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H_∞ Model based control via mixed sensitivity for 2DOF control of twin rotor MIMO system with experimental validation

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Abstract

This article provides a comprehensive study about optimization and real-time implementation of H_∞ model-based controller for uncertain systems (having external perturbations) using robust control methods. The complicated air vehicle modeling is quite difficult because of highly nonlinear behavior, un-modeled states, and cross-coupling effect. The helicopter flight control also depends on two rotors generating continuous thrust simultaneously in two different directions. Simple linear feedback controllers cannot ensure the stability of highly nonlinear systems like a helicopter. The controllability and observability full rank matrix output ensure the credibility of the system before a controller design. The design of a robust controller for the desired flight control, demands the availability of all its states and varying parameters for real-time robust feedback control. However, its prototype, Twin-rotor MIMO System (TRMS) facilitates the researchers and control engineers in terms of the real-time environment of a helicopter flight control. Various control strategies are presented to elaborate the behavior of the MIMO system and the limitations of robust control methods. Each control strategy has been comprehensively discussed with the stability analysis and the design requirements. The considered control techniques, such as the loop-shaping control and H_∞ model-based control augmented with mixed sensitivity control function, make use of control in such a way that robust stability and robust performance are ensured for highly nonlinear systems. Model-based H_∞ optimization technique elaborates the robustness and stability of the system by reducing tracking error of all its states towards origin and thereafter system becomes stable for an infinite period. The loop-shaping control performance as compared to H_∞ model-based control has some limitations to ensure closed-loop robustness and real-time implementation under perturbations. The performance evaluation is done at the end which shows that the H_∞ model-based control gives optimal robust performance including perturbations and coupling effect. The real-time implementation on *MATLAB/Simulation* via personal computer validates the worth of the optimization method in the presence of uncertainties (matched, unmatched, coupling effect) of a linearized state-space model of the TRMS system.

Keywords: MIMO system; coupling effect; Nonlinear Dynamic Inversion; loop-shaping; mixed sensitivity; H_∞ model-based control .

Table 1: Nomenclature.

Begin of Table			
Symbol	Description	Symbol	Description
θ	Pitch (elevation) angle	ϕ	yaw (azimuth) angle
τ_1	momentum of main rotor	τ_2	momentum of tail rotor
I_1	Main rotor inertia	I_2	Tail rotor inertia
$x \in R$	(real number) of states	$u \in R$	input signal
ρ	Euclidean error norm	$'D'$	diagonal matrix
Y	control vector	P	projection matrix
a_1	constant	b_1	constant
a_2	constant	b_2	constant
M_g	Gravitational Momentum	$B_{1\theta}$	Frictional parameter
$B_{2\theta}$	Frictional parameter	$B_{1\varphi}$	Frictional parameter
$B_{2\varphi}$	Frictional parameter	k_{gy}	Gyroscopic Parameter
k_1	Gain of Main Motor	k_2	Gain of Tail Motor
T_{11}	Denominator Constant of main motor	T_{10}	Denominator Constant of tail motor
T_{21}	Denominator Constant of Main Motor	T_{20}	Denominator Constant of tail motor
u_h	Horizontal axis control input.	u_v	Vertical axis control input.
I_v	Inertial momentum of main rotor	I_h	Inertial momentum of tail rotor.
$e_z(t)$	Tracking of pitch and yaw angles.	$2DOF$	Two degree of freedom
k_{H_h}, k_{H_v}	Velocity gains.	k_{f_h}, k_{f_v}	Frictional momentum.
R_V	Returned torque of rotors.	G_d	Disturbance of plant.
G_u	Transfer matrix of control signal.	K_y	Feedback matrix functions.
K_r	Transfer function matrix of pre-filter.	r, d	Reference input, input disturbance.
Δ_F	Fictitious perturbation.	S_o	Output sensitivity matrix.
End of Table			

1. Introduction and Background

The helicopter in real-time representation is lifted and moved by one or more even rotors. The essential preference of a helicopter is that how rotors control the lift and direction by adjusting their speed through motors. Consequently, helicopters are regularly utilized in congested or disconnected territories where altered-wing airplanes can't take off or land. The closed-loop system under control action must show

the required response by manipulating the controlling variables. The experimental setup of a helicopter prototype known as Twin Rotor MIMO System (TRMS) and all parameters with their symbols are defined in table 1. The coupling effect of the TRMS, elaborated with modeling in [1]. This setup is a highly nonlinear system with a limited degree of freedom. The basic parts are enlisted as one is the mechanical (main rotor and tail rotor) and another one is an electrical part (DC motors for both rotors). The focused part is the mechanical part with all possible perturbations. The main rotor (pitch angle) covers motion along the vertical axis (vertical plane) while the tail rotor (yaw angle) covers motion along the horizontal axis (horizontal plane). A linear controller with all its attributes is implemented to regulate the TRMS in the presence of coupling effect as expressed in [2]. The Proportional Integral Derivative (PID) control based on the degree of freedom is elaborated in [3], as a fractional controller and PID also explained with the degree of freedom for the coupled system in [4]. An estimation control with nonlinear control for a highly nonlinear system expressed in [5]. The decoupled system is optimized via swarm optimization, based on the PID control strategy elaborated in [6]. The classical control elaborated through an efficient output response of TRMS defined in [7, 8]. The combination of the nonlinear control with a classical control represented in [9, 10]. The coupling effect is reduced to zero by using the decoupler (static decoupling or dynamic decoupling). In [11], neural networks are introduced in adaptive MPC design to solve the problems of parametric uncertainty. Further, a learning-based adaptive MPC has been represented by a researcher in [12, 13], where a modified Extended Kalman Filter (EKF) is used to perform the joint state and parametric estimation.

Another alternative way to handle the parametric uncertainty is a multiple model second level adaptation (MALSA) scheme described in [14, 15]. An intelligent control strategy implemented (discrete-time), designed according to the prototype nonlinear behavior expressed in [16]. The strategy with its two different types (Sliding Mode Control (SMC) and Integral SMC) are designed to overcome the nonlinear response of the states after using a decoupler for the system represented in [17, 18]. The Takagi-Sugeno fuzzy model has been proposed in [19]. The angles (pitch angle and yaw angle) show the controlled response by using a fuzzy, Linear Quadratic Regulator (LQR) controller designed for the decoupled system. The implementation of adaptive model inversion technique to control cross-coupled dynamics of a nonlinear system by using artificial neural network approach was proposed in [20]. The inversion model is implemented for the pitch channel of the system. The H_∞ control strategy is implemented without using decoupling process for the TRMS as elaborated in [21, 22], after a linearization process. The main reason for the cross-coupling between the pitch and yaw dynamics is the fast and immediate change in the rotors speed. Some states of the system cannot be evaluated so the mathematical modeling of the system becomes difficult due to the reason that the system is no longer flat. The LMI based neural network controller has been designed in [23]. Non-linear Dynamic Inversion (NDI) based controller design to mitigate the complexity behind the inversion of the motion equations and make sure that the model complexity is reduced. The controllability and observability matrix calculation is performed to ensure the stability of the system. A full rank matrix of the order 6×10 and 10×6 is obtained to verify that the system is controllable and observable. The weighted function (tuning parameter) is used to reject the disturbance at the output and track the desired response.

For the multiplicative model uncertainty, smooth response (robust response) obtained by the function S/T . The contribution of the paper is,

- To avoid the pole-zero cancelation on the right half-plane, the H_∞ loop-shaping design procedure (LSDP) is presented. LSDP gives the relaxation on the existence of the number of poles on the right half-plane within the nominal model and the controller.
- The model-based H_∞ control augmented with mixed sensitivity design. The controller uses the disturbance rejection property via weighted functions and the coupling effect is reduced to have desired stability. The infinitive norm with weighting matrix ensures excellent robustness against the perturbations and minimizes the norm value.
- The noise matrix and the disturbance matrix are used to check the perturbations threshold. This property shows that how much magnitude of internal and external disturbance can be managed by technique in the presence of all other perturbations such as coupling effect, parametric uncertainty, and unavailable states of the system.
- The real-time evaluation of the considered optimization strategy provides efficient robustness and stability against all considered perturbations.
- Some important suggestions for control engineers are provided in conclusion.

The remaining of this paper is organized as follows. The mathematical modelling and stability analysis are presented in section 2. The H_∞ LSDP is elaborated with its mixed-sensitivity function in section 3. The H_∞ mixed-sensitivity introduction with its limitations are elaborated in section 4. The H_∞ model-based control strategy is elaborated in section 5 and the robust performance via MATLAB-simulation is explained in section 6. The experimental setup of the TRMS and the control strategy simulations are provided in section 7. The final section of the conclusion is provided via a comprehensive comparison of the simulation's response table.

2. Mathematical Model

The modeling of a higher-order nonlinear system can be derived after some suitable, compromising assumptions as mentioned in [24]. A highly nonlinear system with a finite degree of freedom is explained in figure 1, with important parameters in this section. The experimental prototype with both rotors (main and tail rotor) is described through the mathematical state equations as,

$$\frac{d\dot{\theta}}{dt} = \frac{a_1}{I_1}\tau_1^2 + \frac{b_1}{I_1}\tau_1 - \frac{M_g}{I_1} \sin(\theta) + \frac{0.0326}{2I_1} \sin(2\theta) \dot{\varphi}^2 - \frac{B_{1\theta}}{I_1} \Psi - \frac{k_{gy}}{I_1} \cos(\theta) \dot{\varphi} (a_1\tau_1^2 + b_1\tau_1) \quad (1)$$

$$\frac{d\dot{\varphi}}{dt} = \frac{a_2}{I_2}\tau_2^2 + \frac{b_2}{I_2}\tau_2 - \frac{B_{1\varphi}}{I_2} \dot{\varphi} - \frac{k_c}{I_2} 1.75 (a_1\tau_1^2 + b_1\tau_1) \quad (2)$$

Fig. 1: Basic schematic sketch of TRMS.

For main motor

$$\dot{\tau}_1 = \frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1 \quad (3)$$

For tail motor

$$\dot{\tau}_2 = \frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}}u_2 \quad (4)$$

The state equations 1 to 4 are converted into the linear state equations. Such equations mean there will be no term having the highest degree more than unity. The linearization is the process to obtain the linear equations with some compromised assumptions. The model can be described as,

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5)$$

$$y(t) = Cx(t) \quad (6)$$

here $x \in R$ (real number) of the states, $u \in R$ represents the input signal and $y \in R$ is the measured

output. The state vectors of the TRMS given below as,

$$x = \begin{bmatrix} \theta & \dot{\theta} & \varphi & \dot{\varphi} \end{bmatrix}^T \quad (7)$$

$$y = \begin{bmatrix} \theta & \varphi \end{bmatrix}^T \quad (8)$$

where θ is the pitch (elevation) angle, ϕ is the yaw (azimuth) angle, τ_1 is the momentum of the main rotor, and τ_2 represents the momentum of the tail rotor. Nonlinear Dynamic Inversion (NDI) is used to simplify the system by taking some simple mathematical operations. The states of the system linearized at origin, $x(t) = x(0)$,

$$J = \frac{\delta f(x)}{\delta x} \Big|_{x=0} \quad (9)$$

$$J = \begin{bmatrix} \partial f_1 / \partial x_1 & \dots & \partial f_1 / \partial x_6 \\ \vdots & \ddots & \vdots \\ \partial f_6 / \partial x_1 & \dots & \partial f_6 / \partial x_6 \end{bmatrix} \quad (10)$$

Some mathematical operations applied to get the simplified matrices at origin $(0, 0, 0)$ shown as,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{M_g}{I_1} & -\frac{B_{1\theta}}{I_1} & 0 & 0 & \frac{b_1}{I_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{B_{1\varphi}}{I_2} & -\frac{k_c}{I_2} 1.75 & \frac{b_2}{I_2} \\ 0 & 0 & 0 & 0 & -\frac{T_{10}}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{T_{20}}{T_{21}} \end{bmatrix} \quad (11)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_1}{T_{11}} & 0 \\ 0 & \frac{k_2}{T_{21}} \end{bmatrix} \quad (12)$$

The block diagram of the TRMS with the coupling effect is represented in figure 2. There are two output states called the pitch angle and yaw angle. The coupling effect of the main rotor and the tail rotor is also provided in a diagram.

Table 2: Parameters of TRMS.[?]

Symbol	Description	Units
I_v	1/6100	kgm^2
I_h	1/37000	kgm^2
J_v	3.00581×10^{-2}	kgm^2
k_1	5.00576×10^{-2}	Nm
k_2	9.0036×10^{-2}	Nm
k_3	2.12485×10^{-2}	Nms^2/rad^2
k_4	$2.3790412485 \times 10^{-2}$	kgm^2
k_5	3.00962×10^{-3}	kgm^2
k_{f_h}	5.88996×10^{-3}	$Nm - s/rad$
k_{f_v}	1.27095×10^{-2}	$Nm - s/rad$
k_{h_v}	4.17495×10^{-3}	Nm
k_{v_h}	-1.7820×10^{-2}	Nm
l_m	0.202	m
l_t	0.216	m

2.1. TRMS Stability Analysis

The system under consideration is being checked by a mathematical process which is controllability and observability. The controllability property of the system involves the system matrices A and B . C_c is a matrix of a linear system that is said to be controllable form if it is possible to find some input $u(t)$. The input will transform the state's $x(t_0)$ to the origin at a finite time. If there exist some input $u(t_1)$ and gives $x(t) = 0$, admitted for all initial times and state's, then it is verified for controller represented in [25]. The controllability elaborated as,

- (i) (A, B) is controllable,
- (ii) The controllability matrix can be found as,

$$C_c = [B \ AB \ A^2B \ A^3B \ A^4B \ \dots \ A^{\alpha-1}B] \quad (13)$$

where, α is the positive integer which depends on the order of the system matrix A . The order of the matrix in the above equation is $\alpha = 6$. The system will be considered controllable if its determinant is non-zero. The matrix shows the full rank property so, the system is controllable.

The full rank of the matrix as mentioned above, provide strong validation towards the controllable system. The calculated controllability matrix is,

$$C_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.0014 & 0 & -0.0014 & 0 & -0.0052 & 0 \\ 0 & 0 & 0.0014 & 0 & -0.0014 & 0 & -0.0052 & 0 & 0.0058 & 0 \\ 0 & 0 & 0 & 0 & 0.0016 & 0.0036 & -0.0096 & -0.0216 & 0.0491 & 0.1116 \\ 0 & 0 & -0.0016 & 0.0036 & 0.0096 & 0.0216 & 0.0491 & 0.1116 & 0.2468 & 0.5616 \\ 1 & 0 & 0.0009 & 0 & 0.0008 & 0 & 0.0008 & 0 & 0.0007 & 0 \\ 0 & 0.8 & 0 & 0.0008 & 0 & 0.0008 & 0 & 0.0008 & 0 & 0.0008 \end{bmatrix} \quad (14)$$

Fig. 2: Block Diagram of System .

The excellent validation of the full rank system can be viewed from the above matrix, which shows the valid property of a full rank matrix. The system having full rank property can be verified by finding the determinant of the system matrix which should not equal to zero. All the states of the system converge to the origin validates the system observability. The observability of the system can be verified as,

- (i) (A, C) is observable,
- (ii) The observability matrix can be found here,

$$O_O = [C \ C A \ C A^2 \ C A^3 \ C A^4 \ C A^{\alpha-1}]^T \quad (15)$$

where α represents the order of the system matrix A and value in the above equation is, $\alpha = 6$. The system will be considered controllable if its determinant is non-zero. The matrix shows the full rank property so, the system is observable. The calculated observability matrix is given below. The full rank property can be verified from the above matrix and ensure the observability of the system. The stability analysis provides a strong platform to design a suitable controller. The next section elaborates the controller constraints according to the system dynamics.

Fig. 3: Standard LSC configuration .

$$O_O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -4.7059 & -0.0882 & 0 & 1.3588 & 0 & 0 \\ 0 & 0 & 0 & -5 & 1.6170 & 4.5 \\ 0.4151 & -4.6534 & 0 & 0 & -1.3543 & 0 \\ 0 & 0 & 0 & 25 & -9.5532 & -27 \\ 22.1089 & 0.8294 & 0 & 0 & -8.6574 & 0 \\ 0 & 0 & 0 & -125 & 49.1258 & 139 \\ -3.9032 & 22.0356 & 0 & 0 & 5.8103 & 0 \\ 0 & 0 & 0 & 663 & -246 & -702 \end{bmatrix} \quad (16)$$

3. H_∞ Loop-Shaping Design Procedure (LSDP)

The nominal performance and the robust stabilization can be achieved from the specific cost function of the LSDP problem as a sub-optimization shown in figure 3.

Any highly nonlinear closed-loop system must require a controller gain K , to provide the smooth output with manipulating control input U . G represents a general nonlinear system with reference input r . For the robust controller design, there are several formulas available, S/T and S/KS weighted method. Optimal result can be obtained by S/KS , here the variable S is the function of sensitivity and the output gain of the controller is represented by K . This function is the rejection of the output disturbance and tracking the desired response, and gives robust stability to the model additive perturbation. For the multiplicative model uncertainty robust performance can be achieved by the function S/T , where T is the complementary sensitive function. Both robust controllers design methods are useful or applicable but the restrictions come due to the representation of the model uncertainty that is limited by the number of poles on the right half complex plane in [26]. There may be the possibility of pole-zero cancelation with a nominal model and design

a controller. On the other way, we can introduce an alternative way to introduce the perturbations. For such perturbations, robust stabilization design methods are presented. To avoid the pole-zero cancelation on the right half-plane, the H_∞ LSDP is presented. LSDP provides the relaxation on the existence of the number of poles on the right half-plane within the nominal model and the controller. The representation of the coprime factorization in terms of the matrices, $(\tilde{M}, \tilde{N}) \in H_\infty^+$ elaborate the function along space, for a prototype model G , if and only if,

- $\tilde{M} = 0, \det(\tilde{M}) \neq 0$
- Represents prototype model $G = \tilde{M}^{-1}\tilde{N}$
- The value of $(\tilde{V}, \tilde{N}) \in H_\infty^+, \quad \tilde{M}\tilde{V} + \tilde{N}\tilde{U} = I$

The plant is normalized for the left coprime factorization, if and only if,

$$\tilde{M}\tilde{M} + \tilde{N}\tilde{N} = I \quad (17)$$

The state equations for normalized factorization obtained through equation as,

$$(A - BD^t R^{(-1)}C)Z + Z(A - BD^t R^{(-1)}C) - ZC^T R^{(-1)}CZ + B(I - D^T R^{(-1)}D)B^T = 0 \quad (18)$$

where $R = I + DD^T$ and $D \geq 0$ is the unique stabilizing solution.

$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} A + HC & B + HD & H \\ N^{-1/2}C & R^{-1/2}DR^{-1/2} \end{bmatrix} \quad (19)$$

Required result for the normalized factorization of $G = \tilde{M}^{-1}\tilde{N}$ Similarly, normalized factorization for the right-half coprime factorization can be calculated as,

$$(A - BS^{(-1)}D^T C)^T X + X(A - BS^{(-1)}D^T C) - XBS^{(-1)}B^T X + C^T(I - DS^{(-1)}D^T D)C = 0 \quad (20)$$

here $s = i + D^T D$ and $X \geq 0$. Representation of a the plant along perturbation shown as,

$$G_{\Delta} = (\tilde{M} + G_{\Delta}) (\tilde{M} + \Delta_{\tilde{M}})^{-1} (\tilde{N} + \Delta_{\tilde{N}})^{-1} \quad (21)$$

where $(\Delta_{\tilde{M}}, \Delta_{\tilde{N}})$ represents the unstructured perturbation. From the Hankel approximation the controller K , can be achieved by the following,

$$K = \left[\begin{array}{c|c} \frac{A + BF + \gamma^2 (L^T - 1) ZC^T (C + DF)}{B^T X} & \gamma^2 (L^T - 1) ZC^T \\ \hline & -D^T \end{array} \right] \quad (22)$$

where, $F = -S^{(-1)}(D^T C + B^T X)$ and $L = (1 - \gamma^2)I + XZ$ it would behave like singular. The implementation of such condition is not possible. To get solution of such problem, the controller gain can be calculated

Fig. 4: Singular Value Response of TRMS

as

$$K = \left[\begin{array}{c|c} -L^T s + L^T (A + BF) + \gamma^2 Z C^T (C + DF) & \gamma^2 Z C^T \\ \hline B^T X & -D^T \end{array} \right] \quad (23)$$

The optimization task is to get rid of the structured, unstructured perturbations and smooth output without overshoot response. An optimal gain achieved for the system and also resolves the singularity issues as well as reduces the value of the cost function. The mixed sensitivity controller implementation is also important to validate the behavior of such systems. For the mixed sensitivity controller design expressed below as,

$$[K_1, CL_1, GAM_1] = \text{mixsyn}(G, W_1, W_3) \quad (24)$$

By this controller, the performance of the system and the stability robustness can be combined into a single infinity norm, $\|T_{yu}\|_\infty \leq 1$. For that purpose set the robustness and performance bound W_1 and W_2 . Compute controller K_1 , loop L_1 , sensitivity function S_1 and complementary function T_1 , then plot the step response of the plant. The simulation results show the reliable and acceptable performance for the applied controller. The states of the TRMS required to be controlled are θ , denoting the pitch (horizontal) angle and ϕ denote the yaw (azimuth) angle. The open-loop singular value response of the highly nonlinear system is represented in figure 4. The robust performance and stability analysis of any controller must provide the desired value in the presence of perturbations. The reason to design a closed-loop system is to get rid of conflicting goals. The controller task is to find low-frequency ranges, where perturbation attenuation is suppressed in figure 5 and figure 6 respectively. The optimal value of the controller gain must stabilize the

Fig. 5: Singular value response of controllers

TRMS in the presence of a specific noise weighting function. The validity of any controller depends upon the robustness performance criteria, which must reject perturbations and keep a smooth output response.

The singular value of the controller gain shown in figure 7, which satisfies the required optimal gain response. Remember that the quantities $1/\sigma(S)$, represents the singular values of the matrix. The desired frequency response must fulfill the following requirements as,

- Robust stability
- Robust performance
- Crossover frequency and roll off.

The LSDP step response ensures the convergence of the pitch angle and yaw angle towards the zero (origin) in figure 8 to figure 9. Robust performance, as well as real-time implementation response, validates the drawback of achieving robust performance. The controller task can be furnished by H_∞ model-based control strategy which is discussed in the next section.

4. H_∞ Mixed Sensitivity Control Design

A system is said to be robust if it achieves the desired results and stable performance in the presence of disturbances (matched and mismatched). The mixed sensitivity control calculates the minimized value of the cost function with sensitivity attributes at low-frequency in [27]. The robust controller is designed for the closed-loop system. The H_∞ technique which is developed in the last two decades has shown effective

Fig. 6: Robust performance of closed-loop system

Fig. 7: Singular value gain response of controller

Fig. 8: Pitch angle under LSH

Fig. 9: Yaw angle under LSDP

and efficient results. The various nominal and robust stability considerations were developed to minimize the problem of the infinitive norm H_∞ norm. Hence H_∞ solves the nominal performance and robust stabilization problems in [26]. A robust optimization strategy must approach towards the level of satisfactory smooth output. The standard way to design a controller according to the problem is elaborated. We take a transfer function of general plant $G(s)$ with closed-loop transfer function is represented as,

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (25)$$

$$z = F_l(P, K)w \quad (26)$$

$$z = [g_{11} + g_{12}K(I - g_{22})^{-1}]g_{21} = F_l(P, K)w \quad (27)$$

where $F_l(P, K)w$ verify the representation of fractional transform with P and K . Task for considered controller design will be as,

$$K_{stabilizing}^{min} F_l(G, K)w_\infty \quad (28)$$

The controller have ability to minimize infinity norm which must show the smooth controller gain K . Gain responsibility is to reduce,

$$F_l(G, K)w_\infty = \max_w \bar{\sigma}(F_l(G, K)(jw)) \quad (29)$$

Sub-optimal and the interconnected plant problem can be formulated as,

$$F_l(G, K)w_\infty < \gamma \quad (30)$$

$$G = \begin{bmatrix} I & -M \\ 0 & I \\ I & -M \end{bmatrix} \quad (31)$$

The mixed sensitivity methodology through the H_∞ technique has a drawback that the robust stability and robust performance are not ensured in closed-loop system implementation. The robust performance and robust stability calculation violate the stable threshold value which is near 0.6051 (60%). . The mixed sensitivity function control is not suitable for this type of highly nonlinear system. Such fact limits the applications of the control strategy. The H_∞ model-based control ($2DOFH_\infty$ control) augmented with the mixed sensitivity control, which is a generalized form of the mixed sensitivity approach. The considered control optimization provides great benefits towards robust stability and robust performance. The task of

Fig. 10: Block diagram of input/output scheme.

the controller is to fulfill the requirements of the closed-loop real-time system performance which is being discussed in the next section.

5. H_∞ Model Based Control with 2DOF

The specification design for the control system is tracking the desired signal and block diagram with input-output scheme shown in figure 10. The output of the designed control system should follow the preselected signal. According to the behavior of the nonlinear system, we must have a H_∞ model-based controller rather than other controllers as discussed in [28, 29]. By this technique, two controllers are designed one is the robust stability, internal stability, and the rejection of disturbance represented in figure 11, while the other controller is designed for the minimization of error between the reference signal and the actual response of the model. The uncertain plant has some basic requirements in the presence of perturbations like an internal and external disturbance.

$$u = \begin{bmatrix} K_r & K_y \end{bmatrix} \begin{bmatrix} r & -y_c \end{bmatrix}^T = K_r r - K_y y_c \quad (32)$$

where K_y represents the feedback matrix function and K_r is transfer function matrix of pre-filter. The closed-loop model (uncertain TRMS) shown in figure 12, represents the controller feedback response, performance requirement and disturbance matrix of noise function. The variables like r, d and n represent the reference input, input disturbance, and noise respectively. The output angles as the yaw angle α_h and the pitch angle α_v are required to control (measure) under all kinds of perturbations (noise, parametric). The output

tracking control signals e_y and e_u are error tracking signals. The output feedback vector $y_c = y + W_n n$, is the vector-matrix having measured noise n and W_n filter for noise shaping. The following weighted system in table 3, are required error tracking output (e_y, e_u) equation must satisfy the condition,

$$\begin{bmatrix} e_y \\ e_u \end{bmatrix} = \begin{bmatrix} W_p(S_o G_u K_r - M) & W_p S_o G_d & -W_p S_o G_u K_y W_n \\ W_u S_i K_r & -W_u S_i K_y G_d & -W_u S_i K_y W_n \end{bmatrix} \begin{bmatrix} r \\ d \\ n \end{bmatrix} \quad (33)$$

while $S_i = (I + K_y G_u) - 1$ and $S_o = (I + G_u K_y) - 1$ shows input, output sensitivity matrix function respectively.

Table 3: Weighted Functions.

Functions	Description
$W_p(S_o G_u K_r - M)$	Weighting difference
$W_p S_o G_d$	Weighted sensitivity to disturbance
$W_p S_o G_u K_y W_n$	Weighted sensitivity to noise
$W_u S_i K_r$	Weighted control action due to reference
$W_u S_i K_y G_d$	Weighted control action due to disturbance
$W_u S_i K_y W_n$	Weighted control action due to noise

Fig. 11: Closed-loop system with performance requirements.

The performance criterion requires the transfer function matrix from the exogenous input signals r, d and n to the output signals e_y and e_u to be small, for all possible uncertain plant model G . The transfer function matrices W_p and W_u are used to reflect the relative importance of different frequency ranges for

which the performance requirements should be fulfilled. The transfer function matrices which constitute the transfer function matrix between the inputs and outputs of the extended system are described in table 2. The controller design task is to regulate required output,

$$K = [K_r \quad K_y] \quad (34)$$

That must elaborate and satisfy the enlisted properties under perturbations. The robust stability under perturbations must meet the required response by satisfying closed-loop nominal performance and robust response conditions. The condition for nominal performance,

$$\left[\begin{array}{ccc} W_p (S_{o,nom} G_{u,nom} K_r - M) & W_p S_{o,nom} G_{d,nom} & -W_p S_{o,nom} G_{u,nom} K_y W_n \\ W_u S_{i,nom} K_r & -W_u S_{i,nom} K_y G_{d,nom} & -W_u S_{i,nom} K_y W_n \end{array} \right]_{\infty} < 1 \quad (35)$$

The condition for the robust performance,

$$\left[\begin{array}{ccc} W_p (S_o G_u K_r - M) & W_p S_o G_d & -W_p S_o G_u K_y W_n \\ W_u S_i K_r & -W_u S_i K_y G_d & -W_u S_i K_y W_n \end{array} \right]_{\infty} < 1 \quad (36)$$

The above conditions must be satisfied for G . The weighting functions W_p and W_u are reflecting the trade-off among characteristics of *and/or* for signals.

Fig. 12: Model based control strategy.

The functions K_r and K_y are transfer function of system matrix which can be easily obtained as,

$$\begin{bmatrix} z_1 \\ z_2 \\ e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -W_p M & W_p & | & W_p G \\ 0 & 0 & | & W_u \\ \hline I & 0 & | & 0 \\ 0 & I & | & G \end{bmatrix} \begin{bmatrix} r \\ d \\ u \end{bmatrix} \quad (37)$$

The closed loop system transfer function can be obtained as,

$$T_{zw} = \begin{bmatrix} W_p (S_o G_u K_r - M) & W_p S_o \\ W_u S_i K_r & W_p K_y S_o \end{bmatrix} \quad (38)$$

The controller required task is to minimize the cost function value ($H_\infty norm$) of $T_z w$ and get stable gain. The weighting functions are selected to regulate system required output response as the weighting function matrix chooses are given below,

$$w_m = \begin{bmatrix} w_{m_{11}} & w_{m_{12}} \\ w_{m_{21}} & w_{m_{22}} \end{bmatrix} \quad (39)$$

This is a model matrix for ideal model.

$$w_p = \begin{bmatrix} w_{p_{11}} & w_{p_{12}} \\ w_{p_{21}} & w_{p_{22}} \end{bmatrix} \quad (40)$$

This is a matrix for performances.

$$w_u = \begin{bmatrix} w_{u_1} & 0 \\ 0 & w_{u_2} \end{bmatrix} \quad (41)$$

This is a matrix for control action.

$$w_n = \begin{bmatrix} w_{n_1} & 0 \\ 0 & w_{n_2} \end{bmatrix} \quad (42)$$

This is a matrix for sensor noise. The controller design steps with their calculation mechanism, expressed via flow chart as given in figure 13. The unsatisfactory conditions for upper bound and lower bound can revert to the computation of weighting function (tuning function or parameter). The cost function value must be less than one to ensure the efficient credibility of a controller design. The closed-loop plant interconnection including weighting matrix can be created as and the singular value plot shown in the simulation result. The values achieved show that the system can be stabilized and robust.

The gamma values achieved by using H_∞ controller is $(\gamma) = 0.1006$, this value is less than one so the stability and the robust performance can be achieved.

The range of gamma (γ) iterations are considered from 0.1 to 10 with a tolerance of 0.001, the result shows that for which values of gamma are accepted and which are not. Then we have to design the controller gain K . The transient response and control action inputs response are shown in the figures below.

6. MATLAB Simulations Response

The simulations are carried out to understand the control behavior of both angles, in order to test the robustness of implemented techniques with respect to the matched and unmatched perturbations. The

h!

Fig. 13: Flow chart of H_∞ design.

perturbed model of the TRMS is used to carry out the simulations.

The controller synthesis is done for the several iterative performances (selection) weighting matrix (tuning factor). However, the settling time for this is more than LSDP but this has advantages to other techniques in terms of robust performance and stability validation with real-time implementation.

The singular value plot for the robustness and performance bound is shown in figure 14 and figure 15 respectively. By using this controller technique we can also reject the matched and unmatched uncertainties. The simulations are carried out to access the control behavior of both the angles, in order to test the robustness of implemented technique with respect to the matched and unmatched perturbations. The perturbed model of the TRMS is used to carry out the simulation. The model-based H_∞ controller design based on weighting matrix (weighting function) synthesis is done for the several performances weighting matrix. However, the settling time for this is more than LSDP but this has an advantage over other techniques. The property of the singular value function of the controller gain is elaborated in figure 16 and the robust sensitivity of the uncertain system is shown in figure 17. The complimentary sensitivity is elaborated by a diagram in figure 18. The pitch angle (azimuthal plane) and yaw (horizontal plane) angle response with their control input is represented in figure 19 to figure 22. The considered controller technique also reject the matched and unmatched uncertainties as,

$disturbance(1) = 0.2$ and $disturbance(2) = 0.1$, $noise(1) = 0.1$ and $noise(2) = 0.1$.

Fig. 14: Singular Values under Disturbance.

Fig. 15: Magnitude achieved for robust performance.

Fig. 16: Magnitude achieved for singular value.

Fig. 17: Magnitude of sensitivity uncertain system.

Fig. 18: Complimentary sensitivity performance.

Fig. 19: Pitch angle response of TRMS.

Fig. 20: Control action response of TRMS.

Fig. 21: Yaw angle response of TRMS.

Fig. 22: Control action response of TRMS.

According to the simulations response of both angles, we can conclude that the stability or convergence time of required angles is almost the same with a minor delay of yaw angle response. The coupling effect performs almost zero disturbances during simulation results. While in real-time implementation, it can never be zero due to the highly nonlinear behavior of the system with other perturbations (matched, unmatched).

7. Experimental Setup and System Connections

In this section, we elaborate concept of real-time implementation and system interconnections through system integrated circuits. A brief discussion is also provided concerning the response of the valid output results. The structure of the open-loop system with all input-output (8/8) ports is represented in figure 23. The internal structure of the system is also labeled with ports, to understand implementation more precisely for the reader.

The schematic diagram of the closed-loop system with important variables description elaborated in figure 24 and the number of input-output ports also provided to understand internal structure easily.

The model-based H_∞ optimization is done for the several performances weighting matrix (tuning factor). The control action for both rotors via H_∞ model-based control elaborated through simulation response. The application of the controller allows the unwanted signals to reject perturbations like parametric measurements, disturbance torque, thrust, and external uncertainty. To validate the excellent robust performance, the model being disturbed through 10percent parametric uncertainty with disturbance noise signals on both rotors would be a worse case of robustness in real-time implementation.

$disturbance(1) = 0.2$ and $disturbance(2) = 0.2$, $whitenoise(1) = 0.1$ and $whitenoise(2) = 0.1$.

Fig. 23: System internal structure of open loop TRMS.

Fig. 24: System internal structure of closed loop TRMS.

Fig. 25: Flow chart TRMS Laboratory Setup Implementation.

Fig. 26: Experimental pitch angle response of TRMS.

The real-time implementation with the help of a robust designed controller validates the controller worth under disturbances (noise signal, un-modeled states, parametric, coupling effect). The experimental processing of the TRMS with all required steps is mentioned in figure 25 to understand the laboratory hardware prototype implementation. The limited varying speed provided to validate the system robust response with stability credibility of the controller. The experimental output response of the pitch angle with their control action shown in figure 26 and figure 27 that validates the system's sharp response towards convergence within limited variation in the range of input control voltage. Similarly, yaw angle with its control action are shown in figure 28 and figure 29. The pitch angle overshoot in amplitude is very small from 1.00 to 1.32 with efficient robustness and tracking control. Small attenuation in amplitude response as compared to yaw angle response is due to highly nonlinear behavior and perturbations (noise, parametric). Comparatively yaw angle shows more overshoot in its amplitude response. The factor behind this comparatively high attenuation during tracking is the coupling effect generated by the main rotor as we have described in the 2nd section. Similarly, a sharp variation in the control action of the yaw angle can be observed. The experimental results describe that linearized systems have the almost same response as compared to nonlinear system responses. A high level of noise (disturbance), causes a serious problem with the actuators and input control signal as an error. To get actual actuator input the first-order filter is based on the butterworth filter used.

8. Conclusion

The work reported in this paper is the outcome of several attempts to design the robust performance controller as well as robust stability for the TRMS, which is a prototype model of a helicopter. The TRMS

Fig. 27: Experimental response of TRMS.

Fig. 28: Experimental yaw angle response of TRMS.

Fig. 29: Experimental control action response of TRMS.

model is a highly nonlinear system having significant coupling effect between pitch and yaw rotors which are being controlled by each DC motor. The design of a suitable controller for the robust optimization of the TRMS is a challenging task. Due to nonlinearities in the system, some assumptions have been made while deriving its mathematical model. Two phases were adopted for a controller design. The first phase covers the linearization process (NDI process) and stability analysis based on observability and controllability matrix calculation. In the 2nd phase, we design H_∞ LSDP and H_∞ model-based control augmented with mixed sensitivity control to get the required optimized response. For the robust control method, the weights have been selected in such a way that high gains have been achieved for the low frequency and low gain achieved for the high frequency. The weights have been selected iteratively through the stability and robustness performance of simulation based results. All controllers were implemented in the Simulink/MATLAB environment. The real-time implementation of H_∞ model-based control ensures the robust behavior of the controller in the presence of matched and mismatched perturbations. Under such conditions, the system must be controlled with the efficient sharp response, which is the objective of this work, and a comparative study in table 4 provided to understand the controller's worth. Some suggestions for control engineers are also evaluated from the experimental results given below.

- The disturbance torque of the tail rotor cannot reduce to zero through the decoupling process in the real-time performance evaluation.
- The experimental validation of robust control optimization shows that the TRMS behavior in real-time implementation is very sensitive depending on the exact tuning, selected weighting functions (tuning

parameters).

- The controller report verifies the robust stability as well as robust performance to the modeled perturbations (uncertainty). The maximum tolerance ability against perturbation is more than 550
- The noise signal with high amplitude causes serious contamination for the input actuators and high range frequency.

Table 4: Control strategies comparison.

Performance description	$H_\infty LSC$	Model based H_∞ Controller
Settling time pitch angle (rad/s)	1	1
Settling time yaw angle (rad/s)	2.5	2
Max Overshoot	zero overshoot	zero overshoot
Robust against parametric uncertainty	Yes	Yes
Robust against perturbations.	Not satisfactory	Satisfactory response
Real-time implementation robustness	Poor response	Satisfactory response

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