Recursive Integral Terminal Sliding Mode Control of MEMS Gyroscopes via Composite Neural Learning and Disturbance Observer

Rui Zhang (✉ ruizhangdyx@hotmail.com)

Southeast University <https://orcid.org/0000-0002-4554-7718>

Bin Xu

Northwestern Polytechnical University

Hai Wang

Murdoch University

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Rui Zhang · Bin Xu · Hai Wang

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Abstract The recursive integral terminal sliding mode control based on disturbance observer and composite neural learning is studied in this paper to control the dynamics of MEMS gyroscopes in the presence of system uncertainties and external disturbances. To achieve more accurate approximation accuracy of system uncertainties, the composite neural learning is employed, where the updating law of the weight vector is designed by the tracking error and the prediction error constructed by serial-parallel estimation model. To further improve the system robustness, the recursive integral terminal sliding mode controller is utilized, where faster convergence can be obtained by forcing the system state to start from the sliding mode manifold at the initial time. To deal with external disturbances with unknown upper bounds, the disturbance observer is designed. Furthermore, simulation results are demonstrated to verify that faster convergence and higher tracking accuracy can be achieved under the proposed method.

Keywords MEMS gyroscope, recursive integral terminal sliding mode control, serial-parallel estimation model, composite neural learning, disturbance observer.

1 Introduction

The gyroscopes using micro electro mechanical systems (MEMS) technology are widely utilized to measure the angular velocity in various fields such as electronics, robots, aeronautics and astronautics. There are two control frames for MEMS gyroscopes: the conventional “one axis drive and one axis detection” control frame using weak signal detection technology to detect the angular velocity, and the “biaxial drive” control frame [1] using an adaptive law to estimate the angular velocity online. Since weak signal detection technology is difficult to be further improved, the detection limit of angular velocity under the first control frame can not be broken. In response to this problem, advanced control methods under the “biaxial drive” control frame have received more attention.

Under the “biaxial drive” control frame, the proportional-integral (PI) control is widely used in practical applications. However, due to the inevitable existence of system uncertainties and external disturbances in the dynamics of MEMS gyroscopes, the tracking accuracy of PI control is reduced. For the system uncertainties, the adaptive method is introduced in [2] to estimate the uncertainties, which is further fed to the controller for compensation. Owing to the ability of radial basis function (RBF) neural networks (NN) for the approximation of every nonlinear function with any accuracy [3,4], the RBF NN is employed to approximate system uncertainties in [5,6]. To further improve the approximation performance of NN, finite time learning [7] and multi-loop recurrent NN [8] are proposed. For the external disturbances, when the upper bounds of the disturbances are known, a robust controller is designed in [9,10] to suppress the influence of disturbances. When the upper bounds of the disturbances exist but are unknown, the disturbance observer (DOB) based on Lyapunov method is designed in [11–13]. Furthermore, the active disturbance rejection control is pro-

R. Zhang (✉)
School of Automation, Southeast University, Nanjing 210096, China
E-mail: ruizhangdyx@hotmail.com

B. Xu
School of Automation, Northwestern Polytechnical University, Xi'an 710072, China

H. Wang
Discipline of Engineering and Energy, Murdoch University, Perth, WA 6150, Australia
Centre of Water, Energy & Waste, Harry Butler Institute, Murdoch University, Perth, WA6150, Australia

posed in [14, 15], where the lumped disturbances including system uncertainties and external disturbances are set as a new state and further introduced to the extended state observers, such that the lumped disturbances are accurately estimated. To improve the tracking accuracy of control system in the applications, more advanced intelligent control methods and disturbance observation control theories need to be studied.

Considering the second order dynamic model of MEMS gyroscopes, the back-stepping control scheme is applied to control MEMS gyroscopes in [16], where the second order dynamics can be decomposed into two first order subsystems and the virtual controllers are proposed step by step. To avoid the parameters expansion, the dynamic surface controllers are further studied in [17, 18]. However, the layered design of back-stepping control is too complex. Since sliding mode variable is able to contain the errors of different orders, it is introduced to the control system of MEMS gyroscopes in [19, 20]. Following with that, various modified sliding mode control (SMC) are studied. The terminal SMC (TSMC) [21, 7] and the super-twisting SMC [22] are addressed to force the sliding mode variable to achieve finite time convergence. In [23, 24], the nonsingular TSMC is discussed to avoid the singular problem of conventional TSMC. In addition, various intelligent control methods and disturbance observation control theories are combined with advanced modified SMC to improve the tracking performance of MEMS gyroscopes with system uncertainties and external disturbances. Overall, the improvement of robustness and convergence is the key issue of SMC for MEMS gyroscopes, which will continue to receive attention in the future research.

Motivated by the above studies on the controller design of MEMS gyroscopes, this paper places emphasis on how to efficiently approximate the compound nonlinearity and how to improve the convergence rate of control system. A high precision and robust control scheme is designed to control the MEMS gyroscopes. The recursive integral terminal sliding mode controller (RTSMC) is utilized to achieve faster convergence and higher tracking accuracy, while the composite neural learning and DOB are designed to handle the system uncertainties and external disturbances. Compared with previous works, the main contributions are listed as follows.

- The RTSMC is designed to control the dynamics of MEMS gyroscopes in the presence of system uncertainties and external disturbances. When the control system is forced to start from the sliding surface at the initial time, faster convergence and higher tracking accuracy can be obtained.
- The composite neural learning works together with the DOB to approximate the lumped uncertainty including system uncertainties and external disturbances. Through

updating the NN weight vector by the adaptive law, which is designed by the tracking error and the prediction error constructed by the serial-parallel estimation model (SPEM), higher approximation accuracy is obtained.

The rest of this paper is organized as follows. The dynamics of MEMS gyroscopes and problem formulation are addressed in Section 2. The RTSMC using composite neural learning and DOB is designed in Section 3. The system stability analysis is demonstrated in Section 4. The simulation results are shown in Section 5 to verify the expected faster convergence and higher tracking accuracy. The conclusions are given in Section 6.

2 Problem Formulation

2.1 MEMS Gyroscopes Dynamics

MEMS gyroscopes are essentially mechanical gyroscopes, whose dynamics can be described as a mass-stiffness-damping system shown in Fig.1, where x denotes the drive axis, y denotes the sensitive axis, ϑ_x and ϑ_y denote dimensionless positions, ς_{xx} and ς_{yy} denote dimensionless damping coefficients, ψ_{xx} and ψ_{yy} denote dimensionless stiffness coefficients, ψ_{x3} and ψ_{y3} denote dimensionless nonlinear coefficients.

Fig. 1 Structure design of MEMS gyroscopes.

Considering the parameters uncertainties and the external disturbances, the dimensionless dynamics of MEMS gyroscopes with the mechanical coupling caused by machining errors is given as follows [25]

$$\ddot{\vartheta}_i = f_i + d_i + \tau_i, i = x, y \quad (1)$$

with

$$\begin{aligned} f_x &= -\varsigma_{xx} \dot{\vartheta}_x - (\varsigma_{xy} - 2\Omega) \dot{\vartheta}_y - (\psi_{xx} - \Omega^2) \vartheta_x \\ &\quad - \psi_{xy} \vartheta_y - \psi_{x3} \vartheta_x^3 \\ f_y &= -\varsigma_{yy} \dot{\vartheta}_y - (\varsigma_{yx} + 2\Omega) \dot{\vartheta}_x - (\psi_{yy} - \Omega^2) \vartheta_y \\ &\quad - \psi_{yx} \vartheta_x - \psi_{y3} \vartheta_y^3 \end{aligned} \quad (2)$$

where ς_{xy} and ς_{yx} denote dimensionless damping coupling coefficients, ψ_{xy} and ψ_{yx} denote dimensionless stiffness coupling coefficients, d_x and d_y denote dimensionless external

disturbances, τ_x and τ_y denote dimensionless control inputs, Ω denotes the dimensionless angular velocity.

Defining the dimensionless reference signal as ϑ_{ri} , the tracking error is given as

$$e_i = \vartheta_i - \vartheta_{ri} \quad (3)$$

Assumption 1 To avoid the mechanical damage, the external disturbance is assumed to be bounded. In other words, there exists constants $d_{mi} > 0$ and $\bar{d}_{mi} > 0$, such that $|d_i| \leq d_{mi}$, $|\dot{d}_i| \leq \bar{d}_{mi}$.

Remark 1 Considering the parameters uncertainties caused by environmental changes such as temperature, atmospheric pressure, etc., there are $\varsigma_{l_1 l_2} = \varsigma_{l_1 l_2}^0 + \Delta \varsigma_{l_1 l_2}$, $\psi_{l_1 l_2} = \psi_{l_1 l_2}^0 + \Delta \psi_{l_1 l_2}$ and $\psi_{l_1 3} = \psi_{l_1 3}^0 + \Delta \psi_{l_1 3}$ with $l_1 = x, y$ and $l_2 = x, y$, where $\varsigma_{l_1 l_2}^0$, $\psi_{l_1 l_2}^0$ and $\psi_{l_1 3}^0$ are nominal terms, $\Delta \varsigma_{l_1 l_2}$, $\Delta \psi_{l_1 l_2}$ and $\Delta \psi_{l_1 3}$ are uncertain terms.

2.2 Composite Neural Learning

Since the parameters of dynamics are changing with environmental factors such as temperature, atmospheric pressure, etc., the nonlinear dynamics f_i is constantly changing. Considering the RBF NN can approximate every nonlinear function with any accuracy [26], it is employed to approximate f_i as

$$f_i(\zeta_{in}) = \varpi_i^T \theta(\zeta_{in}, \eta, \kappa) + \varepsilon_i \quad (4)$$

where $\zeta_{in} = [\vartheta_x, \vartheta_y, \dot{\vartheta}_x, \dot{\vartheta}_y]^T \in \mathbb{R}^{4 \times 1}$ is the input vector, $\varpi_i \in \mathbb{R}^{n \times 1}$ is the optimal weight vector, $\theta(\cdot) \in \mathbb{R}^{n \times 1}$ is basis function vector with $\eta \in \mathbb{R}^{4 \times 1}$ as the center and κ as the width, ε_i is the approximation error with $|\varepsilon_i| \leq \varepsilon_m$ and $\varepsilon_m > 0$.

From the dynamics (1), the SPEM is constructed as

$$\ddot{\vartheta}_i = \hat{\varpi}_i^T \theta + d_i + \tau_i + c_i z_i \quad (5)$$

where $\hat{\varpi}_i$ is the estimation of ϖ_i , $c_i > 0$ is a user-defined constant, z_i is the prediction error implying the approximation error which is constructed as

$$z_i = \dot{\vartheta}_i - \hat{\vartheta}_i \quad (6)$$

where $\hat{\vartheta}_i$ is the estimation of ϑ_i .

The updating law of $\hat{\varpi}_i$ is designed as

$$\dot{\hat{\varpi}}_i = (\varsigma_{1i} e_i + \varsigma_{2i} z_i) \theta - \iota_i \hat{\varpi}_i \quad (7)$$

where $\varsigma_{1i} > 0$, $\varsigma_{2i} > 0$ and $\iota_i > 0$ are user-defined constants.

Remark 2 The conventional updating law of NN weight is designed as $\dot{\hat{\varpi}}_i = \varsigma_{1i} e_i \theta - \iota_i \hat{\varpi}_i$. It is observed that only the tracking error e_i is utilized to update $\hat{\varpi}_i$, where the approximation error ε_i is ignored. Therefore, the approximation performance of NN cannot be evaluated. In response to this problem, the SPEM-based prediction error is constructed, where ε_i is implied in z_i . Hence, the approximation accuracy can be improved by designing the updating law (7) using e_i and z_i .

2.3 Control Goal

The goal is to design a high precision and robust control scheme such that the system state can be forced to follow the dimensionless reference trajectory ϑ_{ri} when system uncertainties caused by environmental factors and external disturbances exist. Under the proposed controller, faster convergence and higher tracking accuracy of system tracking are expected.

3 Controller Design

For the dimensionless dynamics of MEMS gyroscopes (1), the RTSMC is designed to obtain higher tracking accuracy, while faster convergence is achieved by forcing the control system to start from the sliding surface at the initial time.

Considering the nonlinear dynamics including system uncertainties, RBF NN is adopted in this work. Different from the previous works where the updating law is constructed by the tracking error [5, 6], the updating law of composite neural learning is designed by the tracking error and the prediction error constructed by SPEM. To deal with the external disturbances, DOB is further developed. The diagram of the control scheme is presented in Fig. 2.

The recursive integral terminal sliding mode variable [27–29] is constructed as

$$s_i = \chi_i + \rho_i \chi_i^I \quad (8)$$

with

$$\begin{aligned} \chi_i &= \dot{e}_i + \ell_i e_i + \lambda_i |e_i|^{\alpha_i} \text{sign}(e_i) \\ \dot{\chi}_i^I &= |\chi_i|^{\beta_i} \text{sign}(\chi_i) \end{aligned} \quad (9)$$

where $\alpha_i > 1$, $0 < \beta_i < 1$, $\rho_i > 0$, $\ell_i > 0$ and $\lambda_i > 0$ are user-defined constants.

The derivative of s_i is calculated as

$$\begin{aligned} \dot{s}_i &= \dot{\chi}_i + \rho_i \dot{\chi}_i^I \\ &= \ddot{e}_i + \ell_i \dot{e}_i + \alpha_i \lambda_i |e_i|^{\alpha_i-1} \dot{e}_i + \rho_i \dot{\chi}_i^I \\ &= \ddot{\vartheta}_i - \ddot{\vartheta}_{ri} + \ell_i \dot{e}_i + \alpha_i \lambda_i |e_i|^{\alpha_i-1} \dot{e}_i + \rho_i \dot{\chi}_i^I \\ &= \varpi_i^T \theta(\zeta_{in}, \eta, \kappa) + \varepsilon_i + d_i + \tau_i - \ddot{\vartheta}_{ri} \\ &\quad + \ell_i \dot{e}_i + \alpha_i \lambda_i |e_i|^{\alpha_i-1} \dot{e}_i + \rho_i \dot{\chi}_i^I \end{aligned} \quad (10)$$

The controller is designed as

$$\tau_i = \tau_{ai} + \tau_{di} + \tau_{ni} + \tau_{si} \quad (11)$$

with

$$\begin{aligned} \tau_{ai} &= -\hat{\varpi}_i^T \theta \\ \tau_{di} &= -\hat{D}_i \\ \tau_{ni} &= \ddot{\vartheta}_{ri} - \ell_i \dot{e}_i - \alpha_i \lambda_i |e_i|^{\alpha_i-1} \dot{e}_i - \rho_i \dot{\chi}_i^I \\ \tau_{si} &= -\mu_{1i} s_i - \mu_{2i} |s_i|^{\nu_i} \text{sign}(s_i) \end{aligned} \quad (12)$$

Fig. 2 Control scheme of the RTSMC for MEMS gyroscopes with composite learning and DOB.

where $\mu_{1i} > 0$, $\mu_{2i} > 0$ and $0 < v_i < 1$ are user-defined constants, and $\hat{D}_i$ is the estimation of the compound disturbance $D_i = d_i + \varepsilon_i$.

From the controller (11), $\dot{s}_i$ is calculated as

$$\dot{s}_i = \tilde{\omega}_i^T \theta + \tilde{D}_i - \mu_{1i} s_i - \mu_{2i} |s_i|^{v_i} \text{sign}(s_i) \quad (13)$$

where $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$, $\tilde{D}_i = D_i - \hat{D}_i$.

The prediction error z_i is constructed as

$$z_i = \dot{\hat{g}}_i - \hat{\dot{g}}_i \quad (14)$$

where $\dot{\hat{g}}_i$ is the estimation of $\dot{g}_i$, and $\hat{\dot{g}}_i$ is given by the following SPEM:

$$\ddot{\hat{g}}_i = \hat{\omega}_i^T \theta + \hat{D}_i + \tau_i + c_i z_i \quad (15)$$

The derivative of z_i is calculated as

$$\begin{aligned} \dot{z}_i &= \ddot{\hat{g}}_i - \ddot{\hat{g}}_i \\ &= \tilde{\omega}_i^T \theta + D_i + \tau_i - \left(\hat{\omega}_i^T \theta + \hat{D}_i + \tau_i + c_i z_i \right) \\ &= \tilde{\omega}_i^T \theta + \tilde{D}_i - c_i z_i \end{aligned} \quad (16)$$

The updating law is proposed as

$$\dot{\hat{\omega}}_i = (\varsigma_{1i} s_i + \varsigma_{2i} z_i) \theta - \iota_i \hat{\omega}_i \quad (17)$$

The DOB is designed as

$$\begin{aligned} \hat{D}_i &= \Gamma_i (\dot{\hat{\Phi}}_i - \Phi_i) \\ \hat{\Phi}_i &= \hat{\omega}_i^T \theta + \hat{D}_i + \tau_i - \frac{\varsigma_{1i}}{\Gamma_i} s_i - \frac{\varsigma_{2i}}{\Gamma_i} z_i \end{aligned} \quad (18)$$

where $\Gamma_i > 0$ is an user-defined constant, and Φ_i is the intermediate variable.

The derivative of $\tilde{D}_i$ is calculated as

$$\begin{aligned} \dot{\tilde{D}}_i &= \dot{D}_i - \dot{\hat{D}}_i \\ &= \dot{D}_i - \Gamma_i (\ddot{\hat{g}}_i - \ddot{\hat{g}}_i) \\ &= \dot{D}_i - \Gamma_i (\tilde{\omega}_i^T \theta + D_i + \tau_i) \\ &\quad + \Gamma_i \left(\hat{\omega}_i^T \theta + \hat{D}_i + \tau_i - \frac{\varsigma_{1i}}{\Gamma_i} s_i - \frac{\varsigma_{2i}}{\Gamma_i} z_i \right) \\ &= \dot{D}_i - \Gamma_i \tilde{\omega}_i^T \theta - \Gamma_i \tilde{D}_i - \varsigma_{1i} s_i - \varsigma_{2i} z_i \end{aligned} \quad (19)$$

Remark 3 In the controller (11), τ_{ai} is the adaptive control using composite neural learning to approximate the system nonlinearity, τ_{di} is the DOB designed to deal with the external disturbances, τ_{ni} is the nominal feedback controller, τ_{si} is the RTSMC signal to achieve finite time convergence.

Remark 4 Since the approximation error ε_i and its derivative $\dot{\varepsilon}_i$ are bounded, the boundedness of the compound disturbance D_i and its derivative $\dot{D}_i$ can be guaranteed under Assumption1.

Remark 5 It is observed from the sliding mode manifold (8) that $s_i(0) = 0$ when $\chi_i^I(0) = -\chi_i(0)/\rho_i$, where $s_i(0)$, $\chi_i^I(0)$ and $\chi_i(0)$ are the initial values of s_i , χ_i^I and χ_i , respectively. That implies faster convergence will be achieved when the initial value of χ_i^I is chosen as $\chi_i^I(0) = -[\dot{e}_i(0) + \ell_i e_i(0) + \lambda_i |e_i(0)|^{\alpha_i} \text{sign}(e_i(0))]/\rho_i$, because the control system is forced to start from the sliding surface at the initial time.

4 Stability Analysis

Lemma 1 For the dynamics (1), if there exists the Lyapunov function V satisfying $\dot{V} + \gamma_1 V + \gamma_2 V^{\gamma_m} \leq 0$, the system stability can be guaranteed in the finite time T_0 [30], with $\gamma_1 > 0$, $\gamma_2 > 0$, $0 < \gamma_m < 1$, and

$$T_0 = \frac{1}{\gamma_1(1-\gamma_m)} \ln \left(\frac{\gamma_1 V^{1-\gamma_m}(0) + \gamma_2}{\gamma_2} \right) \quad (20)$$

where $V(0)$ is the initial value of V .

Theorem 1 Considering the dimensionless dynamics of MEMS gyroscopes described by (1), if the controller (11), the composite learning (17) and the DOB (18) are designed, the signals in (21) are bounded, and the tracking error e_i will converge to the region $\mathfrak{R}_i$ in the finite time T_i .

Proof: The Lyapunov function candidate is selected as

$$V = \sum_{i=x,y} V_i \quad (21)$$

where $V_i = V_{s_i} + V_{z_i} + V_{\tilde{\theta}_i} + V_{\tilde{D}_i}$, with

$$V_{s_i} = \frac{\varsigma_{1i}}{2} s_i^2, V_{z_i} = \frac{\varsigma_{2i}}{2} z_i^2, V_{\tilde{\theta}_i} = \frac{1}{2} \tilde{\theta}_i^T \tilde{\theta}_i, V_{\tilde{D}_i} = \frac{1}{2} \tilde{D}_i^2 \quad (22)$$

The derivative of V_{s_i} is calculated as

$$\begin{aligned} \dot{V}_{s_i} &= \varsigma_{1i} s_i \dot{s}_i \\ &= \varsigma_{1i} s_i \left[\tilde{\theta}_i^T \theta + \tilde{D}_i - \mu_{1i} s_i - \mu_{2i} |s_i|^{v_i} \text{sign}(s_i) \right] \end{aligned} \quad (23)$$

$$= \varsigma_{1i} s_i \left(\tilde{\theta}_i^T \theta + \tilde{D}_i \right) - \varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1}$$

The derivative of V_{z_i} is calculated as

$$\begin{aligned} \dot{V}_{z_i} &= \varsigma_{2i} z_i \dot{z}_i \\ &= \varsigma_{2i} z_i \left(\tilde{\theta}_i^T \theta + \tilde{D}_i - c_i z_i \right) \\ &= \varsigma_{2i} z_i \left(\tilde{\theta}_i^T \theta + \tilde{D}_i \right) - \varsigma_{2i} c_i z_i^2 \end{aligned} \quad (24)$$

The derivative of $V_{\tilde{\theta}_i}$ is calculated as

$$\begin{aligned} \dot{V}_{\tilde{\theta}_i} &= \tilde{\theta}_i^T \dot{\tilde{\theta}}_i \\ &= -\tilde{\theta}_i^T \dot{\tilde{\theta}}_i \\ &= -\tilde{\theta}_i^T \left[(\varsigma_{1i} s_i + \varsigma_{2i} z_i) \theta - \iota_i \tilde{\theta}_i \right] \end{aligned} \quad (25)$$

The derivative of $V_{\tilde{D}_i}$ is calculated as

$$\begin{aligned} \dot{V}_{\tilde{D}_i} &= \tilde{D}_i \dot{\tilde{D}}_i \\ &= \tilde{D}_i \left(\dot{D}_i - \Gamma_i \tilde{\theta}_i^T \theta - \Gamma_i \tilde{D}_i - \varsigma_{1i} s_i - \varsigma_{2i} z_i \right) \end{aligned} \quad (26)$$

Then we obtain

$$\begin{aligned} \dot{V}_i &= \dot{V}_{s_i} + \dot{V}_{z_i} + \dot{V}_{\tilde{\theta}_i} + \dot{V}_{\tilde{D}_i} \\ &= \varsigma_{1i} s_i \left(\tilde{\theta}_i^T \theta + \tilde{D}_i \right) - \varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} \\ &\quad + \varsigma_{2i} z_i \left(\tilde{\theta}_i^T \theta + \tilde{D}_i \right) - \varsigma_{2i} c_i z_i^2 \\ &\quad - \tilde{\theta}_i^T \left[(\varsigma_{1i} s_i + \varsigma_{2i} z_i) \theta - \iota_i \tilde{\theta}_i \right] \\ &\quad + \tilde{D}_i \left(\dot{D}_i - \Gamma_i \tilde{\theta}_i^T \theta - \Gamma_i \tilde{D}_i - \varsigma_{1i} s_i - \varsigma_{2i} z_i \right) \\ &= -\varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} - \varsigma_{2i} c_i z_i^2 \\ &\quad + \iota_i \tilde{\theta}_i^T \tilde{\theta}_i + \tilde{D}_i \left(\dot{D}_i - \Gamma_i \tilde{\theta}_i^T \theta - \Gamma_i \tilde{D}_i \right) \end{aligned} \quad (27)$$

Since the following facts exist

$$\begin{aligned} \tilde{\theta}_i^T \tilde{\theta}_i &\leq \frac{1}{2} \|\tilde{\theta}_i\|^2 - \frac{1}{2} \|\tilde{\theta}_i\|^2 \\ \tilde{D}_i \dot{\tilde{D}}_i &\leq \frac{1}{2} \tilde{D}_i^2 + \frac{1}{2} \dot{\tilde{D}}_i^2 \\ -\tilde{D}_i \tilde{\theta}_i^T \theta &\leq \frac{1}{2} c_m c_\theta^2 \tilde{D}_i^2 + \frac{1}{2 c_m} \|\tilde{\theta}_i\|^2 \end{aligned} \quad (28)$$

where $\|\theta\| \leq c_\theta$, and $c_m > 0$ is a constant.

$\dot{V}_i$ is rewritten as

$$\begin{aligned} \dot{V}_i &\leq -\varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} - \varsigma_{2i} c_i z_i^2 + \frac{\iota_i}{2} \|\tilde{\theta}_i\|^2 \\ &\quad - \frac{\iota_i}{2} \|\tilde{\theta}_i\|^2 + \frac{1}{2} \tilde{D}_i^2 + \frac{1}{2} \dot{\tilde{D}}_i^2 + \frac{1}{2} \Gamma_i c_m c_\theta^2 \tilde{D}_i^2 \\ &\quad + \frac{1}{2 c_m} \Gamma_i \|\tilde{\theta}_i\|^2 - \Gamma_i \tilde{D}_i^2 \\ &\leq -\varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} - \varsigma_{2i} c_i z_i^2 \\ &\quad - \frac{1}{2} \left(\iota_i - \frac{\Gamma_i}{c_m} \right) \|\tilde{\theta}_i\|^2 + \frac{1}{2} (\Gamma_i c_m c_\theta^2 - 2\Gamma_i + 1) \tilde{D}_i^2 \\ &\quad + \frac{\iota_i}{2} \|\tilde{\theta}_i\|^2 + \frac{1}{2} \dot{\tilde{D}}_i^2 \\ &= -\varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} - \varsigma_{2i} c_i z_i^2 \\ &\quad - \frac{1}{2} C_{\tilde{\theta}_i} \|\tilde{\theta}_i\|^2 - \frac{1}{2} C_{\tilde{D}_i} \tilde{D}_i^2 + C_V \end{aligned} \quad (29)$$

where $C_{\tilde{\theta}_i} = \iota_i - \Gamma_i/c_m$, $C_{\tilde{D}_i} = 2\Gamma_i - \Gamma_i c_m c_\theta^2 - 1$, $C_V = \iota_i \|\tilde{\theta}_i\|^2/2 + \dot{\tilde{D}}_i^2/2$. Parameters ι_i , Γ_i , c_m are selected to force $C_{\tilde{\theta}_i} \geq 0$ and $C_{\tilde{D}_i} \geq 0$.

Furthermore, it is concluded that

$$\dot{V}_i \leq -\varsigma_{2i} c_i z_i^2 - \frac{1}{2} C_{\tilde{\theta}_i} \|\tilde{\theta}_i\|^2 - \frac{1}{2} C_{\tilde{D}_i} \tilde{D}_i^2 + C_V \quad (30)$$

Hence, z_i , $\tilde{\theta}_i$ and $\tilde{D}_i$ are bounded with $|z_i| \leq \sqrt{C_V/\varsigma_{2i} c_i}$, $\|\tilde{\theta}_i\| \leq \sqrt{2C_V/C_{\tilde{\theta}_i}}$ and $|\tilde{D}_i| \leq \sqrt{2C_V/C_{\tilde{D}_i}}$.

Define $C_b = \tilde{\theta}_i^T \theta + \tilde{D}_i$. Since $\tilde{\theta}_i$ and $\tilde{D}_i$ are bounded, there exists an unknown constant $B_m > 0$ such that $|C_b| \leq B_m$. From (23), it is concluded that

$$\begin{aligned} \dot{V}_{s_i} &= \varsigma_{1i} s_i C_b - \varsigma_{1i} \mu_{1i} s_i^2 - \varsigma_{1i} \mu_{2i} |s_i|^{v_i+1} \\ &\leq -\varsigma_{1i} \left(\mu_{1i} - \frac{B_m}{2|s_i|} \right) s_i^2 \\ &\quad - \varsigma_{1i} \left(\mu_{2i} - \frac{B_m}{2|s_i|^{v_i}} \right) |s_i|^{v_i+1} \end{aligned} \quad (31)$$

If $s_i \neq 0$, it is obtained that

$$\dot{V}_{s_i} \leq -\bar{\mu}_{1i} V_{s_i} - \bar{\mu}_{2i} V_{s_i}^{\frac{v_i+1}{2}} \quad (32)$$

with

$$\begin{aligned} \bar{\mu}_{1i} &= 2 \left(\mu_{1i} - \frac{B_m}{2|s_i|} \right) \\ \bar{\mu}_{2i} &= 2 \left(\frac{2}{\varsigma_{1i}} \right)^{\frac{v_i-1}{2}} \left(\mu_{2i} - \frac{B_m}{2|s_i|^{v_i}} \right) \end{aligned} \quad (33)$$

From Lemma 1, it is known that $|s_i|$ will converge to the region $\Lambda_i = \min\{\Lambda_{1i}, \Lambda_{2i}\}$ in finite time T_{si} , with

$$\begin{aligned}\Lambda_{1i} &= \left\{ |s_i| \leq \frac{B_m}{2\mu_{1i}} \right\} \\ \Lambda_{2i} &= \left\{ |s_i| \leq \left(\frac{B_m}{2\mu_{2i}} \right)^{\frac{1}{v_i}} \right\} \\ T_{si} &\leq \frac{2}{\bar{\mu}_{1i}(1-v_i)} \ln \left(\frac{\bar{\mu}_{1i}V_{si}^{\frac{1-v_i}{2}}(0) + \bar{\mu}_{2i}}{\bar{\mu}_{2i}} \right)\end{aligned}\quad (34)$$

where $V_{si}(0)$ is the initial value of V_{si} .

When $s_i = 0$, there exists $\chi_i = -\rho_i \chi_i^I$, such that χ_i is with the same convergence time as χ_i^I . Define $V_{\chi_i^I} = (\chi_i^I)^2/2$. It is calculated that

$$\begin{aligned}\dot{V}_{\chi_i^I} &= \chi_i^I \dot{\chi}_i^I \\ &= \chi_i^I |\chi_i|^{\beta_i} \text{sign}(\chi_i) \\ &= \chi_i^I |s_i - \rho_i \chi_i^I|^{\beta_i} \text{sign}(s_i - \rho_i \chi_i^I)\end{aligned}\quad (35)$$

When $|\rho_i \chi_i^I| \geq |s_i| + C_{si}$, where $C_{si} > 0$ is an unknown constant, $\dot{V}_{\chi_i^I}$ can be rewritten as

$$\begin{aligned}\dot{V}_{\chi_i^I} &= -\chi_i^I |s_i - \rho_i \chi_i^I|^{\beta_i} \text{sign}(\chi_i^I) \\ &\leq -|\chi_i^I| \left(|\rho_i \chi_i^I|^{\beta_i} - |s_i|^{\beta_i} \right) \\ &\leq -C_{si}^{\beta_i} |\chi_i^I| \\ &= -\sqrt{2} C_{si}^{\beta_i} V_{\chi_i^I}^{\frac{1}{2}}\end{aligned}\quad (36)$$

Referring to [31, 12], it is concluded that χ_i^I will converge to the region $\Lambda_{\chi_i^I}$ in finite time T_{χ_i} when $|s_i|$ is in the region Λ_i , where

$$\begin{aligned}\Lambda_{\chi_i^I} &= \left\{ |\chi_i^I| \leq \frac{\Lambda_i + C_{si}}{\rho_i} \right\} \\ T_{\chi_i} &\leq \frac{V_{\chi_i^I}^{1-\frac{1}{2}}(0)}{\sqrt{2} C_{si}^{\beta_i} (1-\frac{1}{2})} = \frac{\sqrt{2} V_{\chi_i^I}^{\frac{1}{2}}(0)}{C_{si}^{\beta_i}}\end{aligned}\quad (37)$$

Hence, $|\chi_i|$ will converge to the region $\Lambda_{\chi_i} = \Lambda_i + C_{si}$ in finite time T_{χ_i} . Similar to [31, 12], it is easy to verify that the above finite time convergence still can be guaranteed when $|\rho_i \chi_i^I| < |s_i| + C_{si}$.

Defining $V_{e_i} = e_i^2/2$, we have

$$\begin{aligned}\dot{V}_{e_i} &= e_i \dot{e}_i \\ &= e_i [\chi_i - \ell_i e_i - \lambda_i |e_i|^{\alpha_i} \text{sign}(e_i)] \\ &= e_i \chi_i - \ell_i e_i^2 - \lambda_i |e_i|^{\alpha_i+1} \\ &\leq -\left(\ell_i - \frac{\Lambda_{\chi_i}}{2|e_i|} \right) e_i^2 - \left(\lambda_i - \frac{\Lambda_{\chi_i}}{2|e_i|^{\alpha_i}} \right) |e_i|^{\alpha_i+1}\end{aligned}\quad (38)$$

If $e_i \neq 0$, it is obtained that

$$\dot{V}_{e_i} \leq -E_{1i} V_{e_i} - E_{2i} V_{e_i}^{\frac{\alpha_i+1}{2}} \quad (39)$$

with

$$\begin{aligned}E_{1i} &= 2 \left(\ell_i - \frac{\Lambda_{\chi_i}}{2|e_i|} \right) \\ E_{2i} &= 2^{\frac{\alpha_i+1}{2}} \left(\lambda_i - \frac{\Lambda_{\chi_i}}{2|e_i|^{\alpha_i}} \right)\end{aligned}\quad (40)$$

From Lemma 1, it is known that $|e_i|$ will converge to the region $\aleph_i = \min\{\aleph_{1i}, \aleph_{2i}\}$ in finite time T_{ei} when $|\chi_i|$ is in the region Λ_{χ_i} , with

$$\begin{aligned}\aleph_{1i} &= \left\{ |e_i| \leq \frac{\Lambda_{\chi_i}}{2\ell_i} \right\} \\ \aleph_{2i} &= \left\{ |e_i| \leq \left(\frac{\Lambda_{\chi_i}}{2\lambda_i} \right)^{\frac{1}{\alpha_i}} \right\} \\ T_{ei} &\leq \frac{2}{E_{1i}(1-\alpha_i)} \ln \left(\frac{E_{1i} V_{ei}^{\frac{1-\alpha_i}{2}}(0) + E_{2i}}{E_{2i}} \right)\end{aligned}\quad (41)$$

where $V_{ei}(0)$ is the initial value of V_{ei} .

Thus, e_i converges from any initial condition to the region $\aleph_i$ in the finite time $T_i = T_{si} + T_{\chi_i} + T_{ei}$.

5 Simulation

Note that the proposed controller (11) is denoted as ‘‘CL-RTSMC’’. The highlights of ‘‘CL-RTSMC’’ are the RTSMC (8) and the composite neural learning (17). To verify faster convergence and higher tracking accuracy of ‘‘CL-RTSMC’’, it is compared with ‘‘CL-TSMC’’ (the controller not using the RTSMC) and ‘‘NN-TSMC’’ (the controller not using the RTSMC and the composite neural learning) in simulations, while ‘‘CL-TSMC’’ and ‘‘NN-TSMC’’ are presented as follows:

(a) ‘‘CL-TSMC’’: The sliding mode manifold is designed as

$$s_i = \dot{e}_i + \ell_i e_i + \lambda_i |e_i|^{\alpha_i} \text{sign}(e_i) \quad (42)$$

The controller is designed as

$$\tau_i = \tau_{ai} + \tau_{di} + \tau_{ni} + \tau_{si} \quad (43)$$

with

$$\begin{aligned}\tau_{ai} &= -\hat{\omega}_i^T \theta \\ \tau_{di} &= -\hat{D}_i \\ \tau_{ni} &= \ddot{\vartheta}_{ri} - \ell_i \dot{e}_i - \alpha_i \lambda_i |e_i|^{\alpha_i-1} \dot{e}_i \\ \tau_{si} &= -\mu_{1i} s_i - \mu_{2i} |s_i|^{v_i} \text{sign}(s_i)\end{aligned}\quad (44)$$

where the DOB is designed as (18) and the updating law of NN weight is designed as

$$\dot{\hat{\omega}}_i = (\varsigma_{1i} s_i + \varsigma_{2i} z_i) \theta - \iota_i \hat{\omega}_i \quad (45)$$

with $z_i = \dot{\vartheta}_i - \hat{\vartheta}_i$ constructed by the dynamics (1) and the SPEM (15).

(b) “NN-TSMC”: The sliding mode manifold is designed as (42). The controller is designed as (43). The DOB is designed as

$$\begin{aligned}\hat{D}_i &= \Gamma_i (\dot{\vartheta}_i - \Phi_i) \\ \dot{\Phi}_i &= \hat{\omega}_i^T \theta + \hat{D}_i + \tau_i - \frac{\zeta_{1i}}{\Gamma_i} s_i\end{aligned}\quad (46)$$

and the updating law of NN weight is designed as

$$\dot{\hat{\omega}}_i = \zeta_{1i} s_i \theta - l_i \hat{\omega}_i \quad (47)$$

The dimensionless dynamics parameters [12] are given as $\zeta_{xx}^0 = 0.075$, $\zeta_{xy}^0 = 0.0075$, $\zeta_{yx}^0 = 0.0075$, $\zeta_{yy}^0 = 0.0075$, $\psi_{xx}^0 = 14207$, $\psi_{xy}^0 = 877$, $\psi_{yx}^0 = 877$, $\psi_{yy}^0 = 12564$, $\psi_{x3}^0 = 6.25 \times 10^{-4}$, $\psi_{y3}^0 = 6.25 \times 10^{-4}$ and $\Omega = 0.005$. The parameter uncertainties $\Delta \zeta_{xx}$, $\Delta \zeta_{xy}$, $\Delta \zeta_{yx}$ and $\Delta \zeta_{yy}$ are set as 5% of the nominal parts ζ_{xx}^0 , ζ_{xy}^0 , ζ_{yx}^0 and ζ_{yy}^0 respectively. The parameter uncertainties $\Delta \psi_{xx}$, $\Delta \psi_{xy}$, $\Delta \psi_{yx}$, $\Delta \psi_{yy}$, $\Delta \psi_{x3}$ and $\Delta \psi_{y3}$ are set as 0.25% of the nominal parts ψ_{xx}^0 , ψ_{xy}^0 , ψ_{yx}^0 , ψ_{yy}^0 , ψ_{x3}^0 and ψ_{y3}^0 , respectively. The external disturbances are set as $d_x = 0.7448 + 0.98 \sin(10.5t)$ and $d_y = 0.7448 + 0.98 \sin(11t)$. The reference signals are selected as $\vartheta_{rx} = 6.2 \sin(3.7685t + \pi/3)$ and $\vartheta_{ry} = 5 \sin(3.5447t - \pi/6)$.

Fig. 3 System tracking: (a) ϑ_{rx} and ϑ_x . (b) ϑ_{ry} and ϑ_y .

For “NN-TSMC”, the control parameters are set as $\mu_{1i} = 200$, $\mu_{2i} = 150$, $\alpha_i = 2.4$, $v_i = 0.9$, $\ell_i = 10$, $\lambda_i = 10$, $\Gamma_i = 20$, $l_i = 4$ and $\zeta_{1i} = 810$. For “CL-TSMC”, the same parameters are set as “NN-TSMC”, while $\zeta_{2i} = 54$ and $c_i = 2$. For “CL-RTSMC”, the same parameters are set as “NN-TSMC” and “CL-TSMC”, while $\beta_i = 0.6$ and $\rho_i = 1000$. To further obtain faster convergence, the initial value of χ_i^I is chosen as $\chi_i^I(0) = -[\dot{e}_i(0) + \ell_i e_i(0) + \lambda_i |e_i(0)|^{\alpha_i} \text{sign}(e_i(0))] / \rho_i$.

Fig. 4 Tracking errors of drive axis: (a) e_x . (b) $e_x \in [0.2, 4]$.

Fig. 5 Tracking errors of sensitive axis: (a) e_y . (b) $e_y \in [0.2, 4]$.

The simulation results are demonstrated in Fig.3~Fig.10 and Table 1. It is observed from Fig.3 that system state ϑ_i can effectively follow the reference signal ϑ_{ri} under “NN-TSMC”, “CL-TSMC” and “CL-RTSMC”, faster convergence and higher tracking accuracy are obtained under “CL-RTSMC” in Fig.4~Fig.5. To clearly analyze the tracking performance of controllers, the transient time, the average of steady-state errors and the root mean squared error (RMSE) of steady-state errors are presented in Table 1.

Through comparing the tracking performance index of “CL-TSMC” and “NN-TSMC”, it is observed that, under

Table 1 Tracking performance index of control schemes.

Tracking error	Method	Transient time	Average of steady-state errors	RMSE of steady-state errors
e_x	NN-TSMC	0.5	0.0740	0.7604
	CL-TSMC	0.5	0.0246	0.2558
	CL-RTSMC	0.42	5.1836×10^{-4}	0.0028
e_y	NN-TSMC	0.5	0.0144	0.6019
	CL-TSMC	0.5	0.0092	0.1838
	CL-RTSMC	0.4	3.9260×10^{-4}	0.0024

Fig. 6 Estimation of lumped uncertainty F_x : (a) F_x and $\hat{F}_x$ under “NN-TSMC”. (b) F_x and $\hat{F}_x$ under “CL-TSMC”. (c) F_x and $\hat{F}_x$ under “CL-RTSMC”. (d) Estimation error $\tilde{F}_x$.

“CL-TSMC”, the average of steady-state errors in drive axis and sensitive axis are shortened by 66.76% and 36.11% respectively, while the RMSE of steady-state errors are shortened by 66.36% and 69.46% respectively. Higher tracking accuracy is obtained under “CL-TSMC” because the prediction error z_i constructed from the SPEM is introduced to the updating law (17), such that the approximation error ε_i of NN is indirectly considered. The estimation of the lumped uncertainty $F_i = f_i + d_i$ shown in Fig.6~Fig.7 further confirms the above conclusion, where smaller estimation error $\tilde{F}_i = \tilde{\omega}_i^T \theta + \tilde{D}_i$ is obtained under “CL-TSMC”.

Through comparing the tracking performance index of “CL-RTSMC” and “CL-TSMC”, it is observed that, under “CL-RTSMC”, the transient time is shortened by 16%, the average of steady-state errors in drive axis and sensitive axis are shortened by 99.30% and 97.27% respectively, and the RMSE of steady-state errors are shortened by 99.63% and 99.60% respectively. Higher tracking accuracy and faster convergence are achieved under RTSMC.

The norm of the RBF NN weight vector $\hat{\omega}_i$ is shown in Fig.8, which is bounded in the steady state. From the s-

Fig. 7 Estimation of lumped uncertainty F_y : (a) F_y and $\hat{F}_y$ under “NN-TSMC”. (b) F_y and $\hat{F}_y$ under “CL-TSMC”. (c) F_y and $\hat{F}_y$ under “CL-RTSMC”. (d) Estimation error $\tilde{F}_y$. **Fig. 8** Weight vector norm $\|\hat{\omega}_i\|$: (a) $\|\hat{\omega}_x\|$. (b) $\|\hat{\omega}_y\|$.

liding mode manifold s_i presented in Fig.9, it is observed that the system states under “CL-RTSMC” are closer to the sliding mode. The control input τ_i under two methods are depicted in Fig.10, where τ_i has reached 100000 in some cases. It is noted that τ_i is the dimensionless control amount rather than the control force directly implemented on the MEMS gyroscopes. The dimensionless control amount τ_i is transformed from the control force τ_i^* to avoid the effect of the dimension on gyroscopic motion characteristics by using the reference frequency $\omega_0 = 1\text{kHz}$, reference length

Fig. 9 Sliding mode manifold s_i : (a) s_x . (b) s_y .

Fig. 10 Control input τ_i : (a) τ_x . (b) τ_y .

$\vartheta_0 = 10^{-5}\text{m}$ and the mass $m = 5.7 \times 10^{-9}\text{kg}$, where the deducing process is presented in [25]. Thus, it is referred that $\tau_i^* = m\omega_0^2\vartheta_0\tau_i = 5.7 \times 10^{-8}\tau_i$. When τ_i reaches 100000, the control forces are calculated as $\tau_i^* = 5.7 \times 10^{-3}\text{N}$. For MEMS gyroscopes, the actual control force τ_i^* meets the requirement. Hence, τ_i reaching 100000 in some cases is reasonable. Thus the simulation results verified that the expected higher tracking accuracy of the proposed RTSMC using the composite neural learning and the DOB can be obtained.

6 Conclusions

In this paper, the RTSMC has been designed to control the dynamics of MEMS gyroscopes with faster convergence and higher tracking accuracy in the presence of system uncertainties and external disturbances. In the proposed method, system states have been forced to start from the sliding mode surface at the initial time. The composite neural learning based on SPEM has been proposed to improve the approximation accuracy of system nonlinearity, such that higher tracking accuracy can be obtained. The DOB has been employed to handle the external disturbances. Simulation results have been demonstrated to verify that the proposed control can nicely achieve faster convergence and higher accuracy, compared with the schemes of “NN-TSMC” and “CL-TSMC”.

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Data Availability Statement

My manuscript has no associated data.

Declarations

Data Availability Statement The authors declare that they have no conflict of interest.

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