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Checkerboard solid of dipolar excitons in a two-dimensional lattice

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The Hubbard model constitutes one of the most celebrated theoretical frameworks of condensed-matter physics. It describes strongly correlated phases of interacting quantum particles confined in a lattice potential [1, 2]. For bosons, in the last two decades the Hubbard Hamiltonian has been deeply scrutinised in the regime of short-range on-site interactions [3–6]. On the other hand, extensions to longer-range off-site couplings between neighbouring lattice sites have remained mostly elusive experimentally [7]. Entering this regime constitutes a well identified research frontier where quantum matter phases can spontaneously break the lattice symmetry [8–12]. Here we unveil one of such phases, precisely the long-sought-after checkerboard solid. It is accessed by confining semiconductors dipolar excitons in a two-dimensional square lattice. The exciton checkerboard is signalled by a strongly minimised compressibility at half-filling of the lattice sites, in quantitative agreement with theoretical expectations. Our observations thus highlight that dipolar excitons enable controlled implementations of extended Bose-Hubbard Hamiltonians.

The extended Bose-Hubbard (BH) Hamiltonian is controlled by three main physical parameters – the strength of on-site interactions U , the strength of tunnelling t , and the strength of interactions between nearest neighbouring lattice sites V [1]. While the standard regime where V plays a negligible role has been thoroughly studied [3–6], exploring the situation where V controls the many-body ground-state continues to seriously challenge experimental research in condensed-matter physics [7]. In this regime, long-range interactions induce quantum phases spontaneously breaking the lattice symmetry [8–14], like density waves manifesting checkerboard (CB) or stripe solids, but also lattice supersolids strikingly combining the physical properties of Mott insulators (MIs) and superfluids [15].

Theoretical works have established that dipolar bosons in a lattice provide an ideal platform to implement the extended BH model [10–12]. Here, we follow this path by confining dipolar excitons in a two-dimensional square lattice potential. We then evidence two incompressible phases, at unity and half-filling of the lattice sites. The former case marks the MI regime where lattice sites are all occupied by a single exciton [24]. At half-filling, the lattice symmetry is spontaneously broken and an exciton density wave is realised. Importantly, by comparing spectroscopically the two incompressible phases we directly extract the interaction strength between one exciton and its 4 nearest neighbours. This allows us to confirm theoretically, relying on both mean-field and exact diagonalisation techniques, that excitons realise a checkerboard solid at half-filling of the lattice sites.

Figure 1.a schematically illustrates dipolar excitons confined in our 250 nm period lattice. In recent years, different techniques for engineering tunable lattice potentials in excitonic systems have been developed, including optical [16] or Moiré lattices [17–22]. Here, we follow the approach devised in Refs. [23–26], where the lattice is engineered by an array of gate electrodes. These imprint a periodically varying electric field perpendicular to the plane of a double GaAs quantum well where electrons and holes are spatially separated [27–30]. The potential energy of dipolar excitons, made by the Coulomb attraction between opposite charge carriers, varies then sinusoidally in space (Methods). Here, the lattice depth is set to around 300 μeV . Excitons then have access to two confined Wannier states (WS) separated by about 150 μeV (WS are labeled 1 and 2 in Fig.1.b). Moreover, we optically inject excitons, using a 100 ns long pulsed laser excitation repeated at 1 MHz. We then analyze the reemitted photoluminescence (PL) 300 ns after termination of the loading laser pulse (Methods).

We first illustrate the main physical properties of dipolar excitons in the lattice, by relying on the PL emitted when the mean density $\bar{n}$ is around 1.3 excitons per lattice site. Figure 1.c shows the PL spectrum measured at a bath temperature $T = 330$ mK and averaged over around 100 lattice sites (Methods). We then note that the PL consists of two main contributions. The first and most intense one, at the lowest energy, is due to the radiative recombination of excitons confined in the lattice potential. To fit it, we sum two PL lines separated by 150 μeV with a full-width-at-half-maximum ($\sim 150\mu\text{eV}$) set by our spec-

tral resolution (red and gray in Fig.1.c). Each line is assigned to excitons populating one WS [24]. We suitably adjust their amplitudes, that translate into the mean occupation fraction $\bar{p}$ of the corresponding WS. The red profile in Fig.1.c shows that the spectrum is then quantitatively reproduced. The second emission (dark gray in Fig.1.c) occurs at too high energies to be possibly attributed to lattice confined states. In fact, it marks the contribution from excitons occupying the continuum of states accessible above the lattice potential (blue in Fig.1.b).

Following the approach detailed in Ref. [24], we computed the strength of on-site dipolar interactions between excitons (see also Methods). We assumed that the two WS correspond to s and p orbitals respectively [31]. Then, we deduced that two-body scatterings have a strength greatly exceeding the lattice depth, for excitons confined in the same WS, $U_{1,1} \sim 1$ meV and $U_{2,2} \sim 370$ μ eV for the WS 1 and 2. The on-site interaction strength between excitons confined in distinct WS is also large, $U_{1,2} \sim 175$ μ eV (see Fig.1.b and Methods). As a result, lattice sites cannot be doubly occupied, and on-site interactions easily “expel” excitons into the continuum. The energy separation between the WS and the occupied continuum, around 500 μ eV (see Fig.1.c), quantitatively agrees with the order of magnitude of on-site dipolar interaction strengths.

Figure 1.d presents the phase diagram calculated for our experiments, in the (μ, T) parameter space where μ denotes the chemical potential. The diagram was computed for an extended two-band BH Hamiltonian in the mean-field approximation, combining theoretically calculated and experimentally deduced parameters (see Methods). Exact diagonalization (ED) studies of the full Hamiltonian further confirm the accuracy of mean-field calculations. For our experimental situation, theoretical predictions first show that a MI is energetically favoured around unity filling of the lattice sites. This phase is marked by a minimized compressibility and by excitons uniformly occupying the same WS in every lattice site [24]. On the other hand, both ED (Fig.S1) and mean-field theory (Fig.S2) strikingly predict that, at half-filling of the lattice sites, V is sufficiently large to stabilize a checkerboard (CB) solid. For this incompressible density wave the exciton distribution is such that nearest neighbour interactions are fully avoided. Finally, in Fig.1.d we recover that MI and CB are surrounded by a normal fluid phase (NF). Possible superfluid phases are only accessible at much lower temperatures, below 20 mK for $\bar{n} \sim 1$.

To experimentally evidence MI and CB phases, we measured the exciton compressibility κ . For that we relied on the spectral profile of the PL that allows us to extract the occupation fractions p of each WS (see Ref.[24]). By repeating each measurement 10 times, under fixed conditions for every experimental setting, we computed the mean occupation $\bar{p}(i)$ and its standard deviation $\sigma(p(i))$ for each WS ($i = 1, 2$). Hence, we deduce κ since $\sigma(p(i))/\bar{p}(i)$ is proportional

to $(\kappa k_B T)^{1/2}$ according to the fluctuation-dissipation theorem [4].

Figure 2.a presents the variation of $\sigma(p(1))/\bar{p}(1)$ at $T = 330$ mK, as a function of the average power of our loading laser P that controls the mean occupation of lattice sites $\bar{n}$. For two specific regions, $P \sim 8$ and 17 nW, Fig.2.a shows that κ strongly decreases, by around two-fold compared to the level of Poissonian fluctuations (gray region). Furthermore, for $P = 17$ nW we expect that $\bar{n} \sim 1$ (Methods). Noting that PL intensities differ by around two-fold between $P = 8$ and $P = 17$ nW (Fig.2.b), we deduce that $\bar{n} = 1/2$ for the former excitation. Figure 2.a therefore underlines two incompressible states at unity and half-filling of the lattice sites, qualitatively reproducing the phase diagram shown in Fig.1.d. This suggests that excitons realize MI and CB phases respectively.

The bottom panel of Fig.2.b quantifies the profile of the PL spectrum emitted for $\bar{n} = 1/2$. We note that 80 % of the PL is due to the radiative recombination of excitons occupying the lowest energy WS (red area). Moreover, the inset in Fig.2.b underlines that, for our optical resolution of around 1 μ m, the PL intensity and $\sigma(p(1))/\bar{p}(1)$ vary weakly in the 3 μ m extended region where the spectrum is averaged. This behaviour is also observed for $\bar{n} = 1$, but the top panel of Fig.2.b evidences that this time the maximum of the PL does not coincide with any WS energy, which are shown by the two vertical lines.

We interpret the energy difference between the maxima of the spectra measured for $\bar{n} = 1/2$ and $\bar{n} = 1$ as a manifestation of the interaction strength between one exciton and its 4 nearest neighbours ($4V$). Indeed, for a checkerboard solid, excitons do not have first nearest neighbours, whereas for a Mott phase each exciton interacts with 4 nearest neighbours (Fig.1.d). In the latter regime, the energy of excitons occupying the first WS is then increased by $4V$. From Fig.2.b we thus deduce that $4V \sim (100 \pm 15)$ μ eV. To reproduce the spectrum measured for $\bar{n} = 1$, we then summed two PL contributions. The first one marks the fraction (75%) of the ideal Mott phase characterised by excitons experiencing dipolar repulsions with 4 nearest neighbours (blue area). The second contribution marks the weight (25%) of defects within the MI, which correspond to excitons with less than 4 nearest neighbours (red area). In the same way, for $\bar{n} = 1/2$, we sum the dominant contribution given by excitons with no nearest neighbours corresponding to the ideal CB (red), with a small fraction (10%) of defects corresponding to excitons interacting with 3 or 4 nearest neighbours (blue). Furthermore, the PL spectrum is most accurately reproduced by including a 10% fraction of excitons occupying the second WS (gray).

Theoretically, MI and CB phases are protected by energy gaps of the order of U and V respectively. To further confirm that Fig.2 evidences these phases, we measured the thermal dependence of the exciton compressibility κ . For $\bar{n} = 1/2$, Fig.3.a reveals that increasing T rapidly enhances κ which follows the (classical) variation given by Poissonian fluctuations

(blue curve) for $T \gtrsim 550$ mK. Qualitatively, this behaviour agrees very well with our theoretical expectations (Fig.S2), revealing that the CB is hardly stable above our lowest bath temperature. This conclusion is actually directly supported by analysing the profile of the PL spectrum. Indeed, Fig.3.b shows that at 330 mK the PL is only due to the recombination of excitons with no nearest neighbours, as expected for a CB solid. On the contrary, increasing the bath temperature we observe that the contribution 3-4V above the energy of the lowest WS grows rapidly (bottom panel of Fig.3.b). This higher energy PL, dominant for $T \gtrsim 750$ mK, evidences thermal excitations of the CB, so that excitons tunnel between lattice sites and then interact with 3 or 4 nearest neighbours. Let us finally note that our theoretical model predicts that the CB melts into a normal fluid above a critical temperature $T_m \sim V/k_B$ (see Fig.1.d and Fig.S2). Monte-Carlo calculations for a one-band model of hard-core bosons previously concluded that $T_m \sim 0.6V/k_B$ at the thermodynamic limit [32]. We attribute this small discrepancy partly to the finite system size. Indeed, our ED calculations are performed on a cell restricted to 8 sites, while PL spectra are averaged over around 100 lattice sites (Methods).

For $\bar{n} = 1$, Fig.3.d shows that κ slowly increases towards the level given by Poissonian fluctuations, reached for $T \gtrsim 750$ mK. In this range, we deduce that the MI has melted into a normal fluid. Such a high critical temperature was somewhat expected given the on-site interaction strength [33–35]. In fact, our observations are quantitatively supported by theoretical calculations predicting that the Mott phase remains stable below around 750 mK (Fig.1.d and Fig.S2). As for the CB, the PL profile highlights the mechanism beyond the melting of the MI. Precisely, Fig.3.e signals that the fraction of excitons with less than 4 nearest neighbours increases from 19 to 30% between 330 and 750 mK. At the same time, the contribution of excitons that are not confined by the lattice potential is enhanced from 4 to 10 % (gray in Fig.3.e). These variations manifest a large fraction of defects, i.e. empty sites, in the Mott phase. We attribute them to the very strong on-site interaction strength, which efficiently couples WS to the continuum (Fig.1.b-c), so that thermal excitations expel particles from the lattice, thereby annihilating the order of the MI.

To conclude, our studies provide compelling evidence for the buildup of CB and MI phases, for dipolar excitons confined in the lowest energy WS of a 250 nm period lattice. Lowering the exciton temperature to around 10 mK, which is within experimental reach, the occurrence of superfluid properties, in particular the long-sought-after lattice supersolid, is theoretically predicted. For the device studied here, this phase would buildup in the lowest energy WS, but more exotic configurations seem accessible in the parameter space explorable with dipolar excitons. Particularly, for shallower lattices our theoretical calculations predict that CB, and lattice supersolids, can simultaneously form in both *s*-like and *p*-like orbitals. Such

multi-component symmetry breaking collective states would provide a novel realm for research of quantum matter.

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Data availability

Source data supporting all the conclusions raised in this manuscript are available for download upon request.

Methods

A. Device and experimental procedure

The 250 nm period electrostatic lattice has been realized using the heterostructure described in Ref. [24]. It is based on two 8 nm wide GaAs quantum wells, separated by a 4 nm AlGaAs barrier. The quantum wells are positioned 200 nm below the surface of the field effect device where they are embedded, and

150 nm above a conductive layer that serves as electrical ground. The lattice potential was engineered using the procedure detailed in the Supplementary Informations of Ref. [24]. Then, we designed and realized gate electrodes deposited at the surface of the field-effect device. These were polarised at around 1 V in our experiments, yielding a periodically varying electric field perpendicular to the quantum wells plane. This field imprints the excitons lattice potential due to its interaction with the excitons permanent electric dipole.

In our studies electronic carriers are injected in the lattice potential using a laser excitation at resonance with the direct exciton absorption of each quantum well. The laser excitation is set with a rectangular profile at the surface of our device, with an area equal to around $10 \times 4 \mu\text{m}^2$. We then study spectrally the PL reemitted by dipolar excitons in the $3 \times 2.5 \mu\text{m}^2$ central region (Figs.2-3), the horizontal extension being set by the width of our spectrometer's slit given our optical magnification. This area corresponds then to around 100 lattice sites. The PL is acquired during a 100 ns long time interval, starting 300 ns after extinction of the loading laser pulse while the excitons radiative lifetime exceeds 700 ns in our device [24].

To calibrate the average exciton density, as in Ref.[24], we monitored the dynamic of the PL energy following the laser excitation. For that we used a region without lattice potential so that dipolar excitons realize a homogeneous gas. Comparing the PL energy detected 300 ns after the laser pulse, to the one at much longer delays, we deduce the magnitude of repulsive dipolar interactions that translates into the average exciton density. Thus, for $P \sim 33 \text{ nW}$ we observed that the energy difference is bound to 150 μeV , so that the exciton density is about $6 \times 10^9 \text{ cm}^{-2}$. In a 250 nm period lattice potential this translates into $\bar{n} \sim 2$ excitons per lattice site. Accordingly, we deduce that $\bar{n} \sim 1$ for $P = 17 \text{ nW}$.

B. Physical parameters of the extended Bose-Hubbard Hamiltonian

Most generically, the extended two-band BH Hamiltonian reads $H = \sum_i h_i + \sum_{\langle i,j \rangle} h_{ij}$, (see Ref. [12]), with the on-site terms $h_i = \sum_{\alpha,\beta,\delta,\gamma} U_{\alpha\beta\delta\gamma} b_{i\alpha}^\dagger b_{i\beta}^\dagger b_{i\gamma} b_{i\delta} - \mu \sum_{\alpha} n_{i\alpha}$, and the terms between nearest neighbour sites $\langle i,j \rangle$ given by $h_{ij} = \sum_{\alpha} -t_{\alpha} (b_{i\alpha}^\dagger b_{j\alpha} + \text{h.c.}) + \sum_{\alpha,\beta,\delta,\gamma} V_{\alpha\beta\delta\gamma} b_{i\alpha}^\dagger b_{j\beta}^\dagger b_{j\gamma} b_{i\delta}$, where Greek (Latin) indices represent band (site) degrees of freedom, $b_{i\alpha}$ ($b_{i\alpha}^\dagger$) are bosonic annihilation (creation) operators, and $n_{i\alpha} = b_{i\alpha}^\dagger b_{i\alpha}$. To evaluate the strength of the different on-site and inter-site interactions, U and V respectively, as in Ref. [24] we separate the dipolar potential into short-range and long-range parts, introducing a cut-off distance $r_c = 20 \text{ nm}$, which amounts to around two times the excitons' Bohr radius. Relying on the spatial profiles of the Wannier wave-functions expected for our lattice potential with 300 μeV depth, we estimate the magnitudes quoted in

the main text. The dominant terms are the density-density on-site interactions, namely $U_{1111} \equiv U_{1,1} \sim 1 \text{ meV}$ and $U_{2222} \equiv U_{2,2} \sim 370 \mu\text{eV}$ for the WS 1 and 2, which exceed the excitons confinement depth in the lattice and can then not be measured. Also, we find that $U_{1221} \equiv U_{1,2} \sim 175 \mu\text{eV}$. Density-density interactions are also the dominant inter-site interactions, and for the first WS our calculations yield $V_{1111} \equiv V \sim 10 - 14 \mu\text{eV}$ for the expected excitons electric dipole moment ($d \sim 12 - 14 e \cdot \text{nm}$ where e is the electron charge). Such magnitude for V is about 2 times smaller than the value deduced from the experiments shown in Fig.2. This discrepancy may first result from the profile of our lattice potential, possibly differing from the ideal sinusoidal shape and effectively confining excitons more weakly than expected. In this case, the overlap between Wannier wave-functions localized in neighbouring lattice sites would increase rapidly, resulting in larger values of V . Moreover, our expectations for V may mark the limit of our theoretical treatment of dipolar interactions for short period lattices. For these, a more elaborate description relying on Coulomb interactions between spatially separated electrons and holes is in fact expected to provide sizeable corrections to the dipolar potential. Finally, for the tunnelling strength between nearest neighbouring lattice sites, we obtain $t_1 \sim 0.4 \mu\text{eV}$ and $t_2 \sim 5 \mu\text{eV}$ for the first and second WS respectively.

C. Exact diagonalisation calculations

For the theoretical description we consider the extended two-band Bose-Hubbard Hamiltonian, relying on the theoretically expected parameters for t_{α} , $U_{\alpha\beta\gamma\delta}$, and $V_{\alpha\beta\gamma\delta}$, but with increased amplitude of density-density inter-site interactions to match the experimentally measured value of V in the first WS.

We obtain the full eigenspectrum of this Hamiltonian by applying exact diagonalization on a supercell, spanned by $\mathbf{L}_1 = (2, 2)a$ and $\mathbf{L}_2 = (2, -2)a$, a denoting the lattice period, which contains 8 sites of the square lattice. In the ED calculation, we fix the particle number to 4 excitons (half filling). From the eigenspectrum, we calculate thermal expectation values of the observables of interest. A hallmark of CB order is a peak of the structure factor at $\mathbf{k}_{\text{CB}} \equiv (\frac{\pi}{a}, \frac{\pi}{a})$. Indeed, the structure factor in the first WS, defined as $S_1(\mathbf{k}) \sim \sum_{ij} (\langle n_{i1} n_{j,1} \rangle - \langle n_{i1} \rangle \langle n_{j,1} \rangle) e^{-i\mathbf{k} \cdot \mathbf{R}_{ij}}$, with $\mathbf{R}_{ij}$ the lattice vector connecting sites i and j , is found to exhibit a pronounced peak at $\mathbf{k}_{\text{CB}}$ (see Fig.S1.a), with a value $|S_1(\mathbf{k}_{\text{CB}})|$ which remains more than twice as large as any other value of $|S_1(\mathbf{k})|$ up to temperatures as large as $T \sim 400 \text{ mK}$ (see Fig.S1.b).

D. Mean-field calculations

Larger system sizes are studied in the mean-field approximation which reduces the Hamiltonian to a sum of single-site terms, $H^{\text{MF}} = \sum_i (h_i + h_i^{\text{NN}})$, with

$h_i^{\text{NN}} = \sum_j [-\sum_\alpha t_\alpha (b_{i\alpha}^\dagger \langle b_{j\alpha} \rangle - \langle b_{i\alpha}^\dagger \rangle \langle b_{j\alpha} \rangle + \text{h.c.}) + \sum_{\alpha,\beta} V_{\alpha\beta\beta\alpha} (n_{i\alpha} \langle n_{j\beta} \rangle - \langle n_{i\alpha} \rangle \langle n_{j\beta} \rangle)]$. The sum in j includes all nearest neighbouring sites of i , and for simplicity we restrict ourselves to density-density interactions. On every site, the 4 mean-field values $\langle b_{i1} \rangle$, $\langle b_{i2} \rangle$, $\langle n_{i1} \rangle$, and $\langle n_{i2} \rangle$ have to be chosen such that they self-consistently match the corresponding thermal expectation values obtained from the solution of the mean-field Hamiltonian. In our numerical calculation, the self-consistency loop sweeps through a 4x4 lattice. From the solution of the self-consistent mean-field Hamiltonian, we calculate the phase diagram, shown in Fig.1.d, and the compressibility κ , shown in Fig.S2. Fixing the chemical potential such that the filling corresponds to $\bar{n} = 1$ or $\bar{n} = 1/2$, the temperature-dependence of κ shown in Figs. S2.a and S2.b matches very well the experimental data, shown in Fig.3.

In Fig.S2.c, we plot the compressibility as a function of the chemical potential μ , in analogy to Fig.2.a in the main text showing compressibility as a function of the laser power. At sufficiently low temperatures, we find two distinct regimes of small compressibility, corresponding to the MI and the CB phase. Finally, we provide a direct order parameter for the CB phase, namely the population mismatch $|\langle n_A - n_B \rangle|$ between the two sub-lattices A and B . CB order is associated with a non-zero order parameter. It is found within a finite interval of the chemical potential which shrinks as the temperature is increased (see Fig.S2.d), and entirely vanishes at the critical temperature (≈ 360 mK), in good agreement with the experimental observation and with the estimate from ED.

Fig. 1: Strongly interacting dipolar excitons in a lattice. **a** An array of polarized surface electrodes (gold) imprint a 250 nm period lattice for dipolar excitons, made by electrons (gray) and holes (white) spatially separated in a double quantum well (DQW). **b** In the lattice dipolar excitons are confined in two WS (1 and 2). The strength of two-body on-site interactions, $U_{1,1}$, $U_{2,2}$ and $U_{1,2}$, exceed the confinement depth, thereby expelling excitons towards the higher energy continuum (blue). V marks the strength of dipolar repulsions between nearest neighbouring lattice sites. **c** PL spectrum averaged in a $3 \times 2.5 \mu\text{m}^2$ region of the lattice for a mean density $\bar{n}=1.3$ per lattice site. The solid red line provides the profile reproduced by setting 54 and 46 % occupations for the 1st and 2nd WS respectively (see red and light gray shaded regions), while the dark gray area marks the contribution from excitons occupying the continuum of states above the lattice potential. **d** Phase diagram computed using a two-band mean-field model, highlighting that excitons can realise checkerboard (CB), Mott insulator (MI), and normal fluid (NF) phases in our experiments. CB and MI configurations are illustrated on the left side while the vertical line highlights the lowest bath temperature accessible to our studies. Measurements shown in **c** were performed at $T=330$ mK, error bars displaying the Poissonian noise.

Fig. 2: Incompressible phases at unity and half filling. **a** Fluctuations of the first WS occupation as a function of the average filling controlled by the power P of the laser excitation, set with a precision below by the size of the experimental points. Experimental results are obtained by analysing statistically 10 realisations for every experimental condition. **b** (Bottom) Photoluminescence spectrum measured for $P=8$ nW, corresponding to an average $\bar{n}=1/2$ exciton per lattice site. The solid red line shows the modelled profile assigning occupations of 80% and 10 % for the lower WS with no nearest neighbours (red), and with 3 or 4 nearest neighbours (blue), together with a 10 % occupation for the second WS (gray). (Top) Photoluminescence spectrum measured for $P=17$ nW, corresponding to an average $\bar{n}=1$ exciton per lattice site. The spectrum is reproduced by summing the contributions from excitons confined in the 1st WS with less than 4 (red), and with exactly 4 nearest neighbours (blue), with corresponding weights equal to 25 and 75 % respectively. In both panels the vertical lines mark the energy position of the two WS while error bars display the Poissonian precision. Insets display the spatial variation of $\overline{p(1)}$ (black line) and $\sigma(p(1))/\overline{p(1)}$ (bars) where vertical lines show the $3 \mu\text{m}$ wide region where spectra are averaged. Measurements were all carried out at $T=330$ mK.

Fig. 3: Melting of checkerboard and Mott insulator phases. Measured (red) vs. classical compressibility deduced from Poissonian fluctuations (blue), for $\bar{n}=1/2$ (a) and $\bar{n}=1$ (d), as a function of the bath temperature. **b-e** PL spectra measured at $T=330$ and 750 mK for $\bar{n}=1/2$ (b) and $\bar{n}=1$ (e). In (b) the red area marks the contributions from excitons with no nearest neighbours, and with 3 to 4 nearest nearest neighbours (blue). In (e) the blue and red areas mark the fraction of excitons with exactly 4, and with less than 4 nearest neighbours, respectively. The gray area marks the contribution from excitons occupying states in the continuum above the lattice potential. The variations of all these contributions with the bath temperature are shown in **c-f**. Measurements were obtained by statistically averaging 10 repetitions performed under fixed conditions for every experimental setting.

Fig. S1: Structure factor at filling 1/2. Exact diagonalization results for a 8 site square lattice (Betts cluster) with periodic boundary conditions. Shown is the lowest band structure factor $S_1(\mathbf{k})$ that facilitates the identification of density order. **a** The structure factor at $T=100$ mK has a dominant peak at quasi-momentum $\mathbf{k} = (\pi/a, \pi/a)$, which is a characteristic signature of CB order. A second strongly suppressed quasi-peak is visible (due to finite size effects) at $\mathbf{k} = (0, 0)$, corresponding to a homogenous liquid without any density order. **b** The peak value $|S_1(\pi/a, \pi/a)|$ (blue) and the second highest value $|S_1(0, 0)|$ (red) are plotted vs. temperature T . Up to $T \sim 400$ mK, the structure factor signalling CB order remains at least twice as large as the structure factor for a homogeneous liquid.

Fig. S2: Exciton compressibility and CB order parameter. Temperature variation of the compressibility κ for the CB and MI regimes, red points in panels **a** and **b** respectively, deduced from mean-field calculations. In both panels, the blue curve presents the temperature variation of κ for the normal fluid phase. The compressibility characteristically rises in the insulating regimes upon a temperature increase. For the CB (**a**), the vertical line at $T \sim 350$ mK marks the transition from an insulating phase to a homogeneous liquid (see also **d**). In the MI regime, the melting is not sharp but rather a crossover, starting at around 750 mK, as shown by the vertical line. **c** Compressibility vs. chemical potential for different temperatures. The lower temperatures (blue and red curve) support two different insulating phases at half- and integer-filling, as indicated by the red and light-blue shaded background. At half filling, the insulating phase is identified by a compressibility plateau (blue) or a compressibility minimum (red). These features disappear at higher temperature (black), where CB order vanishes. At integer filling, the insulating phase exhibits a compressibility plateau for any of the applied temperatures. **d** CB order parameter given by the population difference between the two sub-lattices of the square lattice ($n_A - n_B$), vs the chemical potential μ . The extent of the CB phase shrinks with temperature, and entirely vanishes for $T \gtrsim 350$ mK.

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