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Tunable Plasmonic Modes in Graphene-loaded Plasma Media

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Abstract: This paper investigates tunable magneto-plasmons in graphene-based structures combined with gyro-electric layers. In the general waveguide, each graphene sheet has been sandwiched between two different gyro-electric layers. The whole structure has been exposed in the presence of a magnetic bias. An accurate analytical model based on all propagating modes has been proposed and led to closed-form complicated relations. As a special case of the multi-layer waveguide, a hybrid graphene-gyroelectric structure is studied in this paper. It has been illustrated that the propagating features of the exemplary waveguide can be tuned by changing the external magnetic bias and the chemical potential of graphene, which makes it a new tunable platform for the design of non-reciprocal plasmonic devices in the THz region.

Key-words: Multi-layer structure, anisotropic graphene, gyro-electric medium, analytical model

1. Introduction

Nowadays, graphene is known as a fascinating two-dimensional material for the fabrication and development of innovative photonic components, due to its unique properties [1-4]. Among these features, graphene conductivity, a tunable parameter via electrostatic or magnetostatic bias, has created a new emerging science known as “Graphene Plasmonics”. Based on the propagating properties of Surface Plasmon Polaritons (SPPs) on graphene, various plasmonic components have been designed and investigated in the THz frequencies such as sensors [5, 6], couplers [7-9], filters [10-12], resonators [13-15], and circulators [16-19]. Among these devices, graphene-based waveguides play a remarkable role in graphene plasmonics, which are divided into various platforms such as planar [20-28], cylindrical [29-33], and elliptical structures [34-37]. It should be noted that graphene plasmonics is mostly used in the mid-infrared region whereas metal-based plasmonics is a promising candidate in near-infrared frequencies for various applications [38-42].

One of the familiar methods to enhance the performance of graphene-based devices is the integration of graphene with various smart materials such as chiral materials [43-50], and non-linear materials [51-61]. For instance, the authors in [44] have introduced a new platform for THz sensing applications by integrating graphene sheets with chiral materials. The integration of graphene with new materials will lead to new plasmonic wave phenomena and effects and also will give more degrees of freedom to the designer for the tune of the propagation features of SPPs.

In this paper, we aim to study the plasmonic features of hybrid graphene-gyroelectric structures by using a new analytical method. To the author’s knowledge, no published work has been reported for the analytical model of gyro-electric graphene-based multi-layer waveguides. In the general structure, each graphene sheet has been sandwiched between two different gyro-electric layers, with an anisotropic permittivity tensor magnetized by a magnetic bias. This general configuration can be a promising platform for the design of non-reciprocal THz devices because its properties can be altered via magnetic bias and chemical potential. The proposed analytical model considers all propagating modes inside the structure. It gives closed-form relations for the field contributions of all SPP waves. Hence, our analytical model is accurate. Moreover, it can calculate the modal properties of the gyro-electric graphene-based structure faster than any software. Because all electromagnetic software utilizes numerical methods, which need the discretization of the problem.

The remainder of the paper is organized as follows. In section 2, the general structure and its theoretical model will be studied. Then, to show the high accuracy of the model, the effective index prepared by our model will be

compared with the results of [62] for a familiar waveguide in section 3. Then, a new gyro-electric graphene-based waveguide will be introduced and investigated in this section. Finally, section 4 concludes the article.

2. The Proposed General Structure and its Analytical Model

Fig. 1 illustrates the cross-section of the proposed general structure. In this configuration, each graphene sheet has been sandwiched between two different magnetized gyro-electric layers with the following permittivity tensor ($\bar{\bar{\epsilon}}_N$):

$$\bar{\bar{\epsilon}}_N = \epsilon_0 \begin{pmatrix} \epsilon_N & j\epsilon_{a,N} & 0 \\ -j\epsilon_{a,N} & \epsilon_N & 0 \\ 0 & 0 & \epsilon_{||,N} \end{pmatrix} \quad (1)$$

Fig. 1. Cross-section of a general gyro-electric multi-layer structure containing graphene sheets. The external magnetic bias is applied in the z-direction.

The elements of permittivity tensor (relation (1)) are considered as follows:

$$\epsilon_N = \epsilon_{\infty,N} \left(1 - \frac{\omega_{p,N}^2 (\omega + j\nu_N)}{\omega \left[(\omega + j\nu_N)^2 - \omega_c^2 \right]} \right) \quad (2)$$

$$\epsilon_{a,N} = \epsilon_{\infty,N} \left(\frac{\omega_{p,N}^2 \omega_c}{\omega \left[(\omega + j\nu_N)^2 - \omega_c^2 \right]} \right) \quad (3)$$

$$\epsilon_{||,N} = \epsilon_{\infty,N} \left(1 - \frac{\omega_{p,N}^2}{\omega (\omega + j\nu_N)} \right) \quad (4)$$

In (2)-(4), ν_N , $\epsilon_{\infty,N}$ are the effective collision rate and the background permittivity, respectively. Furthermore, the plasma and the cyclotron frequencies are defined as follows [63]:

$$\omega_{p,N} = \sqrt{\frac{n_s e^2}{\epsilon_0 \epsilon_{\infty,N} m^*}} \quad (5)$$

$$\omega_c = \frac{e B_0}{m^*} \quad (6)$$

In (5)-(6), e , m^* and n_s are the charge, effective mass, and the density of the carriers, respectively. The conductivity of magnetically biased graphene sheet in the N -th layer is expressed as follows [64]:

$$\bar{\bar{\sigma}}_N(\omega, \mu_c(\mathbf{E}_0), \Gamma, T, \mathbf{B}_0) = \begin{pmatrix} \sigma_{O,N} & \sigma_{H,N} \\ -\sigma_{H,N} & \sigma_{O,N} \end{pmatrix} \quad (7)$$

In this relation, ω is the radian frequency, Γ is the phenomenological electron scattering rate ($\Gamma = 1/\tau$, where τ is the relaxation time), T is the temperature, μ_c is the chemical potential that can be altered by chemical doping or electrostatic bias $\mathbf{E}_0$, and $\mathbf{B}_0$ is the applied magnetostatic bias field. Besides, $\sigma_{O,N}$, $\sigma_{H,N}$ are the direct and indirect (Hall) conductivities of the graphene, which are obtained by using Kubo's relations reported in [64].

Consider Maxwell's equations in the frequency domain (suppose $e^{i\omega t}$):

$$\nabla \times \mathbf{E} = -j\omega \mu_N \mathbf{H} \quad (8)$$

$$\nabla \times \mathbf{H} = j\omega \bar{\bar{\epsilon}}_N \cdot \mathbf{E} \quad (9)$$

The z-components of the electromagnetic fields inside the gyro-electric media can be obtained as:

$$\left(\nabla_{\perp}^2 + \frac{\epsilon_{\parallel,N}}{\epsilon_N} \frac{\partial^2}{\partial z^2} + (k_0^2 \epsilon_{\parallel,N} \mu_N) \right) E_z + \frac{k_0 \mu_N \epsilon_{a,N}}{\epsilon_N} \frac{\partial H_z}{\partial z} = 0 \quad (10)$$

$$\left(\nabla_{\perp}^2 + \frac{\partial^2}{\partial z^2} + (k_0^2 \epsilon_{\perp,N} \mu_N) \right) H_z - \frac{k_0 \epsilon_{\parallel,N} \epsilon_{a,N}}{\epsilon_N} \frac{\partial E_z}{\partial z} = 0 \quad (11)$$

where

$$\nabla_{\perp}^2 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \quad (12)$$

$$\epsilon_{\perp,N} = \epsilon_N - \frac{\epsilon_{a,N}^2}{\epsilon_N} \quad (13)$$

By considering the radial modes inside the gyro-electric layer,

$$H_{z,N}(r, \varphi, z) = \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{\infty} H_{z,N}^m(z) \exp(-jm\varphi) J_m(\beta r) d\beta \quad (14)$$

$$E_{z,N}(r, \varphi, z) = \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{\infty} E_{z,N}^m(z) \exp(-jm\varphi) J_m(\beta r) d\beta \quad (15)$$

and substituting these equations into (10) and (11), the following characteristics equation for the N -th layer is achieved:

$$s^4 + A_{1,N} s^2 + A_{2,N} = 0 \quad (16)$$

with the following coefficients,

$$A_{1,N} = \left(\frac{\epsilon_N}{\epsilon_{\parallel,N}} \right) \left((k_0^2 \epsilon_{\parallel,N} \mu_N - \beta^2) + (k_0^2 \epsilon_{\perp,N} \mu_N - \beta^2) \frac{\epsilon_{\parallel,N}}{\epsilon_N} + k_0^2 \mu_N \epsilon_{\parallel,N} \left(\frac{\epsilon_{a,N}}{\epsilon_N} \right)^2 \right) \quad (17)$$

$$A_{2,N} = \left(\frac{\varepsilon_N}{\varepsilon_{\parallel,N}} \right) \left[\left(k_0^2 \varepsilon_{\parallel,N} \mu_N - \beta^2 \right) \cdot \left(k_0^2 \varepsilon_{\perp,N} \mu_N - \beta^2 \right) \right] \quad (18)$$

In (14)-(15), m is an integer and β is the propagation constant of plasmonic waves. Now, the roots of the characteristics equation are obtained for the N -th layer,

$$k_{z,2N-1} = \sqrt{\frac{-A_{1,N} + \sqrt{A_{1,N}^2 - 4A_{2,N}}}{2}} \quad (19)$$

$$k_{z,2N} = \sqrt{\frac{-A_{1,N} - \sqrt{A_{1,N}^2 - 4A_{2,N}}}{2}} \quad (20)$$

Suppose that each gyro-electric layer has the following propagation constants:

$$k_z = \begin{cases} k_{z,1}, k_{z,2} & i = 1, 2 \quad ; \text{ for layer } N = 1 \\ \dots & \dots \\ k_{z,2N-1}, k_{z,2N} & i = 2N - 1, 2N \quad ; \text{ for layer } N \\ k_{z,2N+1}, k_{z,2N+2} & i = 2N + 1, 2N + 2 \quad ; \text{ for layer } N + 1 \end{cases} \quad (21)$$

In (21), N, i shows the number of the layer and the index of the root for that layer, respectively. Now, the electromagnetic fields for various regions are written as follows:

$$H_z^m(z) = \begin{cases} A_{m,1,1}^+ e^{+k_{z,1}z} + A_{m,2,1}^+ e^{+k_{z,2}z} & z < 0 \\ A_{m,3,2}^+ e^{+k_{z,3}z} + A_{m,4,2}^+ e^{+k_{z,4}z} + \\ A_{m,3,2}^- e^{-k_{z,3}z} + A_{m,4,2}^- e^{-k_{z,4}z} & 0 < z < d_2 \\ \dots & \dots \\ A_{m,2N+1,N+1}^- e^{-k_{z,2N+1}z} + \\ A_{m,2N+2,N+1}^- e^{-k_{z,2N+2}z} & z > \sum_{k=2}^N d_k \end{cases} \quad (22)$$

$$E_z^m(z) = \begin{cases} T_{1,1}^+ A_{m,1,1}^+ e^{+k_{z,1}z} + T_{2,1}^+ A_{m,2,1}^+ e^{+k_{z,2}z} & z < 0 \\ T_{3,2}^+ A_{m,3,2}^+ e^{+k_{z,3}z} + T_{4,2}^+ A_{m,4,2}^+ e^{+k_{z,4}z} + \\ T_{3,2}^- A_{m,3,2}^- e^{-k_{z,3}z} + T_{4,2}^- A_{m,4,2}^- e^{-k_{z,4}z} & 0 < z < d_2 \\ \dots & \dots \\ T_{2N+1,N+1}^- A_{m,2N+1,N+1}^- e^{-k_{z,2N+1}z} + \\ T_{2N+2,N+1}^- A_{m,2N+2,N+1}^- e^{-k_{z,2N+2}z} & z > \sum_{k=2}^N d_k \end{cases} \quad (23)$$

and

$$T_{i,N}^{\pm} = \frac{\pm 1}{k_0 \varepsilon_{\parallel,N} \left(\frac{\varepsilon_{a,N}}{\varepsilon_N} \right)} \left(k_{z,i} + \frac{1}{k_{z,i}} \left(k_0^2 \varepsilon_{\perp,N} \mu_N - \beta^2 \right) \right) \quad i = 2N - 1, 2N, \quad N = 1, 2, 3, \dots \quad (24)$$

The transverse components of electromagnetic fields can be derived:

$$\begin{pmatrix} E_{r,i}^\pm \\ H_{r,i}^\pm \end{pmatrix} = \bar{\mathbf{Q}}_{i,N}^{Pos,\pm} \frac{\partial}{\partial r} \begin{pmatrix} E_{z,i}^\pm \\ H_{z,i}^\pm \end{pmatrix} + \frac{m}{r} \bar{\mathbf{Q}}_{i,N}^{Neg,\pm} \begin{pmatrix} E_{z,i}^\pm \\ H_{z,i}^\pm \end{pmatrix} \quad i = 2N-1, 2N \quad (25)$$

$$j \begin{pmatrix} E_{\varphi,i}^\pm \\ H_{\varphi,i}^\pm \end{pmatrix} = \bar{\mathbf{Q}}_{i,N}^{Neg,\pm} \frac{\partial}{\partial r} \begin{pmatrix} E_{z,i}^\pm \\ H_{z,i}^\pm \end{pmatrix} + \frac{m}{r} \bar{\mathbf{Q}}_{i,N}^{Pos,\pm} \begin{pmatrix} E_{z,i}^\pm \\ H_{z,i}^\pm \end{pmatrix} \quad i = 2N-1, 2N \quad (26)$$

Where

$$\bar{\mathbf{Q}}_{i,N}^{Pos,\pm} = \frac{1}{2} \left[\frac{1}{k_{z,i}^2 + k_0^2 \varepsilon_{+,N} \mu_N} \begin{pmatrix} \pm k_{z,i} & -\omega \mu_0 \mu_N \\ \omega \varepsilon_0 \varepsilon_{+,N} & \pm k_{z,i} \end{pmatrix} + \frac{1}{k_{z,i}^2 + k_0^2 \varepsilon_{-,N} \mu_N} \begin{pmatrix} \pm k_{z,i} & \omega \mu_0 \mu_N \\ -\omega \varepsilon_0 \varepsilon_{-,N} & \pm k_{z,i} \end{pmatrix} \right] \quad (27)$$

$$\bar{\mathbf{Q}}_{i,N}^{Neg,\pm} = \frac{1}{2} \left[\frac{1}{k_{z,i}^2 + k_0^2 \varepsilon_{+,N} \mu_N} \begin{pmatrix} \pm k_{z,i} & -\omega \mu_0 \mu_N \\ \omega \varepsilon_0 \varepsilon_{+,N} & \pm k_{z,i} \end{pmatrix} - \frac{1}{k_{z,i}^2 + k_0^2 \varepsilon_{-,N} \mu_N} \begin{pmatrix} \pm k_{z,i} & \omega \mu_0 \mu_N \\ -\omega \varepsilon_0 \varepsilon_{-,N} & \pm k_{z,i} \end{pmatrix} \right] \quad (28)$$

are the Q-matrices used in (25)-(26) and,

$$\varepsilon_{\pm,N} = \varepsilon_N \pm \varepsilon_{a,N} \quad (29)$$

We apply the boundary conditions for the graphene sheets ($N = 1, 2, 3, \dots$):

$$E_{r,N} = E_{r,N+1}, E_{\varphi,N} = E_{\varphi,N+1} \quad (30)$$

$$H_{r,N+1} - H_{r,N} = -\sigma_{H,N} E_{r,N} + \sigma_{O,N} E_{\varphi,N} \quad (31)$$

$$H_{\varphi,N+1} - H_{\varphi,N} = -(\sigma_{O,N} E_{r,N} + \sigma_{H,N} E_{\varphi,N}) \quad (32)$$

Now, the final matrix is obtained:

$$\bar{\bar{S}}_{4N+4, 4N+4} \cdot \begin{pmatrix} A_{m,1,1}^+ \\ A_{m,2,1}^+ \\ \dots \\ A_{m,2N+1,N+1}^- \\ A_{m,2N+2,N+1}^- \end{pmatrix}_{4N,1} = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{pmatrix}_{4N,1} \quad (33)$$

In (33), the matrix $\bar{\bar{S}}$ is

$$\bar{\bar{S}} = \begin{pmatrix} P_{1,1,1}^+ & P_{2,1,1}^+ & \dots & 0 & 0 \\ R_{1,1,1}^+ & R_{2,1,1}^+ & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & R_{2N+2,N+1,2}^- e^{-k_{z,2N+2} \left(\sum_{k=2}^N d_k + d_S \right)} \end{pmatrix} \quad (34)$$

and the following relations have been used in (34),

$$\begin{pmatrix} P_{i,N,1}^\pm(\mathbf{r}) \\ P_{i,N,2}^\pm(\mathbf{r}) \end{pmatrix} = \left[\bar{\mathbf{Q}}_{i,N}^{Pos,\pm} \beta J'_m(\beta r) + \frac{m}{r} \bar{\mathbf{Q}}_{i,N}^{Neg,\pm} J_m(\beta r) \right] \begin{pmatrix} T_{i,N}^\pm \\ 1 \end{pmatrix} \quad i = 2N-1, 2N \quad (35)$$

$$\begin{pmatrix} R_{i,N,1}^{\pm}(\mathbf{r}) \\ R_{i,N,2}^{\pm}(\mathbf{r}) \end{pmatrix} = -j \left[\bar{\mathbf{Q}}_{i,N}^{Neg,\pm} \beta J'_m(\beta r) + \frac{m}{r} \bar{\mathbf{Q}}_{i,N}^{Pos,\pm} J_m(\beta r) \right] \begin{pmatrix} T_{i,N}^{\pm} \\ 1 \end{pmatrix} \quad i = 2N-1, 2N \quad N = 1, 2, 3, \dots \quad (36)$$

Now, the propagation constant and other propagating features such as the effective index can be derived by setting $\det(\bar{\mathbf{S}}) = 0$.

It is worth to be mentioned that the present model gives closed-form mathematical relations for all plasmonic modes propagating inside the waveguide. In all electromagnetic software, the structure should be discretized by numerical methods. Thus, compared to the common software, our model is faster than any numerical method. To be brief, the theoretical expressions presented in this section have two main privileges: first, they can compute the electromagnetic field distributions and dispersion relation for all propagation modes, and second, they have high accuracy and speed compared to any numerical software. In what follows, we will investigate the analytical results of our model for a hybrid graphene-gyroelectric waveguide.

3. Results and Discussion

In the previous section, we proposed an analytical model for general gyro-electric multi-layer structures, where each graphene sheet was sandwiched between two different gyro-electric layers. To show the richness of the proposed structure and the analytical model, we will investigate the analytical results for a gyro-electric structure, as a special case of the general waveguide. It is obvious that the study of all possible structures is impossible and is outside the scope of this article. Therefore, we apply the analytical model for a specific graphene-based gyro-electric waveguide.

Before embarking on presenting analytical results for a graphene-based gyro-electric waveguide, the accuracy of the analytical model should be considered. For this reason, we have depicted the effective index ($n_{eff} = Re[\beta]/k_0$) of TM mode for a famous graphene-based waveguide, as seen in Fig. 2. In this structure, a graphene sheet has been deposited on SiO₂-Si layers with the permittivities of 2.09 and 11.9, respectively. Furthermore, the chemical potential of graphene is 0.23 eV and the relaxation time is 0.5 ps. The thickness of the SiO₂ layer is 5 μm . One can see that the effective index increases with the frequency increment. The important point observed in this figure is the good agreement of our analytical model and numerical results prepared by [62], which validates the accuracy of our analytical model.

Fig. 2. The Comparison of our analytical model and the results of [62] for the effective index of TM mode for the waveguide constituting Air-Graphene-SiO₂-Si layers.

Consider the gyro-electric graphene-based waveguide (as a special case of the general structure of Fig. 1), as demonstrated in Fig. 3. In this configuration, the magnetic bias is applied in the z-direction. The magnetized gyro-electric is supposed to be the n-type InSb, with a thickness of $s = 50 \text{ nm}$ and parameters of $\epsilon_{\infty} = 15.68$, $m^* =$

$0.022m_e$, $n_s = 1.07 \times 10^{17}/cm^3$, $\nu = 0.314 \times 10^{13}s^{-1}$ where m_e is the electron's mass. The thickness of the SiO_2 layer is $t = 200\text{ nm}$. In this waveguide, graphene has the following parameters: $\mu_c = 0.45\text{ eV}$, $\tau = 0.34\text{ ps}$, $T = 300\text{ K}$.

In Fig. 4, the modal properties of the gyro-electric graphene-based waveguide have been depicted as a function of the frequency at the magnetic bias of 4T. To consider the effect of non-reciprocity, the propagation features also have been calculated for the reverse direction (i.e. $B_0 = -4T$). As observed in this figure, the proposed structure of Fig. 3 is nonreciprocal due to the change of plasmonic properties by reversing the direction of the external magnetic bias. It is evident from Fig. 4(a) that the nonreciprocity occurs in the vicinity of 6.5 THz, where propagation loss has a maximum point at this frequency (see Fig. 4 (b)).

Fig. 3. The cross-section of the gyro-electric graphene-based waveguide, constituting Graphene-InSb-SiO₂-Si layers.

Fig. 4. Analytical results for (a) the effective index and (b) the propagation losses versus frequency for various external magnetic fields ($B = -4, 4\text{ T}$). The chemical potential of the graphene layer is supposed to be $\mu_c = 0.45\text{ eV}$.

To investigate the performance of the plasmonic waveguides, one of the main parameters is Figure of Merit (defined as $FOM = \text{Re}(k_p)/2\pi\text{Im}(k_p)$) [65]. In Fig. 5, FOM has been shown as a function of external magnetic bias at various frequencies. It should be noted that our studied frequency window, in which nonreciprocity occurs, is 2-10 THz (see Fig. 4). However, in Fig. 5, FOM also has been calculated at a higher frequency ($f = 15\text{ THz}$) to exactly consider the performance of the structure at the outside of this window. It is obvious from this figure that near resonance frequency (6.5 and 7 THz), FOM diagram depends on the value of the external magnetic field. In these frequencies, there are valleys (or minimum points) for the external magnetic bias, which FOM reaches its minimum value.

The strong variations of FOM on external magnetic field near non-reciprocity frequency range originates from the main role of non-diagonal elements in permittivity tensor of the gyro-electric layer at the resonance frequency. As the frequency increases and it moves away from resonance frequency (see Fig. 5, the analytical result of FOM at the frequency of 15 THz), the magnitude of non-diagonal elements in permittivity tensor becomes smaller compared to diagonal elements and thus the FOM does not depend strongly on the magnetic bias at the frequency of 15 THz.

Fig. 5. FOM as a function of magnetic bias at various frequencies. The chemical potential of the graphene layer is supposed to be $\mu_c = 0.45 \text{ eV}$.

Fig. 6. FOM as a function of chemical potential at various frequencies. The applied magnetic bias is $B = 4 \text{ T}$.

The tunability of the proposed structure via the chemical potential of graphene is demonstrated in Fig. 6, which represents the variations of FOM as a function of the chemical potential at various frequencies. At lower frequencies, i.e. frequency range of non-reciprocity effect (2-10 THz), FOM increases as the chemical potential increases (see Fig. 6, the analytical result of FOM at the frequency of 10 THz). As frequencies increase and keep out of the non-reciprocity range, the FOM strongly depends on chemical potential. For instance, consider the frequency of 16 THz in this figure. This diagram has a minimum point in the vicinity of 0.18 eV and thus the structure should not be designed at this chemical potential.

As a final point, we perform a parametric study to investigate the influence of InSb and Silica thicknesses on FOM. Table 1 reports FOM for various values of InSb thickness at 6.5 THz (resonance frequency) and 10 THz. The chemical potential of graphene and applied DC bias in this table are supposed to be 0.2 eV and 1 T, respectively. One can see that as the thickness of the gyroelectric layer increases, FOM decreases. It happens because the field confinement inside the InSb decreases by the thickness increment. However, the slope of FOM variations is different at various frequencies. For instance, at 6.5 THz (resonance frequency), the slope of changes is greater compared to 10 THz. The main reason behind this is the high oscillation of SPPs near resonance frequency. In Table 2, FOM for various values of silica thicknesses is given at 6.5 THz (resonance frequency) and 10 THz. Again, it is assumed that the chemical potential of graphene and applied DC bias in this table are 0.2 eV and 1 T, respectively. Moreover, other parameters remain fixed. It is evident from this table that the increment of silica thickness will decrease FOM.

Furthermore, FOM variations are negligible here, compared to Table 1. This matter is concerned with the energy concentration inside the gyroelectric layer, especially on the graphene sheet. Hence, the change of InSb thickness will alter FOM more in comparison to variations of silica thickness.

Table 1. FOM for Various Thicknesses of Gyroelectric Layer (InSb) @ $B = 1\text{ T}$, $\mu_c = 0.2\text{ eV}$, $t = 200\text{ nm}$

S (nm)		50	60	70	80	90	100
FOM	@ 6.5 THz	4.10	3.91	3.72	3.45	3.34	3.09
	@ 10 THz	8.50	8.43	8.35	8.29	8.19	8.05

Table 2. FOM for Various Thicknesses of Silica @ $B = 1\text{ T}$, $\mu_c = 0.2\text{ eV}$, $S = 50\text{ nm}$

t (nm)		200	210	220	230	240	250
FOM	@ 6.5 THz	4.10	4.08	4.06	4.00	3.93	3.84
	@ 10 THz	8.50	8.49	8.48	8.46	8.42	8.36

4. Conclusion

In this article, we proposed a novel analytical model for gyro-electric multi-layer graphene-based structures. In our general waveguide, each graphene sheet was located between two different magnetized gyro-electric layers. To demonstrate the richness of the general structure and also the high accuracy of our model, a new gyro-electric graphene-based waveguide, constituting Graphene-InSb-SiO₂-Si layers, was studied. It was shown that the proposed gyro-electric waveguide is non-reciprocal, i.e. its propagation properties vary as the direction of external magnetic bias reverses. FOM, an important parameter to evaluate the performance of the structure, was depicted as a function of magnetic bias and chemical potential. The FOM was strongly dependent on the external magnetic bias at the cyclotron frequency. Furthermore, it was observed that the gyro-electric graphene-based waveguide is a tunable device and its FOM can be adjusted via the chemical potential of graphene. Harnessing gyro-electric media together with graphene sheets can be utilized for the design of innovative plasmonic devices at the THz frequencies.

Declarations

Ethics Approval: Not Applicable.

Consent to Participate: Not Applicable.

Consent to Publish: Not Applicable.

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Availability of Data and Materials: Not Applicable.

Code availability: Not Applicable.

Authors Contributions: M. B. Heydari performed the analytical modeling, conducted the numerical simulations, and wrote the manuscript.

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