Spiking-Free HGO-Based Dynamic Surface Control for Flexible Joint Manipulator with Unmeasurable States

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Title page

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ORIGINAL ARTICLE (or REVIEW)

Spiking-free HGO-based Dynamic Surface Control for Flexible Joint Manipulator with Unmeasurable States

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Abstract: For the flexible joint manipulator control system (FJMCS) with unmeasurable states, a novel partial states feedback control (PSFC) is proposed. Firstly, the unmeasurable states and the uncertainties are observed by a high-gain observer (HGO) simultaneously. Then, a dynamic surface controller is proposed based on the output of the HGO. The newly proposed controller has several advantages over existing methods. First, the proposed controller not only uses the estimate states to avoid using unmeasurable states, but also uses the estimation of uncertainties to enhance the robustness of FJMCS. Second, a novel spike suppression function (SSF) is developed to avoid the estimation spike problem in the existing HGO-based controllers. The closed-loop system stability is proved by the Lyapunov theory. Simulation results demonstrate the effectiveness of the proposed controller.

Keywords: Flexible joint manipulator • Partial states feedback • High-gain observer • Mismatched uncertainties • Dynamic surface control • Spike suppression

1 Introduction

With the development of mechanical, control and sensor technology, the design of robot has made great progress and gradually replacing people in the fields of repetitive labor, emergency rescue and home service, etc. [1-4]. In robot system, the robot manipulator has a very important role as the end-effector of the robot which interacts with the external environment [5]. For the robot manipulator, many studies on control strategies for different kinds of robot manipulator components have been completed. And, among these strategies, the disturbance attenuation control takes an

important role since the robot manipulator system is a typical nonlinear uncertain system, and the uncertainties can be caused by the unmodeled dynamics, external environment and parameter uncertainties [6-10].

Compared with the conventional rigid robot manipulator (RRM), the flexible joint manipulator (FJM) has become a research hotspot increasingly since it has several advantages, such as smaller actuator, light weight, lower cost and better maneuverability, etc. [11-14]. However, the flexibility of FJM brings more complex uncertain characteristics of dynamic model. The dynamic model of conventional RRM is a low-order (two-order) matched uncertain system. But the dynamic model of FJM is a typical high-order (four-order) mismatched uncertain system.

To solve the mismatched uncertain problem, the backstepping control method (BCM) is a good candidate [15]. The main advantage of BCM is that it can suppress the mismatched uncertainties through a step-by-step recursive process [16]. Some BCM-based methods for FJM have been developed, such as BCM-based adaptive control [17], BCM-based fault-tolerant control [18], BCM-based neural network control [19], BCM-based intelligent control [20] and BCM-based L2-gain control [21]. For conventional BCM, the computations needed for back-stepping is very tedious and complicated, this is so called the problem of "differential explosion". To solve this problem, the dynamic surface method (DSM) was proposed [22-23]. The DSM not only can suppress the mismatched uncertainties by the step-by-step recursive process like BCM, but also adopt a low-pass filter to avoid the explosiveness of backstepping. Then an adaptive fault-tolerant control for FJM based on DSM was proposed [24], and the complex differential calculation for

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backstepping in [17-21] can be avoid by the low-pass filter. However, in order to complete the step-by-step recursive process, the BCM-based and DSM-based controllers in [17-21,24] not only need to measure the states of link, but also need the states of motor for feedback. Thus, the measurement systems for the controllers in [17-21] are much more complex than the measurement system for the conventional RRM controller which only needs the states of link.

As a control method which only needs the measurements of partial states, the output feedback control (OFC) scheme can simplify the measurement system for high-order systems [25-28], significantly. Some control schemes for FJM based on OFC have been developed to simplify the measurement system. A control scheme which only requires the feedback of position states was developed for FJM [29], in which an observer was design to estimate the unmeasurable link and the actuator velocity, but the uncertainties of FJM were not considered in [29]. The unmeasurable velocities of link and motor were estimated by an approximate differential observer, and a complex adaptive control law was proposed to eliminate the effect of the uncertainties [30]. A high-gain observer was designed to generate pseudo signals for unmeasurable states, and an adaptive law of the projection type was developed to estimate the unknown feed-forward term [31].

To guarantee the observation and control precision, the observer gains of the observers in [29-31] should be chosen as the high gains. Thus, the observers used in [29-31] are typical high-gain observer (HGO). However, the undesirable spike problem can be caused by the high observer gains when the initial values of estimate and true states are not equal [32-33]. The spike problem may cause the saturation of control input and even lead to the instability of whole system. In addition, for the schemes in [30,31], on the basis of using observer, an additional robust control scheme should be adopted to eliminate the effect of uncertainties, thus the structures of controllers in [30,31] are very complex.

In this paper, a new partial states feedback control (PSFC) scheme based on DSM and HGO is proposed for the FJM. The main contributions of this paper are summarized as follows:

(1) The spike problem in the existing HGO-based schemes are solved: a novel spike suppression function (SSF) is adopted to solve the estimation spike problem in the existing HGO such as the observers in [29-31]. And, since the spike problem can be avoided even the observer gain is very high, a high observer gain can be adopted in the proposed scheme to guarantee the high control precision.

(2) Only partial states of FJM need to be measured: The

proposed method only needs the same measurement states as conventional RRM by using the HGO to estimate the unknown states.

(3) The structure of proposed controller is simple: The unmeasurable states and uncertainties are estimated by HGO simultaneously. Then the uncertainties of FJM are compensated by using the estimation of uncertainties without requiring the additional robust control scheme such as the adaptive schemes used in [30,31]. Thus, the structure of proposed control scheme is simpler than the schemes in [30,31].

The remaining parts of this paper are as follows. In section 2, the dynamic model, the design objective, and the motivation of this paper are expounded. The main results are presented in Section 3. In section 3.1, a high-gain observer (HGO) is developed. In section 3.2, a novel spike suppression function (SSF) is developed, and then a dynamic surface controller based on HGO and SSF is proposed. In section 4, a simulation verifies the effectiveness of proposed HGO, SSF and controller. In section 5, the conclusion of the whole paper is presented.

2 Problem Formulation

2.1 Dynamic Model of Flexible Joint Manipulator

Figure 1 Structure of flexible manipulator

The flexible joint manipulator is shown schematically in Figure 1. The dynamic model of the flexible manipulator control system (FMCS) can be described as follows [34]:

$$\begin{cases} (I + \Delta I) \ddot{q} + (K + \Delta K)(q - \theta) \\ + (MgL + \Delta MgL) \sin(q) + d_1 = 0 \\ (J + \Delta J) \ddot{\theta} - (K + \Delta K)(q - \theta) + d_2 = \tau \end{cases} \quad (1)$$

Where q and $\dot{q}$ are the rotation angle and angular velocity of link, respectively. θ and $\dot{\theta}$ are the rotation angle and angular velocity of motor, respectively. I and J are the moment of inertia of link and motor, respectively. M and L represent the mass and the distance between the joint and the centroid of the link, respectively. g is the

gravity acceleration. K represents the elastic stiffness of the flexible joint. τ is the motor torque. ΔI , ΔK , ΔJ and ΔMgL represents the unmodeled dynamics of the FMCS. d_1 and d_2 denote the disturbance from the environment.

Define the lumped uncertainties $\bar{d}_1$ and $\bar{d}_2$ as

$$\begin{cases} \bar{d}_1 = \Delta I \ddot{q} + \Delta K(q - \theta) + \Delta MgL \sin(q) + d_1 \\ \bar{d}_2 = \Delta J \ddot{\theta} - \Delta K(q - \theta) + d_2 \end{cases} \quad (2)$$

Then, Equation (1) can be rewritten as

$$\begin{cases} \ddot{q} + \frac{K}{I}(q - \theta) + \frac{MgL}{I} \sin(q) + \frac{\bar{d}_1}{I} = 0 \\ \ddot{\theta} - \frac{K}{J}(q - \theta) + \frac{1}{J} \bar{d}_2 = \frac{1}{J} \tau \end{cases} \quad (3)$$

The states are defined as $x_1 = q - q_d$, $x_2 = \dot{q} - \dot{q}_d$, $x_3 = K\theta/I$, $x_4 = K\dot{\theta}/I$, and let the control input $u = \tau$, then the FJMCS can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + f_1(x_1) + \bar{\Delta}_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = bu + k_1 x_1 + k_2 x_3 + \bar{\Delta}_2 \end{cases} \quad (4)$$

Where

$$\begin{cases} f_1(x_1) = -\frac{MgL}{I} \sin(x_1 + q_d) - \frac{K(x_1 + q_d)}{I} - \ddot{q}_d \\ k_1 = \frac{K^2}{IJ}, k_2 = -\frac{K}{J} \\ b = \frac{K}{IJ}, \bar{\Delta}_1 = -\frac{\bar{d}_1}{I}, \bar{\Delta}_2 = -\frac{K}{IJ} \bar{d}_2 \end{cases} \quad (5)$$

The control objective is to design the control input u such that the tracking error x_1 approaches to zero, while ensuring that all closed loop signals are bounded.

2.2 Problem Description and Purposes of this paper

For the traditional control scheme for FJM in [17-21, 24], the link states $q, \dot{q}$ and the motor states $\theta, \dot{\theta}$ are both assumed to be measurable, but the RRM controllers only need the link states q and $\dot{q}$. Thus, the measurement system for the controller in [17-21, 24] is much more complex than the measurement system for RRM. This motivates the first research topic of this paper, that is,

designing a control scheme which only needs the same measurement states as RRM to simplify the measurement system of FJM. Thus, we have the following assumption:

Assumption 1: For the FJM system (1), q and $\dot{q}$ are measurable, θ and $\dot{\theta}$ are unmeasurable.

To solve the unmeasurable problem, some schemes in [26-28] have adopted an observer to estimate unmeasurable states. And a high observer gain should be chosen to guarantee the observation precision. However, an undesirable spike problem may be caused by the high observer gain (see [29-30]). To explain the spike problem, the high-gain observer in [28] can be give as following:

$$\dot{\hat{y}} = A_y \hat{y} + k(\hat{y}_1 - y_1) \quad (6)$$

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix}, A_y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, k = \begin{bmatrix} -2k_0 \\ -k_0^2 \end{bmatrix} \quad (7)$$

where y_1 denotes the angle state, $\hat{y}_1$ and $\hat{y}_2$ are the estimate angle state and the estimate angular velocity state, respectively. k_0 is the observer gain.

Let estimate errors $\bar{y}_1 = \hat{y}_1 - y_1$, $\bar{y}_2 = \hat{y}_2 - \dot{y}_1$, we have

$$\bar{y} = A_y \bar{y} + \delta \quad (8)$$

$$\bar{y} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix}, A_y = \begin{bmatrix} -2k_0 & 1 \\ -k_0^2 & 0 \end{bmatrix}, \delta = \begin{bmatrix} 0 \\ \ddot{y}_1 \end{bmatrix} \quad (9)$$

The solutions for the differential equation (10) can be easily obtained as

$$\bar{y}(t) = e^{A_y t} \bar{y}(0) + \int_0^t e^{A_y(t-v)} \ddot{y}_1 dv \quad (10)$$

where e is the e constant. t is the time and the initial time is 0 in this paper. Since the eigenvalues of A_y both are $\lambda(A_y) = -k_0$, for the small time $t = 1/k_0$, $e^{A_y t} \bar{y}(0)$ can be rewritten as

$$e^{A_y/k_0} \bar{y}(0) = \begin{bmatrix} e^{-1} \bar{y}_2(0) / k_0 \\ -k_0 e^{-1} \bar{y}_1(0) + 2e^{-1} \bar{y}_2(0) \end{bmatrix} \quad (11)$$

It can be known that $k_0 e^{-1} \bar{y}_1(0)$ is a very large value if k_0 is a high gain and the initial estimate state error $\bar{y}_1(0) = \hat{y}_1 - y_1 \neq 0$ is not equal to 0. The large values $k_0 e^{-1} \bar{y}_1(0)$ may lead to the saturation of control input and even the instability of system, this is the undesirable spike problem. Actually, the initial values of true state $y_1(0)$

cannot be known in advance for the most of cases. Thus, the initial estimate state error $\bar{y}_1(0)$ maybe not equal to 0 and even a large value. If the initial value of true states $y_1(0)$ is unknown, to avoid the potential spike problem, the HGO-based controllers in [29-31] cannot adopt the enough high observer gain to guarantee the control precision. This motivates the second research topic of this paper, that is, design a new scheme not only can guarantee high precision, but also eliminate the undesirable spike problem in any cases.

On the other hand, from Equation (4), it is clear that FJMCS is a typical mismatched uncertain system. To suppress the matched and mismatched uncertainties, the conventional control schemes in [30,31] for FJMCS need an additional robust scheme. Thus, the structures of the controllers in [30,31] are very complex. This motivates the third research topic of this paper, that is, avoiding using additional robust scheme. The motivation of this paper is summarized as follows:

Motivation of This Paper: (1) To avoid using the unmeasurable states, the unknown states will be estimated by designing a HGO in following section 3.1. (2) To use the high observer gain, while avoiding the spike problem, a novel SSF will be proposed in the following section 3.2. (3) To suppress the uncertainties without using additional complex method, the proposed HGO are used to estimate not only the unmeasurable state but also the uncertainties.

3 Main Result

3.1 High-gain Observer

For the system (13), since θ and $\dot{\theta}$ are unmeasurable, the states x_3 and x_4 are unknown. Combining unknown states with uncertainties to construct the new states. The new states are defined as $p_1 = x_1$, $p_2 = x_2$, $p_3 = x_3 + \bar{\Delta}_1$, $p_4 = x_4 + \bar{\Delta}_1$ and the uncertainty $d_f = \bar{\Delta}_2 + \bar{\Delta}_1 - k_2 \bar{\Delta}_1$. Then the FJMCS (14) can be rewritten as

$$\begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = p_3 + f_1(p_1) \\ \dot{p}_3 = p_4 \\ \dot{p}_4 = bu + k_1 p_1 + k_2 p_3 + d_f \end{cases} \quad (12)$$

Since $p_1 = x_1$, the control objective can be equivalent to $p_1 \rightarrow 0$.

If the states p_3 and p_4 are known, the system (12) is a matched uncertain system. Let the matched uncertainty $d_f = p_5$, where p_5 is the extended state. The system (12)

can be extended as

$$\dot{p} = A_{p1}p + A_{p2}p + Bu + f_p + d \quad (13)$$

where

$$A_{p1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_{p2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ k_1 & 0 & k_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}, \quad f_p = \begin{bmatrix} 0 \\ f_1(p_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \\ 0 \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dot{d}_f \end{bmatrix} \quad (15)$$

The following assumption is needed:

Assumption 2: The uncertainty is bounded and satisfies $\|d_f\| \leq d_{\max}$ and $\|\dot{d}_f\| \leq \bar{d}_{\max}$, where $d_{\max}$ and $\bar{d}_{\max}$ are positive constants.

To satisfy the Assumption 1, we can know that the lumped uncertainties $\bar{d}_1$ and $\bar{d}_2$ in (1) should satisfy the condition: $\bar{d}_1^{(j)}$ and $\bar{d}_1^{(i)}$ are bounded, where $j = 0, 1, 2, 3$ and $i = 0, 1$.

Then a five-order HGO is proposed in the following theorem to estimate unknown states p_3 , p_4 and uncertainty d_f , simultaneously.

Theorem 1: For the system (12), a HESO is constructed as

$$\dot{\hat{p}} = A_{p1}\hat{p} + A_{p2}\hat{p} + Bu + f_p + C(p_1 - \hat{p}_1) \quad (16)$$

where

$$\hat{p} = [\hat{p}_1 \quad \hat{p}_2 \quad \hat{p}_3 \quad \hat{p}_4 \quad \hat{p}_5]^T \quad (17)$$

$$C = [5c_0 \quad 10c_0^2 \quad 10c_0^3 \quad 5c_0^4 \quad c_0^5]^T \quad (18)$$

If assumption 1 and 2 are valid and the observer gain satisfies $c_0 > \bar{k}$, where $\bar{k} = \|A_{p2}^T + A_{p2}\|/2$, then the estimate error $\bar{p} = p - \hat{p}$ will converge to following small region:

$$\|\bar{p}\| \leq \frac{\bar{d}_{\max}}{(c_0 - \bar{k})}, \text{ if } t \rightarrow \infty \quad (19)$$

Proof (Proof of Theorem 1): Define the estimate error vector as

$$\bar{p} = p - \hat{p} \quad (20)$$

$$\bar{p} = [\bar{p}_1 \quad \bar{p}_2 \quad \bar{p}_3 \quad \bar{p}_4 \quad \bar{p}_5]^T \quad (21)$$

Differentiating $\bar{p}$ gives

$$\dot{\bar{p}} = \dot{p} - \dot{\hat{p}} \quad (22)$$

Substituting (13) and (13) into (22) yields

$$\begin{aligned} \dot{\bar{p}} &= A_{p1}p + A_{p2}p + Bu + f_p + d - A_{p1}\hat{p} \\ &\quad - A_{p2}\hat{p} - Bu - f_p - C(p_1 - \hat{p}_1) \\ &= A_{p1}(p - \hat{p}) - C(p_1 - \hat{p}_1) + A_{p2}(p - \hat{p}) + d \\ &= A\bar{p} + A_{p2}\bar{p} + d \end{aligned} \quad (23)$$

where

$$A = \begin{bmatrix} -5c_0 & 1 & 0 & 0 & 0 \\ -10c_0^2 & 0 & 1 & 0 & 0 \\ -10c_0^3 & 0 & 0 & 1 & 0 \\ -5c_0^4 & 0 & 0 & 0 & 1 \\ -c_0^5 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

Construct Lyapunov function V_1 as

$$V_1 = \frac{1}{2} \bar{p}^T \bar{p} \quad (25)$$

Then calculating the time derivative of V_1 along the trajectories of Equation (23), we have

$$\begin{aligned} \dot{V}_1 &= \frac{1}{2} (\dot{\bar{p}}^T \bar{p} + \bar{p}^T \dot{\bar{p}}) \\ &= \frac{1}{2} \left((A\bar{p} + A_{p2}\bar{p} + d)^T \bar{p} + \bar{p}^T (A\bar{p} + A_{p2}\bar{p} + d) \right) \\ &= \frac{1}{2} (\bar{p}^T A^T \bar{p} + \bar{p}^T A_{p2}^T \bar{p} + d^T \bar{p} + \bar{p}^T A \bar{p} + \bar{p}^T A_{p2} \bar{p} + \bar{p}^T d) \\ &= \frac{1}{2} (\bar{p}^T (A^T + A) \bar{p} + \bar{p}^T (A_{p2}^T + A_{p2}) \bar{p} + d^T \bar{p} + \bar{p}^T d) \end{aligned} \quad (26)$$

Since the largest eigenvalue $\lambda_{\max}(A) = \lambda_{\max}(A^T) = -c_0$, then Equation (26) can be rewritten as

$$\dot{V}_1 \leq -c_0 \bar{p}^T \bar{p} + \|A_{p2}^T + A_{p2}\| \bar{p}^T \bar{p} + \|d\| \|\bar{p}\| \quad (27)$$

Since k_1 and k_2 are known system parameters, $\|A_{p2}^T + A_{p2}\|/2$ is known. Let $\bar{k} = \|A_{p2}^T + A_{p2}\|/2$, we have

$$\dot{V}_1 \leq -c_0 \bar{p}^T \bar{p} + \bar{k} \bar{p}^T \bar{p} + \|d\| \|\bar{p}\| \quad (28)$$

Since Assumption 2 is valid, Equation (26) can be rewritten as

$$\begin{aligned} \dot{V}_1 &\leq -c_0 \bar{p}^T \bar{p} + \bar{k} \bar{p}^T \bar{p} + \bar{d}_{\max} \|\bar{p}\| \\ &= -((c_0 - \bar{k}) \|\bar{p}\| - \bar{d}_{\max}) \|\bar{p}\| \end{aligned} \quad (29)$$

Let $c_0 - \bar{k} > 0$, then $\|\bar{p}\|$ is always bounded as

$$\|\bar{p}\| \leq p_{g\max}, \text{ if } t > 0 \quad (30)$$

where $p_{g\max}$ is a unknown positive constant.

From Equation (29), it can be known that $\dot{V}_1 < 0$ if $\|\bar{p}\| > \bar{d}_{\max}/(c_0 - \bar{k})$ and $\|\bar{p}\| \neq 0$, thus $\|\bar{p}\|$ will converge to following region:

$$\|\bar{p}\| \leq \frac{\bar{d}_{\max}}{(c_0 - \bar{k})}, \text{ if } t \rightarrow \infty \quad (31)$$

Remark 3.1: From Equation (31), it can be known that the observer gain c_0 should be chosen large enough to ensure the estimation error is small enough. Thus, the proposed observer (20) also is a typical HGO. The undesirable spike problem may be caused by the high observer gain as mentioned above in section 1.2. To solve this potential problem, a novel spike suppression function (SSF) will be proposed in the following control design.

3.2 Arm Dynamic Surface Controller Based on HGO and SSF

Based on the estimate states $\hat{p}_3$ and $\hat{p}_4$, and the estimation of uncertainty $\hat{p}_5$, a DSM control scheme for the FJMCS (12) will be proposed by following four backstepping steps. And a novel SSF is adopted in the proposed controller to deal with the aforementioned spike problem of HGO.

Step 1: Define a tracking error as $s_1 = p_1$, the virtual control of p_2 is designed as

$$p_{2d} = -\varepsilon_1 s_1 \quad (32)$$

Define a low-pass filter as

$$\tau_2 \dot{p}_{2c} + p_{2c} = p_{2d} \quad (33)$$

where ε_1 and τ_2 are positive constant.

Step 2: Define $s_2 = p_2 - p_{2c}$, the virtual control of p_3 is taken as

$$p_{3d} = -\varepsilon_2 s_2 - f_1(p_1) + \dot{p}_{2c} \quad (34)$$

Define a low-pass filter as

$$\tau_3 \dot{p}_{3c} + p_{3c} = p_{3d} \quad (35)$$

where ε_2 and τ_3 are positive constant.

Step 3: Define $s_3 = p_3 - p_{3c}$, the virtual control of p_4 is taken as

$$p_{4d} = -\varepsilon_3 (\hat{p}_3 - p_{3c}) + \dot{p}_{3c} \quad (36)$$

Define a low-pass filter as

$$\tau_4 \dot{p}_{4c} + p_{4c} = p_{4d} \quad (37)$$

where ε_3 and τ_4 are positive constant.

Step4: Define $s_4 = p_4 - p_{4c}$, the controller is designed as

$$u = b^{-1} [-\varepsilon_4 \chi_1(t) (\hat{p}_4 - p_{4c}) - k_1(p_1 + q_d) - k_2 \chi_2(t) \hat{p}_3 - k_2(1 - \chi_2(t)) p_{4c} - \chi_3(t) \hat{p}_5 + \chi_4(t) \dot{p}_{4c}] \quad (38)$$

where $\chi_i(t)$ ($i=1,2,3,4$) is the spike suppression function (SSF). The SSF $\chi_i(t)$ is defined as

$$\chi_i(t) = (1 - \kappa_i e^{-\eta_i t})^{\psi_i} \quad (39)$$

The constant parameters are chosen as $0 < \kappa_i < 1$, $\eta_i > 0$ and $\psi_i > 1$. Then, for $i=1,2,3,4$, it can be easily obtained that

$$\dot{\chi}_i(t) = \psi_i (1 - \kappa_i e^{-\eta_i t})^{\psi_i - 1} \kappa_i \eta_i e^{-\eta_i t} > 0 \quad (40)$$

$$0 \leq \chi_i(t) \leq 1 \quad (41)$$

Remark 3.2: As mentioned in section 2.2, the spike problem of HGO can be caused by a high observer gain c_0 if $p_1(0) \neq \hat{p}_1(0)$ in the initial small time period. From Equation (38), it is clear that the spike values of HGO can be taken into the control input by $\hat{p}_4$, p_{4c} , $\hat{p}_3$, $\hat{p}_5$, $\dot{p}_{4c}$ which are contain the outputs of HGO directly or indirectly. Thus, for the proposed controller (38), $\chi_1(t)$, $\chi_2(t)$, $\chi_3(t)$ and $\chi_4(t)$ are adopted to suppress the spike values of $\hat{p}_4 - p_{4c}$, $\hat{p}_3$, $\hat{p}_5$ and $\dot{p}_{4c}$. From Equation (39), it can be known that $\chi_i(t)$ is small enough in the initial small time

period by chosen the parameters reasonably, the spike values taken by $\hat{p}_4$, p_{4c} , $\hat{p}_3$, $\hat{p}_5$, $\dot{p}_{4c}$ can be suppressed by the proposed SSF.

Remark 3.3: Since $\chi_i(t) \rightarrow 1$ if $t \rightarrow \infty$, the stability and control precision of the proposed controller will not be affected by the SSF. The detail stability and control precision analysis of the proposed controller with SSF will be given in the Theorem 2.

Before giving the Theorem 2, the following assumption which is commonly used in DSM (such as in the section 3.2 in [22], the author assumed that the derivative of virtual control input is bounded as a positive constant) should be assumed to be valid throughout this paper:

Assumption 3: The vector $\mathbf{p}_f = [\dot{p}_{2d} \ \dot{p}_{3d} \ \dot{p}_{4d}]^T$ is bounded, and satisfies $\|\mathbf{p}_f\| \leq p_{f\max}$, where $p_{f\max}$ is a unknown positive constant.

Theorem 2: Consider the FMCS (12) with the controller (38), if assumption 1, 2 and 3 are valid, and the parameters are set reasonably, then the convergence of the tracking error s_j ($j=1,2,3,4$) to a small neighborhood of zero can be guaranteed.

Proof (Proof of Theorem 2): Define the errors of low-pass filter as

$$\begin{cases} e_2 = p_{2c} - p_{2d} \\ e_3 = p_{3c} - p_{3d} \\ e_4 = p_{4c} - p_{4d} \end{cases} \quad (42)$$

From Equations (33), (35) and (37), we have

$$\begin{cases} \dot{e}_2 = -\tau_2^{-1} e_2 - \dot{p}_{2d} \\ \dot{e}_3 = -\tau_3^{-1} e_3 - \dot{p}_{3d} \\ \dot{e}_4 = -\tau_4^{-1} e_4 - \dot{p}_{4d} \end{cases} \quad (43)$$

Then calculating the time derivative of s_j ($j=1,2,3,4$) along the trajectories of Equation (12), we get

$$\begin{cases} \dot{s}_1 = p_2 + p_{2d} - p_{2d} \\ \dot{s}_2 = p_3 + f_1(p_1) - \dot{p}_{2c} + p_{3d} - p_{3d} \\ \dot{s}_3 = p_4 - \dot{p}_{3c} + p_{4d} - p_{4d} \\ \dot{s}_4 = bu + k_1 p_1 + k_2 p_3 + d_f - \dot{p}_{4c} \end{cases} \quad (44)$$

Substituting Equations (32), (34), (36) and (38) into (44), we have

$$\begin{cases} \dot{s}_1 = -\varepsilon_1 s_1 + p_2 - p_{2d} \\ \dot{s}_2 = -\varepsilon_2 s_2 + p_3 - p_{3d} \\ \dot{s}_3 = -\varepsilon_3 (\hat{p}_3 - p_{3c}) + p_4 - p_{4d} \\ \dot{s}_4 = -\varepsilon_4 \chi_1(t) (\hat{p}_4 - p_{4c}) - k_2 \chi_2(t) \hat{p}_3 - k_2 (1 - \chi_2(t)) p_{3c} \\ \quad - \chi_3(t) \hat{p}_5 + \chi_4(t) \dot{p}_{4c} + k_2 p_3 + d_f - \dot{p}_{4c} \end{cases} \quad (45)$$

Then Equation (45) can be rewritten as

$$\begin{cases} \dot{s}_1 = -\varepsilon_1 s_1 + p_2 - p_{2c} + p_{2c} - p_{2d} \\ \dot{s}_2 = -\varepsilon_2 s_2 + p_3 - p_{3c} + p_{3c} - p_{3d} \\ \dot{s}_3 = -\varepsilon_3 (p_3 - p_{3c} - (p_3 - \hat{p}_3)) + p_4 - p_{4c} + p_{4c} - p_{4d} \\ \dot{s}_4 = -\varepsilon_4 \chi_1(t) ((p_4 - p_{4c}) - (p_4 - \hat{p}_4)) + k_2 \chi_2(t) \\ \quad (p_3 - \hat{p}_3) + k_2 (1 - \chi_2(t)) (p_3 - p_{3c}) + \chi_3(t) \\ \quad (d_f - \hat{p}_5) - (1 - \chi_4(t)) \dot{p}_{4c} + (1 - \chi_3(t)) d_f \end{cases} \quad (46)$$

Combining Equations (46) with (42) yields

$$\begin{cases} \dot{s}_1 = -\varepsilon_1 s_1 + s_2 + e_2 \\ \dot{s}_2 = -\varepsilon_2 s_2 + s_3 + e_3 \\ \dot{s}_3 = -\varepsilon_3 s_3 + \varepsilon_3 \bar{p}_3 + s_4 + e_4 \\ \dot{s}_4 = -\varepsilon_4 \chi_1(t) s_4 + \varepsilon_4 \chi_1(t) \bar{p}_4 + k_2 \chi_2(t) \bar{p}_3 + \\ \quad k_2 (1 - \chi_2(t)) s_3 + \chi_3(t) \bar{p}_5 - (1 - \chi_4(t)) \dot{p}_{4c} \\ \quad + (1 - \chi_3(t)) d_f \end{cases} \quad (47)$$

Construct Lyapunov function V_2 as

$$V_2 = \frac{1}{2} s^T s + \frac{1}{2} e^T e \quad (48)$$

where

$$s = [s_1 \quad s_2 \quad s_3 \quad s_4]^T \quad (49)$$

$$e = [e_2 \quad e_3 \quad e_4]^T \quad (50)$$

Calculating the time derivative of Equation (48) along the trajectories of Equations (43) and (47) yields

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2} (\dot{s}^T s + s^T \dot{s}) + \frac{1}{2} (\dot{e}^T e + e^T \dot{e}) \\ &= (\dot{s}_1 s_1 + \dot{s}_2 s_2 + \dot{s}_3 s_3 + \dot{s}_4 s_4) + (\dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4) \\ &= -\varepsilon_1 s_1^2 + s_1 s_2 + s_1 e_2 - \varepsilon_2 s_2^2 + s_2 s_3 + s_2 e_3 - \varepsilon_3 s_3^2 \\ &\quad + \varepsilon_3 s_3 \bar{p}_3 + s_3 s_4 + s_3 e_4 - \varepsilon_4 \chi_1(t) s_4^2 + \varepsilon_4 \chi_1(t) s_4 \bar{p}_4 \\ &\quad + k_2 \chi_2(t) s_4 \bar{p}_3 + \chi_3(t) s_4 \bar{p}_5 - \tau_2^{-1} e_2^2 - \dot{p}_{2d} e_2 - \tau_3^{-1} e_3^2 \\ &\quad - \dot{p}_{3d} e_3 - \tau_4^{-1} e_4^2 - \dot{p}_{4d} e_4 + k_2 (1 - \chi_2(t)) s_3 s_4 + \\ &\quad (1 - \chi_3(t)) d_f s_4 - (1 - \chi_4(t)) \dot{p}_{4c} s_4 \end{aligned} \quad (51)$$

Using Young's inequality [22-24], obtains

$$\begin{aligned} \dot{V}_2 &\leq -\varepsilon_1 s_1^2 + (s_1^2 + s_2^2)/2 + (s_1^2 + e_2^2)/2 - \varepsilon_2 s_2^2 + (s_2^2 + s_3^2)/2 + \\ &\quad (s_2^2 + e_3^2)/2 - \varepsilon_3 s_3^2 + \varepsilon_3 (s_3^2 + \bar{p}_3^2)/2 + (s_3^2 + s_4^2)/2 + (s_3^2 + e_4^2)/2 \\ &\quad - \varepsilon_4 \chi_1(t) s_4^2 + \varepsilon_4 \chi_1(t) (s_4^2 + \bar{p}_4^2)/2 + k_2 \chi_2(t) (s_4^2 + \bar{p}_3^2)/2 + \chi_3(t) \\ &\quad (s_4^2 + \bar{p}_5^2)/2 - \tau_2^{-1} e_2^2 + (\dot{p}_{2d}^2 + e_2^2)/2 - \tau_3^{-1} e_3^2 + (\dot{p}_{3d}^2 + e_3^2)/2 - \tau_4^{-1} e_4^2 \\ &\quad + (\dot{p}_{4d}^2 + e_4^2)/2 + k_2 (1 - \chi_2(t)) (s_3^2 + s_4^2)/2 + (1 - \chi_3(t)) (d_f^2 + s_4^2)/2 \\ &\quad + (1 - \chi_4(t)) (\dot{p}_{4c}^2 + s_4^2)/2 \\ &= -(\varepsilon_1 - 1) s_1^2 - (\varepsilon_2 - 3/2) s_2^2 - (\varepsilon_3/2 - 3/2 - k_2 (1 - \chi_2(t))/2) s_3^2 \\ &\quad - (\varepsilon_4 \chi_1(t)/2 - 3/2 + \chi_4(t)/2) s_4^2 - (\tau_2^{-1} - 1) e_2^2 - (\tau_3^{-1} - 1) e_3^2 - \\ &\quad (\tau_4^{-1} - 1) e_4^2 + (\varepsilon_3/2 + k_2 \chi_2(t)/2) \bar{p}_3^2 + \varepsilon_4 \chi_1(t) \bar{p}_4^2/2 + \chi_3(t) \bar{p}_5^2/2 + \\ &\quad \dot{p}_{2d}^2/2 + \dot{p}_{3d}^2/2 + \dot{p}_{4d}^2/2 + (1 - \chi_3(t)) d_f^2/2 + (1 - \chi_4(t)) \dot{p}_{4c}^2/2 \end{aligned} \quad (52)$$

Define the functions $f_3(t)$ and $f_4(t)$ as

$$\begin{aligned} f_3(t) &= \varepsilon_3/2 - 3/2 - k_2 (1 - \chi_2(t))/2 \\ &= \varepsilon_3/2 - 3/2 - k_2 \left(1 - (1 - \kappa_2 e^{-\eta_2 t})^{\eta_2}\right)/2 \end{aligned} \quad (53)$$

$$\begin{aligned} f_4(t) &= \varepsilon_4 \chi_1(t)/2 - 3/2 + \chi_4(t)/2 \\ &= \varepsilon_4 (1 - \kappa_1 e^{-\eta_1 t})^{\eta_1}/2 - 3/2 + (1 - \kappa_4 e^{-\eta_4 t})^{\eta_4}/2 \end{aligned} \quad (54)$$

Then we have

$$\begin{aligned} \dot{V}_2 &\leq -(\varepsilon_1 - 1) s_1^2 - (\varepsilon_2 - 3/2) s_2^2 - f_3(t) s_3^2 - f_4(t) s_4^2 \\ &\quad - (\tau_2^{-1} - 1) e_2^2 - (\tau_3^{-1} - 1) e_3^2 - (\tau_4^{-1} - 1) e_4^2 + \\ &\quad (\varepsilon_3/2 + k_2 \chi_2(t)/2) \bar{p}_3^2 + \varepsilon_4 \chi_1(t) \bar{p}_4^2/2 + \\ &\quad \chi_3(t) \bar{p}_5^2/2 + \dot{p}_{2d}^2/2 + \dot{p}_{3d}^2/2 + \dot{p}_{4d}^2/2 + \\ &\quad (1 - \chi_3(t)) d_f^2/2 + (1 - \chi_4(t)) \dot{p}_{4c}^2/2 \end{aligned} \quad (55)$$

Differentiating $f_3(t)$ and $f_4(t)$, we have

$$\dot{f}_3(t) = k_2 \psi_2 \left(1 - \kappa_2 e^{-\eta_2 t}\right)^{\psi_2 - 1} \kappa_2 \eta_2 e^{-\eta_2 t} / 2 \geq 0 \quad (56)$$

$$\begin{aligned} \dot{f}_4(t) &= \varepsilon_4 \psi_1 \left(1 - \kappa_1 e^{-\eta_1 t}\right)^{\psi_1 - 1} \kappa_1 \eta_1 e^{-\eta_1 t} / 2 + \\ &\quad \psi_4 \left(1 - \kappa_4 e^{-\eta_4 t}\right)^{\psi_4 - 1} \kappa_4 \eta_4 e^{-\eta_4 t} / 2 \geq 0 \end{aligned} \quad (57)$$

Then we have

$$-f_3(t) s_3^2 \leq -f_3(0) s_3^2 = -\left(\varepsilon_3 / 2 - 3 / 2 - k_2 \left(1 - (1 - \kappa_2)^{\psi_2}\right) / 2\right) s_3^2 \quad (58)$$

$$-f_4(t) s_4^2 \leq -f_4(0) s_4^2 = -\left(\varepsilon_4 \left(1 - \kappa_1\right)^{\psi_1} / 2 - 3 / 2 + \left(1 - \kappa_4\right)^{\psi_4} / 2\right) s_4^2 \quad (59)$$

Construct Lyapunov function V_3 as

$$V_3 = \dot{p}_{4c}^2 / 2 \quad (60)$$

Calculating the time derivative of V_3 along the trajectories of (37) and considering the Assumption 2 is valid, yields

$$\begin{aligned} \dot{V}_3 &= \dot{p}_{4c} \ddot{p}_{4c} \\ &= \dot{p}_{4c} \tau_4^{-1} (\dot{p}_{4d} - \dot{p}_{4c}) \\ &\leq -\tau_4^{-1} |\dot{p}_{4c}|^2 + |\dot{p}_{4c}| p_{f \max} \\ &= \tau_4^{-1} |\dot{p}_{4c}| (-|\dot{p}_{4c}| + \tau_4 p_{f \max}) \end{aligned} \quad (61)$$

From (61), it is clear that $\dot{V}_3 < 0$ if $-|\dot{p}_{4c}| + \tau_4 p_{f \max} < 0$, then we have

$$|\dot{p}_{4c}| \leq \max\{\dot{p}_{4c}(0), \tau_4 p_{f \max}\}, \text{ if } t > 0 \quad (62)$$

$$|\dot{p}_{4c}| \leq \tau_4 p_{f \max}, \text{ if } t \rightarrow \infty \quad (63)$$

From Equation (34), it can be obtained that $\bar{p}_3$, $\bar{p}_4$ and $\bar{p}_5$ are bounded. And Considering the Assumption 1, Assumption 2, Equations (41) and (62) are valid, we have

$$\begin{aligned} &(\varepsilon_3 / 2 + k_2 \chi_2(t) / 2) \bar{p}_3^2 + \varepsilon_4 \chi_1(t) \bar{p}_4^2 / 2 + \\ &\chi_3(t) \bar{p}_5^2 / 2 + \dot{p}_{2d}^2 / 2 + \dot{p}_{3d}^2 / 2 + \dot{p}_{4d}^2 / 2 + \\ &(1 - \chi_3(t)) d_f^2 / 2 + (1 - \chi_4(t)) \dot{p}_{4c}^2 / 2 \leq c_2, \text{ if } t > 0 \end{aligned} \quad (64)$$

where c_1 is a unknown positive constant.

Define the constants ϕ_1 , ϕ_2 and $\phi_{1\min}$ as

$$\begin{cases} \phi_1 = 2 \min\{\varepsilon_1 - 1, \varepsilon_2 - 2, f_3(0), f_4(0)\} \\ \phi_2 = 2 \min\{\tau_2^{-1} - 1, \tau_3^{-1} - 1, \tau_3^{-1} - 1\} \\ \phi_{1\min} = \min\{\phi_1, \phi_2\} \end{cases} \quad (65)$$

where $\min\{\bullet\}$ is the minimum function.

From Equations (58), (59), (64) and (65), Equation (55) can be rewritten as

$$\begin{aligned} \dot{V}_2 &\leq -\phi_{1\min} \left(\left(\frac{1}{2} s^T s \right) + \left(\frac{1}{2} e^T e \right) \right) + c_1 + c_2 \\ &= -\phi_{1\min} V_2 + c_1 + c_2 \end{aligned} \quad (66)$$

From Equation (62), we have

$$V_2 \leq V_2(0) e^{-\phi_{1\min} t} + (c_1 + c_2) (1 - e^{-\phi_{1\min} t}) / \phi_{1\min}, \text{ if } t > 0 \quad (67)$$

Since $\phi_{1\min}$ is a bounded constant, it can be known from Equation (67) that $V_2(t_k)$ is bounded for an arbitrary finite time $t = t_k$.

For the finite time $t = t_k$, the parameters are chosen as

$$\begin{cases} \varepsilon_1 - 1 > \phi_3 / 2, \varepsilon_2 - 2 > \phi_3 / 2 \\ f_3(t_k) > \phi_3 / 2, f_4(t_k) > \phi_3 / 2 \\ \tau_2^{-1} - 1 > \phi_4 / 2, \tau_3^{-1} - 1 > \phi_4 / 2, \tau_4^{-1} - 1 > \phi_4 / 2 \end{cases} \quad (68)$$

Where the constants $\phi_3 > 0$ and $\phi_4 > 0$. Define the constants $\phi_{2\min}$ as

$$\phi_{2\min} = \min\{\phi_3, \phi_4\} > 0 \quad (69)$$

Considering Equations (56), (57) and (68), Equation (55) can be rewritten as

$$\begin{aligned} \dot{V}_2 &\leq -\phi_{2\min} \left(\frac{1}{2} s^T s + \frac{1}{2} e^T e \right) + (\varepsilon_3 / 2 + k_2 \chi_2(t) / 2) \bar{p}_3^2 \\ &\quad + \varepsilon_4 \chi_1(t) \bar{p}_4^2 / 2 + \chi_3(t) \bar{p}_5^2 / 2 + \dot{p}_{2d}^2 / 2 + \dot{p}_{3d}^2 / 2 \\ &\quad + \dot{p}_{4d}^2 / 2 + (1 - \chi_3(t)) d_f^2 / 2 + (1 - \chi_4(t)) \dot{p}_{4c}^2 / 2, \text{ if } t \geq t_k \end{aligned} \quad (70)$$

Define the constants $\phi_{3\max}$ as

$$\phi_{3\max} = \max\{\varepsilon_3 / 2 + k_2 \chi_2(t) / 2, \varepsilon_4 \chi_1(t) / 2, \chi_3(t) / 2\} \quad (71)$$

Then Equation (70) can be rewritten as

$$\dot{V}_2 \leq -\phi_{2\min} \left(\frac{1}{2} s^T s + \frac{1}{2} e^T e \right) + \phi_{3\max} \|\bar{p}\| + \|p_f\| / 2 + (1 - \chi_3(t)) d_f^2 / 2 + (1 - \chi_4(t)) \dot{p}_{4c}^2 / 2, \text{ if } t \geq t_k \quad (72)$$

From Equation (31), (41), Assumption 1, Assumption 2 and Equation (72) can be rewritten as

$$\dot{V}_2 \leq -\phi_{2\min} V_2 + \phi_{3\max} \|\bar{p}\| + p_{f\max} / 2 + (1 - \chi_3(t)) d_{\max}^2 / 2 + (1 - \chi_4(t)) (\tau_4 p_{f\max})^2 / 2, \text{ if } t \geq t_k \quad (73)$$

Then we have

$$\dot{V}_2 \leq -\phi_{2\min} (V_2 - \bar{V}(t)), \text{ if } t \geq t_k \quad (74)$$

where

$$\bar{V} = \left(\phi_{3\max} \|\bar{p}\| + p_{f\max} / 2 + (1 - \chi_3(t)) d_{\max}^2 / 2 + (1 - \chi_4(t)) (\tau_4 p_{f\max})^2 / 2 \right) / \phi_{2\min}$$

Equation (74) implies that $\dot{V}_2 < 0$ if $V_2 > \bar{V}$ and $t \geq t_k$. Considering $V_2(t_k)$ is bounded, it can be known that V_2 will converge to following region if $t \rightarrow \infty$:

$$V_2 < \bar{V}, \text{ if } t \rightarrow \infty \quad (75)$$

Considering $\chi_i(t) \rightarrow 1 (i=1,2,3,4)$ if $t \rightarrow \infty$ and Equation (31) is valid, then we have

$$\bar{V} = (\phi_{3\max} \bar{d}_{\max} / (c_0 - \bar{k}) + p_{f\max} / 2) / \phi_{2\min}, \text{ if } t \rightarrow \infty \quad (76)$$

Combining Equation (75) with Equation (68), then we have

$$V_2 < (\phi_{3\max} \bar{d}_{\max} / (c_0 - \bar{k}) + p_{f\max} / 2) / \phi_{2\min}, \text{ if } t \rightarrow \infty \quad (77)$$

If large enough c_0 is chosen, then $d_{\max} / (c_0 - \bar{k})$ is small enough. If small enough $\tau_i (i=2,3,4)$ and large enough $\varepsilon_i (i=2,3,4)$ are chosen, then $\phi_{2\min}$ is large enough (see Equation (68)). And $p_{f\max}$ is constants. Thus, it can be known from (77) that the bounded range can be made arbitrarily small through choosing large enough c_0 and small enough $\tau_i (i=2,3,4)$.

Remark 3.4: From the proof of Theorem 2, it is clear that the stability and control precision are not affected by the

proposed SSF.

Remark 3.5: From the above research in section 3.1 and 3.2, the three purposes of this paper mentioned in section 2.2 are achieved: 1) From (38), it is clear that the proposed controller only needs the same measurement states as RRM; 2) By designing a novel SSF, the spike problem can be solve, and the control effect also will be discussed in the following simulation section; 3) Since HGO (16) estimates not only the unmeasurable state $p_i (i=1,2,3,4)$ but also the uncertainty d_f , the roust of controller can be guaranteed without using additional complex method.

4 Simulation Results

In this section, to illustrate the effectiveness of the proposed control scheme, the mathematical simulation is presented. Consider the parameters for FJMCS system (1) are selected as

$$MgL = 5, I = 1, J = 1, K = 15 \quad (78)$$

The initial states are set as $[q \ \dot{q} \ \theta \ \dot{\theta}]^T = [0 \ 0 \ 0 \ 0]^T$. In addition, the maximum limit of the control input is selected as $|u| \leq 500\text{Nm}$.

Case1 (The effectiveness of SSF): This case shows that the effectiveness of the proposed SSF. The unmodeled dynamics and the disturbance from the environment are chosen as

$$\Delta I = 0.1I, \Delta K = 0.1K, \Delta MgL = 0.1MgL, d_1 = d_2 = 0.1 \sin(t/3) \quad (79)$$

The parameters of controller (38) are chosen as

$$\varepsilon_1 = 2, \varepsilon_2 = 2, \varepsilon_3 = 7, \varepsilon_4 = 12, \tau_2 = \tau_3 = \tau_4 = 0.01 \quad (80)$$

The desired rotation angle of link is chosen as

$$q_d = 1 \text{ rad} \quad (81)$$

To ensure the estimation error is small enough, a high observer gain of HGO is selected as

$$\omega_0 = 100 \quad (82)$$

The initial value of HGO is set as

$$[\hat{p}_1 \ \hat{p}_2 \ \hat{p}_3 \ \hat{p}_4 \ \hat{p}_5]^T = [0 \ 0 \ 0 \ 0 \ 0]^T \quad (83)$$

It is clear that $\hat{p}_1(0) \neq p_1(0)$. Consider the following two control schemes in Case 1:

(1) controller (38) without SSF: $\chi_i(t)=0, i=1,2,3,4$.

(2) controller (38) with SSF :
 $\chi_i(t)=(1-0.97e^{-4.5t})^5, i=1,2,3,4$.

Figures 2~9 show the simulation results for case1. From Figures 2, for the controller without SSF, it is clear that the tracking error converges slowly and an undesirable large overshoot phenomenon can be observed. As mentioned before, the reason is that the spike problem of HGO can be caused by choosing a large gain if $p_1(0) \neq \hat{p}_1(0)$. And the spike values from HGO can be taken into the control input, directly or indirectly. The spike values from HGO can be observed from Figures 4~8. And Figures 3 shows that the control input of the controller without SSF is saturated under the spike value. From Figures 2~3, it is also clear that the spike values can be suppressed effectively by the proposed controller with SSF. An excellent control performance still can be guaranteed by the proposed controller with SSF. From Figure 9, it can be seen that the SSF $\chi_i(t)$ is small enough in the initial time, thus the spike values can be suppressed. And $\chi_i(t) \rightarrow 1$ if the time is large enough, then control precision will not be affected by the SSF.

Figure 2 Tracking error p_1 (Case 1)

Figure 3 Control input u (Case 1)

Figure 4 Estimate error $\bar{p}_1$ (Case 1)

Figure 5 Estimate error $\bar{p}_2$ (Case 1)

Figure 6 Estimate error $\bar{p}_3$ (Case 1)

Figure 7 Estimate error $\bar{p}_4$ (Case 1)

Figure 8 Estimate error $\bar{p}_5$ (Case 1)

Figure 9 SSF $\chi_i(t)$

Case2 (The control performance of partial states feedback): To shows the performance of the proposed partial states feedback scheme, the following two controllers are compared in this case:

(1) Proposed partial state feedback controller (PSFC): Like case 1, the expression of controller is given in (38). The parameters of PSFC and observer are given in (80) and (82). And the SSF also is chosen as $\chi_i(t) = (1 - 0.97e^{-4.5t})^5$, $i = 1, 2, 3, 4$.

(2) Full state feedback controller (FSFC): It is assumed that all states are measurable, which means that it is unnecessary to design the observer to estimate the unmeasurable states. Then, only based on the conventional DSM [22-24], the FSFC for FJMCS (4) can be designed as

$$\begin{cases} u = b^{-1}(-\varepsilon_{x4}(x_4 - x_{4c}) - k_1(x_1 + q_d) - k_2x_3 - \sigma_2 \text{sigmf}(x_4 - x_{4c}) + \dot{x}_{4c}) \\ x_{2d} = -\varepsilon_{x1}(x_1 - q_d) \\ \tau_{x2}\dot{x}_{2c} + x_{2c} = x_{2d} \\ x_{3d} = -\varepsilon_{x2}(x_2 - x_{2c}) - f_1(x_1) + \dot{x}_{2c} - \sigma_1 \text{sigmf}(x_2 - x_{2c}) \\ \tau_{x3}\dot{x}_{3c} + x_{3c} = x_{3d} \\ x_{4d} = -\varepsilon_{x3}(x_3 - x_{3c}) + \dot{x}_{3c} \\ \tau_{x4}\dot{x}_{4c} + x_{4c} = x_{4d} \\ \text{sigmf}(x) = 2\left(\frac{1}{1 + e^{-20x}} - \frac{1}{2}\right) \end{cases} \quad (84)$$

Where the parameters are chosen as

$$\begin{aligned} \varepsilon_{x1} = 2, \varepsilon_{x2} = 2, \varepsilon_{x3} = 7, \varepsilon_{x4} = 12, \tau_{x2} = 0.1, \\ \tau_{x3} = 0.05, \tau_{x4} = 0.03, \sigma_1 = 1, \sigma_2 = 400 \end{aligned} \quad (85)$$

And $\text{sigmf}(\bullet)$ is used to avoid the chattering problem and suppress the uncertainties. The explanation of $\text{sigmf}(\bullet)$ can be found in previous work³⁵. The unmodeled dynamics and the disturbance from the environment are chosen as in (79).

The above PSFC and FSFC are used for tracking two kinds of desired angles $q_d = 1$ rad and $q_d = \cos(t)$ rad, respectively. The simulation results for case 2 are shown in the Figures 10~13. From the simulation results, it is clear that both the conventional FSFC and the proposed PSFC can guarantee the fast and accurate convergence of tracking error for the two kinds of desired angles. Thus, compared with the conventional full states feedback scheme, the proposed method of this paper can achieve a similar excellent control performance even with partial states feedback.

Figure 10 Tracking error p_1 (case 2: $q_d = 1$ rad)

Figure 11 Control input u (case 2: $q_d = 1$ rad)

Figure 12 Tracking error p_1 (case 2: $q_d = \cos(t)$ rad)

Figure 13 Control output u (case 2: $q_d = \cos(t)$ rad)

Case3 (The anti-disturbance performance of the proposed partial states feedback scheme): To show the anti-disturbance performance of the proposed scheme, the following three kinds of unmodeled dynamics and disturbances from the environment are chosen as:

(1) Small amplitude and low frequency
 $\Delta I = 0.05I, \Delta K = 0.05K, \Delta MgL = 0.05MgL,$
 $d_1 = d_2 = 0.1\sin(t/6)$ (86)

(2) Large amplitude and low frequency
 $\Delta I = 0.3I, \Delta K = 0.3K, \Delta MgL = 0.3MgL,$
 $d_1 = d_2 = \sin(t/6)$ (87)

(3) Large amplitude and high frequency
 $\Delta I = 0.3\sin(t)I, \Delta K = 0.3\sin(t)K,$
 $\Delta MgL = 0.3\sin(t)MgL, d_1 = d_2 = \sin(t)$ (88)

The parameters of proposed PSFC are given in Equations (84) and (86). And the SSF also is chosen as $\chi_i(t) = (1 - 0.97e^{-4.5t})^5, i = 1, 2, 3, 4$.

Figures 14~15 show the simulation results for case3. From Figures 14 and 15, It is clear that the proposed PSFC all can achieve a good anti-disturbance performance for the three kinds of unmodeled dynamics and disturbances.

Figure 14 Tracking error p_1

Figure 15 Control input u

5 Conclusions

In this paper, a novel PSFC is proposed for the FJM with unmeasurable system states based on DSM and HGO. The main contributions here are that:

- 1) The proposed method only needs the same measurement states as conventional RRM;
- 2) The estimation spike problem in the existing HGO-based control is solved by developing a novel SSF;
- 3) Without using additional complex robust method which is widely used in the conventional PSFC schemes, the proposed control scheme still can suppress the uncertainties by using HGO to estimate the unmeasurable states and uncertainties simultaneously. Finally, mathematical simulation result demonstrated that the performances of the proposed controller is excellent.

6 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Competing interests

The authors declare no competing financial interests.

Consent for publication

Not applicable

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Not applicable

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Figures

Figure 1

Structure of flexible manipulator

Figure 2

Tracking error P1 (Case 1)

Figure 3

Control input u (Case 1)

Figure 4

Estimate error P1 (Case 1)

Figure 5

Estimate error P2 (Case 1)

Figure 6

Estimate error P3 (Case 1)

Figure 7

Estimate error P4 (Case 1)

Figure 8

Estimate error P5 (Case 1)

Figure 9

SSF $\chi_i(t)$

Figure 10

Tracking error P1 (case 2: $q_d = 1$ rad)

Figure 11

Control input u (case 2: $q_d = 1$ rad)

Figure 12

Tracking error p_1 (case 2: $q_d = \cos(t)$ rad)

Figure 13

Control output u (case 2: $q_d = \cos(t)$ rad)

Figure 14

Tracking error P1

Figure 15

Control input u