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Machining accuracy enhancement of five-axis machine tools by a new sensitivity analysis method based on machining error model

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Abstract: Identification of key geometric errors is an essential prerequisite for improving the machining accuracy of five-axis machine tools. This paper presents a new sensitivity analysis (SA) method to extract key geometric errors, and then to improve the machining performance of machine tools by compensating key geometric error components. Development of geometric error prediction model is involved to obtain geometric error values at arbitrary positions at first. Based on the multi-body system theory and flank milling theory, the machining error prediction model is developed, which considers 37 geometric errors. Then, a new SA method is introduced by taking the machining error model as sensitivity analysis model and taking the geometric errors as analytical factors. Meanwhile, a sensitivity index, which has the characteristics of simple expression and clear physical meaning, is proposed, i.e., the peak value of the machining error caused by each geometric error. Moreover, the simulations analysis is carried out to obtain the sensitivity coefficient of each geometric error and the key error components. Finally, the validity and correctness of the proposed method is demonstrated by the experiments. Furthermore, the SA method can be extended to multi-axis machine tools.

Key words: Five-axis machine tools; Machining error model; Key geometric errors; Sensitivity analysis; Sensitivity index

1. Introduction

NC machine tools, as mother machine in the industrial field, have become the embodiment of machinery manufacturing capability and development level for a country [1-3]. The machining accuracy of NC machine tool is an important index to measure machine tools' performance and an important symbol of the country's technological level. Thus, how to improve the machining accuracy of machine tools has become the focus of research. Until now, common methods for improving the performance of machine tools include accuracy optimization design and error compensation.

Accuracy optimization design is a way to improve the machining accuracy by reasonably distributing geometric errors to the key parts of machine tools [4-8]. Due to the degree of influence for each geometric error on machining accuracy has significant differences. If the geometric errors are assigned based on design experience, it will cause the distortion of the optimization results. Therefore, reasonable distribution of weights to each geometric error is the prerequisite for the accuracy optimization design. In addition, error compensation is another way to improve the machining accuracy by compensating the key geometric errors [9-13]. In economic terms, it is unreasonable to compensate all geometric errors. Only the key geometric errors been compensated is a cost-effective manner to improve the machining accuracy. Therefore, it is an important issue to obtain the weight coefficient of each geometric error and make sure which vital geometric errors need to be compensated.

As the most effective way to quantify the influence degree of geometric error on the machining accuracy and extract the key error components, the sensitivity analysis (SA) has been widely applied and developed at present. In the last several decades, the abundant intensive research works about SA have

been carried out.

Niu et al. [14] employed the form of sequential multiplication to establish the global SA model of error components. The influence of each error component on machining precision in a certain stroke was discussed. Wu et al. [15] selected F-value results and the Euclidean norms of parametric test results as sensitivity coefficients to identify the key geometric errors. The identified results provide the references for the accuracy design of machine tools. Based on the error mapping Jacobian matrix, a series of sensitivity indices were presented in [16] to trace 10 key error components and 10 trivial error components. Yang et al. [17] utilized the 3D geometric sensitivity analysis to obtain the influences of the 11 position-independent geometric errors on each of the measurement paths. The Monte Carlo method and kriging were integrated in [18] to develop a SA method for milling accuracy. And then, the impact of each parameter distribution on the milling accuracy was quantified. The local and global SA of reliability was introduced in [19]. The sensitivity coefficients were calculated by taking only one machining point as an example to determine the key error components. Fang et al. [20] developed a new SA model by utilizing the Lie theory. Due to the units of angular error and linear error are different, the sensitivity coefficients of two kinds errors were calculated and compared respectively. The key geometric errors were identified, which applied in accuracy design and error compensation. Zhang et al. [21] established the machining error model based on homogeneous transformation matrices to describe the impact of deformation error on the machining error. Then, the sensitivity coefficient of deformation errors is calculated based on the machining error model. The SA of 37 geometric errors was presented in [22]. The position errors and angular errors were set as fixed value, i.e., $0.1\ \mu\text{m}$ and $0.1\ \mu\text{rad}$ respectively. And the sensitivity coefficients were calculated by taking only three machining points as an example to determine the key error components. Yao et al. [23] taken the derivatives of volumetric error as the sensitivity coefficient of each geometric error. Simultaneity, for computational expediency, assuming all the position errors and angular errors are set as fixed value, i.e., $10\ \mu\text{m}$ and $5''$ respectively. The machining performance of machine tools was improved by reducing the identified key error components. Guo et al. [24] proposed the Morris global sensitivity analysis (GSA) method, and the absolute mean values and the standard deviation are utilized to calculate the sensitivity coefficients of the error parameters. The SA results can provide guidance for error compensation. Tian et al. [25] developed the probabilistic sensitivity analysis method and defined global sensitivity index to extract the key error components. Extended Fourier amplitude sensitivity test method was utilized in [26] to establish GSA model. The crucial error components were identified based on the GSA model, and the machining accuracy of CNC machine tools was improved by compensating crucial error components.

Although the above-mentioned researches have obtained some positive achievements, the existing researches are plagued with the following problems:

- (1) Some of these SA methods based on the assumption that all the angular errors and linear errors have the same value, which cannot reflect the key factor that the geometric errors vary with machining positions and can make the SA results inaccurate or even unreliable. Thus, the SA results failed to correctly guide the accuracy optimization design and error compensation of machine tools.
- (2) The units of angular error and linear error are different, which need to calculate its sensitivity separately. Hence, such scenario will cause these SA methods involve too much calculation.
- (3) The sensitivity index of each geometric error is defined unreasonable, which lead to these SA methods with poor universality.

To rectify the insufficiency of above-mentioned researches, this paper presents a new SA method to quantify the influence degree of each geometric error on the machining accuracy and extract the key

error components as outlined in the logical flowchart shown in Fig. 1. First, in order to effectively figure out each geometric error at any machining position, the geometric error prediction models are established. In addition, the machining error prediction model is developed based on the multi-body system theory and flank milling theory. Then, the sensitivity analysis model is established based on the machining error prediction model. For computational expediency, the peak value of the machining error caused by each geometric error is taken as the sensitivity index, which has the characteristics of simple expression and clear physical meaning. Meanwhile, the sensitivity of each geometric error and the key error components are obtained. Finally, an experiment is carried out to prove the validity and correctness of the proposed method.

Henceforth, the structure of this paper is organized as follows. Section 2 establishes the machining error model. In Section 3, the new SA method is developed to extract the key geometric errors. The simulation verification is conducted in Section 4. For demonstrating the correctness of the proposed method, the experimental validation is carried out in Section 5. Finally, some conclusions are given in Section 6.

Nomenclature

SA	Sensitivity analysis	${}^i_j T_p$	The ideal static transformation matrices between the adjacent bodies of machine tool
GSA	Global sensitivity analysis	${}^i_j T_{pe}$	The actual static transformation matrices between the adjacent bodies of machine tool
PIGEs	Position-independent geometric errors	${}^i_j T_s$	The ideal motion transformation matrices between the adjacent bodies of machine tool
PDGEs	Position-dependent geometric errors	${}^i_j T_{se}$	The actual motion transformation matrices between the adjacent bodies of machine tool
MBS	Multi-body system	$E_v (v = x, y, z)$	The volumetric error in the v direction
W	Workpiece	$e_v (v = x, y, z)$	The machining error in the v direction
T	Tool	$\mathbf{r}_{wjk}^i$	Ideal position vector of the tool center point corresponding to the k -th cutting point in the workpiece coordinate system
CMM	Coordinate measuring machine	$\mathbf{r}_{tjk}$	Position vector of the tool center point corresponding to the k -th cutting point in the tool coordinate system
S_i	Sensitivity	$\mathbf{n}_{sjk}$	Unit normal vector of the workpiece at the k -th cutting point
SN_i	The sensitivity coefficient	$\mathbf{r}_{wjk}$	Actual position vector of the tool center point corresponding to the k -th cutting point in the workpiece coordinate system
$\mathbf{G}$	The geometric error vector	$\mathbf{u}_{jk}$	Actual cutting point of machined workpiece
$\mathbf{D}$	Position vector of each moving axis	$\mathbf{H}$	Position vector of the body reference coordinate system origin of each body
l	Length of the cutting tool	$\mathbf{u}_{jk}^i$	Ideal cutting point of machined workpiece
$\mathbf{E}$	The volumetric error model	$\mathbf{r}_w$	Position vector of the tool center point in the workpiece coordinate system in the actual machining process
$\mathbf{e}_i$	Machining error caused by i -th geometric error	$\mathbf{r}_t$	Position vector of tool center point in tool coordinate system
d	The tool radius	$\mathbf{r}_w^i$	Position vector of the tool center point in the workpiece coordinate system in the ideal machining process
$-l_k$	The tool length corresponding to the k -th cutting point

Fig. 1 Flowchart of the sensitivity analysis of five-axis machine tool

2. Machining error modeling

2.1 Geometric error modeling

Geometric errors, which are caused by the manufacturing errors and assembly errors of machine tools' key parts, are one of the key influence factors for machine tools' machining accuracy. According to the ref. [27], a five-axis machine tool with three linear axes and two rotary axes has 37 geometric errors, which consisted of position-independent geometric errors (PIGEs) and position-dependent geometric errors (PDGEs), as listed in Table 1. Figs. 2-3 displays the schematic diagram of PDGEs for X-axis and C-axis respectively. Fig. 4 (a) shows the schematic diagram of PIGEs for linear axis, and (b)

represents PIGEs for rotary axis.

Due to the geometric errors vary with machining positions, assuming all the angular errors and linear errors have the same value will lead to the sensitivity analysis results with poor universality. Therefore, obtaining geometric error values at any position in the entire workspace is an essential step to do sensitivity analysis. Since the geometric error modeling method proposed in [28] has the merits of good universality, great efficiency, and stronger applicability, it was applied to calculate geometric error values in this research. Due to limited space, only the final modeling results of geometric errors for X-axis and C-axis are displayed in Eqs. (A1) and (A2) in Appendix A, respectively.

Fig. 2 Schematic diagram of PDGEs for a linear axis (X-axis)

Fig. 3 Schematic diagram of PDGEs for a rotary axis (C-axis)

Fig. 4 The schematic diagram of PIGEs: (a) linear axis, (b) rotary axis (C-axis).

Table 1

Geometric errors of the MAMT.

		Linear error			Angular error		
		Along X	Along Y	Along Z	Around X	Around Y	Around Z
PDGEs	X axis	$\delta_x(x)$	$\delta_y(x)$	$\delta_z(x)$	$\varepsilon_x(x)$	$\varepsilon_y(x)$	$\varepsilon_z(x)$
	Y axis	$\delta_x(y)$	$\delta_y(y)$	$\delta_z(y)$	$\varepsilon_x(y)$	$\varepsilon_y(y)$	$\varepsilon_z(y)$
	Z axis	$\delta_x(z)$	$\delta_y(z)$	$\delta_z(z)$	$\varepsilon_x(z)$	$\varepsilon_y(z)$	$\varepsilon_z(z)$
	C axis	$\delta_x(c)$	$\delta_y(c)$	$\delta_z(c)$	$\varepsilon_x(c)$	$\varepsilon_y(c)$	$\varepsilon_z(c)$
	B axis	$\delta_x(b)$	$\delta_y(b)$	$\delta_z(b)$	$\varepsilon_x(b)$	$\varepsilon_y(b)$	$\varepsilon_z(b)$
PIGEs					$\varepsilon_{yz}, \varepsilon_{yc}, \varepsilon_{bz}$	$\varepsilon_{xz}, \varepsilon_{xc}$	$\varepsilon_{xy}, \varepsilon_{xb}$

2.2 Volumetric error modeling

To date, the multi-body system (MBS) theory is the most famous and widely used volumetric error modeling method because of its higher applicability and generality [29-30]. In this research, a gantry type five-axis NC milling machine is taken as research objective to establish the volumetric error modeling, which consists of X-, Y-, Z-, B-, and C-axis, as displayed in Fig. 5. Based on MBS theory, the simplified analysis of gantry type five-axis NC milling machine is carried out. And the topological structure of the machine tool is established, which consists of tool branch and workpiece branch, as shown in Fig. 6. Then, the static and motion transformation matrices between the adjacent bodies of machine tool are established based on the homogeneous coordinate transformation, as shown in Tables 2-3 in Appendix B, respectively. The position vector of the body reference coordinate system origin of each body is listed in Table 4 in Appendix B.

In the actual machining process, the position vector of the tool center point in the workpiece coordinate system can be obtained, as shown in Eq. (1).

$$\mathbf{r}_w = \left({}^1T_p \ {}^1T_{pe} \ {}^1T_s \ {}^1T_{se} \right)^{-1} {}^1T_p \ {}^1T_{pe} \ {}^1T_s \ {}^1T_{se} \ {}^2T_p \ {}^2T_{pe} \ {}^2T_s \ {}^2T_{se} \ {}^3T_p \ {}^3T_{pe} \ {}^3T_s \ {}^3T_{se} \ {}^4T_p \ {}^4T_{pe} \ {}^4T_s \ {}^4T_{se} \ {}^5T_p \ {}^5T_{pe} \ {}^5T_s \ {}^5T_{se} \ {}^6T_p \ {}^6T_{pe} \ {}^6T_s \ {}^6T_{se} \mathbf{r}_t \quad (1)$$

where 1-6 denote the key parts of gantry type five-axis NC milling machine, as displayed in Fig. 5; w denotes the workpiece; t denotes the tool; i_jT_p and ${}^i_jT_{pe}$ denote the ideal and actual static transformation matrices between the adjacent bodies of machine tool; i_jT_s and ${}^i_jT_{se}$ denote the ideal and actual motion transformation matrices between the adjacent bodies of machine tool; $\mathbf{r}_t$ denotes position vector of tool center point in tool coordinate system, i.e., $\mathbf{r}_t = (0, 0, -l, 1)^T$, where l denotes the tool length.

In the ideal machining process, i.e., each geometric error value is zero, the position vector of the tool center point in the workpiece coordinate system can be obtained, as shown in Eq. (2).

$$\mathbf{r}_w^i = \left({}^1T_p \ {}^1T_s \right)^{-1} {}^1T_p \ {}^1T_s \ {}^2T_p \ {}^2T_s \ {}^3T_p \ {}^3T_s \ {}^4T_p \ {}^4T_s \ {}^5T_p \ {}^5T_s \ {}^6T_p \ {}^6T_s \mathbf{r}_t \quad (2)$$

Ultimately, based on MBS theory, the volumetric error model can be deduced by subtracting Eq. (2) from Eq. (1), as shown in Eq. (3).

$$\mathbf{E} = \left(E_x, E_y, E_z, 0 \right)^T = \mathbf{r}_w - \mathbf{r}_w^i \quad (3)$$

where E_v ($v = x, y, z$) denotes the volumetric error in the v direction.

By ignoring the high-order terms, the volumetric error model of gantry type five-axis NC milling machine is obtained.

Fig. 5 The structure diagram of the gantry type five-axis NC milling machine (1-machine bed, 2-X axis slide carriage, 3-Y axis slide carriage, 4-Z axis slide carriage, 5-C axis, 6-B axis, W-workpiece, T-cutting tool).

Fig. 6 The topological structure of gantry type five-axis NC milling machine

2.3 Machining error modeling

Flank milling, which processing the machined surface with the tool side edge, is a kind of processing method to process complex curved surface. The schematic diagram of flank milling is shown in Fig. 7. The machining error is determined by the normal distance between the ideal cutting surface and the actual cutting surface, as shown in Fig. 8.

According to Eq. (1) and Fig. 7, the actual position vector of the tool center point corresponding to the k -th cutting point in the workpiece coordinate system can be obtained, as shown in Eq. (4).

$$\mathbf{r}_{ijk} = \left({}^1T_p \quad {}^1T_{pe} \quad {}^1T_s \quad {}^1T_{se} \right)^{-1} {}^1T_p \quad {}^1T_{pe} \quad {}^1T_s \quad {}^1T_{se} \quad {}^2T_p \quad {}^2T_{pe} \quad {}^2T_s \quad {}^2T_{se} \quad {}^3T_p \quad {}^3T_{pe} \quad {}^3T_s \quad {}^3T_{se} \quad {}^4T_p \quad {}^4T_{pe} \quad {}^4T_s \quad {}^4T_{se} \quad {}^5T_p \quad {}^5T_{pe} \quad {}^5T_s \quad {}^5T_{se} \quad {}^6T_p \quad {}^6T_{pe} \quad {}^6T_s \quad {}^6T_{se} \quad {}^7T_p \quad {}^7T_{pe} \quad {}^7T_s \quad {}^7T_{se} \quad {}^8T_p \quad {}^8T_{pe} \quad {}^8T_s \quad {}^8T_{se} \quad {}^9T_p \quad {}^9T_{pe} \quad {}^9T_s \quad {}^9T_{se} \quad {}^{10}T_p \quad {}^{10}T_{pe} \quad {}^{10}T_s \quad {}^{10}T_{se} \quad {}^{11}T_p \quad {}^{11}T_{pe} \quad {}^{11}T_s \quad {}^{11}T_{se} \quad {}^{12}T_p \quad {}^{12}T_{pe} \quad {}^{12}T_s \quad {}^{12}T_{se} \quad {}^{13}T_p \quad {}^{13}T_{pe} \quad {}^{13}T_s \quad {}^{13}T_{se} \quad {}^{14}T_p \quad {}^{14}T_{pe} \quad {}^{14}T_s 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\quad {}^{68}T_{se} \quad {}^{69}T_p \quad {}^{69}T_{pe} \quad {}^{69}T_s \quad {}^{69}T_{se} \quad {}^{70}T_p \quad {}^{70}T_{pe} \quad {}^{70}T_s \quad {}^{70}T_{se} \quad {}^{71}T_p \quad {}^{71}T_{pe} \quad {}^{71}T_s \quad {}^{71}T_{se} \quad {}^{72}T_p \quad {}^{72}T_{pe} \quad {}^{72}T_s \quad {}^{72}T_{se} \quad {}^{73}T_p \quad {}^{73}T_{pe} \quad {}^{73}T_s \quad {}^{73}T_{se} \quad {}^{74}T_p \quad {}^{74}T_{pe} \quad {}^{74}T_s \quad {}^{74}T_{se} \quad {}^{75}T_p \quad {}^{75}T_{pe} \quad {}^{75}T_s \quad {}^{75}T_{se} \quad {}^{76}T_p \quad {}^{76}T_{pe} \quad {}^{76}T_s \quad {}^{76}T_{se} \quad {}^{77}T_p \quad {}^{77}T_{pe} \quad {}^{77}T_s \quad {}^{77}T_{se} \quad {}^{78}T_p \quad {}^{78}T_{pe} \quad {}^{78}T_s \quad {}^{78}T_{se} \quad {}^{79}T_p \quad {}^{79}T_{pe} \quad {}^{79}T_s \quad {}^{79}T_{se} \quad {}^{80}T_p \quad {}^{80}T_{pe} \quad {}^{80}T_s \quad {}^{80}T_{se} \quad {}^{81}T_p \quad {}^{81}T_{pe} \quad {}^{81}T_s \quad {}^{81}T_{se} \quad {}^{82}T_p \quad {}^{82}T_{pe} \quad {}^{82}T_s \quad {}^{82}T_{se} \quad {}^{83}T_p \quad {}^{83}T_{pe} \quad {}^{83}T_s \quad {}^{83}T_{se} \quad {}^{84}T_p \quad {}^{84}T_{pe} \quad {}^{84}T_s \quad {}^{84}T_{se} \quad {}^{85}T_p \quad {}^{85}T_{pe} \quad {}^{85}T_s \quad {}^{85}T_{se} \quad {}^{86}T_p \quad {}^{86}T_{pe} \quad {}^{86}T_s \quad {}^{86}T_{se} \quad {}^{87}T_p \quad {}^{87}T_{pe} \quad {}^{87}T_s \quad {}^{87}T_{se} \quad {}^{88}T_p \quad {}^{88}T_{pe} \quad {}^{88}T_s \quad {}^{88}T_{se} \quad {}^{89}T_p \quad {}^{89}T_{pe} \quad {}^{89}T_s \quad {}^{89}T_{se} \quad {}^{90}T_p \quad {}^{90}T_{pe} \quad {}^{90}T_s \quad {}^{90}T_{se} \quad {}^{91}T_p \quad {}^{91}T_{pe} \quad {}^{91}T_s \quad {}^{91}T_{se} \quad {}^{92}T_p \quad {}^{92}T_{pe} \quad {}^{92}T_s \quad {}^{92}T_{se} \quad {}^{93}T_p \quad {}^{93}T_{pe} \quad {}^{93}T_s \quad {}^{93}T_{se} \quad {}^{94}T_p \quad {}^{94}T_{pe} \quad {}^{94}T_s \quad {}^{94}T_{se} \quad {}^{95}T_p \quad {}^{95}T_{pe} \quad {}^{95}T_s \quad {}^{95}T_{se} \quad {}^{96}T_p \quad {}^{96}T_{pe} \quad {}^{96}T_s \quad {}^{96}T_{se} \quad {}^{97}T_p \quad {}^{97}T_{pe} \quad {}^{97}T_s \quad {}^{97}T_{se} \quad {}^{98}T_p \quad {}^{98}T_{pe} \quad {}^{98}T_s \quad {}^{98}T_{se} \quad {}^{99}T_p \quad {}^{99}T_{pe} \quad {}^{99}T_s \quad {}^{99}T_{se} \quad {}^{100}T_p \quad {}^{100}T_{pe} \quad {}^{100}T_s \quad {}^{100}T_{se} \quad \mathbf{r}_{ijk} \quad (4)$$

where $\mathbf{r}_{ijk}$ represents the position vector of the tool center point corresponding to the k -th cutting point in the tool coordinate system, $\mathbf{r}_{ijk} = (0, 0, -l_k, 1)^T$, $-l_k$ represents the tool length corresponding to the k -th cutting point, as shown in Fig. 7.

According to Eq. (2) and Fig. 7, the ideal position vector of the tool center point corresponding to the k -th cutting point in the workpiece coordinate system can be obtained, as shown in Eq. (5).

$$\mathbf{r}_{wjk}^i = \left({}^1T_p {}^1T_s \right)^{-1} {}^1T_p {}^1T_s {}^2T_p {}^2T_s {}^3T_p {}^3T_s {}^4T_p {}^4T_s {}^5T_p {}^5T_s {}^6T_p {}^6T_s \mathbf{r}_{tjk} \quad (5)$$

According to Eq. (4) and Fig. 8, actual cutting point of machined workpiece can be obtained by taking the point of actual tool axis trajectory plane offset the tool radius along the unit normal vector of the workpiece at the k -th cutting point, as shown in Eq. (6).

$$\mathbf{u}_{jk} = \mathbf{r}_{wjk} \left(d \mathbf{n}_{sjk} \right) \quad (6)$$

where d represents the tool radius; $\mathbf{n}_{sjk}$ represents unit normal vector of the workpiece at the k -th cutting point.

According to Eq. (5) and Fig. 8, ideal cutting point of machined workpiece can be obtained by taking the point of ideal tool axis trajectory plane offset the tool radius along the unit normal vector of the workpiece at the k -th cutting point, as shown in Eq. (7).

$$\mathbf{u}_{jk}^i = \mathbf{r}_{wjk}^i \left(d \mathbf{n}_{sjk} \right) \quad (7)$$

Fig. 7 The schematic diagram of flank milling

Fig. 8 The schematic diagram of cutting point offset

According to Fig. 9, the machining error can be deduced, as shown in Eq. (8).

$$\mathbf{e} = \mathbf{u}_{jk} - \mathbf{u}_{jk}^i \quad (8)$$

Eventually, the machining error model can be established by substituting Eqs. (6) and (7) into Eq. (8) as follows.

$$\mathbf{e} = (\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z, 0)^T \quad (9)$$

where $e_v (v = x, y, z)$ denotes the machining error in the v direction. Therefore, the machining error model of gantry type five-axis NC milling machine is obtained, as shown in Eq. (C) in Appendix C.

Fig. 9 The definition of machining errors

3. Sensitivity analysis of geometric error components

The influence degree of different geometric errors on machining errors are widely different. To quantify the influence degree of each geometric error on machining error and trace the key geometric errors, the sensitivity analysis of geometric error was carried out. However, the existing SA methods, as described in section 1, are plagued with the above-mentioned problems. In order to tackle the above deficiencies, this paper has developed a new SA method, which by taking the machining error model as sensitivity analysis model and taking the geometric errors as analytical factors, to trace the key geometric errors effectively.

A simple sensitivity index, which can truly reflect the influence degree of each geometric error on machining error of machine tools, is defined first. It is the prerequisite to trace the key geometric errors and has a decisive effect on the SA results. In this study, the peak value of the machining error caused by each geometric error is firstly proposed to represent the sensitivity index of geometric error. The greater the sensitivity of each geometric error, the greater the influence degree of each geometric error on machining error.

According to Eq. (9), the general form of machining error model can be established, as shown in Eq. (10).

$$\mathbf{e} = \mathbf{f}(\mathbf{G}, \mathbf{D}, \mathbf{H}, L) = (\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z, 0)^T \quad (10)$$

where $\mathbf{G}$ denotes the geometric error vector composed of n -term geometric errors of NC machine tools, $\mathbf{G} = (g_1, g_2, \dots, g_n)^T$, g_i denotes the i -th geometric error of NC machine tool, $i = 1, 2, \dots, n$; $\mathbf{D}$ denotes position vector of each moving axis of NC machine tool, $\mathbf{D} = (x, y, z, b, c)^T$; $\mathbf{H}$ denotes position vector of the body reference coordinate system origin of each body, $\mathbf{H} = (h_{jx}, h_{jy}, h_{jz}, 1)^T$, j denotes the j -th body of NC machine tool; L denotes the tool length.

According to Eq. (10), the machining error caused by each geometric error can be obtained as:

$$\mathbf{e}_i = \mathbf{f}(g_i, \mathbf{D}, \mathbf{H}, L) = (e_{ix}, e_{iy}, e_{iz}, 0)^T \quad (11)$$

where i represents the i -th geometric error, i.e., $i = 1, 2, 3, \dots, 37$.

Therefore, according to Eq. (11), the sensitivity of each geometric error can be defined as:

$$S_i = \max |e_i| = \max \left(\sqrt{(e_{ix})^2 + (e_{iy})^2 + (e_{iz})^2} \right) \quad (12)$$

In order to quantify the influence degree of geometric error on the machining accuracy and extract the key geometric errors more intuitively, the sensitivity coefficient of each geometric error is obtained by normalizing the sensitivity of each geometric error, which is shown as:

$$SN_i = \frac{|S_i|}{\sum_{i=1}^{37} |S_i|} \quad (13)$$

Therefore, the sum of the normalized sensitivity index of all the error components is 1, as shown in Eq. (14).

$$\sum_{i=1}^{37} SN_i = 1 \quad (14)$$

4. Simulations

To obtain the SA results, a simulation is carried out on the researched gantry type five-axis NC milling machine. First, the influence degree of each geometric error on machining error can be obtained by employing the machining error model (i.e., Eq. (11)). Then, the SA is carried out to extract the key geometric errors.

4.1 The influence degree of each geometric error on machining error

The “S” shaped test specimen, possesses the characteristics of the twist angle, open-close angle conversion, and variable curvature, is widely applied for characterization of complex curved surface. Therefore, the “S” shaped test specimen is selected as the machined workpiece in this research, and its profile error can really represent the machining error of machine tools.

To qualitatively analyze the influence degree of each geometric error on machining error for the researched machine tool, simulation analysis is conducted by MATLAB R2016b. The flowchart of the simulation is outlined in Fig. 10. And, the procedures to fulfill machining errors prediction are shown as follows:

(1) To obtain the NC machining instruction of “S” shaped test specimen. First, the 3D model of “S” shaped test specimen is built by UG NX10.0, as shown in Fig. 11(a). Then, the cutter location file is attained through the UG NX CAM processing module, as shown in Fig. 11(b). Finally, on the basis of the cutter location file, the NC machining instruction is obtained by the post processor of UG NX 10.0. Part of NC machining instruction codes are displayed in Fig. 11(c).

(2) To calculate the geometric errors. According to the BS ISO 8636-2:2007 [31], the tolerance parameters of the researched machine tools’ key parts are given first, as listed in Table 5. Then, on the basis of geometric error model established in Section 2.1, the geometric errors at arbitrary positions can be calculated.

(3) To predict the profile errors of “S” shaped test specimen by utilizing the machining error model and the NC machining instruction.

The key parameters of the researched machine tool are listed in Table 6. Based on the above-mentioned analysis, the influence law of each geometric error on machining error for the researched machine tool can be obtained. Due to limited space, only the influence law of X-axis geometric errors on machining errors are displayed in Fig. 12. From Fig. 12, it can be clearly seen that the influence law

changes with different geometric errors and machining positions. Thus, assuming all the angular errors and linear errors have the same value can make the SA results inaccurate or even unreliable.

Fig. 10 The flowchart of the simulation

Fig. 11 The flow chart of NC instruction code generation for "S" shaped test specimen

Table 5

Tolerance parameter values of the researched machine tools' key parts

Tolerance	Definition	Value
t_1	Straightness tolerance of X-axis guideway in XOZ plane	0.07mm
t_2	Straightness tolerance of X-axis guideway in XOY plane	0.08mm
t_3	Positioning tolerance of X-axis lead screw	0.006mm
t_4	Parallelism tolerance between X-axis guide rail pairs	0.05mm
t_5	Straightness tolerance of Y-axis guideway in YOZ plane	0.06mm
t_6	Straightness tolerance of Y-axis guideway in XOY plane	0.06mm
t_7	Positioning tolerance of Y-axis lead screw	0.005mm
t_8	Parallelism tolerance between Y-axis guideway pairs	0.04mm
t_9	Straightness tolerance of Z-axis guideway in XOZ plane	0.05mm
t_{10}	Straightness tolerance of Z-axis guideway in YOZ plane	0.04mm
t_{11}	Positioning tolerance of Z-axis lead screw	0.005mm
t_{12}	Parallelism tolerance between Z-axis guideway pairs	0.04mm
t_{13}	Tolerance of B-axis rotation angle	0.07/1000
t_{14}	Radial circular runout tolerance of B-axis bearing hole	0.008mm
t_{15}	Axial runout tolerance of B-axis	0.006mm
t_{16}	Tolerance of C-axis rotation angle	0.1/1000
t_{17}	Radial circular runout tolerance of C-axis bearing hole	0.01mm
t_{18}	Axial runout tolerance of C-axis	0.007mm
t_{19}	Perpendicularity tolerance between X-axis and Z-axis	0.01/1000
t_{20}	Perpendicularity tolerance between X-axis and Y-axis	0.02/1000
t_{21}	Perpendicularity tolerance between Y-axis and Z-axis	0.02/1000
t_{22}	Perpendicularity tolerance between X-axis and B-axis	0.03/500
t_{23}	Perpendicularity tolerance between Z-axis and B-axis	0.03/500
t_{24}	Perpendicularity tolerance between X-axis and C-axis	0.02/500
t_{25}	Perpendicularity tolerance between Y-axis and C-axis	0.02/500

Table 6

The key parameters of the researched machine tool

Key parameters	Value
Stroke of linear axis (L_x, L_y, L_z) /mm	6700, 3000, 1000
Stroke of rotation axis (R_c, R_b) /mm	$\pm 360^\circ, \pm 120^\circ$
Maximum speed of spindle /r · min ⁻¹	20000
Feed rate of linear axis /m · min ⁻¹	≥ 30
$h_{3x}, h_{4x}, h_{5x}, h_{6x}$ /mm	-200, -350, 0, 0
$h_{3y}, h_{4y}, h_{5y}, h_{6y}$ /mm	200, 0, 0, 0,
$h_{3z}, h_{4z}, h_{5z}, h_{6z}$ /mm	150, -900, -410, -280
l /mm	100

Fig. 12 The influence law of X-axis geometric errors on machining errors

4.2 Identification of key geometric errors

On the basis of the influence law of each geometric error on machining error displayed in Section 4.1 and Eq. (13), the sensitivity coefficient of each geometric error can be obtained, as listed in Table 7. In addition, in order to reflect the SA results more intuitively, the sensitivity coefficient of each geometric error is sorted, as shown in Fig. 13. From Fig. 13 and Table 7, the sensitivity coefficients of the 10th, 17th, 22nd, 24th, 37th, 6th and 21st geometric error are relatively large (i.e., 0.1527, 0.1510, 0.1392, 0.1300, 0.0498, 0.0388 and 0.0379, respectively), accounting for 69.93% of total. Therefore, it can be concluded that they are the key geometric errors affecting the machining accuracy of the researched machine tool.

Fig. 13 The sensitivity coefficient of each geometric error

Table 7

The sensitivity coefficient of tolerance parameters

Serial number	Geometric error	Sensitivity coefficient	Serial number	Geometric error	Sensitivity coefficient	Serial number	Geometric error	Sensitivity coefficient
1	$\delta_x(x)$	0.0036	14	$\delta_y(z)$	0.0013	27	$\delta_z(c)$	0.0010
2	$\delta_y(x)$	0.0231	15	$\delta_z(z)$	0.0001	28	$\varepsilon_x(c)$	0.0318
3	$\delta_z(x)$	0.0063	16	$\varepsilon_x(z)$	0.0116	29	$\varepsilon_y(c)$	0.0261
4	$\varepsilon_x(x)$	0.0094	17	$\varepsilon_y(z)$	0.1510	30	$\varepsilon_z(c)$	0.0084
5	$\varepsilon_y(x)$	0.0243	18	$\varepsilon_z(z)$	0.0003	31	ε_{xy}	0.0113
6	$\varepsilon_z(x)$	0.0388	19	$\delta_x(b)$	0.0045	32	ε_{xz}	0.0048
7	$\delta_x(y)$	0.0038	20	$\delta_y(b)$	0.0053	33	ε_{yz}	0.0027
8	$\delta_y(y)$	0.0023	21	$\delta_z(b)$	0.0379	34	ε_{xb}	0.0091
9	$\delta_z(y)$	0.0088	22	$\varepsilon_x(b)$	0.1392	35	ε_{bz}	0.0026
10	$\varepsilon_x(y)$	0.1527	23	$\varepsilon_y(b)$	0.0106	36	ε_{xc}	0.0256
11	$\varepsilon_y(y)$	0.0010	24	$\varepsilon_z(b)$	0.1300	37	ε_{yc}	0.0498
12	$\varepsilon_z(y)$	0.0011	25	$\delta_x(c)$	0.0294			
13	$\delta_x(z)$	0.0059	26	$\delta_y(c)$	0.0246			

5 Experimental validation

To demonstrate the correctness and validity of the SA method presented in this study, the machining and measurement experiment of “S” shaped test specimen are carried out. The machining experiment is implemented on the researched machine tool. The measurement experiment is conducted on the coordinate measuring machine (CMM) named ZEISS PRISMO navigator. The flowchart of the experiment is shown in Fig. 14.

Compensating the key geometric errors that were extracted by utilizing the SA method proposed in

this paper was defined as the compensation scheme 1. Compensating all geometric errors was defined as the compensation scheme 2. Before machining, the corrected NC machining instruction codes based on two different compensation schemes are generated by employing the actual inverse kinematics method proposed in [32]. Additionally, the semifinished products of “S” shaped test specimens are the same. A $\Phi 20$ mm cylindrical milling cutter is selected to finish milling. Then, based on above-mentioned conditions, the machining experiment is conducted, as displayed in Fig. 15. Subsequently, two “S” shaped test specimens are obtained, as illustrated in Fig. 16. Finally, the profile errors of two “S” shaped test specimens are measured by the CMM. The measurement scene is shown in Fig. 17.

Fig. 14 The flowchart of the experiment

Fig. 15 The machining scene

Fig. 16 “S” shaped test piece obtained by different compensation strategies

Fig. 17 The measurement scene

According to Ref. [33], for the sake of simplicity, three detection lines (i.e., L1, L2, and L3) along the Z direction of the “S” shaped edge strip are selected to test the profile errors of “S” shaped test specimen, as shown in Fig. 18. 25 detection points are distributed uniformly in each detection line. In addition, the indoor ambient temperature is set at $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$ [34]. For improving the stability of measurement results, the measurement experiment was repeated 5 times. The average value of 5 times measurement results was taken as the final profile errors of the machined “S” shaped test specimen, as displayed in Figs. 19–21.

Fig.18 The error detection points position of the “S” shaped test piece

In order to verify the correctness of the proposed SA method more intuitively, the comparisons of the average profile errors for two “S” shaped test specimens are listed in Table 8. From Table 8, in contrast to compensation scheme 1, the average profile error obtained by compensation scheme 2 is reduced from 0.041 to 0.036mm, 0.036 to 0.032mm, and 0.044 to 0.038mm at L1, L2, and L3, respectively. In other words, the average profile error obtained by compensation scheme 2 has decreased by 0.005mm, 0.004mm, and 0.006mm at L1, L2, and L3, respectively. Summarized above, the comparison results show that there is no significant difference between the profile errors obtained by compensation scheme 1 and compensation scheme 2. The experimental results proved that the key geometric errors extracted by the proposed SA method have significant effect on the machining accuracy of the researched machine tool. Therefore, it is obvious that the presented SA method is correct and effective.

Fig. 19 The profile errors of the “S” shaped test specimen based on two different compensation schemes at L1.

Fig. 20 The profile errors of the “S” shaped test specimen based on two different compensation schemes at L2.

Fig. 21 The profile errors of the “S” shaped test specimen based on two different compensation schemes at L3.

Table 8

The comparison of average profile errors for “S” shaped test specimen

	The average profile errors (mm)		Reduced profile errors (mm)
	Compensation scheme 1	Compensation scheme 2	
L1	0.041	0.036	0.005
L2	0.036	0.032	0.004
L3	0.044	0.038	0.006

6. Conclusions

This paper proposes a new SA method to quantify the influence degree of geometric error on the machining accuracy and extract the key geometric error components. By comparison with the existing SA methods [14-26], the superiorities of the new SA method are listed as follows: (1) A novel sensitivity index is proposed to solve the problem that the SA results of angular error and linear error need compared separately, which significantly simplified the SA procedures and improves the computational efficiency. (2) The proposed SA model takes into account the characteristics that geometric errors change with the change of machining positions. The model solved the problem of SA results with poor universality, which caused by assuming all the angular errors and linear errors have the same value.

To verify the practicability and validity of the presented SA method, experiments on a gantry type five-axis NC milling machine are conducted. The experimental results show that the average profile errors obtained by compensation scheme 2 has decreased by 0.005mm, 0.004mm, and 0.006mm at L1, L2, and L3 respectively when compared with compensation scheme 1. The comparison results show that there is no significant difference between the profile errors obtained by two compensation schemes. Thus, there is no doubt that the key geometric errors identified by the proposed SA method have significant effect on the machining accuracy of the researched machine tool. It is therefore reasonable to conclude that the presented SA method is correct and effective. This method is universal and provides a reference for key geometric error identification of multi-axis NC machine tool.

Appendix A

$$\left\{ \begin{array}{l} \delta_x(x) = \frac{t_3}{2} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x}{\lambda_x}\right) + xa_x \\ \delta_y(x) = \frac{t_2 \left(\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i+1}}{\lambda_x}\right) + \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i-1}}{\lambda_x}\right) \right)}{4} \\ \delta_z(x) = \frac{t_1 \left(\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i+1}}{\lambda_x}\right) + \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i-1}}{\lambda_x}\right) \right)}{4} \\ \varepsilon_x(x) = \frac{t_4 \left(\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i+1}}{\lambda_x}\right) + \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i-1}}{\lambda_x}\right) \right)}{4L_x} \\ \varepsilon_y(x) = \frac{t_1 \left(\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i+1}}{\lambda_x}\right) - \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i-1}}{\lambda_x}\right) \right)}{2D_x} \\ \varepsilon_z(x) = \frac{t_2 \left(\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i+1}}{\lambda_x}\right) - \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x_{i-1}}{\lambda_x}\right) \right)}{2D_x} \end{array} \right. \quad A1$$

$$\left\{ \begin{array}{l} \delta_x(c) = \frac{t_{17}}{2} \sum_{n=1}^{\infty} \frac{1}{n} \sin \theta_c \cos \theta_c \\ \delta_y(c) = \frac{t_{17}}{2} \sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \theta_c \\ \delta_z(c) = \frac{t_{18}}{2} \sum_{n=1}^{\infty} \frac{1}{n} \sin \theta_c \\ \varepsilon_x(c) = \frac{\delta_y(c)}{H_c} = \frac{t_{17} \sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \theta_c}{2H_c} \\ \varepsilon_y(c) = \frac{\delta_x(c)}{H_c} = \frac{t_{17} \sum_{n=1}^{\infty} \frac{1}{n} \sin \theta_c \cos \theta_c}{2H_c} \\ \varepsilon_z(c) = \frac{t_{16}}{2} \sum_{n=1}^{\infty} \frac{1}{n} \sin \theta_c \end{array} \right. \quad \text{A2}$$

Appendix B

Table 2

Ideal and actual static homogeneous transformation matrices for the adjacent bodies

Adjacent body	Ideal static homogeneous transformation matrices	Actual static homogeneous transformation matrices
1-W (Workpiece)	${}^1_w T_p = \begin{bmatrix} 1 & 0 & 0 & h_{wx} \\ 0 & 1 & 0 & h_{wy} \\ 0 & 0 & 1 & h_{wz} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^1_w T_{pe} = \mathbf{E}_{4 \times 4}$
1-2 (X-axis)	${}^1_2 T_p = \begin{bmatrix} 1 & 0 & 0 & h_{2x} \\ 0 & 1 & 0 & h_{2y} \\ 0 & 0 & 1 & h_{2z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^1_2 T_{pe} = \mathbf{E}_{4 \times 4}$
2-3 (Y-axis)	${}^2_3 T_p = \begin{bmatrix} 1 & 0 & 0 & h_{3x} \\ 0 & 1 & 0 & h_{3y} \\ 0 & 0 & 1 & h_{3z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^2_3 T_{pe} = \begin{bmatrix} 1 & -\varepsilon_{xy} & 0 & 0 \\ \varepsilon_{xy} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
3-4 (Z-axis)	${}^3_4 T_p = \begin{bmatrix} 1 & 0 & 0 & h_{4x} \\ 0 & 1 & 0 & h_{4y} \\ 0 & 0 & 1 & h_{4z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^3_4 T_{pe} = \begin{bmatrix} 1 & 0 & \varepsilon_{xz} & 0 \\ 0 & 1 & -\varepsilon_{yz} & 0 \\ -\varepsilon_{xz} & \varepsilon_{yz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
4-5 (C-axis)	${}^4_5 T_p = \begin{bmatrix} 1 & 0 & 0 & h_{5x} \\ 0 & 1 & 0 & h_{5y} \\ 0 & 0 & 1 & h_{5z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^4_5 T_{pe} = \begin{bmatrix} 1 & 0 & \varepsilon_{xc} & 0 \\ 0 & 1 & -\varepsilon_{yc} & 0 \\ -\varepsilon_{xc} & \varepsilon_{yc} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
5-6 (B-axis)	${}^5_6 T_p = \begin{bmatrix} 1 & 0 & 0 & h_{6x} \\ 0 & 1 & 0 & h_{6y} \\ 0 & 0 & 1 & h_{6z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^5_6 T_{pe} = \begin{bmatrix} 1 & -\varepsilon_{bz} & 0 & 0 \\ \varepsilon_{bz} & 1 & -\varepsilon_{xb} & 0 \\ 0 & \varepsilon_{xb} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
6-T (Tool)	${}^6_t T_p = \mathbf{E}_{4 \times 4}$	${}^6_t T_{pe} = \mathbf{E}_{4 \times 4}$

Table 3

Ideal and actual motion homogeneous transformation matrices for the adjacent bodies

Adjacent body	Ideal motion homogeneous transformation matrices	Actual motion homogeneous transformation matrices
1-W (Workpiece)	${}^1T_s = E_{4 \times 4}$	${}^1T_{se} = E_{4 \times 4}$
1-2 (X-axis)	${}^2T_s = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^2T_{se} = \begin{bmatrix} 1 & -\varepsilon_z(x) & \varepsilon_y(x) & \delta_x(x) \\ \varepsilon_z(x) & 1 & -\varepsilon_x(x) & \delta_y(x) \\ -\varepsilon_y(x) & \varepsilon_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
2-3 (Y-axis)	${}^3T_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^3T_{se} = \begin{bmatrix} 1 & -\varepsilon_z(y) & \varepsilon_y(y) & \delta_x(y) \\ \varepsilon_z(y) & 1 & -\varepsilon_x(y) & \delta_y(y) \\ -\varepsilon_y(y) & \varepsilon_x(y) & 1 & \delta_z(y) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
3-4 (Z-axis)	${}^4T_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^4T_{se} = \begin{bmatrix} 1 & -\varepsilon_z(z) & \varepsilon_y(z) & \delta_x(z) \\ \varepsilon_z(z) & 1 & -\varepsilon_x(z) & \delta_y(z) \\ -\varepsilon_y(z) & \varepsilon_x(z) & 1 & \delta_z(z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
4-5 (C-axis)	${}^5T_s = \begin{bmatrix} \cos(c) & -\sin(c) & 0 & 0 \\ \sin(c) & \cos(c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^5T_{se} = \begin{bmatrix} 1 & -\varepsilon_z(c) & \varepsilon_y(c) & \delta_x(c) \\ \varepsilon_z(c) & 1 & -\varepsilon_x(c) & \delta_y(c) \\ -\varepsilon_y(c) & \varepsilon_x(c) & 1 & \delta_z(c) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
5-6 (B-axis)	${}^6T_s = \begin{bmatrix} \cos(b) & 0 & \sin(b) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(b) & 0 & \cos(b) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	${}^6T_{se} = \begin{bmatrix} 1 & -\varepsilon_z(b) & \varepsilon_y(b) & \delta_x(b) \\ \varepsilon_z(b) & 1 & -\varepsilon_x(b) & \delta_y(b) \\ -\varepsilon_y(b) & \varepsilon_x(b) & 1 & \delta_z(b) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
6-T (Tool)	${}^6T_s = E_{4 \times 4}$	${}^6T_{se} = E_{4 \times 4}$

Table 4

Position vector of the body reference coordinate system origin of each body

The body reference coordinate system origin of each body	Position vector
B_w	$\mathbf{h}_w = (h_{wx}, h_{wy}, h_{wz}, 1)^T$
B_2	$\mathbf{h}_2 = (h_{2x}, h_{2y}, h_{2z}, 1)^T$
B_3	$\mathbf{h}_3 = (h_{3x}, h_{3y}, h_{3z}, 1)^T$
B_4	$\mathbf{h}_4 = (h_{4x}, h_{4y}, h_{4z}, 1)^T$
B_5	$\mathbf{h}_5 = (h_{5x}, h_{5y}, h_{5z}, 1)^T$
B_6	$\mathbf{h}_6 = (h_{6x}, h_{6y}, h_{6z}, 1)^T$

Appendix C

$$\begin{aligned}
 e_x = & (\delta_x(x) + \delta_x(y) - l_k \cos(b))(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz}) + h_{4z}(\varepsilon_y(x) + \varepsilon_y(y)) \\
 & - (\varepsilon_y(b) + \delta_y(c))\sin(c) - \varepsilon_z(x)h_{3y} - h_{4y}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy}) + \varepsilon_{xz}z + \delta_x(c)\cos(c) \\
 & + h_{5z}(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xz}) + h_{6z}\left(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz}\right) + \varepsilon_y(x)z \\
 & - h_{5y}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) - l_k \cos(b)\cos(c)(\varepsilon_y(b) + \varepsilon_y(c) + \varepsilon_x(c)) - \varepsilon_{xy}y \\
 & + \delta_x(b)\cos(b)\cos(c) + (\delta_z(b) - l_k)\cos(c)\sin(b) - \varepsilon_z(x)y - \varepsilon_{bz}l_k \cos(b)\sin(c) + \varepsilon_y(y)z \\
 & - (\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy})(h_{6y}\cos(c) + h_{6x}\sin(c)) + \varepsilon_y(x)h_{3z} + \delta_x(z) \\
 & - \varepsilon_x(b)l_k \sin(c) + (\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xb} + \varepsilon_{xy})l_k \sin(b)\sin(c) \mathbf{d}n_{sjk} \\
 e_y = & (\delta_y(x) + \delta_y(y) + \delta_y(z) + h_{5x}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy} + \varepsilon_z(z)) + h_{4x}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy}) \\
 & + \delta_z(b)\sin(b)\sin(c) + \varepsilon_z(x)h_{3x} - \varepsilon_{yz}z + h_{6x}\cos(c)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) \\
 & - h_{5z}(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yz}) - \varepsilon_x(x)z - \varepsilon_x(y)z + (\varepsilon_{bz} - \varepsilon_x(c))l_k \cos(b)\cos(c) \\
 & - h_{6z}(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz} + \varepsilon_x(c)\cos(c) - \varepsilon_y(c)\sin(c)) + \delta_y(b)\cos(c) \\
 & - l_k \cos(b)(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz}) + \cos(b)\sin(c)(\delta_x(b) + \varepsilon_y(c) - \varepsilon_y(b)) \\
 & - h_{6y}\sin(c)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) - h_{4z}(\varepsilon_x(x) + \varepsilon_x(y)) + l_k \varepsilon_x(b)\cos(c) \\
 & + \delta_y(c)\cos(c) + \delta_x(c)\sin(c) - l_k \cos(c)\sin(b)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xb} + \varepsilon_{xy}) \\
 & - \varepsilon_x(x)h_{3z} \mathbf{d}n_{sjk} \\
 e_z = & (\delta_z(c) + \varepsilon_x(x)y - \varepsilon_y(x)h_{3x} + l_k \cos(c)\sin(b)(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz}) \\
 & + \sin(b)\left(\varepsilon_y(b)l_k + \varepsilon_y(c)l_k - \delta_x(b)\right) - l_k \sin(c)\left(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz}\right) - h_{5x}(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xz}) \\
 & + h_{6y}(\varepsilon_x(c) + \varepsilon_{yc}\cos(c) + \sin(c)(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz})) - h_{4x}(\varepsilon_y(x) + \varepsilon_y(y)) \\
 & + h_{5y}(\varepsilon_{yz} + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_x(x)) + \delta_z(x) + \delta_z(y) + \delta_z(z) + h_{4y}(\varepsilon_x(x) + \varepsilon_x(y)) \\
 & + h_{6x}\left(\sin(c)(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz}) - \cos(c)\right) + \varepsilon_x(b)\varepsilon_y(z)\varepsilon_{xb}l_k \cos(c) \\
 & + \delta_z(b)\cos(b) + \varepsilon_x(x)h_{3y} \mathbf{d}n_{sjk}
 \end{aligned}
 \tag{C1}$$

Author's contribution

Haohao Tao: Resources, Supervision, Methodology, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing. Tongjie Li: Project administration, Funding acquisition. Feng Chen: Conceptualization, Investigation. Jinwei Fan: Project administration, Funding acquisition. Ri Pan: Data Curation. Zhen Yu: Software.

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Availability of data and material

The authors confirm that the data supporting the findings of this study are available within the article.

Code availability

All code of this study is available from the corresponding author on reasonable request.

Declarations

Ethics approval

According to the *Submission guidelines*, this section is not applicable because this manuscript is used for non-life science journals.

Consent to participate

According to the *Submission guidelines*, this section is not applicable because this manuscript is used for non-life science journals.

Consent for publication

All the authors agreed to publish this paper.

Conflicts of interest

The authors declare no competing interests.

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