. ELASTIC ENERGY FORMULATIONS

Consider a cube with edge length r of an elastic rock that is subjected to a load increase by three principal stresses, σ_1 , σ_2 , and σ_3 . In a linear elastic solid, the strain components, ε_1 , ε_2 , and ε_3 , are related to the loading stresses by the elastic constant of Young modulus, E , and Poisson's ratio, ν . The force acting on the cube face normal to σ_1 is $\sigma_1 r^2$, the displacement associated with this stress is $\varepsilon_1 r$. The work associated with gradual, simultaneous increase of the stress and strain in the x_1 direction is,

$$\frac{1}{2} \sigma_1 \varepsilon_1 r^3,$$

where r^3 is the unit volume. This work is accumulated as the potential elastic energy within the rock cube, with the elastic energy density defined as,

$$U = \frac{1}{2} [\sigma_1 \cdot \varepsilon_1 + \sigma_2 \cdot \varepsilon_2 + \sigma_3 \cdot \varepsilon_3].$$

By considering the stress-strain relations of a linear elastic solid⁸,

$$U = \frac{1}{2} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)] / E$$

Recently, Barsanescu³² (and personal communication) investigated this total elastic strain equation, and showed that as

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = (\sigma_1 + \sigma_2 + \sigma_3)^2 - 2(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) = I_1^2 - 2I_2$$

where $I_1 = (\sigma_1 + \sigma_2 + \sigma_3)$ and $I_2 = (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)$ are the first and second invariants of the principal stresses. Thus, the elastic energy density can be expressed as,

$$U = 0.5 [I_1^2 - 2(1 + \nu) I_2] / E$$

The formulation of the elastic energy by stress invariants factors appears very effective in the present analysis.

We explored the relationships between the critical elastic density, $CEFF$ (hypothesis A, text), and the energy dissipation during experimental faulting^{28,44}. In rock failure experiments, the dissipated work during faulting is evaluated by integrating the area under the stress-displacement curve after the peak stress (Fig. S1). This measured mechanical work is considered the shear fracture energy^{28,44,45}. We examine here the results of Hakami⁴⁶ and Hakami and Stephansson²⁸ who presented all the data needed for dissipation calculations: sample dimensions, fault angle, shear fracture energy, G , and fault slip during failure, Δu (Table S1). The triaxial experiments were conducted on Stripa granite while using intact samples and samples with well healed fracture (column 2, Table S1). The determined fracture energy in these experiments ranges $G = 0.36\text{-}8.12 \cdot 10^4 \text{ Jm}^{-2}$ (column 5, Table S1) which is similar to the values reported by Wong⁴⁴ of $G = 0.3\text{-}7.3 \cdot 10^4 \text{ Jm}^{-2}$ for experimental measurements (there Table 2).

Hakami and Stephansson²⁸ used cylindrical samples of 21.8 mm diameter and twice the height. In the experiments the confining pressure was first increased to an initial selected level (column 3A, Table S1), and then the confining chamber was locked allowing the pressure to rise with the increasing axial load (initial and final confining pressures in column 3, Table S1). The images of the faulted samples (Figs. 5.18-5.23 in Hakami⁴⁶) indicate that the faults cut across the central part of the samples (Fig. S2A-E). Thus, the area of the faults is in the range of 0.5-0.75 of the largest possible fault (Fig. S2F). We calculated the fault area (column 9) by using the fault angle (column 4, Table S1), and sample dimensions; the 0.5-0.75 range was used to estimate the area error. The total dissipated energy during each experiment (column 10A, Table S1) was calculated by using the fracture energy, G (column 5, Table S1), and the estimated fault area. Then the dissipated energy per the volume is calculated by using equation 4 (text),

$$W_D = G \cdot A / V$$

where V is the sample volume. The faulting dissipated energies are listed as total energy during failure (column 10A, Table S1), and as energy density (column 10B, Table S1). Fig. 4A (text) displays the elastic energy density, $CEFF$, (column 8, Table S1), and the dissipated energy density, W_D , (column 10B, Table S1) due to the formation of the sample-crossing fault. The significance of the good agreement between the two sets of values, which were independently calculated, is discussed in the main text.

Hakami and Stephansson²⁸ also reported the faults' slip during failure, Δu (their Table 1, column 6A in Table S1). This central measurement can be compared to our calculated slip along the preferred faults. Our calculations are based on hypothesis B (fault strain equals the elastic strain), and the preferred fault satisfies this strain requirement with the best-fit minimum slip, D_{ST} (Methods). The slip magnitude that we calculated for the nine experiments indicate $D_{ST} = 1.78\text{-}6.78 \text{ mm}$. However, as our strain and energy calculations are in SI units, the results are implicitly related to slip within a 1m^3 rock-body. Thus, we scaled these slip values to the dimensions of a given sample by using the relations,

$$\text{Expected fault slip } (\Delta u_{\text{expected}}) = \text{calculated slip } (D_{ST}) \cdot (\text{sample height}) / 1\text{m}$$

The scaling to the 0.0436 m sample height of Stripa experiments returned values of $\Delta u_{\text{expected}} = 0.08\text{-}0.30 \text{ mm}$ (column 6B, Table, S1). The average measured slip is crudely twice the calculated slip (column 6, Table S1), and both the measured and calculated slip display irregular scatter with a weak increasing trend with σ_1 (Fig. S3). We suggest that this agreement, within a

factor of two, between the measured and predicted slip values supports the validity of our approach. More refined agreement may be achieved in dedicated experimental work.

Table S1. Experimental and calculated parameters of the triaxial failure experiments on Stripa granite samples. Experimental data from Hakami⁴⁶ and Hakami and Stephansson²⁸; expected parameters were calculated in the present study.

1	2	3A, 3B		4	5	6A, 6B		7	8	9	10A, 10B	
Sample#	Sample type ¹	Confinment (MPa)		Fault angle (°)	G, 10 ⁴ J/m ²	Fault slip Δu, (mm)		σ ₁ at failure (MPa)	Elastic energy ² MJ/m ³	Crossing fault area ³ (10 ⁻³ m ²)	Dissipated energy on crossing fault	
		Initial	Final			Meas ured	calcul ated				Total (J) ⁴	Density ⁵ (MJ/m ³)
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Mean values					3.08	0.40	0.20		1.25			1.01

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3. Area of likely single fault in the sample (see Fig. S3).
4. Total dissipated energy = $G \cdot$ fault area
5. Dissipated energy density is the total dissipated energy per volume of the sample

Fig. S1. Method of measuring fracture energy in faulting experiment. Left: Schematic view of the stress-strain relations of a sample in a typical triaxial failure test. Right: The fracture energy, $G = 0.5 \cdot (\tau_p - \tau_f) \cdot \Delta u$ in the integrated (grey) area; the peak (τ_p) and final (τ_f) shear stresses vs the fault displacement, Δu (converted from the sample curve, left).

Fig. S2. Faulted samples of Stripa granite (Figs. 5.18-23 in Hakami⁴⁶). A-C. Intact samples, tests SG15, SG13, SG11, respectively. D-E. Samples with pre-existing fracture, tests SG6, SG5, respectively. In all the cases, the fault cut the center of the sample, and the fault surface area (yellow in F) is estimated as 0.5-0.75 of the maximum possible fault (red ellipses).

Fig. S3. Experimentally measured fault slip⁴⁶, and calculated fault slip to these experiments (OrthoFault code, this study) as function the experimental axial stress.

TEXT S3: COMPARING EXPERIMENTALLY OBSERVED FAULTS WITH PREDICTED PREFERRED FAULTS

We compare the experimentally observed fault orientations during failure of intact cubic samples of Berea sandstone²² (Fig. S4 A, B) with the orientations of the preferred fault calculated with OrthoFault (text). In these experiments, intact cubic samples were loaded under polyaxial strain state with principal axes perpendicular to the cubes faces (Fig. S4 A) were loaded under polyaxial stress state with principal axes perpendicular to the cubes faces. We mapped eight faults on the exterior faces of three faulted cubes (Fig. S4 B), and calculated the fault inclinations in the principal axes coordinates. The orientations of the preferred faults predicted by OrthoFault for the three experiments are displayed together with the experimental observations in a combined plot of stereonet and rose diagram (Figs. S4 C, 4D).

688 Fig. S4. Fault systems that developed during failure of intact cubic samples of Berea sandstone
 689 that were loaded under polyaxial stress state with principal axes perpendicular to the cubes
 690 faces²². A. Photos of the exterior faces of three faulted cubes. All faults in the 12 rock faces trend
 691 diagonally to the sides of cubes, excluding faults at the cubes' edges due to edge effect. B. Traces
 692 of the eight main faults in these experiments that were used to calculate the fault inclinations in
 693 the principal axes coordinates. C. A combined plot of stereonet and rose diagram (NW quarter) of
 694 the observed faults (in A and B) and the preferred faults predicted by OrthoFault. The stereonet
 695 shows: Poles to observed faults in three experiments (A) (red dots), and poles to preferred faults
 696 predicted by OrthoFault for these three experiments (purple dots). The rose diagram shows the
 697 poles to dip-direction of the same data (experimental faults in blue sectors, and preferred faults in
 698 red sectors).
 699

TEXT S4: STRENGTH DEPENDENCE ON SAMPLE SIZE

The critical-energy concept leads to another interesting deduction. The analysis refers to the elastic-energy-density that is total elastic energy, U_T , per unit volume. Consider a rock-body with characteristic dimension, L (Fig. S2F). The elastic energy density is proportional to the applied stresses, however, while the total energy depends on the volume, $U_T \propto L^3$, the dissipated energy on the fault surface is proportional to L^2 . Consider now doubling the sample characteristic size from L to $2L$, and maintaining the same stresses, shape and material. The elastic energy density remains the same, the total energy increases by a factor of 8, but the area of a cutting fault increases by a factor of 4; thus, the total elastic energy available for faulting increases by a factor of two. Therefore, as relatively more energy is available for faulting in the larger sample, it is expected that failure could occur under lower stresses. This strength dependence on sample size was recognized by Sih⁴⁸ who stated: “Specimen volume or size has been known to affect the mechanical and fracture properties of metals. Tests have shown that the apparent strength of a larger size specimen is lower than a smaller size specimen of the same geometry and material.” The inverse relationship between strength and size was also demonstrated in rock-mechanics experimentally (Fig. S5); yet, one should note the experimental difficulty to experiment with large rock samples, e.g., $L > 20$ cm¹.

We also note that the inverse strength-size relation may be explained differently. Following Griffith³⁴, it is assumed that fracture grows from the most critically stressed flaw, and that the local stress amplification is proportional to the flaw size. As already demonstrated by Griffith, a larger sample is likely to contain larger flaws, and thus it is reasonable that larger samples would fail under lower stresses. We thus conclude that the strength-size relations can be attributed to two effective mechanisms: larger flaws in larger samples and more available energy in the larger sample.

Fig. S5. Sample size effect on rock strength. Experimental data indicate up to 70% weakening per decade of length increase of the tested sample (after Fig. 9 in Lockner¹).

TEXT S5: EXAMPLES OF ORTHOFAULT SIMULATIONS RESULTS.

Figs. S6-9 display the graphic output of OrthoFault (Methods) calculations of Westerly granite, KTB amphibolite, Maha-Sarakham salt and Mount-Scott granite, respectively. The legend of all figures is the same as for Extended Data Fig. 2.

A. Stereonet plot of the preferred faults (red great circles) with the related slip axis (black dot). The orthorhombic fault set (blue circles) is displayed with the corresponding slip direction (light blue dots) is produced by rotation and mirror of the calculated preferred one (text). The conjugate fault set of Coulomb criterion is shown by two black circles.

B. The 'critical energy', U_c , as a function of the maximum normal stress, σ_1 , with fitted power relationship, $U_c = a (\sigma_1)^b$, and the calculated dissipated energy by fault slip and dilation.

C. The calculated strength envelope as a shear-stress vs normal-stress (red line) for the 37 experiments (red dots). The Coulomb strength envelope for these experiments (black line) was calculated by using σ_1 vs σ_3 relations shown in I.

D. Mohr plot of the loading principal stresses at failure.

E. The orthorhombic fault set (four dashed blue circles) of for one selected preferred fault (blue great circle) with the corresponding slip direction (pink arrows).

F. The calculated orthorhombic fault set (dashed blue in E) is shown in 3-D block view with corresponding strain-state displayed by three pairs of arrows in the center, and the corresponding strain and stress Mohr diagrams (G).

Westerly-granite-Haimson