Superconducting Quantum Circuit of NOR in Quantum Annealing

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Abstract

One possible way to improve the applicability of quantum annealing to various problems is to express the Hamiltonian using a circuit satisfiability problem. The NOR logic function serves as a basic unit for implementing the Hamiltonian for the superconducting quantum circuit. Here we demonstrate quantum annealing with “native” implementation of a Boolean logic Hamiltonian for a quantum circuit composed of superconducting flux qubits. This circuit is robust and can be easily expanded to scale up the analysis. We present an experiment with complete logic operation of NOR, where the circuit produces results with a high probability of success for arbitrary combinations of inputs. Features of the quantum circuit agree qualitatively with theory, especially the mechanism for an operation under external flux modulation. With calibration of bias conditions to compensate for the offset flux from the surrounding circuit, the quantum circuit also agrees quantitatively with theory.

Introduction

Quantum annealing (QA) is a unique and simple architecture that provides the most practical approach to quantum computation in the near term^{1–11}. When expressing problems to be solved, QA uses a Hamiltonian with a time-dependent term for initializing the ground state. At the end of the evolution, the ground state represents the lowest-energy configuration for the Hamiltonian, and thus a solution to the optimization problem^{12–15}. We have proposed QA with native implementation of the Hamiltonian for a superconducting quantum circuit, likely as an application-specific integrated circuit in a semiconductor¹⁶. The problem Hamiltonian, which has a set of ground states consistent with a given truth table, is implemented for the circuit with no redundant qubits. This direct implementation of the original Hamiltonian is essential for obtaining solutions with high accuracy because the original energy relationship in the Hamiltonian is preserved^{16,17}. Using this unique method, we have demonstrated QA in the Hamiltonian of logic gates and a multiplier¹⁶. If the problem Hamiltonian is described with an expression for the circuit satisfiability (SAT) problem^{18,19}, both a highly accurate solution and expandability of scaling can be achieved using this method. In this paper, we present the operation of a basic logic gate in QA with high accuracy. The mechanism can be explained quantitatively by theory, indicating reliable controllability.

Results

Features of NOR and NAND operation. NOR is known to be a versatile computing unit. In our method, the Hamiltonian is designed to minimize energy for logic components of NOR. The superconducting quantum circuit is constructed by directly implementing the Hamiltonian shown in Supplementary Fig. 1(a). Our designed instances are NORs composed of three qubits, corresponding to A and B for inputs and R for the logic result, with critical currents (I_c) of 6.25 mA (NOR1) and 3.75 mA (NOR2). The sample configuration is described in the Methods section. The logic components of NOR, corresponding to the

four combinations of (A, B, R) with the minimum energy, appear at a degeneracy point after QA. Theoretically, the degeneracy point is expressed as

$$\begin{aligned} I_{h2} &= \frac{M_{23}}{M_{31}} \cdot I_{h1}, \\ I_{h3} &= \frac{M_1}{M_3} \cdot \frac{M_{23}}{M_{12}} \cdot I_{h1}, \end{aligned} \quad (1)$$

where I_{hi} ($i = 1-3$) is the external bias of qubit i (corresponding to labels of A, B , and R), M_i ($i = 1-3$) is the mutual inductance between qubit i and the external bias line, and M_{ij} ($i = 1-3, j = 1-3$) is the mutual inductance between qubits i and j . The process for deriving Eq. (1) is described in the Supplementary Methods. The inductances of the qubits and the mutual inductances between them are extracted from the circuit layout (see Methods). The theoretical degeneracy points of NOR1 and NOR2 are estimated as $(I_{h1}, I_{h2}, I_{h3}) = (1.4, 1.4, 3.1)$ and $(1.3, 1.3, 2.8)$ [mA], respectively. Figure 1(a)-(c) respectively shows state diagrams obtained from theory, from simulation using a Josephson integrated circuit simulator (JSIM)²⁰ with $I_{h3} = 2.0$ mA, and from an experiment with $I_{h3} = 2.0$ mA carried out at 10 mK. Detailed methods of the JSIM and the experiment are presented in the Methods section and in the “Experimental configuration” section of the Supplementary Methods, respectively. In the JSIM analysis, a thermal noise current is neglected in order to emphasize the trend of the boundary condition in each logic component. A degeneracy point, where every logic component in NOR appears, is found around a current condition of $(I_{h1}, I_{h2}, I_{h3}) = (1.8, 1.8, 2.0)$ [mA] both in experiments and in JSIM analysis at NOR1. Supplementary Fig. 2 shows the frequency distribution of logic components in experiments carried out at the degeneracy point. Logic components corresponding to the minimum energy of the Hamiltonian are selectively generated. In the state diagram, the boundary along the diagonal direction is found, which we call “ladder” for the sake of convenience. The ladder rising diagonally in the left direction is generated when I_{h3} decreases from the degeneracy point (Fig. 1(d)-(f)). On the other hand, the ladder rising diagonally in the right direction is generated when I_{h3} increases from the degeneracy point (Fig. 1(g)-(i)). These trends agree qualitatively with theory, JSIM analysis, and experiments. At the experimentally obtained degeneracy point, the logic components of NOR randomly occur (see Supplementary Fig. 3 and the “Detailed characteristics of the NOR operation” section of the Supplementary Note). Note that we can produce a desirable logic component by applying an appropriate offset current (a) against the degeneracy point. For example, the logic component of $(A, B) = (0, 1)$ can be considered by applying an external flux bias of $(I_{h1}', I_{h2}') = (I_{h1} - a, I_{h2} + a)$. This corresponds to adopting a along a diagonal direction from the degeneracy point. By applying an appropriate value of a , NOR logic can be reproduced with high accuracy (see Supplementary Fig. 4). We emphasize that a flux injection to one of the qubits by adopting a in the initial condition restricts the state of the other qubit because the qubits interact with each other to minimize the energy after QA. Using this property, input combinations can be clarified from the output value, corresponding to the inverse operation of NOR (see Supplementary Fig. 5). Moreover, this quantum circuit behaves as NAND when I_h is supplied with a negative sign¹⁶. In the state diagram of NAND, the absolute value of the

degeneracy point is almost the same as that of NOR. The boundary of each logic component is modulated by I_{h3} , as it similarly is in NOR (see Supplementary Fig. 6 and the “NAND operation” section of the Supplementary Note). Each logic component of NAND is reproduced with a probability of success up to 100% by adopting an appropriate value of I_{h3} (see Supplementary Fig. 7). QA in NOR1 shows a high probability of success in NOR and NAND operation, but its degeneracy point is different among theory, JSIM analysis, and experiments.

Gray zone evaluation. In NOR2, the barrier height in the energy potential of each qubit is reduced compared with that in NOR1 due to the reduction of I_c . Supplementary Fig. 8 shows the frequency distribution of each logic component with a current condition of $(I_{h1}, I_{h2}) = (1.6, 1.6)$ [mA] and a modulation of I_{h3} between 0 and 9 mA. Around an I_{h3} of 2.8 mA, all candidate logic components in NOR occur. Figure 2(a) and 2(b) show state diagrams with 2D and 3D images at an I_{h3} of 2.8 mA. The experimental degeneracy point is close to the theoretical one. Note that the boundary of each logic component drastically changes around this point. The experimental degeneracy point of NOR2 is $(I_{h1}, I_{h2}, I_{h3}) = (1.6, 1.6, 2.8)$ [mA]. For the sake of convenience, we define the transient width between two different logic regions as a “gray zone.” Two types of gray zones exist: Type I is generated between neighboring regions, such as “100”-“001” and “001”-“010”, and Type II occurs in the same diagonal direction as the ladder. Theoretically, the width of the ladder monotonically decreases with external bias I_{h3} before the degeneracy point. Later, it monotonically increases with I_{h3} . Types I and II can be evaluated from four kinds of line profiles (L_1 - L_4) and the two profiles L_5 and L_6 , respectively (Fig. 2(c)-2(f)). In L_5 and L_6 , four logic components of NOR are identified. Type I gray zones depend on the annealing time (T_a) (see Supplementary Fig. 9 and the “Feature of the Type I gray zone” section of the Supplementary Note). As T_a becomes shorter, the spread of the gray zone becomes wider. These gray zones are clarified in the case of JSIM analysis with the thermal noise current (see Supplementary Fig. 10). Figure 3(a) and 3(b) show the Type I gray zones evaluated in experiments and in JSIM analysis, respectively. The minimum width of the gray zone differs between experiments and JSIM analysis. The impact of flux generated by surrounding circuits appears differently between JSIM and experiments, which results in a difference in the minimum width of the gray zone. However, the consideration of the equidistant current step in the evaluation of the gray zone contributes to suppressing the effect of minor logic component generation. Gray zones between “100” and “001” and between “001” and “010” tend to be large. These trends correspond to the fact that a boundary position is likely to change due to I_{h3} in Fig. 1, indicating an ease in changing the energy state. On the other hand, the values are small in cases of boundaries between “110” and “010” and between “100” and “110”. These trends correspond to the fact that the values of I_{h1} and I_{h2} do not change with modulation of I_{h3} in Fig. 1, indicating difficulty in changing the energy state. These relationships are also confirmed regardless of the value of T_a (see Supplementary Fig. 9). Figure 3(c) and 3(d) show a Type II gray zone with two trends in experiments and JSIM analysis, respectively. The first is a monotonical response against the absolute value of I_{h3} starting from the degeneracy point. This trend agrees with the prediction of the theory. The second is the gray zone spreading slightly wider with the decrease of I_{h3} before the degeneracy point than with the increase of I_{h3} after the degeneracy point.

These trends correspond to the result shown in Fig. 1, where occupation of the “001” region modulates widely with a decrease of I_{h3} compared with the case of an increase of I_{h3} . JSIM analysis also reproduces the same trends seen in experiments. Note that trends change for a thermal noise current above 2.5 pA/ $\sqrt{\text{Hz}}$ in JSIM analysis. Under 2.0 pA/ $\sqrt{\text{Hz}}$, trapping to the local minimum state occurs (see the “Gray zone analysis in JSIM” section of the Supplementary Methods). The logic in NOR and NAND can be realized with high accuracy by tuning the current condition with values of a above 1 mA along a diagonal direction from the degeneracy point, contributing to the avoidance of the gray zone.

Discussion

Here, we discuss why the theoretical and experimental degeneracy points differ. Degeneracy points agree between theory and JSIM analysis when the thermal noise current is applied. This relates to an escape from trapping to a local minimum energy state in JSIM analysis. We focus on the difference between the experimental results and the JSIM analysis. Here, an offset magnetic flux by a surrounding circuit affects the state transition of a qubit in experiments. We can identify the impact of the offset flux from the trend in the state-1 probability (see Supplementary Fig. 11 and the “Calibration of the degeneracy point in experiment” section of the Supplementary Note). The calibrated degeneracy points are $(I_{h1}, I_{h2}, I_{h3}) = (1.6, 1.7, 3.0)$ [mA] in NOR1 and $(1.6, 1.6, 2.9)$ in NOR2 [mA]. Since the smallest half-width of the gray zone is about 0.2 mA, the degeneracy point is almost consistent between theory and experiments. Qubits consisting of NOR1 and NOR2 are designed as the value of a dimensionless factor $b_L = 2\pi L I_c / \Phi_0$ (Φ_0 is the flux quantum) of 4.2 and 2.5, respectively, which is designed to construct an energy potential without a local minimum state. As a result, by considering the calibration of the offset flux, the experimental degeneracy point is quantitatively close to the theoretical one. Since the experimental degeneracy point is predictable from consideration of the theoretical one and the offset flux, both high accuracy and controllability are possible with the combination of this superconducting quantum circuit of NOR.

In order to achieve true QA, the suppression of noise effects, including thermal noise, magnetic flux noise from the environment, and electric noise, is vital. The impact of thermal noise is suppressed for experiments carried out at 10 mK. Our experimental setup supports the evaluation of state transitions in the Josephson junction (JJ) with a switching current of 0.28 mA, which is small compared with I_c in NOR1 and NOR2. The primary origin of noise that affects the Type I gray zone is the noise floor from the dc power supply used for the current source. Supplementary Fig. 12 represents the expanding width of the gray zone up to 1 mA when the experiment is performed using a dc power supply with a three-orders-of-magnitude higher noise floor compared with the one used in Fig. 1 (see the “Suppression of electric noise toward true quantum annealing” section of the Supplementary Note). In this case, the state transition does not show a dependence on annealing time (see Supplementary Fig. 11(f)). By responding to eliminate the undesirable effect from the electric noise of the instrument, QA contributes to the search for a global minimum state in the Hamiltonian, resulting in high accuracy.

It is known that optimization problems that can be effectively handled by QA can be formulated as circuit SAT problems^{18,19}. The large scale of the Hamiltonian can be treated by expanding the versatile logic elements of NOR and NAND with a connection qubit. Note that a carry transfer without any error is possible by considering the inductance of the connection qubit (see Supplementary Fig. 13 and the “Expandability of the circuit” section of the Supplementary Note). This means that a superconducting quantum circuit with a wide range of applications will be possible with high expandability in QA.

We have fabricated the superconducting quantum circuit of NOR with a complete logic operation, where the circuit produces arbitrary combinations of inputs with a high probability of success in QA. Using the connection qubit, the superconducting quantum circuit is easy to expand. With offset flux calibration, operations in experiments quantitatively agree with theory. These results contribute to extending the use of quantum computing by providing highly accurate computation for solving Hamiltonians of circuit SAT problems, which are widely applicable to real-world problems.

Methods

Superconducting flux qubit. The qubits used in this experiment are superconducting compound JJ rf-SQUID flux qubits, which is a similar configuration to that described by Harris et al^{9,10}. We fabricate the superconducting quantum circuit using a process that creates four Nb layers and a JJ with a critical current density of 1 mA/mm². When a flux of F_0 is applied to an inserted small loop with two JJs (the switching current of single JJ is defined as I_c), the rf-SQUID takes two bistable states with persistent current flowing clockwise or counterclockwise through the main loop shown in Supplementary Fig. 1(b). These two states correspond to logical 1 and 0 states in the qubit. Measurement details are described in the “Experimental configuration” section of the Supplementary Methods.

Design of the NOR. Inductances (L) and mutual inductances (M) are extracted from the layout of the superconducting quantum circuit using InductEX²¹ (see Supplementary Table 1). In the qubit, a bistable energy state can be achieved by coordinating the value of b_L . NOR is composed of three superconducting flux qubits, which have all-to-all connectivity. The two types of NOR, consisting of the same superconducting circuits ($L = 110$ pH) with different I_c , are prepared. I_c is tuned so that it is thermally annealed at 220°C.

JSIM analysis. The NOR circuit model is constructed and analyzed by a JSIM²⁰. Owing to the time constraint, T_a is settled in 1 ms. The thermal noise current, which overcomes a trap to local minimum energy, is used. Each current condition is performed with 300 and 1000 iterations for the evaluation of the state diagram of NOR and the state transition in a single qubit, respectively. Details of the gray zone analysis are given in the “Gray zone analysis in JSIM” section of the Supplementary Methods.

Declarations

Data availability

All data generated or analyzed during this study are included in this published article (and its supplementary information files).

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Author contributions

D.S. came up with the initial concept and wrote the manuscript. K.I. and D. S. considered the theory. D.S. considered and designed the device configuration. D.S. created the circuit model. D. S. and K. M. performed simulations with the JSIM. Y.Y. developed the software interface for the measurement apparatus. M.H. selected the devices for testing. D.S. conducted the experiments at 10 mK and analyzed the results. All authors reviewed and approved the manuscript.

Competing interests

The authors declare no competing interests.

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Figures

Figure 1

Trends in state diagrams of NOR operation.

State diagrams for **(a-c)** $I_{h3} = 2.0$ mA, **(d-f)** $I_{h3} = 1.0$ mA, **(g-i)** $I_{h3} = 3.0$ mA: (a, d, f) theory, (b, e, h) JSIM analysis without a thermal noise current, and (c, f, i) a 10 mK experiment.

Figure 2

Analysis of state diagrams in NOR operation.

(a) Two and (b) three-dimensional state diagrams of NOR2 in a 10 mK experiment at $I_{h3} = 2.8$ mA. In order to analyze the boundary width between two logic regions (defined as a gray zone), line profiles of L_1 - L_6 , depicted in (a), are evaluated. Line profiles of (c) L_1 , (d) L_2 , (e) L_5 , and (f) L_6 .

Figure 3

Gray zone analysis.

Type I gray zone analysis in **(a)** 10 mK experiment and **(b)** JSIM analysis. The inset of (a) represents the theoretical analysis. In the JSIM analysis of (b), a thermal noise current of 3.0 pA/√Hz is adopted. Type II gray zone analysis in **(c)** a 10 mK experiment and **(d)** JSIM analysis. NOR2 is utilized.

Supplementary Files

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