Stellar Initial Mass Function Varies with Metallicities and Time

Chao Liu (✉ liuchao@nao.cas.cn)

National Astronomical Observatories <https://orcid.org/0000-0002-1802-6917>

Jiadong Li

National Astronomical Observatories, CAS <https://orcid.org/0000-0002-3651-5482>

Zhi-yu Zhang

Nanjing University <https://orcid.org/0000-0002-7299-2876>

Hao Tian

National Astronomical Observatories, CAS

Xiaoting Fu

Kavli Institute of Astronomy and Astrophysics, Peking University

Jiao Li

National Astronomical Observatories, CAS

Physical Sciences - Article

Keywords:

Posted Date: January 24th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1278194/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Stellar Initial Mass Function Varies with Metallicities and Time

Jiadong Li^{1,2,3,*}, Chao Liu^{1,2,3,*}, Zhi-Yu Zhang^{4,5}, Hao Tian¹, Xiaoting Fu⁶, and Jiao Li¹

¹Key Lab of Space Astronomy and Technology, National Astronomical Observatories, Beijing, 100101, China

²Institute for Frontiers in Astronomy and Astrophysics of Beijing Normal University, Beijing, 100875, China

³University of Chinese Academy of Sciences, Beijing, 100049, China

⁴School of Astronomy and Space Science, Nanjing University, Nanjing, 210000, China

⁵Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, People's Republic of China

⁶Kavli Institute of Astronomy and Astrophysics, Peking University, Beijing, 100871, China

*corresponding author, liuchao@nao.cas.cn

ABSTRACT

Most structural and evolutionary properties of our Milky Way and external galaxies strongly rely on the stellar initial mass function (IMF), namely the distribution of the initial stellar mass formed in one episode of stellar population^{1–3}. Both constant and variable IMFs across different environments have been claimed in our Galaxy and in external galaxies^{3–8}, despite a large number of theoretical and observational efforts. However, most of these studies either depend on indirect models or are biased to relatively small sample sizes, leading to high uncertainties and inconclusive results. Here we report a large-sample star-counting statistical result based on $\sim 93,000$ M-dwarf stars in the 100–300 parsec Solar neighbourhood selected from the LAMOST⁹ and Gaia surveys¹⁰, and find unambiguous evidence of a variable IMF distribution that depends on metallicity and stellar age. Specifically, the stellar population formed at the early time contains fewer low-mass stars compared to the canonical IMF¹, independent of stellar metallicities. In present days, on the other hand, the proportion of low-mass stars increases with stellar metallicity. Such non-universality of the IMF, especially for star-forming systems at low-metallicity environments, implies that the history of the Milky Way and bulk evolution of the Universe have to be reconsidered.

The mass distribution of stars when they were born shapes the stellar population in all stellar systems of our Universe. The pioneering study was first presented by Salpeter¹¹, who derived the distribution of the initial mass of stars, named as the stellar initial mass function (IMF), by counting stars with masses from 0.4 to 10 $M_{\odot}$ in the Solar neighbourhood. The IMF turns out to be one of the most fundamental concepts for many fields of modern astronomy, including star formation, galaxy evolution, dark matter, black holes, planets, etc. Because low-and-intermediate-mass stars contribute the majority of stellar mass, while massive stars contribute most of the emission energy, the IMF behaves as the key to extrapolating from observables of massive stars to the dim unobservable low-mass stars.

Whether the IMF is universal is a question. Despite a large number of efforts in theoretical study, which are possibly associated with turbulence, gas temperature, gas metallicity, cosmic rays, and cosmic microwave background temperature, etc., the primary physical mechanisms that regulate the IMF are still unclear¹². Several observational evidence in external galaxies, on the other hand, traced by mass-to-light ratio, chemical abundances, H α /UV colours, etc., suggest that the IMF is not universal^{5–8, 13–18}. These studies, however, are strongly dependent on the assumptions of mass-to-light ratios, spectral synthesis

Fig. 1. (a) Colour-magnitude distribution of the samples of M-type stars. The colour bar indicates the median stellar metallicity, $[M/H]$, in each colour-magnitude bin. The dashed lines indicate the stellar mass from PARSEC stellar models. (b) The stellar counts as a function of stellar mass and distance to the Sun. We chose stars inside the black dashed box, within which the coverage of the stellar mass is roughly uniform along the distance. Hence, the selected stars are barely affected by any volume selection effect. (c) The stellar counts as a function of stellar mass and metallicity, $[M/H]$. The two horizontal dashed lines indicate the selected stellar mass range, $0.3 M_\odot < M^* < 0.7 M_\odot$. The stellar counts showing here have not been corrected the observational selection effects.

Fig. 2. (a) IMF estimates at different stellar metallicities. All IMFs are normalised at $0.5 M_\odot$ for comparison. The black dotted line denote the expected IMF with $[M/H] = +0.25$. The black dash-dotted line indicates the Kroupa IMF with the power exponents $\alpha_1 = 1.3$ at $< 0.5 M_\odot$ and $\alpha_2 = 2.3$ at $> 0.5 M_\odot$. The dashed line indicates the Salpeter IMF with the power exponent of 2.35. (b) The IMF power exponent, α , as a function of stellar metallicity. The filled rectangles display α at different $[M/H]$ bins. The grey solid lines represent α - $[M/H]$ relation, after correcting the binary star fractions²⁵, with the grey shade representing the $1-\sigma$ uncertainty. The vertical shaded region emphasizes the metallicity range of $-0.3 < [M/H] < +0.1$, in which α tightly increases with $[M/H]$.

models, and/or star-formation history^{7,14,16–19}.

Direct star counting, which provides the most direct and least model-dependent evidence, is essential to verify the IMF variation in our Milky Way. Although this approach has been applied in IMF studies for decades, the sample sizes were limited to a few simple stellar populations or a few thousand field stars with distance less than 10 pc^{3,20}. Recent large surveys such as Gaia¹⁰ and Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST)⁹ offers a new opportunity to reassess the IMF with the star-counting technique²¹.

We focus on the low-mass stars in the Milky Way by counting field stars with a stellar mass range of $0.3 \sim 0.7 M_{\odot}$, which are mostly M-dwarf stars. Their lifespans are longer than the Universe, so none has evolved out of the main sequence. Therefore, their current mass distribution, i.e., the present-day mass function, is essentially identical to their IMF, irrespective of the star formation history.

First, we selected $\sim 300,000$ M-dwarf stars with effective temperature (T_{eff}) and metallicity ($[M/H]$) obtained from a catalogue based on LAMOST and Gaia data²². We derived their stellar masses with the typical uncertainty of $0.01 M_{\odot}$ by comparing the stellar parameters with the PAdova and TRieste Stellar Evolution Code (PARSEC) stellar model (see the Methods for details). Fig. 1 (a) displays the colour-magnitude diagram of the stellar samples, with theoretical grids of stellar mass overlaid. Second, we chose stars with distance to the Sun from 100 to 300 pc and stellar mass between 0.3 and $0.7 M_{\odot}$ to avoid the selection effect of the stellar mass at different distances (see Fig. 1 b). The mass coverage is also essentially constant in different metallicities, as displayed in Fig. 1 (c). Third, we measured the stellar density of sub-samples in each metallicity-mass bin after correcting the selection effect²³ (see Methods) of the LAMOST spectroscopic survey by comparing it with Two Micron All Sky Survey (2MASS) photometric data²⁴. Last, we corrected the bias from binary populations based on the observed binary information from the literature²⁵ (see Methods).

To count stars with different masses in the Solar vicinity, we assume that the IMF of a given metallicity should be the same everywhere in the studied volume. We then adopted a hierarchical Bayesian model for deriving the posterior probability density distribution of α , which is the power-law exponent of the IMF, from the stellar density profiles with different stellar masses. In this model, we adopted a single exponential vertical stellar density profile²⁶, up to the largest distance of our selected samples, 300 pc, from the Sun.

Our result shows that the stellar IMF is obviously not universal (see Fig 2 a). The IMF slopes vary with metallicities between the canonical IMF and Salpeter IMF. As the $[M/H]$ increases from -0.8 to $+0.1$, the IMF slope continuously moves from 2.0 to 2.5. This trend is quantitatively displayed in Fig 2 (b), in which α is directly associated with metallicity. While α does not vary with metallicity when $[M/H] < -0.3$, it is tightly correlated with metallicity at $-0.3 < [M/H] < +0.1$. The most bottom-heavy (biased to low-mass stars) IMF occurs at $[M/H] \sim +0.1$.

When $[M/H]$ changes from -0.8 to $+0.1$, α increases by a factor of ~ 0.6 . Such an amplitude of variation is in well agreement with theoretical predictions^{4,15} and models²⁷, which suggest that the $d\alpha/d[M/H] \simeq 0.5$ for $m < 0.7 M_{\odot}$. It also reconciles other observational results, e.g., younger clusters with larger $[M/H]$ have more bottom-heavy IMFs²⁸ and metal-poor old thick-disc stars ($[Fe/H] = -0.6$) and Galactic globular clusters ($[Fe/H] < -1$) have more bottom-light IMFs^{29,30}.

However, the sharp turnoff between $[M/H] = +0.1$ and $[M/H] = +0.25$ dex (α drops from 2.50 ± 0.06 to 2.00 ± 0.17) significantly deviates from the increasing trend of α - $[M/H]$ in the regime of $[M/H] \leq 0.1$. The majority of stars with $[M/H] > +0.1$ are likely migrated from a few kpc closer to the Galactic centre than the Sun in the inner Galactic disc^{31,32}. Stars with $[M/H] \sim +0.25$ are likely from relatively ancient populations, as found in previous studies³². A reasonable inference is that the IMF is not only related to metallicity but also to the star-forming environment where the migrated stars were born. For instance, they

may be formed under a high star formation rate regions in the inner Galactic disc, which may produce top-heavy IMFs¹⁴.

Fig. 3. The IMF power exponent as a function of stellar metallicity, for dynamically-hot stars ($J_z > 20 \text{ kpc km s}^{-1}$, red filled circles) and dynamically-cold stars ($J_z < 10 \text{ kpc km s}^{-1}$, blue triangles), respectively. The red and blue solid lines represent α , after correcting their binary star fractions, of dynamically-hot and dynamically-cold stars, respectively²⁵. The red and blue shades represent their $1\text{-}\sigma$ uncertainty regions. Other symbols are similar to Fig 2 (b).

We further examine the IMF variation against their dynamic property of vertical action (J_z), which could roughly trace stellar ages. Stars with larger J_z have experienced more scattering with molecular clouds and thus are more likely old, while those with small J_z are not necessarily young, since old stars may also have small J_z if they were not effectively scattered over a long time³³. Therefore, we split the samples into dynamically-hot and -cold groups with $J_z > 20 \text{ kpc km s}^{-1}$ and $J_z < 10 \text{ kpc km s}^{-1}$ (see Methods), respectively (see Fig 3).

When $[M/H] \leq -0.4 \text{ dex}$, where most stars are older than four billion years (see Methods), both dynamically-hot and -cold groups show constant power-law indices of the IMFs (Fig 3). When $[M/H] \geq -0.4 \text{ dex}$, however, the power-law slopes have a strong bimodality between dynamically-hot and -cold groups.

While dynamically-cold stars follow the IMF variation of the whole sample, dynamically-hot stars show a flat α trend with metallicity (see Fig 3). Although the sample contains a limited number of stars with $J_z > 20 \text{ kpc km s}^{-1}$ at $[M/H] = +0.25$, their average J_z biases to larger values, which is consistent with the argument that they are relatively-old migrated populations (see Methods). Therefore, it is not surprising that α drops to a value similar to the dynamically-hot and metal-poor stars.

The fraction of binary stars may also bias the IMF determination. Therefore, we corrected the α estimates by considering scenarios of a general binary fraction from observations²⁵, a high binary fraction³⁴ for dynamically-hot stars (45%), and an extreme case with all stars in binary systems (see Methods). They all end up with the similar α – $[M/H]$ trend with slightly increased α by 0.1–0.2 after the binary correction.

Our findings present a robust observational confirmation of the variability of IMF in the Milky Way, based on star counting in the Solar neighbourhood. The dependencies on stellar metallicity and age

indicate that exotic physical environments, such as variable Jeans mass³⁵, turbulence³⁶, cosmic rays³⁷, and the cosmic microwave background³⁸, etc., may systematically influence star-forming processes and shape the IMF. The IMF variation also calls for an extensive revision of star formation rates and stellar mass in the Milky Way and external galaxies, especially those with extreme physical conditions and in the early Universe. Future studies, such as the large field surveys from the forthcoming Chinese Space Station Telescope (CSST), will provide insights into the IMF variation in larger volumes, lower stellar mass, and probably more stellar clusters.

References

1. Kroupa, P. On the variation of the initial mass function. *Mon. Not. R. Astron. Soc.* **322**, 231–246 (2001).
2. Chabrier, G. Galactic Stellar and Substellar Initial Mass Function. *Publ. Astron. Soc. Pac.* **115**, 763–795 (2003).
3. Bastian, N., Covey, K. R. & Meyer, M. R. A Universal Stellar Initial Mass Function? A Critical Look at Variations. *Annu. Rev. Astron. Astrophys.* **48**, 339–389 (2010).
4. Kroupa, P. The Initial Mass Function of Stars: Evidence for Uniformity in Variable Systems. *Science* **295**, 82–91 (2002).
5. Cappellari, M. *et al.* Systematic variation of the stellar initial mass function in early-type galaxies. *Nature* **484**, 485–488 (2012).
6. Martín-Navarro, I. *et al.* IMF&ndashMetallicity: A Tight Local Relation Revealed by the CALIFA Survey. *Astrophys. J. Let.* **806**, L31 (2015).
7. Li, H. *et al.* SDSS-IV MaNGA: Variation of the Stellar Initial Mass Function in Spiral and Early-type Galaxies. *Astrophys. J.* **838**, 77 (2017).
8. Zhang, Z.-Y. *et al.* Stellar populations dominated by massive stars in dusty starburst galaxies across cosmic time. *Nature* **558**, 260–263 (2018).
9. Zhao, Gang *et al.* LAMOST spectral survey — An overview. *Research in Astronomy and Astrophysics* **12**, 723-734 (2012).
10. Gaia Collaboration *et al.* Gaia Data Release 2. Summary of the contents and survey properties. *Astron. Astrophys.* **616**, A1 (2018).
11. Salpeter, E. E. The Luminosity Function and Stellar Evolution. *Astrophys. J.* **121**, 161 (1955).
12. Offner, S. S. R. *et al.* The Origin and Universality of the Stellar Initial Mass Function. *Protostars and Planets VI*, 53 (2014).
13. van Dokkum, P. G. & Conroy, C. A substantial population of low-mass stars in luminous elliptical galaxies. *Nature* **468**, 940–942 (2010).
14. Bartko, H. *et al.* An Extremely Top-Heavy Initial Mass Function in the Galactic Center Stellar Disks. *Astrophys. J.* **708**, 834–840 (2010).
15. Marks, M. & Kroupa, P. Evidence for top-heavy stellar initial mass functions with increasing density and decreasing metallicity. *Mon. Not. R. Astron. Soc.* **422**, 2246-2254 (2012).
16. Gennaro, M. *et al.* Evidence of a Non-universal Stellar Initial Mass Function. Insights from HST Optical Imaging of Six Ultra-faint Dwarf Milky Way Satellites. *Astrophys. J.* **855**, 20 (2018).

17. Parikh, T. *et al.* SDSS-IV MaNGA: the spatially resolved stellar initial mass function in ~ 400 early-type galaxies. *Mon. Not. R. Astron. Soc.* **477**, 3954–3982 (2018).
18. Smith, R. J. Evidence for Initial Mass Function Variation in Massive Early-Type Galaxies. *Annu. Rev. Astron. Astrophys.* **58**, 577–615 (2020).
19. Conroy, Charlie Modeling the Panchromatic Spectral Energy Distributions of Galaxies. *Annu. Rev. Astron. Astrophys.* **51**, 393–455 (2013).
20. Reid, I. N. & Gizis, J. E. Low-Mass Binaries and the Stellar Luminosity Function. *Astron. J.* **113**, 2246 (1997).
21. Sollima, A. The stellar initial mass function of the solar neighbourhood revealed by Gaia. *Mon. Not. R. Astron. Soc.* **489**, 2377–2394 (2019).
22. Li, J. *et al.* Stellar Parameterization of LAMOST M Dwarf Stars. *Astrophys. J.* **253**, 45 (2021).
23. Liu, C. *et al.* Mapping the Milky Way with LAMOST I: method and overview. *Research in Astronomy and Astrophysics* **17**, 096 (2017).
24. Skrutskie, M. F. *et al.* The Two Micron All Sky Survey (2MASS). *Astrophys. J.* **131**, 1163–1183 (2006).
25. Liu, C. Smoking gun of the dynamical processing of solar-type field binary stars. *Mon. Not. R. Astron. Soc.* **490**, 550–565 (2019).
26. Jurić, M. *et al.* The Milky Way Tomography with SDSS. I. Stellar Number Density Distribution. *Astrophys. J.* **673**, 864–914 (2008).
27. Yan, Zhiqiang *et al.* The optimally sampled galaxy-wide stellar initial mass function. Observational tests and the publicly available GalIMF code. *Astron. Astrophys.* **607**, A126 (2017).
28. Kroupa, P. *et al.* *The Stellar and Sub-Stellar Initial Mass Function of Simple and Composite Populations*, (2013).
29. Reylé, C. & Robin, A. C. Early galaxy evolution from deep wide field star counts. II. First estimate of the thick disc mass function. *Astron. Astrophys.* **373**, 886–894 (2001).
30. Ebrahimi, H. *et al.* New insight into the stellar mass function of Galactic globular clusters. *Mon. Not. R. Astron. Soc.* **494**, 4226–4243 (2020).
31. Hayden, M. R. *et al.* Chemical Cartography with APOGEE: Metallicity Distribution Functions and the Chemical Structure of the Milky Way Disk. *Astrophys. J.* **808**, 132 (2015).
32. Kordopatis, G. *et al.* The rich are different: evidence from the RAVE survey for stellar radial migration. *Mon. Not. R. Astron. Soc.* **447**, 3526–3535 2015.
33. Ting, Y.-S. & Rix, H.-W. The Vertical Motion History of Disk Stars throughout the Galaxy. *Astrophys. J.* **878**, 21 (2019).
34. Kroupa, Pavel *et al.* The formation of a bound star cluster: from the Orion nebula cluster to the Pleiades. *Mon. Not. R. Astron. Soc.* **321**, 699–712 (2001).
35. Larson, R. B. Early star formation and the evolution of the stellar initial mass function in galaxies. *Mon. Not. R. Astron. Soc.* **301**, 569–581 (1998).
36. Padoan, P. & Nordlund, Å. The Stellar Initial Mass Function from Turbulent Fragmentation. *Astrophys. J.* **576**, 870–879 (2002).

37. Papadopoulos, P. P., Thi, W.-F., Miniati, F. & Viti, S. Extreme cosmic ray dominated regions: a new paradigm for high star formation density events in the Universe. *Mon. Not. R. Astron. Soc.* **414**, 1705–1714 (2011).
38. Zhang, Z.-Y. *et al.* Gone with the heat: a fundamental constraint on the imaging of dust and molecular gas in the early Universe. *Royal Society Open Science* **3**, 160025 (2016).

Methods

Sample selection

We select the sample from a public catalogue of $\sim 300,000$ M-dwarf stars²², which have precise spectroscopic stellar parameters estimated from LAMOST low-resolution spectra ($R \sim 1800$). Typical uncertainties of effective temperature (T_{eff}) and stellar metallicity ($[M/H]$) are 60 K and 0.12 dex, respectively. The accuracy of metallicity provided in this catalogue has been well validated in both open clusters and wide binaries²². We also collected the line-of-sight velocities measured by the LAMOST pipeline⁵⁸ and the proper motions from Gaia DR2 of these M-dwarf stars³⁹. We further crossmatch with the catalogue with 2MASS²⁴ by angular distances within $5''$. Then, we adopt the Bayesian distance (r_{est}) derived from the parallax of Gaia³⁹, and measured the absolute magnitude (M_{K_s}) at the K_s band of 2MASS.

Stellar masses

The stellar mass of each star is derived by comparing their T_{eff} and M_{K_s} with PARSEC isochrones^{40,41}, using XGboost algorithms⁴². The specific training procedure uses the root mean square error as the loss function. A 10-fold cross-validation has been conducted, and the overall uncertainty introduced by the algorithm is $\sim 0.03 M_{\odot}$.

The uncertainty of individual stellar mass was estimated via a Monte Carlo technique. Specifically, for each star, we calculated the stellar mass for 1,000 times. Each time we drew a set of random values of M_{K_s} , r_{est} , and T_{eff} from Gaussian distributions, with their measured values and uncertainties as to the mean and standard deviation, respectively. After the 1,000 time run, we obtain a probability density distribution (PDF) of the stellar mass. The resulting median value and standard deviation of each PDF were adopted as the derived stellar mass and the associated uncertainty, respectively. The typical random error (without systematic error) of the stellar mass is $\sim 0.01 M_{\odot}$.

We further compared the derived stellar mass with those determined by the empirical mass-luminosity relation (MLR)⁴³, which was benchmarked using orbital parameters of 62 binary stars, within a stellar mass range of $0.075\text{--}0.70 M_{\odot}$. The mean value of the difference is only $0.01 M_{\odot}$ with a $0.02 M_{\odot}$ dispersion, which will not alter our results dramatically.

In this work, we adopted a single power-law function to describe the IMF in the mass range of $0.3\text{--}0.7 M_{\odot}$. This is because both canonical IMFs (Kroupa and Chabrier) may not be optimal for this mass range. The Kroupa IMF¹ is a segmented power-law function, with two arbitrary broken points at $0.5 M_{\odot}$ and $1 M_{\odot}$. Our mass range goes across the discontinuity at $0.5 M_{\odot}$, which is not driven by physics⁴⁴. The Chabrier IMF adopts a log-normal function² with a characteristic mass (m_c) range of $0.15\text{--}0.25 M_{\odot}$ ². To avoid the degeneracy between m_c and σ of the log-normal function, a single power-law is more favoured when the focused mass range is larger than the characteristic mass⁴⁵.

Stellar density profiles

We selected the M-dwarf stars in a distance range of $100\text{--}300$ pc and an M_{K_s} range of $4.5\text{--}7.0$ mag, to avoid the Malmquist bias²⁸ during star counting in the flux-limited sample. After the selection, the number

of stars is uniformly distributed in the parameter space of distance, $[M/H]$, and $M_{\odot}$ (see Fig 1 a). The final selected sample contains 93,159 M-dwarf stars.

The selection bias, induced by the targeting strategy design, the observation conditions, data process of the LAMOST survey, affects the number density profile measurements. We chose a Bayesian statistical method²³, which was designed for LAMOST to derive the stellar density of each star for selection effect correction.

First, we assume that the 2MASS catalogue is a complete sample for stars with $K_s < 14.3$ mag. Second, in the colour-magnitude plane, we retrieve a selection function at each line of sight, by modelling a number distribution based on 2MASS photometry²³. Last, we obtain the intrinsic stellar density profile by multiplying this selection function with the stellar density profile observed by LAMOST.

Hierarchical Bayesian model

We employed a hierarchical Bayesian model to derive IMF parameters by taking the stellar number density profile of the Galactic disc into account. We model the vertical stellar density with an exponential profile of the thin disc²⁶, which dominates the stellar distribution within 300 pc in the Solar neighbourhood. We neglect the stellar components of the thick disc and the halo. The derived profile model is shown as following:

$$v_i(m_i) = v_0(m_i) \cdot \exp\left(-\frac{|z_i|}{h_0}\right), \quad (1)$$

where $v_i(m_i)$ is the number density of the i^{th} star with a stellar mass of m_i , z_i is the height above/beneath the mid-plane in the cylindrical coordinate system of the Galactic disc ($0 \text{ pc} < z_i < 300 \text{ pc}$), h_0 is the scale height, and $v_0(m_i)$ is the number density at $z_i = 0$. Here we assume that the IMF of a given metallicity does not vary with z_i and h_0 with stellar mass in the studied volume.

To optimise the completeness, we use M-dwarf stars in a mass range of $0.4\text{--}0.6M_{\odot}$ to derive the thin-disc prior. The resulted Gaussian profile is centred at 512 pc with a standard deviation of 25 pc ($p(h_0) \propto \mathcal{N}(512, 25)$).

We then describe the IMF using a single power-law function with a power exponent α and a constant C ,

$$\frac{dn}{dm} = C \cdot m^{-\alpha}. \quad (2)$$

The joint posterior distribution of the IMF parameters (α and C) and the scale height h_0 can be written as:

$$p(\alpha, C, h_0 \mid m_i, v_i, z_i) \propto p(\alpha, C) \cdot p(h_0) \cdot \mathcal{L}(\{v_i\} \mid \{m_i, z_i\}, \alpha, h_0, C). \quad (3)$$

We set flat prior with uniform distributions from 0 to 4 and 3 to 7, respectively, for α and $\log C$. For h_0 we adopt the Gaussian prior derived from Eq 1. Then, we integrated Eq (1) over z in both sides of equation. The total number of stars in the range of stellar mass from m to $m + dm$ can be written as

$$\frac{dn}{dm} = \int_0^{\infty} v_0(m) \cdot \exp(-|z_i|/h_0) dz = v_0 h_0. \quad (4)$$

Combining Eqs (2) with (4), we get

$$v_0(m) = \frac{C \cdot m^{-\alpha}}{h_0}, \quad (5)$$

We then brought Eq (5) back to Eq (1) and obtained the number density of each star:

$$v_{\text{model},i} = \frac{C \cdot m_i^{-\alpha} \cdot \exp(-|z_i|/h_0)}{h_0}. \quad (6)$$

Last, for N observations of the stellar number density (v_i) with associated stellar mass (m_i), height(z_i), and uncertainty of number density (σ_{v_i}), the likelihood in Eq (3) is defined as

$$\mathcal{L}(\{v_i\} | \{m_i, z_i\}, \alpha, h_0, C) = \prod_1^N \exp \frac{[v_{\text{model}}(z_i | m_i, \alpha, C, h_0) - v_i(z_i)]^2}{2\sigma_{v_i}^2}. \quad (7)$$

We separate M-dwarf stars into metallicity bins of -1.2 , -0.7 , -0.5 , -0.3 , -0.2 , -0.1 , $+0.0$, $+0.2$ and $+0.5$ dex, with bin widths of 0.5, 0.2, 0.2, 0.1, 0.1, 0.1 and 0.3 dex, respectively. The non-uniformity of the bin width is to ensure that at least 500 stars are enclosed in each bin. The minimum bin width is close to the typical uncertainty of $[M/H]$, 0.12 dex, which would not significantly smear out the results of the neighbouring bins.

To obtain the posteriors of the IMF parameters, we adopted a Markov Chain Monte Carlo (MCMC) sampling using the `PyMC3`¹ software package⁴⁶. After marginalising $\log C$, the median value and 15% (85%) of the posterior distribution of α were adopted as the IMF slope and its corresponding uncertainty.

To verify whether the IMF depends on the exponential disc model, we further test the results by direct counting. This time we do not consider the disc distribution, and only select a narrow scale-height range of $100 < |z| < 250$ pc, where the observations were less affected by selection effects. The likelihood of each star can be expressed with the number density (v_i) at its location:

$$\mathcal{L}_{\in}(\{v_i\} | \{m_i\}, \alpha, C) = \prod_1^N \exp \frac{(C \cdot m_i^{-\alpha} - v_i)^2}{2\sigma_{v_i}^2}. \quad (8)$$

Although the direct counting method has a slightly larger uncertainty, it is consistent with the results from the Bayesian model with an exponential Galactic disc (see Extended Data Fig 1). The IMF slopes obtained from the two methods differ by a maximum of 0.09 dex, less than the average $1 - \sigma$ uncertainty (~ 0.14 dex), indicating the robustness of our results.

Validation with mock data

We generate 3,563,892 mock stars with *Galaxia*⁴⁷, which adopts -1.6 as the IMF power-law index. Stellar mass, magnitudes, distances, effective temperature, ages and metallicities are created automatically. Binary stars are not considered here. All mock stars have stellar mass $< 1 M_{\odot}$, and they are distributed in a 20-square degree area toward the northern Galactic pole.

¹<https://docs.pymc.io/>

We selected stars inside a 2-kpc radius, with an absolute magnitude range of $3 \text{ mag} < M_{K_s} < 14 \text{ mag}$. This simulated data does not have selection bias, so we do not need to correct their number density. We estimate the stellar density in a moving cube of $50 \times 50 \times 50 \text{ pc}^3$ for each star.

We randomly drew subsets of 500, 1,000, 10,000, and 100,000 mock stars from the sample and derived the number density profile for each subset. Then we applied the same hierarchical Bayesian model to each subset of stars and obtained the posterior distribution of α . We find that as long as a subset contains more than 1,000 mock stars, the accuracy of α is $\lesssim 0.1 \text{ dex}$.

Vertical actions and stellar ages

We took *galpy*⁴⁸ to measure the 3-D location of each star in heliocentric Cartesian coordinates with velocity components (V_R, V_T, V_Z) in the Galactocentric cylindrical coordinate. The 3-D velocities of each star were calculated using line-of-sight velocity from LAMOST, parallax from *Gaia*, and proper motion from *Gaia*. The position of the Sun was adopted as $(X, Y, Z) = (8000, 0, 27) \text{ pc}$. We also adopted the velocity of the local standard of rest (LSR) concerning the Galactic center as $V_{\text{LSR}} = 220 \text{ km} \cdot \text{s}^{-1}$ ⁴⁹ and the Solar motion corresponding to the LSR as $(U_{\odot}, V_{\odot}, W_{\odot}) = (11.1, 12.24, 7.25) \text{ km} \cdot \text{s}^{-1}$ ⁵⁰.

The orbital actions, including the vertical action J_z , were measured in the angle-action coordinate^{51,52} with *galpy*. The actions were estimated under the Stäckel approximation⁵² with the Milky Way potential `MWPotential2014`⁴⁸. In a static or quasi-static axisymmetric gravitational potential, the amplitudes of a star's oscillation parallel and perpendicular to the Galactic plane can be described by the action J_r and J_z ⁵¹. The third action $J_{\phi} \equiv L_z$ ⁵³ stands for the angle momentum with respect to the z axis (L_z). J_z is defined as

$$J_z = \frac{1}{2\pi} \oint dz v_z, \quad (9)$$

where z and v_z are the current orbital position and vertical velocity of a star, respectively.

In general, J_z is correlated with stellar age³³, since it continuously increases by scattering with giant molecular clouds⁵⁴. Although the J_z -age relation is theoretically explicit, the observed J_z -age relation has a large dispersion, making it impossible to determine the exact age for an individual star³³. Nevertheless, a large J_z means that the star has experienced more scattering and thus is more likely to be old. On the other hand, a small J_z does not necessarily mean that the star is young since a star may not be effectively scattered over a long time. Therefore, we can separate the older population by selecting stars with large J_z but not selecting young populations simply from J_z .

We benchmark the old population using red giant stars⁵⁵ whose age is calibrated by asteroseismology, with an uncertainty of $\sim 24\%$. We selected dynamically-hot stars with $J_z > 20 \text{ kpc km s}^{-1}$, mostly older than 4 Gyr (peaking at $\sim 8.5 \text{ Gyr}$), as the probe of old populations. We also selected the dynamically-cold stars with $J_z < 10 \text{ kpc km s}^{-1}$ for reference. Extended Data Fig 2 shows the dynamically-hot red giant stars, while the dynamically-cold stars broadly spread in all ranges of age. We apply the same J_z criteria obtained from red giants to low-mass main-sequence stars because the J_z -age relation would not significantly change between them.

We find that the ratio between dynamically-hot and -cold stars decreases against metallicity (see Extended Data Fig 3). An exception, however, is the metallicity bin of $[M/H] = +0.25$, which contains more dynamically-hot stars than at $-0.3 < [M/H] < 0.1$. This bin contains more old stars than its neighbours, consistent with previous findings that they were migrated from the inner disc^{31,32}.

The impact of binarity in stellar mass estimation

The influence of binary stars are two-fold: a) the unidentified binaries may be overestimated in luminosities and consequently masses, and b) the two stars in a binary system should be all counted in IMF. Here we discuss the first issue and leave the second in the next section.

The stellar mass of a primary star of a binary system might be overestimated due to the increased magnitude contributed by the secondary star. The shift of the absolute magnitude of an unresolved binary star from the single sequence in Hertzsprung-Russel diagram reflects the mass ratio ($q = m_2/m_1$, where m_1 and m_2 are the masses of the primary and secondary stars, respectively) of the binary system.

When $q > 0.7$, the contribution to the luminosity from the secondary star is > 0.1 mag²⁵, higher than the typical photometric uncertainty of ~ 0.07 mag for M_G (absolute magnitude in Gaia G -band).

Therefore, the combined luminosity for such binaries cannot be neglected compared to silaohaongle stars. In this scenario, we assume that 25% of M-dwarfs are in binary systems⁵⁶ and the secondary-to-primary mass ratio is larger than 0.7 for 30% binary stars. These would significantly affect the observed luminosity by > 0.1 mag. Therefore, the absolute magnitudes of $\sim 7.5\%$ stars in our sample are overestimated for more than 0.1 mag, which may lead to an excessive estimation of stellar mass.

We again adopt mock stars generated by *Galaxia* to measure α shifted by the overestimated mass from unidentified binary stars. The hidden secondary stars were randomly simulated for given binary fractions and mass-ratio distributions (in a power-law form with a power exponent, Γ). The M_{K_s} of the mock stars were calculated using the mass-luminosity relation calibrated by dynamical stellar mass⁴³. Moreover, the M_{K_s} of the primary stars were reacquired by eliminating the flux contribution of the secondary stars. Then we recalculate the stellar mass of the primary stars following the same method as the LAMOST M-dwarf stars.

We simulated the mock stars with binary fractions from 15% to 40%. We found that the effect of the overestimated stellar mass is limited for the derivation of α . The change in α by binary stars is less than ~ 0.1 dex.

Binarity correction in IMF

To accurately obtain the IMF, we correct both the binary companions in counting stars and the overestimated stellar masses of the primary stars. We adopted the binary fraction, f_b , in a range of 0.0, 0.25, 0.75 and, 1.0, and the mass ratio distribution of binary stars, Γ , in a range of 0.0, 0.5, 1.5, and 5.0, respectively. For each combination of f_b and Γ , we randomly draw subsets of 500, 1,000, 10,000, and 100,000 mock stars. We then measure α with the Bayesian model for each mock dataset and derive the deviation of α , $\Delta\alpha$, from the ground truth. We generated binary stars randomly ten times in each dataset and obtained the mean $\Delta\alpha$ and its standard deviation (see Extended Fig 3). We find that $\Delta\alpha$ increases with binary fraction. Even if all stars are in binary systems, the derivation of α from the ground truth is less than 0.25 ± 0.1 dex. If 30% M-dwarfs are in binary systems, which is similar to the observed value^{25,57}, $\Delta\alpha$ is ~ 0.08 , which would not significantly change our main results.

We adopted the binary fractions and mass-ratio distributions at different $[M/H]$ measured from stars in the Solar neighbourhood²⁵. Then, we interpolate these two quantities to our $[M/H]$ bins. The adopted values, the corresponding Γ and corrected α are listed in Extended Data Table 1. The binary corrected α is drawn as grey line and shade in Fig.3.

The dynamically-hot stars often have larger velocity dispersion, which may be contaminated by the additional orbital velocity in a close binary system. Therefore, we presume that the dynamically-hot stars may contain more binaries than the dynamically-cold stars. To quantify the binary fraction of the dynamically-hot stars, we compare the maximum radial velocity difference ($\Delta RV_{\max}$, the difference between the highest and the smallest radial velocities of a given star observed with multi-epoch observa-

tions.) distributions between the dynamically-hot and -cold stars. The peak of the $\Delta RV_{\max}$ distribution is dominated by the measurement errors of radial velocities, while the binaries mainly contribute to the distribution tail. Hence, $\Delta RV_{\max}$ between any two epochs for the same object can be used to detect binary stars⁵⁷.

We select stars that have been observed for at least three epochs with LAMOST, which has a radial velocity uncertainty $\sim 11.5 \text{ km s}^{-1}$ ⁵⁸. There are 2803 dynamically-hot and 108 dynamically-cold stars, respectively. Extended Fig 5 compares the $\Delta RV_{\max}$ distributions of the two groups. For stars with $\Delta RV_{\max} > 30 \text{ km s}^{-1}$ show noticeable the difference in distributions between the dynamically-hot and -cold groups. Dynamically-hot stars seem to contain more binaries. We performed a Kolmogorov-Smirnov test obtained a p-value of 0.18, further illustrating that the difference between these two distributions is statistically significant.

We assumed that stars with $\Delta RV_{\max} > 25 \text{ km} \cdot \text{s}^{-1}$ (larger than $2\text{-}\sigma$ of the typical uncertainty of radial velocity, which is 11.5 km s^{-1} of LAMOST M-dwarf stars⁵⁸) are primarily in binary systems. The fractions are 0.15 and 0.1, respectively, for dynamically-hot and- cold samples. The binary fraction of the dynamically-hot group is $\sim 1.5\times$ that of the dynamically-cold group. Therefore, the binary fraction of the dynamically-hot population is $\sim 45\%$, given that the average binary fraction of dynamically-cold stars is 30% over the whole metallicity regime²⁵.

Extended data

Extended Data Fig 1: The IMF power-law index as a function of $[M/H]$ measured with different methods. The filled rectangles are derived by all stars with the hierarchical Bayesian model considering the exponential Galactic disc profile. The brown stars denote the α s derived directly from the observed densities of stars in the $|z|$ range from 100 to 250 pc.

Extended Data Fig 2: Age-metallicity distributions of red giant branch stars⁵⁵ within 300 pc in the solar vicinity. The red and blue contours display the age-metallicity distribution of dynamically-hot ($J_z > 20 \text{ kpc km s}^{-1}$) and -cold ($J_z < 10 \text{ kpc km s}^{-1}$) stars, respectively. Their contour densities are smoothed by the kernel density estimation method. The dotted lines indicate the separation of $[M/H]$ slices used in the M dwarf samples.

Extended Data Fig 3: The ratio of the normalised number of dynamically-hot stars to that of the dynamically-cold stars as a function of [M/H]. $f_{\text{dyn-hot}}$ is the proportion of the number of dynamically-hot stars in each [M/H] bin to all dynamically-hot stars. $f_{\text{dyn-cold}}$ denotes the similar proportion, but for dynamically-cold stars. $f_{\text{dyn-hot}}/f_{\text{dyn-cold}}$ indicate the normalised number ratio between dynamically-hot and -cold stars. The vertical shaded region represents the metallicity range of $-0.3 < [\text{M}/\text{H}] < 0.1$, corresponding to the area of α variation in Fig.2a and Fig.3.

Table 1. Extended Data Table 1: Binary corrections to IMF slopes in different [M/H]

[M/H]	f_b	Γ	$\Delta \alpha$
-0.80	0.30	0.58	0.09 ± 0.04
-0.60	0.30	0.58	0.09 ± 0.04
-0.40	0.30	0.58	0.10 ± 0.04
-0.25	0.33	1.32	0.11 ± 0.04
-0.15	0.33	1.32	0.10 ± 0.04
-0.05	0.29	1.6	0.07 ± 0.03
+0.05	0.23	1.75	0.07 ± 0.04
+0.25	0.24	1.72	0.08 ± 0.04

Extended Data Fig 4. The results of simulations to verify the impact of the binary stars in the IMF. The results show the difference of the estimated α from the true values versus binary fraction with the number of mock stars equal to 500, 1000, 10 000, and 100 000. The different dashed lines with colours represent the binary stars mixed into the mock stars with various power exponents (Γ) of the mass-ratio distribution, which is assumed to be a power law. The dotted line in each panel represents the α shifts when the binary fraction is 30%.

Extended Data Fig 5. The $\Delta RV_{\max}$ distribution of the dynamically-hot M dwarf stars (red solid-line), and dynamically-cold stars (blue dashed-line). The vertical dotted line denotes the typical $1-\sigma$ uncertainty of radial velocity uncertainty of LAMOST M dwarf stars.

References

39. Bailer-Jones, C. A. L. *et al.* Estimating Distance from Parallaxes. IV. Distances to 1.33 Billion Stars in Gaia Data Release 2. *Astron. J.* **156**, 58 (2018).
40. Bressan, A. *et al.* PARSEC: stellar tracks and isochrones with the PAdova and TRieste Stellar Evolution Code. *Mon. Not. R. Astron. Soc.* **427**, 127–145 (2012).
41. Chen, Y. *et al.* Improving PARSEC models for very low mass stars. *Mon. Not. R. Astron. Soc.* **444**, 2525–2543 (2014).
42. Chen, T. *et al.* XGBoost: A Scalable Tree Boosting System. *Proceedings Of The 22nd ACM SIGKDD International Conference On Knowledge Discovery And Data Mining*, 785–794 (2016).
43. Mann, A. W. *et al.* How to Constrain Your M Dwarf. II. The Mass-Luminosity-Metallicity Relation from 0.075 to 0.70 Solar Masses. *Astrophys. J.* **871**, 63 (2019).
44. Miller, G. E. & Scalo, J. M. The Initial Mass Function and Stellar Birthrate in the Solar Neighborhood. *Astrophys. J.* **41**, 513 1979.
45. El-Badry, K. & Weisz, D. R. & Quataert, E. The statistical challenge of constraining the low-mass IMF in Local Group dwarf galaxies. *Mon. Not. R. Astron. Soc.* **468**, 319–332 2017.
46. Salvatier, J. *et al.* Probabilistic programming in Python using PyMC3.. *PeerJ Computer Science*, 55 (2016).
47. Sharma, S. *et al.* Galaxia: A Code to Generate a Synthetic Survey of the Milky Way. *Astrophys. J.* **730**, 3 (2011).
48. Bovy, Jo *et al.* galpy: A python Library for Galactic Dynamics. *Astrophys. J.* **216**, 29 (2015).
49. Bovy, J. *et al.* The Milky Way’s Circular-velocity Curve between 4 and 14 kpc from APOGEE data. *Astrophys. J.* **759**, 131 2012.
50. Schönrich, Ralph *et al.* Local kinematics and the local standard of rest. *Mon. Not. R. Astron. Soc.* **403**, 1829–1833 (2010).
51. Binney, J. & Tremaine, S. *Galactic Dynamics: Second Edition* 2008.
52. Binney, J. Actions for axisymmetric potentials. *Mon. Not. R. Astron. Soc.* **426**, 1324–1327 (2012).
53. Sanders, J. L. & Binney, J. A review of action estimation methods for galactic dynamics. *Mon. Not. R. Astron. Soc.* **457**, 2107–2121 (2016).
54. Jenkins, A. & Binney, J. Spiral heating of galactic discs. *Mon. Not. R. Astron. Soc.* **245**, 305–317 (1990).
55. Wu, Y. *et al.* Mass and age of red giant branch stars observed with LAMOST and Kepler. *Mon. Not. R. Astron. Soc.* **475**, 3633–3643 (2018).
56. Delfosse, X. *et al.* M dwarfs binaries: Results from accurate radial velocities and high angular resolution observations. *Spectroscopically and Spatially Resolving the Components of the Close Binary Stars* **318**, 166–174 (2004).
57. Moe, Maxwell *et al.* The Close Binary Fraction of Solar-type Stars Is Strongly Anticorrelated with Metallicity. *Astrophys. J.* **875**, 61 (2019).
58. Yi, Zhenping *et al.* M Dwarf Catalog of the LAMOST Pilot Survey. *Astrophys. J.* **147**, 33 (2014).

Acknowledgements

This work is supported by the National Key R&D Program of China No. 2019YFA0405500. C.L. thanks the National Natural Science Foundation of China (NSFC) for grant No. 11835057 and the science research grants from the China Manned Space Project with NO.CMS-CSST-2021-A08. Z.Y.Z acknowledges the support of NSFC grants No. 12041305 and No. 12173016 and the Program for Innovative Talents, Entrepreneur in Jiangsu. H.T. acknowledges the support of NSFC grant No.12103062. X.F. acknowledges the support of China Postdoctoral Science Foundation No. 2020M670023, the National Natural Science Foundation of China under grant number 11973001, and the National Key R&D Program of China No. 2019YFA0405504. J.L. acknowledges the science research grants from the China Manned Space Project with NO.CMS-CSST-2021-A10 and NO.CMS-CSST-2021-B05. Guoshoujing Telescope (the Large Sky Area Multi-Object Fiber Spectroscopic Telescope, LAMOST) is a National Major Scientific Project built by the Chinese Academy of Sciences. The National Development and Reform Commission has provided funding for the project. LAMOST is operated and managed by the National Astronomical Observatories, Chinese Academy of Sciences. This work has made use of data from the European Space Agency (ESA) mission Gaia (<https://www.cosmos.esa.int/gaia>), processed by the Gaia Data Processing and Analysis Consortium (DPAC; <https://www.cosmos.esa.int/web/gaia/dpac/consortium>). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

Author contributions statement

Jiadong L. contributed most of the modelling and calculations and wrote the initial manuscript. C.L. provided the ideas to initialise the project, supervised Jiadong L. on the modelling, and revised the manuscript. Z.-Y.Z. and X.F. compared the results with other theoretical and observed work and helped write the manuscript. H.T. ran the calculations to derive vertical actions. J.L. helped with the discussion of the impact of the binary stars. All authors discussed and commented on the manuscript.

Competing interest

The authors declare no competing interests.

Additional information