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Modified pseudo-dynamic bearing capacity of shallow foundations subjected to inclined-eccentric combined loading

Maedeh Akhavan Tavakoli¹, Hessam Fathipour², Meghdad Payan³, Reza Jamshidi Chenari^{4*}, Hadi Ahmadi⁵

ABSTRACT

1 Shallow foundations are commonly subjected to simultaneous inclined and eccentric combined
2 loadings exerted by the overlying superstructure and geo-environmental sources. The
3 performance of such footings in seismic-prone areas is a topic of great interest in geotechnical
4 engineering practice. In this paper, a comprehensive parametric study is conducted to evaluate
5 the seismic bearing capacity of shallow strip foundations overlying dry and cohesionless granular
6 soil under the action of vertical-horizontal-moment combined loading. For this purpose, a
7 systematic combination of the lower-bound theorems of the limit analysis, the finite element
8 method, and the nonlinear programming is implemented. The second-order cone programming
9 (SOCP) is adopted for efficient optimization purposes so as to model the actual nonlinear form
10 of the universal Mohr-Coulomb yield function. In addition, the equilibrium equations associated
11 with the combined loading are incorporated into the lower bound formulations. The seismic
12 condition is simulated by the well-established modified pseudo-dynamic approach by accounting
13 for the significant influence of phase difference as well as the primary and shear waves
14 propagation through applying non-uniform inertia forces along the vertical and horizontal
15 directions, respectively. The employed formulations are rigorously validated against a majority

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of high-quality studies in the literature in the static combined loading condition. The results of the seismic bearing capacity are presented in the forms of spectral responses and failure envelopes for the eccentric and inclined loadings. Accordingly, the influences of non-dimensional frequency, induced seismic acceleration and material damping on the ultimate bearing capacity of eccentrically and obliquely loaded strip footings are thoroughly examined and discussed. The results show that the most critical responses of the shallow foundation are captured in the resonance condition of earthquake excitation. As the seismic intensity decreases and the damping ratio increases, the spectral vertical, horizontal and moment bearing capacities of surface footings become smoother. In addition, the failure envelopes of the shallow foundation subjected to either inclined or eccentric loading significantly shrink with the increase in the earthquake accelerations and decrease in the material damping of the underlying soil mass. The amount of changes in the size of failure envelopes in the normalized $V - H$ and $V - M$ spaces due to the variation of seismic intensity and material damping depends directly on the non-dimensional frequency of the earthquake excitation.

Keywords: Bearing capacity; Modified pseudo-dynamic; Combined loading; Lower bound; Finite element limit analysis (FELA); Second-order cone programming (SOCP)

LIST OF NOTATIONS

$[A]$	Matrix of linear constraints
$\{b\}$	Vector of linear constraints
A^e	The area of each triangular element
a_h	Horizontal earthquake acceleration
a_v	Vertical earthquake acceleration
B	Width of the footing
C_p	Coefficient of MPD approach for vertical earthquake acceleration
C_{pz}	Coefficient of MPD approach for vertical earthquake acceleration
C_s	Coefficient of MPD approach for horizontal earthquake acceleration
C_{sz}	Coefficient of MPD approach for horizontal earthquake acceleration

D	Material Damping ratio
e	Eccentricity with respect to the centre of footing
H	Horizontal Bearing capacity
h	Depth of the medium
k_h	Horizontal acceleration coefficient
k_v	Vertical acceleration coefficient
M	Moment bearing capacity
N	Shape function
Q	Applied load
Q_c^3	Cone quadratic (second-order) constraint
Q_u	Final resultant collapse load
T	Period of harmonic seismic acceleration
S_p	Coefficient of MPD approach for vertical earthquake acceleration
S_{pz}	Coefficient of MPD approach for vertical earthquake acceleration
S_s	Coefficient of MPD approach for horizontal earthquake acceleration
S_{sz}	Coefficient of MPD approach for horizontal earthquake acceleration
T	Dominant period of incident waves
t	Shaking time
x	Horizontal coordinate
y	Vertical coordinate / Distance of element centroid from the ground surface
V	Vertical bearing capacity
V_p	Primary wave velocity
V_s	Shear wave velocity
z	Nodal auxiliary variable
z_{p1}	Coefficient of MPD approach for vertical earthquake acceleration
z_{p2}	Coefficient of MPD approach for vertical earthquake acceleration
z_{s1}	Coefficient of MPD approach for horizontal earthquake acceleration
z_{s2}	Coefficient of MPD approach for horizontal earthquake acceleration
α	Angle of load inclination with respect to the vertical direction
α_e	Angle of each edge of the triangular element with x-axis
γ	Soil unit weight
ψ	Dilatation angle
η	Shape function coefficient
ξ	Shape function coefficient
ζ	Shape function coefficient
σ_x	Horizontal normal stress
σ_y	Vertical normal stress
τ_{xy}	Shear stress
φ	Internal friction angle
ω	Angular frequency of the motion

1. INTRODUCTION

According to the fierce demand for constructing a variety of infrastructures with stringent design codes, the evaluation of the ultimate bearing capacity of shallow foundations subjected to eccentric and inclined combined loading has received great attention amongst geotechnical engineering community. Traditionally, most of the solutions to the bearing capacity problem have been related to the ultimate sustainable vertical load which could be applied concentrically to the strip footing (Jamshidi Chenari *et al.* 2014; Ranjbar Pouya *et al.* 2014; Jamshidi Chenari and Mahigir 2014 & 2017; Veiskarami *et al.* 2017; Pakdel *et al.* 2019; Jamshidi Chenari *et al.* 2019; Foroutan Kalourazi *et al.* 2019; Izadi *et al.* 2019 & 2021a).

As truly admitted by several researchers in most previous studies, a shallow foundation is rarely subjected to solely vertical concentric load. However, it always experiences both inclined and eccentric loadings which are applied by either the overlying superstructures self-weight or various geo-environmental sources, such as earthquake, wind, wave forces, etc. The stability analysis of shallow footings under the action of such loadings and their interactions turns out to be far more complicated as compared to the vertical load bearing capacity assessment. To be more specific, the relatively simple methods previously developed for the vertical bearing capacity analysis would not lead to practically credible and theoretically meticulous solutions. Several studies have been conducted throughout the literature to investigate the bearing capacity of shallow foundations subjected to inclined concentric loading (Saran *et al.* 1971; Yahia-Cherif *et al.* 2017; Zheng *et al.* 2018; Ouahab *et al.* 2020), vertical eccentric loading (Prakash and Saran 1971; Purkayastha and Char 1977; Okamura *et al.* 2002; Taiebat and Carter 2002; Krabbenhoft *et al.* 2012; Sargazi and Hosseininia 2017; Pham *et al.* 2019), and combined eccentric and inclined loading (Saran and Agrawal 1991; Bransby and Randolph

1998; Gottardi *et al.* 1999; Gourvenec 2007; Patra *et al.* 2012a; Ganesh *et al.* 2016; Pham *et al.* 2020; Keawsawasvong *et al.* 2021).

In seismic-prone areas, the shallow foundations and the overlying structure may be subjected to earthquake loading caused primarily by the seismic waves traveling through the earth layers toward the ground surface. The seismic bearing capacity of shallow foundations subjected to vertical loading has been the focus of numerous experimental, theoretical and numerical studies in the past. Numerical studies on the seismic performance of shallow foundations are performed mainly by the effective stress dynamic analyses, displacement-based sliding block analyses, or force-based pseudo-static methods. In the pioneering studies, Meyerhof (1963) proposed a convenient, simple method to evaluate the pseudo-static bearing capacity of shallow foundations, in which equivalent constant seismic coefficients were applied at the centre of gravity of the shallow foundation's failure zone. The method was extensively utilized in several subsequent studies and in this way, many researchers tried to define a design procedure for shallow foundations under pseudo-static loading condition (Sarma *et al.* 1990; Budhu and Al-Karni 1993; Richards *et al.* 1993; Dormieux and Pecker 1995; Paolucci and Pecker 1997; Soubra 1997, Kumar and Mohan Rao 2002; Choudhury and Subba Rao 2005; Cascone and Casablanca 2016; Pane *et al.* 2016; Ghosh and Debnath 2017; Qin and Chian 2018; Conti 2018; Jamshidi Chenari *et al.* 2018; Cinicioglu and Erkli 2018; Izadi *et al.* 2019, 2021a; Haghsheno *et al.* 2020; Zhang *et al.* 2020; Foroutan Kalourazi *et al.* 2021; Nouzari *et al.* 2021; Khazaei *et al.* 2021). Using the classical upper bound limit analysis method, Conti (2018) developed the three-dimensional failure envelopes and derived formulations to evaluate the seismic bearing capacity of eccentrically and obliquely loaded shallow foundations resting on the cohesive-frictional and purely cohesive soils subjected to pseudo-static loading.

78 Despite its great advantages, the so-called pseudo-static method fails to consider the concrete
79 dynamic nature of the seismic excitation, thus resulting in potentially unrealistic predictions of
80 the overall behaviour of the foundations and underlying soil layer subjected to seismic loading.
81 In an attempt to overcome such shortcomings, **Steedman and Zeng (1990)** and **Choudhury and**
82 **Nimbalkar (2005)** developed the conventional pseudo-dynamic method (CPD), so as to
83 incorporate the effects of phase difference and time in seismic analyses. In this approach, the
84 earthquake excitation is simulated through the propagation of shear and primary waves adopting
85 non-constant inertia forces along horizontal and vertical directions, respectively. In order to take
86 into account, the spectral characteristics of site response, **Bellezza (2014)** proposed a modified
87 pseudo-dynamic approach (MPD) based on the visco-elastic constitutive behaviour of the soil
88 medium to present a more realistic response of geo-structures under the effect of seismic waves.
89 The advantages of MPD over pseudo-static and conventional pseudo-dynamic include: *i)*
90 satisfying the traction-free boundary condition at the ground surface, *ii)* assessing the effects of
91 soil dynamics parameters such as material damping and shear wave velocity, and *iii)* considering
92 the effect of the time duration of the earthquake, phase difference, and reasonable amplification
93 of seismic forces. The conventional/modified pseudo-dynamic approach has been well
94 implemented in the literature to explore the seismic bearing capacity of shallow foundations
95 (**Ghosh 2008; Saha and Ghosh 2015; Zhou et al. 2016; Saha and Ghosh 2017; Kurup and**
96 **Kolathayar 2018; Rezai Soufi et al. 2021**). **Saha and Ghosh (2015)** presented a comparative
97 study on the pseudo-dynamic vertical bearing capacity of shallow strip footing using limit
98 equilibrium and limit analysis methods by considering the Coulomb failure mechanism. In
99 another study, **Izadi et al. (2021b)** deployed the limit equilibrium method to assess the influence
100 of the excitation frequency on the spectral seismic bearing capacity of shallow strip footings by

invoking the conventional pseudo-dynamic approach. Recently, **Safardoost Siahmazgi *et al.***
(2021) employed the lower bound limit analysis along with the finite element method to evaluate
the seismic vertical bearing capacity of a strip footing using conventional pseudo-dynamic
(CPD), spectral pseudo-dynamic (SPD), and modified pseudo-dynamic (MPD) approaches.
Krishnan and Chakraborty (2021) also explored the modified pseudo-dynamic bearing
capacity of strip footings over a horizontal ground of cohesionless soil using the lower bound
FELA method by generating the random field and spatial distribution of soil internal friction
angle through the soil stratum. Very recently, **Rezaie Soufi *et al.* (2021)** presented a
comprehensive comparison between the conventional and modified pseudo-dynamic approaches
in bearing capacity estimations for shallow footings. They vividly pinpointed the advantages of
MPD approach over the CPD method in terms of the capability to incorporate site effects and
damping characteristics of the soil deposit underneath shallow footing.

Despite the great number of research studies on the vertical bearing capacity of shallow
foundations in both static and dynamic loading conditions, there exists the lack of a systematic
work on the seismic bearing capacity of footings subjected to combined loading, especially under
the action of pseudo-dynamic loading. Indeed, nearly all the previous studies have been
restricted to vertical bearing capacity whose applicability for the foundations subjected to
combined loading is questionable. To fill this gap in the literature, this paper aims at evaluating
the pseudo-dynamic seismic bearing capacity of shallow foundations resting on a homogenous
cohesionless granular soil layer subjected to combined loading by lower bound finite element
limit analysis (FELA) and second-order cone programming (SOCP). Accordingly, the influence
of various parameters, including seismic accelerations and damping ratio, on the spectral
responses as well as failure envelopes is meticulously examined and discussed.

2. METHOD OF ANALYSIS

In this section, the detailed formulations of the finite element limit analysis (FELA), simulation of the eccentric and inclined loadings applied on the foundation and its incorporation into the FELA formulations, and also the simulation of the modified pseudo-dynamic seismic loading are thoroughly described.

2.1. FELA formulations

2.1.1. Theoretical Background

The lower bound theorem of limit analysis evaluates the lower bound of statically admissible stress field within the soil medium at the ultimate limiting state. Following the progressive evolution of the classical limit analysis over the years, **Lysmer (1970)** and **Anderheggen and Knopfel (1972)** were the first to incorporate the finite element method into the conventional limit analysis theorems so as to introduce the finite element limit analysis (FELA). This new method utilized the numerical procedure to rectify the difficulties associated with the manual simulation of stress field in a stability problem, specifically in the cases of complex geometries and complicated loading scenarios. In this regard, **Sloan (1988)** introduced the general formulations of lower bound FELA for the plane strain problems by means of linear programming. The formulation deals mainly with the ultimate limit state of geo-structures and thus, all the deformations occurring prior to failure and during collapse remain practically insignificant. The corresponding FELA formulation was then used extensively in the literature for the stability analysis of various geotechnical systems using either linear or nonlinear programming (**Basudhar et al. 1979; Sloan 1988; Lyamin and Sloan 2002a, b; Shiau et al. 2008; Veiskarami et al. 2017, 2019; Fathipour et al. 2020, 2021a, b, c; Mirmoazen et al. 2021; Izadi & Jamshidi Chenari 2021a & b).**

2.1.2. Problem definition

Fig. 1 shows the problem under study which consists of a shallow foundation resting on a homogenous soil layer subjected to inclined and eccentric combined loading. The load Q is assumed to be applied with an inclination angle of α with respect to the vertical axis and at an eccentric distance of e from the footing centreline. The whole problem domain was extended sufficiently so as to accommodate potential boundary effects. The soil beneath the foundation is a cohesionless granular material, whose behaviour is considered to be elastic-perfectly plastic, following the Mohr-Coulomb failure criterion with an associated flow rule ($\varphi = \psi$). The internal friction angle of the underlying soil are considered to be $\varphi = 30^\circ$. The lower bound theorems of limit analysis are implemented herein to provide a safe estimate of the ultimate combined loading bearing capacity for a rigid shallow foundation overlying the corresponding granular deposit.

2.1.3. Fundamentals of finite element method

As shown in **Fig. 1**, the whole soil medium underneath the shallow foundation has been discretized by the linear three-noded triangular elements. The nodal normal and shear stresses in this finite element mesh, namely σ_x , σ_y , and τ_{xy} , can be calculated adopting the lower bound theorems of limit analysis for the bearing capacity problem. The unknown stresses at the points other than the nodes could be estimated from the nodal stresses, using proper isoparametric shape functions and adopting the Cartesian coordinate system and the right-handed rule, as follow:

$$\sigma_x = \sum_{i=1}^3 N_i \sigma_{x,i}; \quad \sigma_y = \sum_{i=1}^3 N_i \sigma_{y,i}; \quad \tau_{xy} = \sum_{i=1}^3 N_i \tau_{xy,i} \quad (1)$$

168 In these equations, the linear shape function is characterized by $N_i = (\xi_i + \eta_i x + \zeta_i y)/2A^e$, where
 169 ξ , η and ζ are shape function coefficients and A^e is an area of each triangular element. The shape
 170 function coefficients are defined based on the horizontal and vertical coordinates, as follow:

$$\xi_1 = x_2 y_3 - x_3 y_2; \quad \eta_1 = y_2 - y_3; \quad \zeta_1 = x_3 - x_2 \quad (2a)$$

$$\xi_2 = x_3 y_1 - x_1 y_3; \quad \eta_2 = y_3 - y_1; \quad \zeta_2 = x_1 - x_3 \quad (2b)$$

$$\xi_3 = x_1 y_2 - x_2 y_1; \quad \eta_3 = y_1 - y_2; \quad \zeta_3 = x_2 - x_1 \quad (2c)$$

171 The area of each triangular element in the finite element mesh is also estimated according to
 172 $A^e = \frac{1}{2} |\eta_1 \zeta_2 - \eta_2 \zeta_1|$.

173 According to the comprehensive study conducted by **Sloan (1988)**, the finite element
 174 formulations, as characterized above, could be incorporated into the lower bound theorems of
 175 limit analysis. The resulting formulations of the finite element limit analysis for the assessment
 176 of the bearing capacity of shallow foundations are summarized in the following sections. In this
 177 framework, the matrix forms of different limit analysis constraints, including the element
 178 equilibrium, the discontinuity equilibrium, the boundary conditions, and the yield criterion
 179 enforcement are described and then assembled into the general optimization scheme.

180 **2.1.4. Element equilibrium constraint**

181 According to the element equilibrium constraint, the triangular elements in the problem domain
 182 and on the failure wedge must be in the equilibrium state due to all acting body forces. The
 183 general equilibrium equations in the x and y directions are expressed as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{a_h}{g} \gamma \quad (3a)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \left(1 + \frac{a_v}{g}\right) \gamma \quad (3b)$$

184 In the matrix notation, these equations can be presented as:

$$[A_{equil}^e] \{\sigma\} = \{b_{equil}^e\} \quad (3c)$$

185 where

$$[A_{equil}^e] = \frac{1}{2A^e} [T_1 \quad T_2 \quad T_3], \quad T_i = \begin{bmatrix} \eta_i & 0 & \zeta_i \\ 0 & \zeta_i & \eta_i \end{bmatrix} \quad (3d)$$

$$\{\sigma\} = \{\sigma_{x,1} \quad \sigma_{y,1} \quad \tau_{xy,1} \quad \sigma_{x,2} \quad \sigma_{y,2} \quad \tau_{xy,2} \quad \sigma_{x,3} \quad \sigma_{y,3} \quad \tau_{xy,3}\}^T \quad (3e)$$

$$\{b_{equil}^e\} = \left\{ \frac{a_h}{g} \gamma \quad \left(1 + \frac{a_v}{g}\right) \gamma \right\}^T \quad (3f)$$

186 2.1.5. Discontinuity equilibrium constraint

187 In the finite element limit analysis formulations, one single coordinate may be assigned to the
 188 nodes of different elements. In other words, more than one node may share the same coordinate,
 189 which means that the unknown stresses are generally discontinuous along adjacent edges of the
 190 triangles. In order to satisfy the continuity of stresses throughout the problem domain, the normal
 191 and shear nodal stresses on the permitted discontinuities are bound to be equal on the shared
 192 planes at the common edges of the adjacent triangular elements; i.e. $\sigma_{n1} = \sigma_{n2}$, $\tau_1 = \tau_2$;
 193 $\sigma_{n3} = \sigma_{n4}$, $\tau_3 = \tau_4$ (**Fig. 1**). In the matrix notation, the constraint of discontinuity at each edge
 194 of the triangular element could be described as in **Eq. (4)**, based on the normal and shear stresses
 195 acting on a plane having an angle α_e with respect to the x -axis; i.e. $\sigma_n = \sin^2 \alpha_e (\sigma_x) +$
 196 $\cos^2 \alpha_e (\sigma_y) - \sin 2\alpha_e (\tau_{xy})$ and $\tau = -0.5 \sin 2\alpha_e (\sigma_x) + 0.5 \sin 2\alpha_e (\sigma_y) + \cos 2\alpha_e (\tau_{xy})$.

$$[A_{equil}^d] \{\sigma\} = \{b_{equil}^d\} \quad (4a)$$

197 where

$$[A_{equil}^d] = \begin{bmatrix} T & -T & 0 & 0 \\ 0 & 0 & T & -T \end{bmatrix}, T = \begin{bmatrix} \sin^2 \alpha_e & \cos^2 \alpha_e & -\sin 2\alpha_e \\ -0.5 \sin 2\alpha_e & 0.5 \sin 2\alpha_e & \cos 2\alpha_e \end{bmatrix} \quad (4b)$$

$$\{\sigma\} = \{\sigma_{x,1} \quad \sigma_{y,1} \quad \tau_{xy,1} \quad \sigma_{x,2} \quad \sigma_{y,2} \quad \tau_{xy,2} \quad \sigma_{x,3} \quad \sigma_{y,3} \quad \tau_{xy,3} \quad \sigma_{x,4} \quad \sigma_{y,4} \quad \tau_{xy,4}\}^T \quad (4c)$$

$$\{b_{equil}^d\} = \{0 \quad 0 \quad 0 \quad 0\}^T \quad (4d)$$

198 2.1.6. Boundary conditions

199 The boundary conditions impose particular prescribed shear and normal stresses on the
 200 boundaries of the problem domain, including the marginal surfaces and also the soil-structure
 201 interfaces. In terms of the marginal surfaces, no external (overburden) loading is applied on the
 202 ground surface, thus $\sigma_n = 0$ and $\tau = 0$ (**Fig. 1**). The matrix form of this constraint on the edges
 203 of the horizontal ground surface can be expressed as follows:

$$[A_{bound}^{gs}]\{\sigma\} = \{b_{bound}^{gs}\} \quad (5a)$$

204 where

$$[A_{bound}^{gs}] = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}, \quad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5b)$$

$$\{\sigma\} = \{\sigma_{x,1} \quad \sigma_{y,1} \quad \tau_{xy,1} \quad \sigma_{x,2} \quad \sigma_{y,2} \quad \tau_{xy,2}\}^T \quad (5c)$$

$$\{b_{bound}^{gs}\} = \{0 \quad 0 \quad 0 \quad 0\}^T \quad (5d)$$

205 In terms of boundary condition at the contact surface between the foundation and the underlying
 206 soil deposit, the magnitude of the mobilized shear stress at the soil-foundation interface should
 207 not exceed the shear strength of the interface, i.e. $|\tau_{xy}| \leq -\sigma_y \tan \delta$, where δ is the interface
 208 friction angle. In the matrix form, this can be presented by the following inequality:

$$[A_{bound}^{s-f}]\{\sigma\} \leq \{b_{bound}^{s-f}\} \quad (6a)$$

209 where

$$[A_{bound}^{s-f}] = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}, \quad T = \begin{bmatrix} 0 & \tan \delta & -1 \\ 0 & \tan \delta & 1 \end{bmatrix} \quad (6b)$$

$$\{\sigma\} = \{\sigma_{x,1} \quad \sigma_{y,1} \quad \tau_{xy,1} \quad \sigma_{x,2} \quad \sigma_{y,2} \quad \tau_{xy,2}\}^T \quad (6c)$$

$$\{b_{bound}^{s-f}\} = \{0 \quad 0 \quad 0 \quad 0\}^T \quad (6d)$$

210 2.1.7. Yield criterion enforcement

211 As mentioned earlier, the soil mass beneath the shallow foundation in this study is considered to
 212 be perfectly plastic, following the universal Mohr-Coulomb yield criterion. According to the
 213 fundamental concepts of FELA approach, the mobilized stresses at all nodal points within the
 214 soil medium should not exceed the Mohr-Coulomb yield function, defined as:

$$F = (\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 - [-(\sigma_x + \sigma_y) \sin \varphi]^2 \leq 0 \quad (7)$$

215 In this approach, in order to convert the Mohr-Coulomb yield function into the standard form of
 216 the second-order cone programming, three nodal auxiliary variables are introduced as the
 217 interrelationship between the nodal shear and normal stresses, as: $z_1 = (\sigma_x - \sigma_y)$, $z_2 = 2\tau_{xy}$,
 218 and $z_3 = -(\sigma_x + \sigma_y) \sin \varphi$. The translation into the SOCP optimization scheme is conducted by
 219 adopting the conic quadratic (second-order) constraint (Q_c^3), defined as:

$$Q_c^3 = \left\{ z \in R^3: \sqrt{z_{1,i}^2 + z_{2,i}^2} \leq z_{3,i} \right\} \quad i = 1, 2, 3, \dots, \text{number of nodes} \quad (8)$$

220 The matrix form of the corresponding interrelationship equation is expressed as:

$$[A_{yield}]\{\sigma\} + I\{z\} = \{b_{yield}\} \quad (9a)$$

221 where

$$[A_{yield}] = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & -2 \\ \sin \varphi & \sin \varphi & 0 \end{bmatrix} \quad (9b)$$

$$\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \tau_{xy}\}^T \quad (9c)$$

$$\{z\} = \{z_1 \quad z_2 \quad z_3\}^T \quad (9d)$$

$$\{b_{yield}\} = \{0 \quad 0 \quad 0\}^T \quad (9e)$$

$$I \text{ (identity matrix)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9f)$$

222 The three nodal auxiliary variables, i.e. z_1 , z_2 and z_3 , are related to the nonlinear optimization
223 scheme adopted in the current study, which has been elaborated in the following section.

224 **2.1.8. Simulation of eccentric and inclined combined loading**

225 In most practical cases, the shallow foundation supporting a superstructure is subjected to a
226 combination of vertical (V), horizontal (H) and moment (M) loadings. It is therefore desirable to
227 find a yield envelope as a limiting surface of load combinations beyond which the foundation
228 would fail. The combined load Q is defined by the vertical component ($V = Q \cos \alpha$), the
229 horizontal component, $H = V \tan \alpha$, and an overturning moment, $M = V \times e$, where α and e are
230 the inclination angle and load eccentricity, respectively. The force and moment equilibrium
231 equations are as follow:

$$\sum F_x = 0 : \quad Q \sin \alpha = \int_S \tau \, dx \quad (10a)$$

$$\sum F_y = 0 : \quad Q \cos \alpha = \int_S \sigma_n \, dx \quad (10b)$$

$$\sum M_{kz} = 0 : \quad \int_S x \sigma_n \, dx = 0 \quad (10c)$$

232 where S is the foundation surface area, x is the distance from the point of load application, k is
 233 the point of load application, and σ_n and τ are the normal and shear stresses along the footing
 234 nodes, respectively. **Eqs. (10a) and (10b)** could be divided and simplified into the following
 235 correlation:

$$\sin\alpha \int_S \sigma_n dx = \cos\alpha \int_S \tau dx \quad (11)$$

236 In the matrix form, the equilibrium equations at each node along the soil-foundation interface are
 237 expressed as follow:

$$[A_{inc}]\{\sigma\} = \{b_{inc}\} \quad (12a)$$

$$[A_{ecc}]\{\sigma\} = \{b_{ecc}\} \quad (12b)$$

238 where

$$[A_{inc}] = [0, \sin\alpha, -\cos\alpha] \quad (13c)$$

$$[A_{ecc}] = [0, x, 0] \quad (13d)$$

$$\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \tau_{xy}\}^T \quad (13e)$$

$$\{b_{inc}\} = \{0\} \quad (13f)$$

$$\{b_{ecc}\} = \{0\} \quad (13g)$$

239 Moreover, in order to allow segregation of the footing and underlying soil (which may occur due
 240 to the developed tensile contact stress under eccentric loading condition), a "tension cut-off"
 241 constraint must be applied to the soil-foundation interface nodes for the vertical normal stresses
 242 beneath the foundation at each node being compressive (negative) only ($\sigma_y \leq 0$) as:

$$[A_{bound}^{cut}]\{\sigma\} \leq \{b_{bound}^{cut}\} \quad (14a)$$

$$[A_{equil}^{cut}] = [0 \quad 1 \quad 0] \quad (14b)$$

$$\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \tau_{xy}\}^T \quad (14c)$$

$$\{b_{bound}^{cut}\} = \{0\} \quad (14d)$$

2.1.9. Assembling & Optimization

In the preliminary formulation of FELA presented by **Sloan (1988)**, the nonlinear form of Mohr-Coulomb yield function (**Eq. 7**) was linearized and roughly approximated by interior regular polygons of specific number of sides and vertices. With further evolution of optimization programming in FELA concepts, several attempts were made to simulate the actual nonlinear form of the corresponding yield criterion, with the seminal contributions of **Lyamin and Sloan (2002a, b)**, **Makrodimopoulos and Martin (2005)**, and **Krabbenhoft *et al.* (2008)**. Apart from the superior precision of the obtained solutions, using such optimization can significantly enhance the speed of computation. This issue is more pronounced for the evaluation of the seismic bearing capacity of shallow foundations, which is intrinsically time-consuming because of the high computational demand. The second-order cone programming (SOCP), as a nonlinear optimization scheme, has been proven to be a quick, precise and the most appropriate technique to evaluate the overall stability of various geo-structures (**Makrodimopoulos and Martin 2006; Fathipour *et al.* 2020, 2021a, b, c; Safardoost Siahmazgi *et al.* 2021, Izadi and Jamshidi Chenari 2021a, b**, among others). At the final stage, all the constraints are assembled into the following general matrix form to obtain the unknown stress field using the second-order cone optimization.

Following the general framework of the lower bound FELA, the objective function, which is the vertical load bearing capacity of shallow footing defined as the integral of normal stresses (σ_n) along the soil-footing interface, must be maximized:

$$V = \text{Maximize} \left\{ - \int_s \sigma_n dx \right\} \quad (15)$$

263 Subjected to

$$[A]\{X\} = [B] \quad (16b)$$

$$\{X\} = \{\sigma; z_{socp}\} \quad (16c)$$

$$z_{socp} \in Q_c^3 \quad (16d)$$

264 2.2. Modified pseudo-dynamic seismic loading

265 In the so-called pseudo-dynamic loading, a more realistic dynamic nature of the seismic
 266 excitation is taken into account using various soil dynamic properties and spectral parameters of
 267 the problem under study. In this way, a more viable response of the shallow foundation subjected
 268 to seismic loading could be captured by simulating a visco-elastic material by means of the well-
 269 defined Kelvin-Voigt model. The pseudo-dynamic method considers the influences of shear and
 270 primary wave velocities, amplification of earthquake loading, and period of lateral shaking. It is
 271 also assumed that the shear wave velocity and material damping are constant within the zone of
 272 influence by shallow footing loading and the acceleration phase varies along the soil depth. In
 273 the modified pseudo-dynamic approach, the earthquake accelerations with an angular frequency
 274 of ω and a dominant period of T (where $T = 2\pi/\omega$) are considered as *a priori* uniform
 275 sinusoidal functions of time (t) and depth (y). The seismic horizontal and vertical accelerations
 276 acting on the centroid of each element are in turn evaluated by the following modified pseudo-
 277 dynamic functions so as to be representatives of the one-dimensional shear and primary waves,
 278 respectively, propagating vertically through the soil mass from earthquake sources (**Bellezza**
 279 **2014**). The horizontal and vertical earthquake acceleration components can be calculated as
 280 follow:

$$a_h(y, t) = \frac{k_h g}{C_S^2 + S_S^2} \left[(C_S C_{sy} + S_S S_{sy}) \cos\left(2\pi \frac{t}{T}\right) + (S_S C_{sy} - C_S S_{sy}) \sin\left(2\pi \frac{t}{T}\right) \right] \quad (17a)$$

$$a_v(y, t) = \frac{k_v g}{C_P^2 + S_P^2} \left[(C_P C_{py} + S_P S_{py}) \cos\left(2\pi \frac{t}{T}\right) + (S_P C_{py} - C_P S_{py}) \sin\left(2\pi \frac{t}{T}\right) \right] \quad (17b)$$

281 where for the horizontal seismic acceleration, we have:

$$C_S = \cos(z_{s1}) \cosh(z_{s2}) \quad (18a)$$

$$S_S = -\sin(z_{s1}) \sinh(z_{s2}) \quad (18b)$$

$$C_{sy} = \cos\left(\frac{z_{s1}y}{h}\right) \cosh\left(\frac{z_{s2}y}{h}\right) \quad (18c)$$

$$S_{sy} = -\sin\left(\frac{z_{s1}y}{h}\right) \sinh\left(\frac{z_{s2}y}{h}\right) \quad (18d)$$

282 where

$$z_{s1} = \frac{2\pi h}{TV_s} \sqrt{\frac{\sqrt{1+4D^2}+1}{2(1+4D^2)}} \quad (18e)$$

$$z_{s2} = -\frac{2\pi h}{TV_s} \sqrt{\frac{\sqrt{1+4D^2}-1}{2(1+4D^2)}} \quad (18f)$$

283 In addition, for the vertical seismic acceleration, we have:

$$C_P = \cos(z_{p1}) \cosh(z_{p2}) \quad (18a)$$

$$S_P = -\sin(z_{p1}) \sinh(z_{p2}) \quad (18b)$$

$$C_{py} = \cos\left(\frac{z_{p1}y}{h}\right) \cosh\left(\frac{z_{p2}y}{h}\right) \quad (18c)$$

$$S_{py} = -\sin\left(\frac{z_{p1}y}{h}\right) \sinh\left(\frac{z_{p2}y}{h}\right) \quad (18d)$$

284 where

$$z_{p1} = \frac{2\pi h}{TV_p} \sqrt{\frac{\sqrt{1 + 4D^2} + 1}{2(1 + 4D^2)}} \quad (18e)$$

$$z_{p2} = -\frac{2\pi h}{TV_p} \sqrt{\frac{\sqrt{1 + 4D^2} - 1}{2(1 + 4D^2)}} \quad (18f)$$

285 where C_s , S_s , C_{sz} , S_{sz} , z_{s1} , z_{s2} are the coefficients of modified pseudo-dynamic horizontal
 286 seismic acceleration; C_p , S_p , C_{pz} , S_{pz} , z_{p1} , z_{p2} are the coefficients of modified pseudo-dynamic
 287 vertical seismic acceleration; k_h is the coefficient of horizontal acceleration at base (bedrock)
 288 level; k_v is the coefficient of vertical acceleration at base (bedrock) level; g is the gravitational
 289 acceleration; t is the time of shaking; y is the distance of element centroid from the ground
 290 surface; h is the thickness of the soil deposit beneath the footing; V_s is the shear wave velocity;
 291 V_p is the primary wave velocity, and D is the material damping.

292 As an iterative algorithm, the MPD equations are incorporated into the FELA formulations. In
 293 this regard, for any specific non-dimensional frequency (h/TV_s), the bearing capacity is
 294 evaluated and updated in each iteration for different values of shaking time (t) in incremental
 295 steps of $\Delta t/T = 0.015$. The minimum value of bearing capacity during the shaking time is then
 296 opted as the critical value for the vertical bearing capacity component V optimized in **Eq. (15)**
 297 for each time step. The other two loading components (H and M) are interchangeably calculated
 298 from the optimized vertical loading component at critical shaking time. The joint contribution of
 299 the horizontal and vertical earthquake acceleration coefficients is commonly considered in a
 300 single parameter, namely the seismic angle defined as $\theta = \tan^{-1} (k_h/(1 - k_v))$. Seismic angle
 301 is characterized to represent the angle of deviation of the gravitational forces from the earth
 302 gravitation force direction. Hence, $k_h/(1 - k_v)$ represents the severity of earthquake loading
 303 imposed on the soil medium and the overlying superstructure. In addition, in order to estimate

the modified pseudo-dynamic seismic forces according to the practical assumptions reported by **Payan *et al.* (2017, 2020)**, V_p is considered to be 1.87 times V_s accounting for a typical constant value of Poisson's ratio equal to 0.3.

A step-by-step flowchart of the computational process of the modified pseudo-dynamic formulations integrated with the finite element limit analysis theory is provided in **Fig. 2** to have a vision on the current study conducted methods.

3. RESULTS & DISCUSSION

3.1. Validation of FELA formulation for static combined bearing capacity

In order to verify the accuracy of the current lower bound finite element limit analysis formulations, the failure envelopes of shallow foundations under either inclined or eccentric combined loading in static loading condition reported in the literature have been compared with those obtained in the course of this study using the well-established FELA approach elaborated in the previous section. The selected studies from the literature are the main ones in this area which have utilized various theoretical and experimental approaches and also different numerical methods such as finite element (FE), finite difference (FD), limit equilibrium (LE), FELA, etc. **Fig. 3a** refers to the studies on obliquely-loaded footings, while **Fig. 3b** represents such a comparison for the foundations under eccentric loading. Both types of envelopes have been presented in a normalized fashion, as highly accepted in the literature. It should be noted that the soil underlying shallow foundation in all these studies was cohesionless sand with an internal friction angle of 35° . As it can be observed in the figures, the failure envelopes obtained with the current FELA formulation are in great agreement with those reported in the literature for both types of inclined and eccentric loading, as the envelopes in both cases approximately lie on the

majority of the ones presented in the previous studies. With this superior capability, the FELA formulations adopted in the current study are deemed sufficiently accurate in the prediction of the bearing capacity of shallow foundations subjected to inclined and eccentric combined loading.

3.2. Obliquely-loaded shallow foundations subjected to modified pseudo-dynamic excitation

In this section, a comprehensive analysis has been carried out on the modified pseudo-dynamic bearing capacity of obliquely-loaded shallow foundations. First, the spectral responses of footings and the influences of seismic intensity and material damping ratio are examined. Then, the failure envelopes of obliquely-loaded foundations subjected to modified pseudo-dynamic excitation are presented in terms of various non-dimensional frequencies, seismic intensities and material damping ratios.

3.2.1. Spectral responses

3.2.1.1. Influence of seismic intensity ($k_h/(1 - k_v)$)

Fig. 4 shows the spectral variations of normalized horizontal ($H/V_{max(static)}$) and vertical ($V/V_{max(static)}$) bearing capacity with respect to the non-dimensional frequency (h/TV_s) for obliquely-loaded shallow foundations subjected to modified pseudo-dynamic loading of various seismic intensities ($k_h/(1 - k_v)$) with a constant typical material damping of 0.05. It should be noted that the values of the ultimate vertical and horizontal loads have been normalized with respect to the maximum vertical load which could be sustained by the foundation in the static loading condition. The figures in each row contain information related to a particular seismic intensity. For each constant value of the seismic intensity (0.1, 0.15 and 0.2), the external load is

considered to be applied on the footing at various inclination angles (α) ranging from 0° to 30° by 6° increments.

As highlighted in most pseudo-dynamic studies in the literature, in all cases, the spectral normalized horizontal and vertical bearing capacities start with the pseudo-static loading condition at very low frequencies (high wavelength) and end at very high frequencies (low wavelengths). The corresponding two extremes of infinitely large and infinitesimal wavelengths are associated with the uniform acceleration field in the pseudo-static condition and successively counteracted inertia forces, respectively. In non-dimensional wavelengths between the two extremes, the soil layer and the overlying foundation experience the so-called pseudo-dynamic excitation, where the bearing capacity values could be observed to be more critical in most cases, specifically in the resonance condition. To be more specific, in most non-dimensional wavelengths in the spectral curves, the bearing capacity of shallow foundations could be observed to be much lower than the corresponding values in the pseudo-static loading condition. Hence, seismic analysis of foundations based on pseudo-static loading condition would most probably lead to insecure and non-conservative designs. These deficiencies associated with the pseudo-static approach arise predominantly from failing to consider the real seismic nature of earthquake loading.

As it can be observed, the first mode of resonance occurs at the non-dimensional frequency of approximately 0.25 for all cases irrespective of the input seismic acceleration and load inclination. This is in full conformity to the theoretical estimations for the fundamental frequency of a uniform, damped soil deposit on a rigid rock. In other words, the n^{th} natural frequency of such a layer would be $\omega_n \approx \frac{V_s}{h}(\frac{\pi}{2} + n\pi)$, where n is the mode number ranging from $0, 1, 2, \dots, \infty$; V_s is the shear wave velocity of the soil layer, h is the soil deposit thickness, and ω_n is the n^{th}

372 natural angular frequency of the layer (**Kramer 1996**). Deploying the above mentioned
 373 relationship, the modal non-dimensional frequencies would be $h/TV_s = 1/4 + n/2$, which
 374 clearly explain why the first and second resonant non-dimensional frequencies in this study are
 375 lying around 0.25 and 0.75. In addition, at a given load inclination, as the seismic intensity
 376 increases, more fluctuations in the spectral bearing capacity could be observed and in particular,
 377 the extremum values of horizontal and vertical bearing capacity measures in the resonance state
 378 are more affected. With the increase in the seismic intensity, the horizontal and vertical bearing
 379 capacities expectedly decrease in value in a wide range of non-dimensional frequency. This is
 380 more visible in the resonance condition where the ultimate bearing capacities take some
 381 minimum values at relatively low seismic intensity (0.1 herein), whereas they are nearly zero for
 382 the seismic intensities of 0.15 and 0.2. Influence of seismic intensity on vertical and horizontal
 383 bearing capacities is however further pronounced at the second mode of resonance (i.e., at $h/$
 384 $TV_s=0.75$), where the bearing capacity measures are much greater at lower seismic intensities. In
 385 other words, as the seismic intensity increases, the changes in the normalized bearing capacities
 386 are greater in the second mode of resonance as compared to the first mode.
 387 At a given seismic intensity, as the load inclination increases, the variation of the normalized
 388 vertical bearing capacity with the normalized frequency could be observed to become smoother
 389 and at relatively high α values, say 30° , the wavelength has practically trivial or no influence on
 390 the vertical bearing capacity. More interestingly, the bearing capacity values corresponding to
 391 the static and pseudo-static loadings converge at noticeably high values of α . Moreover, with the
 392 increase in the load inclination, the ultimate vertical bearing capacity decreases with a fairly
 393 constant rate for all seismic intensities and non-dimensional frequencies (**Figs. 4a, 4c, 4e**).
 394 Despite the vertical bearing capacity, the ultimate horizontal load the foundation can sustain

without reaching the failure state does not follow a constant increasing or decreasing trend. Conversely, for all intensities of the incident wave, the horizontal bearing capacity and its corresponding fluctuation increase with the load inclination up to about 12° , beyond which they show reduction (**Figs. 4b, 4d, 4f**). This observation can be attributed to the very fact that the horizontal limit load component is coupled with its vertical counterpart, which undergoes sharp reductions with load inclination. In other words, the horizontal limit load will be a multiplication of two increasing ($\tan \alpha$) and decreasing (V) functions. After a certain load inclination, the notable decline in the vertical limit load component would give rise the horizontal limit load to start decreasing. Indeed, at the two extreme values of load inclinations under study, i.e. 0° and 30° , the variation of the normalized horizontal bearing capacity against non-dimensional frequency becomes almost trivial. According to the above discussion, the approximate angle of 12° can be considered as the optimum load inclination rendering the most horizontal bearing capacity. This value is a ballpark estimate and may be slightly different as all inclination angles have not been shown in this figure. The more accurate optimum value can be captured from the failure envelopes discussed in the following sections. Considering the spectral responses presented in **Fig. 4**, it can be generally inferred that the foundation fails to function efficiently when the load is applied with an inclination more than 30° . This threshold is equal to the adopted internal/interface friction angle, which lends support to the contention that the critical load inclination angle would be the interface friction angle, governed by the Coulomb law for frictional materials.

3.2.1.2. Influence of material damping (D)

Fig. 5 shows the influence of material damping (D) on the spectral variations of normalized horizontal ($H/V_{max(static)}$) and vertical ($V/V_{max(static)}$) bearing capacities against the non-

dimensional frequency (h/TV_s) for obliquely-loaded shallow foundations subjected to modified pseudo-dynamic loading with a constant typical seismic intensity of 0.2. Different values of 0.05, 0.1 and 0.2 have been considered for the material damping of the underlying sand layer in each row of this figure. Once again, the first mode of resonance could be observed to occur at the non-dimensional frequency of approximately 0.25 for all cases with no particular influence from the load inclination and material damping. At a constant load inclination, as the material damping increases, less fluctuation in the spectral bearing capacity as well as higher values of horizontal and vertical bearing capacities could be captured. This observation is especially more pronounced at non-dimensional frequencies higher than 0.5 near the second mode of resonance. The phenomenon could be attributed primarily to the dissipation of incident waves due to the high material damping of the underlying soil which in effect would diminish most of the seismic energy emanated from the earthquake source, thus rendering greater values of vertical and horizontal bearing capacities. For high values of material damping (0.2 herein) and at all load inclination angles, non-dimensional frequency barely affects the spectral responses beyond $h/TV_s=0.5$. As in the case highlighted for the effect of seismic intensity, the influence of material damping on the spectral bearing capacity measures is more pronounced in the second mode of resonance as compared to the first mode. Comparing **Figs. 4** and **5**, it can be generally observed that at the first mode of resonance (i.e., $h/TV_s=0.25$), the seismic intensity has a greater impact on the responses compared to the material damping. On the other hand, at the second mode of resonance (i.e., $h/TV_s=0.75$), the material damping can be seen to have a higher influence on both vertical and horizontal bearing capacities as compared to the seismic intensity. Similar observations described earlier on the effect of load inclination on the spectral responses of both vertical and horizontal bearing capacities could also be observed herein for various

material damping ratios. Similarly, 12° could be captured to be the optimum load inclination angle offering the maximum horizontal bearing capacity for all material damping values. As stated, this value is approximate and the more exact optimum inclination angle can be distinguished from the failure envelopes discussed in the following sections.

3.2.2. Failure envelopes

3.2.2.1. Influence of non-dimensional frequency (h/TV_s)

Fig. 6 demonstrates the failure envelopes of obliquely-loaded shallow foundations subjected to different non-dimensional frequencies (h/TV_s) under a wide range of load inclinations from zero to 36° . The seismic intensity and damping ratio have been considered to be constant and equal to the typical values of 0.2 and 0.05, respectively. From the full spectral curves presented earlier, typical h/TV_s values of extremely low (about zero corresponding to the pseudo-static condition), 0.4, 0.7, 0.85, and extremely high (infinity corresponding to the static condition) have been selected for the illustration of the failure loci. As shown in **Fig. 6a**, the normalized vertical bearing capacity decreases continuously with the inclination angle for all non-dimensional frequencies. In addition, **Fig. 6b** shows that the normalized horizontal bearing capacity for all h/TV_s values initially increases with the load inclination angle up to about 12° to 14° , where it shows reduction thereafter. As elaborated earlier, the horizontal limit load starts diminishing after a certain stage due to the drastically reduced vertical limit load component. The inclination angle at which the maximum $H/V_{max(static)}$ occurs is barely influenced by non-dimensional frequency. This is due primarily to the fact that both the horizontal and vertical limit load components are equally affected by the excitation frequency and their ratio remains almost unaffected by the non-dimensional frequency. Close examination of **Figs. 6a** and **6b** reveals that the influence of non-dimensional frequency on the vertical and horizontal bearing capacities of

surface footing under various modified pseudo-dynamic loading scenarios becomes fairly negligible beyond the load inclination angle of about 30° , as all curves converge afterwards.

As shown in **Fig. 6c**, the general shapes of the yield surfaces in the normalized $V - H$ plane are well described by a parabolic form, as also observed in several previous studies (**Loukidis et al. 2008; Krabbenhoft et al. 2012; Zheng et al. 2019**; among others). In all failure envelopes, $H/V_{max(static)}$ first increases with $V/V_{max(static)}$ until it reaches a maximum value. With further increase of $V/V_{max(static)}$, the horizontal bearing capacity decreases up to where it reaches zero (corresponding to $\alpha = 0$) on the $V/V_{max(static)}$ axis. As it can be observed, the smallest failure envelope among the ones presented herein is related to $h/TV_s = 0.7$. It should be noted that the smallest failure envelope among all non-dimensional frequencies, constituting literally zero resistance of the strip footing against oblique loading in most cases, is related to the first mode of resonance condition, where $h/TV_s = 0.25$ (see **Figs. 4 and 5**). The values of $V/V_{max(static)}$ on the horizontal axis at zero $H/V_{max(static)}$ for different curves shown in **Fig. 6c** also show that the vertical bearing capacity is smaller for the non-dimensional frequency of 0.7. In addition, the value of $V/V_{max(static)}$ where the maximum horizontal bearing capacity of the envelopes occurs is lower for the case of $h/TV_s = 0.7$. On the other hand, the largest failure envelope is not unexpectedly pertaining to the static loading condition (high frequency/low wavelength) in which the maximum horizontal bearing capacity occurs at the relatively higher $V/V_{max(static)}$ value compared to the other loading scenarios. As depicted in the spectral responses of the surface footing in the previous section, with the increase in non-dimensional frequency, the failure envelope becomes either larger or smaller in size depending on the adopted h/TV_s . However, as mentioned above, it is evident that the worst case scenario belongs to the

corresponding resonance condition where the minimum values of vertical and horizontal bearing capacities are acquired.

3.2.2.2. Influence of seismic intensity ($k_h/(1 - k_v)$)

Fig. 7 shows the significant influence of seismic intensity ($k_h/(1 - k_v)$) on the failure envelope of obliquely-loaded shallow foundations subjected to a wide range of load inclinations. In these envelopes, the material damping is assumed to be constant and equal to 0.05 and the soil mass is considered to be subjected to the modified pseudo-dynamic loading at the typical non-dimensional frequencies of 0.4 and 0.7. As illustrated in **Figs. 7a, 7b, 7d and 7e**, as the load inclination increases, $V/V_{max(static)}$ decreases while the $H/V_{max(static)}$ first increases and then decreases past the peak value which occurs at the inclination angles of approximately 12° to 14° ; the angle which is not affected by the seismic intensity. **Figs. 7c and 7f** show that the failure envelope markedly shrinks as the seismic intensity increases from 0.1 to 0.2. The reduction in the size of failure envelopes is more pronounced in the case of $h/TV_s=0.7$, where the size of failure envelope decreases significantly to almost its half size with the increase in the seismic angle from 0.1 to 0.2. It should be emphasized that the two non-dimensional frequencies examined here are typical values and the contribution of the seismic intensity to the amount of reduction of the size of failure envelopes directly depends on the non-dimensional frequency of the earthquake excitation (see **Figs. 4 and 5**). In addition, with the increase of seismic intensity, $V/V_{max(static)}$ at which the maximum normalized horizontal bearing capacity occurs, shifts to lower values, which is more pronounced in the case of $h/TV_s=0.7$. The values of $V/V_{max(static)}$ when $H/V_{max(static)}=0$ demonstrate that the vertical bearing capacity of the foundation decreases when it is subjected to higher seismic intensities.

The effect of seismic intensity is more pronounced at the lower inclination angles and turns out to be insignificant after a specific α value of about 27° to 30° . This observation can be ascribed to the very fact that at high load inclination angles, both the vertical and horizontal limit load components are low and the footing failure is governed by the footing-soil interface failure. Indeed, due to the very low vertical limit load values, the horizontal limit load is governed by the footing sliding at the interface level. This idea has been substantiated by illustrating some sample failure patterns for different load inclination values, as shown in **Fig. 8**. As shown in this figure, at a given seismic intensity in the static and pseudo-static conditions, as the inclination angle increases from 0° to 20° , the failure zone decreases in size. In other words, the size of passive zone in the side of load inclination significantly decreases compared to the other side, thus revealing a reduction in the overall bearing capacity of the foundation. In addition, in the case of vertical loading (zero load inclination angle), the passive failure zone at the side of pseudo-static load application gradually diminishes as the seismic intensity increases from 0 to 0.2. However, as the inclination angle increases, the diminished passive wedge reappears and has the most impact on resisting the oblique load. Moreover, with the increase in the inclination angle, the influence of seismic intensity on the failure zones notably decreases. This phenomenon is more pronounced at relatively high inclination angles (say 20° herein), as the failure zones for different seismic intensities are almost similar. It is worth mentioning that in the cases assumed in **Fig. 8**, the load inclination and the pseudo-static load are applied in the opposite directions. This explains why they have counteracting influence on the failure mechanism underneath the footing. However, the most critical condition will manifestly occur when they are in the same direction.

3.2.2.3. Influence of material damping (D)

Fig. 9 shows the influence of material damping (D) on the failure envelope of obliquely-loaded shallow foundations subjected to a wide range of load inclinations. The seismic intensity is assumed to be constant and equal to 0.2 and the soil mass is considered to be subjected to the modified pseudo-dynamic loading at the typical non-dimensional frequencies of 0.4 and 0.7. As depicted in **Figs. 9a, 9b, 9d** and **9e**, the trends of variations of $V/V_{max(static)}$ and $H/V_{max(static)}$ with inclination angle are quite similar to the previous cases. Once again, the maximum horizontal limit loads for different damping ratios occur at an inclination angle of 12° to 14° . According to the **Figs. 9c** and **9f**, with the increase in the material damping from 0.05 to 0.2, the failure envelope in the normalized $V - H$ plane expands, thereby bearing witness to the greater resistance of the obliquely-loaded footing against seismic loading. The contribution of damping ratio to the increase in the size of failure envelope is more pronounced at the non-dimensional frequency of 0.7. Indeed, when $h/TV_s=0.4$, material damping could be observed to have no significant influence on the failure envelope. Once again, it should be highlighted that the two h/TV_s values examined here are only two typical values and the contribution of material damping to the amount of increase in the size of failure envelopes directly depends on the non-dimensional frequency induced by the earthquake excitation (see **Figs. 4** and **5**).

The contribution of damping ratio to the augmentation of the bearing capacity is also less pronounced at higher inclination angles. To be more specific, beyond a particular inclination angle of approximately 27° , the damping ratio barely affects the failure envelopes. As explained earlier, by increasing the load inclination angle, due to the shallow failure condition as acquired from **Fig. 8**, the site amplification properties pale to insignificance. Furthermore, the $V/V_{max(static)}$ values corresponding to the maximum horizontal bearing capacity increase with the increase in the damping ratio.

3.3. Eccentrically-loaded shallow foundations subjected to modified pseudo-dynamic excitation

The modified pseudo-dynamic bearing capacity of eccentrically-loaded shallow foundations has been examined in this section. Similar to the previous section, first, the influences of seismic acceleration and material damping on the spectral responses of surface footings are discussed. In the second part, the failure loci of eccentrically-loaded shallow foundations under the action of modified pseudo-dynamic excitation are presented accounting for the effects of non-dimensional frequency, seismic intensity and damping ratio.

3.3.1. Spectral responses

3.3.1.1. Influence of seismic intensity ($k_h/(1 - k_v)$)

The influence of seismic intensity on the spectral variations of normalized moment ($M/BV_{max(static)}$) and vertical ($V/V_{max(static)}$) bearing capacity of eccentrically-loaded shallow foundations is illustrated in **Fig. 10**. The eccentricity ratio (e/B) in each seismic intensity has been considered to vary from 0 to 0.4. Once again, the spectral normalized vertical and moment bearing capacities in all cases start with the pseudo-static loading condition at very low frequencies and end at the static loading condition at very high frequencies. The bearing capacity values associated with the static and pseudo-static loadings converge at considerably high values of load eccentricity.

As it can be observed and similar to the case of inclined loading, irrespective of the values of the input seismic acceleration and load eccentricity, the first mode resonance occurs at the non-dimensional frequency of approximately 0.25 for all cases. At a constant load eccentricity, with the increase in the seismic intensity, more fluctuations and relatively smaller spectral vertical and moment bearing capacities could be captured; the trend which is more pronounced in the first

and second modes of resonance state especially at smaller seismic intensities. Indeed, the bearing capacities are practically zero at the first and second modes of resonance at high seismic intensity and thus, the foundation has no resistance against the applied external loading. Furthermore, the effect of seismic intensity on vertical and moment bearing capacities is more pronounced at the second mode of resonance, occurring at approximately $h/TV_s=0.75$, as the changes in the bearing capacities due to the increase of the seismic intensity in the corresponding mode of resonance is much greater compared to the first mode.

At a given seismic intensity, with the increase in the load eccentricity, the ultimate vertical bearing capacity decreases in a fairly constant rate and the spectral variations of $V/V_{max(static)}$ could be observed to become smoother and at relatively high eccentricity ratios (e/B values of approximately 0.4), the non-dimensional frequency has no significant influence on the vertical bearing capacity (**Figs. 10a, 10c, 10e**). On the contrary, at a constant seismic intensity, the normalized moment bearing capacity and its corresponding fluctuations increase with the load eccentricity ratio up to about e/B ratio of 0.2 (to be determined more precisely in the failure envelopes), and decrease thereafter (**Figs. 10b, 10d, 10f**). Likewise the inclination effect, the developed moment bearing capacity is a synergy of both the eccentricity and the vertical limit load per se; indeed, after a certain eccentricity ratio, let's say 0.2, the vertical limit load decreases substantially, thus, rendering an overall decreasing moment bearing capacity. As a result, it can be generally expressed that the surface footing fails to function efficiently when the load is applied with an eccentricity of more than about $0.4B$.

3.3.1.2. Influence of material damping (D)

The influence of material damping on the spectral variations of normalized moment ($M/BV_{max(static)}$) and vertical ($V/V_{max(static)}$) bearing capacity of eccentrically-loaded surface

footings is depicted in **Fig. 11**. Similarly, it can be observed that the first mode of resonance condition occurs at the non-dimensional frequency of approximately 0.25 for all cases irrespective of the material damping and load eccentricity. Considering a given load eccentricity, the increase in the material damping gives rise to the significant reduction of the fluctuations as well as noticeable increase in the vertical and moment bearing capacities, particularly after $h/TV_s=0.5$. In other words, for high values of material damping and at all load eccentricities, non-dimensional frequency bears no particular effect on the vertical and moment bearing capacities of the shallow foundation when subjected to the earthquake excitation with h/TV_s values of more than 0.5. As in the case of the effect of seismic intensity, the influence of material damping on the spectral bearing capacity turns out to be more pronounced in the second mode of resonance as compared to the first mode. With a comparison between **Figs. 10** and **11**, it can be generally concluded that at the first mode of resonance (i.e., $h/TV_s=0.25$), the seismic intensity has a greater influence on the spectral responses as compared to the material damping. At the second mode of resonance (i.e., $h/TV_s=0.75$), in turn, the material damping can be observed to more substantially affect the bearing capacities compared to the seismic intensity.

3.3.2. Failure envelopes

3.3.2.1. Influence of non-dimensional frequency (h/TV_s)

Fig. 12 shows the failure envelopes of eccentrically-loaded surface footings under different load eccentricity ratios ranging from 0 to 0.5 and subjected to various non-dimensional frequencies of 0 (the pseudo-static condition), 0.4, 0.7, 0.85 and 1 (the static condition). As shown in **Figs. 12a** and **12b** and similar to the cases for the inclined loading, the normalized vertical bearing capacity decreases continuously with the load eccentricity for all non-dimensional frequencies, whereas the normalized moment bearing capacity first increases with the eccentricity ratio up to about

1/6, and reduces thereafter. The load eccentricity where the maximum moment bearing capacity occurs is not affected by non-dimensional frequency. The influence of non-dimensional frequency on the vertical and moment bearing capacities turns out to be insignificant beyond the load eccentricity ratio of about 0.4. As shown in **Fig. 12c**, the smallest failure envelope, among the ones examined herein, associated with the minimum resistance of eccentrically-loaded shallow foundation is related to the non-dimensional frequency of 0.7. In addition, as discussed for the spectral responses, as the non-dimensional frequency changes, the failure envelope becomes either larger or smaller in size depending on the imposed non-dimensional frequency.

3.3.2.2. *Influence of seismic intensity ($k_h/(1 - k_v)$)*

Fig. 13 illustrates the influence of seismic intensity on the failure envelope of eccentrically-loaded shallow foundations subjected to the modified pseudo-dynamic loading at the typical non-dimensional frequencies of 0.4 and 0.7. As observed, similar to the inclined loading, with the increase in the seismic intensity from 0.1 to 0.2, the failure envelope significantly shrinks, thus highlighting the substantial influence of earthquake-induced accelerations on the moment bearing capacity of strip footings. As it can be perceived from this figure, the influence of seismic intensity on the failure envelopes is more remarkable at the lower load eccentricities and is almost negligible beyond an eccentricity ratio of approximately 0.35. This trend of variation can be attributed to the very shallow and skewed failure mechanism when the eccentricity ratio rises; hence, lending support to the contention that the shallow failure mechanisms are less affected by the site effects. Moreover, with the increase of seismic intensity, the value of $V/V_{max(static)}$ corresponding to the maximum normalized moment bearing capacity decreases.

Fig. 14 illustrates the failure patterns of eccentrically-loaded shallow foundations under static and pseudo-static loading conditions. As depicted, at a given seismic intensity, as the load

eccentricity increases from 0 to $B/3$, the failure zone significantly shrinks, thereby rendering a noticeable reduction in the overall bearing capacity of the footing. Moreover, the failure wedges shift horizontally with the load eccentricity so as to appear underneath the point of load application. Furthermore, in the case of vertical concentric loading ($e/B=0$), as the seismic intensity increases from 0 to 0.2, the passive failure zone at the side of pseudo-static load application gradually diminishes. However, with the increase in the load eccentricity, the corresponding passive wedge comes into sight again. The impact of seismic intensity on the sizes and shapes of failure zones is almost similar for all load eccentricities. As stated earlier for the impact of load inclination, the direction of the pseudo-static loading in **Fig. 14** is to the contrary of the load eccentricity. This means that their effects on the bearing capacity of the overlying shallow footing would counteract each other. In other words, what is presented in **Fig. 14** is not the most critical case; however, it has been brought up to isolate the two different effects and to better illustrate the way the failure mechanism is transforming underneath the footing due to the application of the eccentric loading under the influence of horizontal pseudo-static loading component.

3.3.2.3. Influence of material damping (D)

Fig. 15 shows the influence of material damping on the failure envelope of eccentrically-loaded shallow foundations subjected to modified pseudo-dynamic loading at the typical non-dimensional frequencies of 0.4 and 0.7. As depicted in **Figs. 15a, 15b, 15d** and **15e**, the maximum horizontal limit loads for different damping ratios occur at an approximate eccentricity ratio of $1/6$. This eccentricity ratio theoretically constitutes no tensile stress formation underneath the footing. Based on the failure envelopes in the normalized $V - M$ plane shown in **Figs. 15c** and **15f**, the increase in the material damping from 0.05 to 0.2 yields the failure envelope

expansion, indicating greater resistance of the eccentrically-loaded shallow foundation against seismic loading. The contribution of damping ratio increase to the expansion of the failure envelope, however, is directly influenced by the non-dimensional frequency imposed by the earthquake excitation (see **Figs. 10 and 11**). Accordingly, at the non-dimensional frequency of 0.4, material damping has no particular influence on the failure envelope, while it has a sound effect on the failure loci at $h/TV_s = 0.7$.

As depicted in **Figs. 15c and 15f**, the failure envelope in the $V/V_{max(static)} - M/BV_{max(static)}$ plane expands as the material damping increases from 0.05 to 0.2. It could also be observed that the effect of damping ratio on the failure loci is more pronounced when the external load is applied on the foundation with a lower eccentricity. In particular, after a specific eccentricity ratio of approximately 0.35 to 0.4, material damping pales to insignificance. Moreover, the $V/V_{max(static)}$ values associated with the maximum moment bearing capacity increase as the damping ratio increases.

4. CONCLUDING REMARKS

In this study, the seismic bearing capacity of shallow strip foundation overlying a cohesionless soil layer subjected to inclined and eccentric combined loading was examined through a comprehensive set of lower bound finite element limit analyses (FELA) using the second-order cone programming (SOCP). The actual nonlinear form of the rigid plastic Mohr-Coulomb yield criterion, the equilibrium equations associated with the applied combined loading and also the well-established modified pseudo-dynamic approach were all incorporated into the FELA formulations. The adopted formulation was rigorously validated against a majority of high-

quality studies in the literature for the static loading condition. From the results of numerical simulations, the following results were drawn:

- The first mode of resonance occurs at the non-dimensional frequency of approximately 0.25 for all cases, irrespective of the imposed seismic acceleration, material damping, load inclination and load eccentricity. In addition, at a given inclination angle or load eccentricity, as the seismic intensity increases, more fluctuations and relatively smaller spectral vertical, horizontal and moment bearing capacities could be captured; the trend which is more pronounced in the first and second modes of resonance state especially at smaller seismic intensities.
- At a given seismic intensity and material damping, as the inclination angle or load eccentricity increases, the variation of the normalized vertical bearing capacity with the non-dimensional frequency could be observed to become smoother. Moreover, with the increase in α or e , the ultimate vertical bearing capacity decreases. Conversely, for all intensities of incident waves, the horizontal or moment bearing capacity and their corresponding fluctuations increase up to a particular value of inclination angle or load eccentricity, respectively, beyond which they exhibit reduction.
- The failure envelopes of the shallow foundation subjected to either inclined or eccentric loading significantly shrink with the increase in the earthquake accelerations and decrease in the material damping of the underlying soil mass. The amount of changes in the size of failure envelopes in the normalized $V - H$ and $V - M$ spaces due to the variation of seismic intensity and material damping depends directly on the non-dimensional frequency of the earthquake excitation.

- In most non-dimensional frequencies in the spectral curves, the bearing capacity of shallow foundations could be observed to be much lower than the corresponding values in the pseudo-static loading condition, especially in the resonance conditions where almost zero bearing capacities were captured in most of the cases studied. Hence, seismic analysis of foundations based on pseudo-static loading condition would most probably lead to insecure and non-conservative designs. These deficiencies associated with the pseudo-static approach arise predominantly from failing to consider the real seismic nature of earthquake loading. The modified pseudo-dynamic approach examined in this study considers a more realistic nature of earthquake shaking, thus it can be a better representative of the seismic loading applied on various geo-structures. Therefore, the bearing capacity measured using this approach and subsequent failure envelopes are more realistic.

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FIGURES

Fig 1. Triangular discretization of the soil medium beneath the shallow foundation subjected to inclined-eccentric combined loading

Fig 2. Flowchart of the detailed computational procedure followed for the estimation of the modified pseudo-dynamic bearing capacity of shallow foundation subjected to inclined-eccentric combined loading

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Fig 3. Comparison of the failure envelopes between the results of the FELA simulations conducted in the current study and those reported in the previous studies using various theoretical/experimental/numerical methods: (a) obliquely-loaded foundation; (b) eccentrically-loaded foundation

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Fig. 4. Influence of seismic intensity ($k_h/(1 - k_v)$) on the spectral modified pseudo-dynamic bearing capacity of obliquely-loaded shallow foundations subjected to a wide range of load inclinations ($D=0.05$)

Fig. 5. Influence of material damping (D) on the modified pseudo-dynamic bearing capacity of obliquely-loaded shallow foundations subjected to a wide range of load inclinations ($k_h/(1 - k_v)=0.2$)

Fig. 6. Failure envelopes of obliquely-loaded shallow foundations subjected to various non-dimensional frequencies under a wide range of load inclinations ($k_h/(1 - k_v)=0.2$, $D=0.05$)

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Fig. 7. Influence of seismic intensity ($k_h/(1 - k_v)$) on the failure envelope of obliquely-loaded shallow foundations subjected to a wide range of load inclinations ($h/TV_s=0.25$, $D=0.05$)

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Fig. 8. Failure patterns of obliquely-loaded shallow foundations under static and pseudo-static loading conditions

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Fig. 9. Influence of material damping (D) on the failure envelope of obliquely-loaded shallow foundations subjected to a wide range of load inclinations ($h/TV_s=0.25$, $k_h/(1-k_v)=0.2$)

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1039 **Fig. 10.** Influence of seismic intensity ($k_h/(1 - k_v)$) on the modified pseudo-dynamic bearing capacity
1040 of eccentrically-loaded shallow foundations subjected to a wide range of load eccentricities ($D=0.05$)
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Fig. 11. Influence of material damping (D) on the modified pseudo-dynamic bearing capacity of eccentrically-loaded shallow foundations subjected to a wide range of load eccentricities ($k_h/(1 - k_v)=0.2$)

Fig. 12. Failure envelopes of eccentrically-loaded shallow foundations subjected to various non-dimensional frequencies under a wide range of load eccentricities ($k_h/(1 - k_v)=0.2$, $D=0.05$)

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Fig. 13. Influence of seismic intensity ($k_h/(1 - k_v)$) on the failure envelope of eccentrically-loaded shallow foundations subjected to a wide range of load eccentricities ($h/TV_s=0.25$, $D=0.05$)

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Fig. 14. Failure patterns of eccentrically-loaded shallow foundations under static and pseudo-static loading conditions

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Fig. 15. Influence of material damping (D) on the failure envelope of eccentrically-loaded shallow foundations subjected to a wide range of load eccentricities ($h/TV_s=0.25$, $k_h/(1-k_v)=0.2$)

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