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A statistical description scheme of global sub-grid topography for numerical models

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Abstract

Slope and aspect are the most prominent and important topographic elements for thermodynamic and dynamics of atmospheric circulation, especially for radiation and precipitation. In order to reduce the amount of computation after considering the sub-grid slope and aspect in numerical models, we propose a statistical method based on trigonometric function transformation to calculate these two factors for describing the orographic characteristics of complex areas over the world. It is found that the transformed conditional probability density function (PDF) conforms to the Gaussian distribution in most of the global areas (~98%), and this feature will not be eliminated with the improvement of resolution. Finally, the reasonability of this scheme in the Tibetan Plateau is tested by the radiation with the most significant topographic feedback. The results show that the surface solar radiation downwards (SSRD) in the Tibetan Plateau has been significantly improved compared with the results from the grid average scheme, especially in autumn. The improvement of Root mean square error (RMSE) is about 18.2 W/m^2 , and the improvement rate is reaching 38.4%. The improvements of maximum and regional-averaged of SSRD over the whole Tibetan Plateau are $\sim 130 \text{ W/m}^2$ and $\sim 44.3 \text{ W/m}^2$ respectively. Although we only consider the correction of solar shortwave radiation, which has a certain bias with the observation data. The improvement effect is enough to prove the rationality of the statistical

41 method.

42 **Keywords:** Numerical models, sub-grid topography, slope and aspect,

43 Tibetan Plateau, Gaussian distribution

1. Introduction

The scientific idea of numerical weather prediction (NWP) pointed out in the early last century and was successfully applied to operational application in the 1950s (Charney et al. 1990; Eugenia, 2003). Nowadays, information on the current state of NWP has been widely used and has become an important tool to predict and simulate the weather and climate. Because of its necessary in prediction and simulation, how to approach the NWP to the objective and quantitatively investigate and make it closer to the actual weather and climate situation is one of the key goals of the present study. Davies and brown (2001) pointed out that under the condition of some model resolutions, it is impossible to completely describe some small-scale topography actually. Therefore, the actual impact of these small-scale topography can not be recognized by the model on atmosphere thermal and dynamic aspects. At the same time, its effects may not be completely reflected in the NWP, resulting in large biases of the simulation in some areas. The role and influence of sub-grid topography can only be considered by parameterization scheme.

Slope and aspect are prominent feature in many regions of the world, and as important influence on complexity terrain. The plain area accounts for less than 20% of the total land of the world. Some studies (Spreen, 1947; Burns, 1953) have shown that, together with slope, aspect and surface elevation, these factors account for about 80% of precipitation in some

mountainous areas. Therefore, there are a lot of research on the parameterization of sub-grid slope and aspect. For example, Vico and Porporato (2009) propose a simple realistic probabilistic description of local slope and aspect as a function of properties of the field of elevation changes. Löwe and Helbig (2012) provide a quasi-analytical method to compute the sub-grid slope and aspect in complex terrain. These schemes describe the statistical characteristics of sub-grid slope and aspect accurately, but only for specific landscapes and not applied to the atmospheric numerical models (ANM, including regional and global weather and climate models).

Topography has an important influence on global atmospheric circulation (Hughes et al. 2009; Torre et al. 2015; Junquas et al. 2015). There has been extensive research into the role of large terrain in NWP (Dörnbrack and Schumann, 1993; Doyle et al. 2013), but there are still a number of deficiencies of sub-grid topography remain unresolved, especially the slope and aspect. Most of previous parameterization schemes were to represent the orographic characteristics of each grid with average (Giorgi et al. 2003; Zhang et al. 2006), standard deviation (Wallace et al. 1983), Laplacian operator of grid topography (Ma et al. 2016) or characteristic coefficient (Dozier and Frew 1990). The advantage of these methods is that it can describe the terrain features of each sub grid in detail. The disadvantage is that it requires a large amount of calculation first. For

global weather and climate models, applying these parameterization schemes at each grid costs too much computation. Taking the range of 100km as an example, if taking the sub-grid terrain of $1\times 1\text{km}^2$ for considering, it needs to calculate 10^5 times. While the transformation of statistical description is only one calculation be required. Therefore, it is not applicable to global area. Secondly, these topographic features have no directionality, that is, they do not consider the factors of slope and aspect, and are not applicable for radiation and topographic dynamic uplift.

Based on the above reasons, previous studies have done a lot of research on slope aspect sub grid parameterization for radiation, such as 3D Monte Carlo photon tracing simulation (Chen et al. 2006; Liou et al. 2007; Lee et al. 2013). The most well-known schemes are employing sub-grid-scale topographic parameters as independent variables, by using 3D Monte Carlo photon tracing simulations to develop a parameterization scheme to establish a multiple linear expression equation applying on solar fluxes (Lee et al. 2011). This scheme makes the simulation results very close to realistic increased surfaces but depends on observation data, and different multiple linear regression models are required under different terrain. Later, sub-grid topographic effects on solar radiation have been recognized and parameterized in a few regional weather and climate models (Hauge and Hole, 2003; Arthur et al. 2018; Gu et al. 2012), global climate models (Lee et al. 2015) and land models (Lee et al. 2019; Hao et

al. 2021).

The above analysis shows that the effects of sub-grid topography is not explicitly included in ANM, especially in dynamic process. Nearly all global climate models neglect the sub-grid slope and aspect effects on dynamical lifting. Most ANM adopt unobstructed horizontal surfaces into dynamical processes. Such simplified parameterizations do not account for sub-grid topographic effects and can lead to large systematic biases in simulating weather and climate processes over complex terrain. Besides the qualitative influence of the slope and aspect on the solar radiation, it has been demonstrated that the impacts on topographic uplift are of great significance. For example, California's mountain fires are largely related to the unique terrain and climate conditions in this region. California is located on the Pacific coast west of the Rocky Mountains in the western United States, with a large height difference. When there is a strong cold and high pressure in the mid-west of the United States going south to the Rocky Mountains, due to the strong pressure gradient, a strong northeast wind will blow along the coast of California. This wind will fall directly from the Rocky Mountains to the Pacific coast. The wind will sink along the hillside across the mountains, forming a dry adiabatic sinking process, resulting in a significant increase in the temperature and low humidity of the air mass after crossing the mountain, forming a "foehn" (Krick 1933; Krick 1934). There are similar "foehn effects" in almost all mountains

around the world, such as the Southern Alps in New Zealand (McGowan and Sturman, 1996), the Andes in Chile (Antico et al. 2021) and the Andes in the United States (Hoinka 1985), causing tremendous human and huge property losses (Richner and Hächler, 2013).

In addition, the sub-grid topography is also of great significance in topographic precipitation (Leung and Ghan, 1995). For large mountains, the precipitation increases on the windward side and the rain shadow forms on the leeward side. This is due to the slope, which forces the air flows to uplift. In steep topographic slope areas, the vertical velocity of this forced updraft is considerable. Due to the topographic forcing, the air flow is lifted to the saturation, and then the saturated air condenses into clouds, increasing precipitation. In the weather prediction system, there has been extensive research into topographic precipitation, such as rainstorm and local extreme precipitation (Zhong et al. 2015; Chen et al. 2021). If there is a low-level jet in front of the slope, a large amount of water vapor is transported to the mountain, and there are huge water droplets in the convective unstable flow, which can increase the rainfall enhancement effect of the topography. Unfortunately, there is little research on sub-grid topography parameterization scheme in dynamic forcing uplift applied in ANM, especially in General Circulation Models (GCMs). Only Shen et al. (2006) proposed a parameterization scheme for the dynamic effects of sub-grid topography and its impacts on air flow and rainfall simulation, and

154 applied it into the P- σ regional climate model of Nanjing University.

155 A simple and efficient parameterization scheme for sub-grid slope and
156 aspect in the global climate models need to account for the effects of
157 dynamic processes, especially the topographic dynamic uplift. However,
158 due to the lack of detailed observation data of vertical velocity, while the
159 radiation has a large number of accurate observation data all over the world,
160 and the most significant topographic feedback. Accordingly, we will use
161 radiation to verify. In this study, we propose a sub-grid statistical
162 description scheme based on Gaussian distribution and apply it to global
163 areas with complex topography. In Section 2, we describe an improved
164 method that utilizes the 1-km resolution topography data and
165 corresponding mathematical transformation of this method, then explain
166 the reasons. In Section 3, we describe its global and regional applicability
167 and reasonability of the method, and introduce its application to the Tibetan
168 Plateau. The deviation of surface solar radiation in mountains from
169 unobstructed horizontal surfaces is analyzed in Section 4. Conclusions and
170 discussion are given in Section 5.

171 **2. Data and method**

172 **2.1 Data**

173 The topography data used in this article is United States Geological
174 Survey (USGS) DEM data with the resolution of 1 km $\times$ 1 km. DEM is

derived from field surveys and airborne remote sensing techniques (Holland et al. 2006; Kornus et al. 2006). The observation data is the dataset of high resolution (3 hours, 10 km) global surface solar radiation (Tang et al. 2019) provided by the National Qinghai Tibet Plateau science data center, from 1989 to 2018 are selected. The result of correctness verification is offline calculation.

2.2 Statistical description

According to the introduction, slope and aspect have important effects on radiation and topographic precipitation. Alpert and Shafir (1989) found that orographic rainfall on the small mesoscale is highly predictable with the adiabatic assumption that the uplift is determined by $V \cdot \nabla Z_s$. In the study of topographic vertical velocity, through the P-coordinate system,

$$\omega_s = \frac{dp_s}{dt} = \frac{\partial p_s}{\partial t} + \vec{V}_s \cdot \nabla p_s. \quad (1)$$

The equation of static equilibrium and deformation formula is as follows,

$$\begin{aligned} \vec{V}_s \cdot \nabla p_s &= -\rho g \vec{V} \cdot \nabla Z_s = -\rho g \cdot |\vec{V}| \cdot \tan \theta_N \cdot \cos(\theta - \varphi_N) \\ &= -\rho g \times \sqrt{u^2 + v^2} \times \tan \theta_N \times (\cos \theta \times \cos \varphi_N + \\ &\quad \sin \theta \times \sin \varphi_N) \end{aligned}$$

(2)

Where θ is wind direction, θ_N is the slope, and φ_N is the aspect.

In addition to dynamics, slope and aspect also play an important role in thermodynamics, that is, radiation. Kondratyev (1977) found that the

dependence of $|\downarrow E_{s,dir}|$ on slope angle was formulated with the help of the geometry factor $\cos(\alpha)$:

$$\begin{aligned}\cos(\alpha) = & \cos(\theta_N) \sin(\theta_S) \\ & + \sin(\theta_N) \cos(\theta_S) \cos(\varphi_S - \varphi_N).\end{aligned}\quad (3)$$

Where α is the angle between the unit vector of the solar direct beam and the normal vector of the surface. The position of the sun is given by the sun elevation angle θ_S and the sun azimuth angle φ_S . $\downarrow E_{s,dir}$ is the amount of solar radiation (also known as shortwave radiation) reaching the surface of the Earth.

Equation 1 and 3 show that slope and aspect are used in the calculation of topographic vertical velocity and radiation. But after studying the two orographic factors of slope and aspect, we found that either of the slope and aspect PDFs is irregular. After a large number of statistical analysis, it is found that while transforming them into trigonometric functions, $\tan(\text{slope}) \times \cos(\text{aspect})$ (call for TC, the same below) and $\tan(\text{slope}) \times \sin(\text{aspect})$ (call for TS, the same below) conform to the Gaussian distribution. At the same time, TC and TS are crucial features for the calculation of radiation and topographic vertical velocity.

In our study, we only consider at the grids with slope $>5^\circ$. This is because in practical application, slope is an indicator to distinguish whether it is an area with complex topography or not. Plateaus with the high altitude and gentle slope are defined as a high-altitude plain rather than an area with

complex topography. Moreover, the weather and climate characteristics of vast plain with very gentle slope are difficult to distinguish. The dynamic and thermal effects of complex topography are easy to cover up in the regions with slope $\leq 5^\circ$ and $\tan(\text{slope}) < 0.1$. These unfavorable conditions for quantitative analyses and researches are hard to conduct in these regions, therefore, only the grids with slope $> 5^\circ$ are discussed.

In order to test the impact of this scheme on radiation, we simplify and deform the method proposed by Mathias and Scherer (2005), and apply it to the topography over the Tibet Plateau. The surface solar radiation downwards (SSRD) is calculated as follows.

$$\downarrow E_{s,\text{dir}}^* = \downarrow E_{s,\text{dir}} \times f_{\text{cor}} \quad (4)$$

f_{cor} is defined as follows,

$$\begin{aligned} f_{\text{cor}} &= 1 + \frac{\tan(\theta_N)}{\tan(\theta_S)} \times \cos(\varphi_S - \varphi_N) \\ &= 1 + \frac{\tan \theta_N \times \cos \varphi_N \times \cos \varphi_S + \tan \theta_N \times \sin \varphi_N \times \sin \varphi_S}{\tan \varphi_S} \end{aligned} \quad (5)$$

where $\downarrow E_{s,\text{dir}}$ is down-welling shortwave radiation on unobstructed horizontal surface, $\downarrow E_{s,\text{dir}}^*$ is the result of radiation correction. It is noted that in this study, we only consider the correction of solar direct radiation, and do not consider the influence of scattered radiation and other additional radiation. This is because compared with SSRD, the diffuse radiation account for less than 10%, it is negligible. The sky viewing angle factor is calculated by taking the average value.

2.3 Skewness and kurtosis test

The Gaussian distribution test is based on tests for Skewness and Kurtosis (Bai and Ng 2005). When the sample n is limited and large ($n \geq 100$), the standard deviation of sample skewness is as follows.

$$s1 = \sqrt{\frac{6(n-2)}{(n+1)(n+3)}} \quad (6)$$

The standard deviation of sample kurtosis is as follows.

$$s2 = \sqrt{\frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}} \quad (7)$$

The k-order central moment is as follows.

$$m_k = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^k \quad (8)$$

The coefficient of skewness and kurtosis are defined as follows.

$$\tau = \frac{\mu_3}{\sigma^3} = \frac{E[(x - \mu)^3]}{E[(x - \mu)^2]^{\frac{3}{2}}} \quad (9)$$

$$\kappa = \frac{\mu_4}{\sigma^4} = \frac{E[(x - \mu)^4]}{E[(x - \mu)^2]^2} \quad (10)$$

According to the symmetry of the normal distribution, assuming the significance level as α , the final significance level should be $\frac{\alpha}{2}$. If the skewness coefficient $\tau_\alpha = s1 \times \kappa - 3$ and the kurtosis coefficient $\kappa_\alpha = s2 \times \tau$ meet the normal distribution with the significance level α , then the sample can be considered from a normal-distribution population.

2.4 Gaussian distribution

The normal Gaussian distribution uses standard deviation (σ) or mean

value to test, mainly to avoid the occurrence of small probability time, but lack of the flexibility. In this study, it is transformed into standard normal distribution (standardized transformation) before processing, and a representative value of several sub-grid topography values in the model grid scale are selected, which can be easily described and applied

An important factor of Gaussian distribution is that it is easy to obtain the percentile of the distribution, which can be calculated according to the probability distribution value P of normal distribution. The advantages are as followed. Assuming that a series of data x conforms to the Gaussian distribution, it is necessary to find the percentile representing 80% values (that is, the representative value of a series of data can represent 80% of the data characteristics). According to the probability P table, the value Z_p corresponding to Z during Z-Scores ($z = \frac{x-\mu}{\sigma}$, μ is the average, σ is the standard deviation) transformation can be found. The representative value y is as follows.

$$y_p = \mu + Z_p * \sigma, \quad (11)$$

Where Z_p can be seen by table that the closest value when $p = 0.8$ is 0.7995, which corresponds to $Z_p = 0.84$. In this way, the representative value (such as average value) of the series of data can no longer be calculated according to the range of value range, which simplifies the problem.

3 The analysis of statistical description of global sub-grid topography

3.1 Statistical description of global sub-grid topography

According to the above method, the spatial distribution of global sub-grid topography after the trigonometric function conversion is shown in Fig.1. It can be seen that in areas with complex topography around world, such as Tibetan Plateau, Rocky Mountains and Andes mountains, steep slopes are calculated in high-resolution numerical forecasting models. As the increasing of horizontal resolution, TC and TS do not change significantly. It's indicated that the increasing of resolution has little impact on the characteristics of this complex orography, and the steepness of the topography and the orientation of the mountains are not covered up in high-resolution simulations.

Table 1 shows the proportions of the grids with the Gaussian distribution of TC and TS at different resolutions. The grids with the Gaussian distribution of TS at 1° resolution show the largest (98.3%) proportion, while those at a resolution of 0.1° show the least proportion of 95.92%; those of TC at 0.5° resolution account for the largest proportion at 98.74%. And the same as TS, at 0.1° resolution, the proportion for TC is the least, about 96.22%. From the resolution of 1° to 0.5° , the total grid number increases by 1.3×10^4 , while the grids that do not conform to the Gaussian distribution increase from about 100 to more than 200, taking up

about 1.5% of the total grids. At 0.1° resolution, only 1.5×10^5 out of 3.68×10^6 grids (account for $\sim 5.1\%$) do not conform to the Gaussian distribution. Although the number of grids that do not conform to the Gaussian distribution increases, the increasing proportion is small. It is shown in Table 1 that the slope and aspect of almost all topography conform to the Gaussian distribution after the trigonometric function transformation. Although the proportion of grids conforming the Gaussian distribution decreases slightly with the increase of the resolution, the reduction has few influences on the global scale (All the grids in Table 1 refer to the grids slope > 5 . If the proportion of land in the unit grid is greater than 10%, the grid is defined as land). The above analysis shows that with the improvement of the resolution, the Gaussian distribution characteristics are stable and the method is feasible. Therefore, we can confirm that the global sub-grid topography conforms to the Gaussian distribution after the statistical description. It is more obviously indicated from Fig. 2 that the Gaussian distribution conforms to the distribution characteristics of global complex topography. The spatial distributions are similar at different resolutions.

3.2 Characteristics of sub-grid statistical description in different regions

The spatial distributions of the proportion of TC and TS Gaussian distribution in the globe are given above. Since different continents have

different topographic and regional characteristics, the spatial distributions of elevation and topography in different continents are provided. Standard deviation of topography is commonly used in climate models, such as CESM1.0.4, to represent the steepness of topography. The turbulent drag from sub-grid-scale topography is represented by the Turbulent Mountain Stress (TMS) scheme using USGS topographic data (Liang et al. 2016). Fig. 3 shows the spatial distribution of standard deviation of sub-grid topographic height on the land of four continents. It is found that the average standard deviation of geography in Asia is the largest, reaching 1925m while in Africa is the smallest, only 86.9m. It is implied that the complex topography in Asia accounts for a large proportion, while the overall topography in Africa is relatively gentle. However, the standard deviation cannot represent the direction of the topography, it is necessary to use slope and aspect for further analysis.

It can be seen that the Gaussian distribution in East Asia (Fig. 4a and 4b) accounts for the largest proportion, and as the altitude increases, the grids of the Gaussian distribution gradually increase, such as in Siberia and Southwest China, especially over the Tibetan Plateau. Except for the peaks of the mountains, the topographic statistical characteristics of most areas over the whole Tibetan Plateau conform to the Gaussian distribution. In addition, the distribution of Gaussian in South Korea and North Korea also accounts for a larger proportion, which may be related to the topography

of Halla Mountain and the large mountain ranges in the northeastern North Korea. However, the proportion of the Gaussian distribution in Africa (Fig. 4d and 4e) is very small. Although the altitude in Southeast Africa is higher than 1000 meters, and there is even the Ethiopian plateau called "Roof of Africa", we find that only a few grids conform to the Gaussian distribution, accounting for only about 5% of all the land grids. Most of other African regions do not conform to this feature, because in those regions the topographic gradients are relatively gentle and less complex. Combined with Fig. 3, it shows that the relationship between complex topography and the altitude is not necessary and sufficient. Areas with complex topography may have the high altitude, but not all areas with the high altitude have complex orography. In South America and North America (Fig. 4c, 4d, 4g and 4f), the Rocky Mountains and the Andes Mountains are the hotspots in previous researches. In these two regions, the topography basically conforms to the Gaussian distribution. But on the Brazilian plateau, the second largest plateau in the world, the land surface is relatively gentle and the topography is less complex. Only a few grids on the east coast conform to the Gaussian distribution. In addition, there are also some complex terrains near the Mount Rey and Mount Roraima in Venezuela and the east coast of Canada, which conform to the distribution characteristics of Gaussian statistics. Fig. 3 shows the standard deviations of topography in Asia, North America, Africa and South America. The correlation between

the Gaussian distribution and the complex-orography distribution in Asia, North America, Africa and South America are 91.1%, 90.6%, 85.6% and 89.9%, respectively. This result proves the reasonability of the sub-grid geostatistical description scheme. In addition, several mountainous regions with typical complex climate characteristics are selected for the further analysis.

As shown in Fig. 4, most of the grids that conform to the Gaussian distribution are in mountainous and plateau areas. We selected Tibetan Plateau, Rocky Mountains and Andes mountains for detailed analyses. Fig. 5a and 5b give the proportions of grids conforming to the Gaussian distribution over the whole world and three mountains at all the land grids (slope $>0^\circ$). Over the world and three typical mountainous landscapes, the proportion of the grids with the Gaussian distribution at 1° resolution is the largest among different resolutions. With the improvement of resolution, the proportions of the grids with the Gaussian distribution of TC and TS decrease. It may be related to the decrease of effective grids, which needs further analyses. Fig. 5c and 5d show the proportion of the effective grids (slope $\geq 5^\circ$) with Gaussian distribution over global and three mountainous landscapes. It has been clearly shown in Table 1 that the proportion of global Gaussian distribution is close to 100%, and the proportion changes little as the resolution improves. As for the three mountainous landscapes, in general, the Gaussian distribution of TC is slightly larger than that of TS,

which is about 8% higher. Both TC and TS show the largest proportions of the Gaussian distribution at the resolution of 0.5° , followed by 0.25° , and that at 0.1° resolution is the least. From a regional perspective, the Gaussian distribution in the Rocky Mountains area accounts for the largest proportion, followed by the Andes Mountains, and the Tibetan Plateau shows the smallest proportion, which is about 10% less than the Rocky Mountains on average. It indicates that there are some differences in the topographic features of the above three mountain landscapes.

4 Results of radiation deviation improvement

The above analysis gives the statistical characteristics of slope and aspect after mathematical transformation in three mountainous landscapes. We further take the radiation over the whole Tibetan Plateau area as an example to verify the reasonability of this scheme. A region of 25°N – 40°N and 70°E – 105°E was selected to cover the Tibetan Plateau. It has the most complex topography surrounded by the highest mountains in the world, such as the Himalayas, Pamir, Kunlun Shan, Hengduan Mountains, and Kailas Range (Chen et al., 2003; Wu et al., 2007). The slope is steep, and the feedback to radiation is significant. This study uses the radiation scheme in CESM (community earth system model) Version 1.2.1 developed by NCAR (National Center for Atmospheric Research). The SSRD of unobstructed horizontal surfaces (CTL) and considering the sub-grid topography after radiometric correction according to Eq. (5) and (6)

are calculated offline (Rad) and compared with the observation data (OBS). The results are shown in Fig. 6. It indicates that the maximum SSRD of CTL in spring (Fig. 6a) is concentrated in the southwest side of Tibetan Plateau is about 280 W/m^2 . The minimum is located on the southeast side, with the spatial distribution characteristics of decreasing from southwest to southeast. While the maximum SSRD of Rad and OBS (Fig. 6b and 6c) is located on the top mountains of Tibetan Plateau, $\sim 280 \text{ W/m}^2$, with the spatial distribution characteristics of decreasing from the top of the mountain to the surroundings. The SSRD is the largest on the summer which may be caused by solar direct point near the Tropic of Cancer, the closest latitude distance to the Tibetan Plateau and the highest solar elevation angle. In this season, the maximum of CTL SSRD (Fig. 6d) is located on the west side of Tibetan Plateau, more than 350 W/m^2 , but the SSRD received by the slope at the south side of the Plateau is very small, only about 100 W/m^2 . At this time, the distribution of Rad and OBS SSRD (Fig. 6e and 6f) is still similar to that in spring, and the maximum is located over Tibetan Plateau, decreasing from the center to all surroundings. Although the maximum of SSRD received by CTL (Fig. 6g) in autumn is located on the Tibetan Plateau, which is similar to the results of OBS and Rad (Fig. 6h and 6i), the magnitude of SSRD is still much smaller, and its maximum is only about 205 W/m^2 , while the maximums of OBS and Rad are about 230 W/m^2 . Since the solar direct point in winter is in the southern

hemisphere, the SSRD over the Tibetan Plateau is very small, we would not analyze it here.

We further make quantitative analysis on the radiation correction results of Rad method. Table 2 gives the advantages of Rad method evaluated in terms of root mean square error (RMSE). It indicates that in spring, summer and winter, the RMSE of Rad is smaller than that of CTL. In the results of CTL, the RMSE is the largest in autumn and the smallest in summer, while in Rad, the RMSE is the largest in spring and the smallest in autumn. Further, the most improved season of RMSE is autumn, with an improvement of about 18.2 W/m^2 , and the improvement rate is reaching 38.4%, which may be due to the largest deviation between CTL and OBS in autumn that makes the improvement effect the best. The improvement effect in spring is close to that in summer, the improvement rate is about 21%.

Fig. 7 shows the spatial distribution of the deviation between Rad, CTL and OBS. Overall, the deviation between the SSRD of CTL and OBS is mainly concentrated on the south side of the Tibetan Plateau and windward slope in summer and autumn, which has been significantly improved after considering the slope and aspect. From the spatial distribution, it can also be found that compared with OBS, the SSRD of CTL in autumn is generally lower (Fig. 7c), about $10\sim 80 \text{ W/m}^2$ lower than OBS. The largest deviation is located at the South and Southeast foothills

of the Tibetan Plateau, reaching 85 W/m^2 . After considering the slope and aspect (Fig. 7f), the deviation between Rad and OBS is significantly reduced, especially in the complex terrain area at the foot of Tibetan Plateau. It is consistent with the fact that the best improvement for RMSE is in autumn. The deviation of SSRD between CTL and OBS in summer (Fig. 7b) is about $-40 \sim 110 \text{ W/m}^2$. The area with the largest difference is located at the southern foot of Tibetan Plateau, reaching 105 W/m^2 . After considering the topography, the deviation is almost completely eliminated (Fig. 7e). In spring, except near the southern foothills of the Tibetan Plateau, the SSRD of CTL is significantly lower than that of OBS in most areas, with a deviation of about $-10 \sim 70 \text{ W/m}^2$ (Fig. 7a). After topographic correction, the deviation between Rad and OBS is also reduced (Fig. 7d), but the improvement effect in spring is not significant, and the reason needs to be further researched.

It can be seen from the above analysis that in spring, summer and autumn, the SSRD of CTL in Tibetan Plateau is almost lower than that of OBS, and Rad effectively increases the SSRD in this region, but the specific areas need to be further analyzed. Fig. 8 shows the spatial distribution of difference between SSRD of Rad and CTL. The difference means $\downarrow E_{s,dir}^* - \downarrow E_{s,dir}$ and the improvement ratio is $(\downarrow E_{s,dir}^* - \downarrow E_{s,dir}) / \downarrow E_{s,dir}$. It can be seen that at the height above 3000m, compared with CTL, SSRD of Rad has almost increased in spring, summer and autumn. The

best improved is in autumn with a range of 130W/m^2 located at the southeast foot of Tibetan Plateau with an improvement rate of more than 130%. The whole windward slope has also been improved by 80-100%, and the improvement on Tibetan Plateau mountains is less, about 10-40%. The area with the best improvement effect in summer (Fig. 8b and 8e) is the south side of the Plateau, with an improvement range of 125 W/m^2 , about 105%, while the SSRD in the north side decreases about 40 W/m^2 , account for ~10%. The improvement in spring (Fig. 8a and 8d) is concentrated on the east and southeast foothills of the Plateau, with an improvement range of 105 W/m^2 and an improvement rate of ~80-100%. The results are consistent to the previous researches (Lee et al. 2013; Lee et al. 2019). In previous studies, more detailed topographic factors such as the sky view factor and the surface albedo are discussed, and only a specific time of a day is considered. In this study, we mainly consider the direct solar shortwave and do not consider the factors such as shading, longwave correction and shortwave scattering, so the reflected fluxes of the radiation are improved more accurate by previous researchers.

In order to focus on the improvement of the statistical scheme on SSRD, we set up a control test using the grid averaged topography (called Mean test) to correct the radiation and an experiment using statistical description scheme, for analyzing the difference in the maximum and regional average SSRD over Tibetan Plateau with different slopes. Fig. 9

shows the difference between the Mean test and the Rad test. These results suggest that for both the Mean test and the Rad test, the regional maximum (Fig. 8a) and average (Fig. 8b) of SSRD over Tibetan Plateau is increasing with the increase of slope. Obviously, the results of Rad test are better than those of Mean test. The difference of maximum SSRD between Rad and Mean test in spring is about 27~43W/m², and that of the regional average SSRD is about 25~30W/m². The divergence of maximum SSRD in summer is about 27~57W/m², and that of the regional average SSRD is about 13~22W/m². The divergence of maximum SSRD in autumn is about 19~34W/m², and that of the regional average SSRD is about 28~43W/m². The above analysis indicates that in summer, the divergence of maximum SSRD is the largest, while it is the second largest in autumn and the least in spring. The difference of regional average SSRD over Tibetan Plateau between Rad and Mean test is the largest in autumn, while it is second in spring and the least in spring. This is consistent with the previous conclusion that RMSE improved best in autumn while the improvement effect in spring is not significant. And compared with spring and autumn, the SSRD in Tibetan Plateau is the largest in summer, so its maximum improved most.

5 Conclusions and Discussion

In this study, a statistical description scheme of slope and aspect based on the trigonometric function is proposed for ANM in complex topography.

We found that the PDF of this scheme conforms to the Gaussian distribution in global areas in complex topography and this distribution feature is also applicable to typical mountainous landscapes locally. We apply this method to the SSRD over Tibetan Plateau. It shows that compared with the unobstructed horizontal surfaces and the method of grid average, the statistical scheme shows better performance in simulating the maximum and regional average SSRD. The improvement is the most significant in autumn, the divergence of the regional average SSRD ranges from 28 to 43W/m² over the Tibetan Plateau.

On the basis of the above resolution, we only analyze the topography in terms of SSRD in the Tibetan Plateau. No research is done in other regions, such as Africa. Since there are very few grids conforming to the Gaussian distribution in Africa, while it is more reasonable to calculate sub-grid orography at each grid need more research. On the other hand, the topography does not belong to the factors of underlying surface, we do not consider the albedo but only consider the variability of radiation caused by slope and aspect and its impact on dynamic uplift. However, it is not clear why the feedback of radiation on terrain is not obvious in Africa.

Moreover, our method highlights the adaptation of the statistical scheme for ANM, it does not depend on the observation data and for both thermodynamics and dynamics. We have proved that it can work in radiation at low resolution. However, compared with previous statistical

methods (Lee et al. 2013), the resolution of topography data is insufficient, and the parameterization scheme only considers the impact of slope and aspect on SSRD. It does not consider the impact of factors such as diffuse-reflected flux, direct-reflected flux and coupled flux et al. on solar radiation, which leads to some deviation between Rad and OBS. Future study based on the Eq (4) and this statistical scheme can applied into ANM for testing, and whether factors such as diffuse-reflected and diffuse flux can be revised in a similar way will be considered.

We propose a universal sub-grid topography parameterization scheme for GCM in thermodynamics and dynamics, but has not been applied to ANM for numerical experiments. Only the correctness of the scheme is verified offline in theory. In addition, sub-grid slope and aspect are also of great significance in dynamics, especially for the topographic vertical motion, such as vertical uplift of topography and foehn effect. Since the improving precipitation simulation in complex terrain areas by parameterization method has always been the key works for NWP (Fonseca et al. 2015), few studies have taken slope and aspect into account in the sub-grid topographic parameterization scheme to simulate complex terrain precipitation. Previous studies only incorporated the parameterization scheme of slope and aspect in GCM to represent surface radiation process which indirectly affected the ability of simulating precipitation (Gu et al. 2020). It should be noted that not only considering

the thermodynamic and dynamic effects of the scheme into GCM
respectively, but also further explore the effect of the scheme on the ability
of topographic precipitation simulation under the combined influence of
thermal and dynamic in the future. In addition, further analyses on a wider
test by introducing this scheme into AGCM, consider the impact of adding
topographic vertical motion on atmospheric circulation, temperature and
precipitation.

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742 **Table1** Proportion of Gaussian distribution of TC and TS at different resolution

Resolution	Gaussian distribution (TS)	Gaussian distribution (TC)	All Grids
1°×1°	4929 (97.8%)	4944 (98.17%)	5036
0.5°×0.5°	18001 (98.3%)	18075 (98.74%)	18305
0.25°×0.25°	65508 (97.56%)	65766 (97.95%)	67142
0.1°×0.1°	353015 (95.92%)	354094 (96.22%)	368018

743

744 **Table 2.** RMSE in different seasons

RMSE	CTL	Rad	Improvement
Spring	42.86	33.85	9.01(21%)
Summer	41.3	32	9.3(22.5%)
Autumn	47.36	29.13	18.2(38.4%)

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Figure Captions

Figure 1. Spatial distribution of global terrain after transformation of trigonometric function at different resolutions. (a) TS and (b)TC for 1° resolution; (c) - (d) As in (a)-(b), but for 0.5° resolution; (e)-(f) As in (a)-(b), but for 0.25° resolution

Figure 2. The spatial distribution of global Gaussian statistics under different resolution. (a) TC and (e) TS for 1° resolution; (b) and (f) As in (a) and (b), but for 0.5° resolution; (c) and (g) As in (a)-(b), but for 0.25° resolution (d) and (h) As in (a) and (b), but for 0.1° resolution. Red dots indicate consistent and blue dots indicate inconsistent

Figure 3. Maps of standard deviation of orography data in (a) Asia; (b) North America; (c) Africa; (d) South America (units: m)

Figure 4. Maps of elevation (grey lines, units: m) and Gaussian distribution in East Asia (a and b), North America (c and d), Africa (e and f) and South

768 America (g and h)

769

770 **Figure 5.** Proportion of Gauss distribution on global and three typical

771 mountainous landscapes at different resolution. (a) TC in all land grids; (b)

772 TS in all land grids; (c) TC in effective grids; (d) TS in effective grids

773

Figure 6. Left panel: SSRD considering Gaussian distribution of slope and aspect in spring (top row), summer (middle row) and autumn (bottom row); Middle panel: same as in the left panel but for unobstructed horizontal surface; Right panel: same as in the left panel but for observation data (unit: W/m^2). The black contour indicate elevation of 3000 meter, the same as below

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783

784 **Figure 7.** Top row: deviations of SSRD between OBS and CTL in spring

785 (left panel), summer (middle panel) and autumn (right panel); Bottom row:

786 same as in the top row but for OBS and Rad (unit: W/m²)

787

Figure 8. Differences between Rad and CTL SSRD (Rad-CTL, top row, units: W/m^2) and the relative difference $((\text{Rad-CTL})/\text{CTL})$, bottom row) in spring (left panel), summer (middle panel) and autumn (right panel)

Figure 9. SSRD distribution of (a) Maximum and (b) regional average with different slopes over Tibetan Plateau in spring (bule lines), summer (red lines) and autumn (green lines). Solid (dotted) lines indicate Rad (Mean) tests (units: W/m²)