Hall effect in transmission lines

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Short Report

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Hall effect in transmission lines

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Abstract

Transmission lines are subject to several undesired effects which are observed during distribution of power. This paper provides elementary proof-of-concept of a nonlinear phenomenon which causes the development of transverse electric potential in a conductor. In a simple setting for a two-cable HVDC transmission line, it is shown that a voltage perpendicular to the flow of current in a cable is generated, which is proportional to the square of current in the nearby cable. It also creates a deflection in electron path by concentrating them towards one side which produces an effect analogous to the skin effect (found with AC supply), but here occurring with DC supply.

1 Introduction

Over decades, researchers have identified several phenomena like Ferranti effect, corona effect, skin effect, proximity effect, sagging, flashover, wind effect, mechanical loading due to ice and dust, and so on, which are known to influence power transition through a transmission line [1–6]. The impact of these effects is observable in the voltage or current measured at either ends of the line [7]. In this paper, another phenomenon is reported and investigated which, interestingly, is perpendicular to the direction of current and exists in the cross-section of the cable.

Due to several advances in high voltage apparatus over the past three decades, high voltage direct current (HVDC) transmission lines have become viable means for power transfer over very long distances [10]. Advantages of HVDC include elimination of system synchronization and stability problems, lower net costs for long distances, elimination of substations, as well as absence of skin effect among others.

The presence of stray magnetism in low-current circuits can lead to Hall effect. For metals, the Hall voltage is of the order of 10^{-20} volts or lower due to

poor Hall coefficient. However, in HV transmission lines, it is speculated that the input voltage is sufficiently high to give rise to non-negligible Hall phenomenon. The objective of this paper is to explore and elaborate this process.

2 Concept and Analysis

Let us consider a two-cable HVDC transmission line as shown in Figure 1. The equivalent two-port network is also given, where $I(z)$ and $V(z)$ are functions of length z of the transmission line, while $V_S = V(0)$ and $I_S = I(0)$ are source voltage and source current respectively [11].

Figure 1: a) Two-cable transmission line, and b) Equivalent two-port network

In the presence of applied electric field, there will be a net overall movement of electrons towards the positive terminal, which means that the charge carriers get an average drift velocity. The drift velocity of electron in cable-1 is represented as

$$\mathbf{v} = -\frac{1}{b} \mathbf{J} = -\frac{1}{b} \frac{I}{\pi R^2} \hat{z} \quad (1)$$

in the cylindrical coordinate system, where b is charge density. If electric field intensity is increased then the electrons gain more drift velocity towards positive potential, that is, in the opposite direction to the field. Figure 2 shows the concentric magnetic field lines generated by the current returning from cable-2, which is given by

$$\mathbf{B} = -\frac{\mu I}{2\pi r} \hat{\phi} \quad (2)$$

where μ is permeability and r is distance from center of cable-2.

Figure 2: Electron in cable-1 under the influence of magnetic field due to cable-2

This magnetic field due to cable-2 interacts with the flow of electrons in cable-1. The Lorentz force acting on the electron in cable-1 is $F = q(\mathbf{E}_H + \mathbf{v} \times \mathbf{B})$ which must be zero. Thus, we have $\mathbf{E}_H = -\mathbf{v} \times \mathbf{B}$ which is the Hall field. Substituting expressions of B and v from above, we get

$$\nabla V_H = \frac{1}{b} \frac{I}{\pi R^2} \hat{z} \times \frac{\mu I}{2\pi r} \hat{\phi} \quad (3)$$

where ∇ is the gradient operator in cylindrical coordinates. The vector cross-product of the right side of the equation gives

$$\nabla V_H = -K \frac{I^2}{R^2 r} \hat{r}. \quad (4)$$

where $K = \frac{\mu}{2\pi^2 b}$ is a constant. The gradient operator provide three derivatives in three directions, two of which are zero while other equates to the $\hat{r}$ direction. Integrating over the entire transmission line, gives the expression:

$$\int_{V_{H+}}^{V_{H-}} dV_H = -\frac{KI^2}{R^2} \int_{D-R}^{D+R} \frac{dr}{r} \quad (5)$$

which simplifies to

$$V_{H \cdot x}(z) = K I^2(z) \frac{\log\left(\frac{D+R}{D-R}\right)}{R^2} \quad (6)$$

as indicated in Figure 3.

Figure 3: Depiction of transverse voltage generated in cable-1 with the electric field pointing left causing a sideward concentration of electrons.

3 Further Discussion and Conclusion

Consider two aluminum cables, each of radius 22.56 mm, which are separated by distance 116 mm. Assuming no temperature effect, we have

$$\begin{aligned} \frac{V_{H.x}(z)}{I^2(z)} &= \frac{4\pi \times 10^{-7} \times 0.34 \times 10^{-10}}{2\pi^2} \times \frac{\log\left(\frac{116+22.56}{116-22.56}\right)}{22.56^2 \times 10^{-6}} \\ &= 1.6756 \times 10^{-15}. \end{aligned}$$

This emphasizes the fact that this effect will only be observable in very long transmission line operating at kilovolts or megavolts. Still the voltage will be small enough to be simply measured by basic electronic voltmeter.

Therefore we can conclude that a potential is generated in the cable perpendicular to the flow of current. This voltage is proportional to the square of current in the nearby cable. Furthermore, its magnitude depends upon distance between cables, radius of the cable, charge density, permeability of atmosphere around the cables, impedance of power line, and it varies with the length of transmission line. The development of this voltage also shifts the electrons from the central path deviating them slightly towards one side. Interestingly, this is an effect analogous to the skin effect that takes place in AC systems.

An ideal transmission line model must transmit power reliably and efficiently, which is not possible in the presence of the undesired effects listed initially. Researchers have developed protection and control devices to ensure successful transmission in the presence of these non-ideal effects [12]. Unlike such effects, the phenomenon described in this work is not entirely undesired. The reason is that it could facilitate easy implementation of devices for remote monitoring and control (M&C) of transmission line, in contrast to the heavy apparatus used currently. For instance, short circuit fault detection can be done by merely using

an electronic circuit to measure $V_{H.x}$ because it strongly depends upon current. Unlike other devices which are placed at the either ends of a transmission line, one can install the above device anywhere on the cable by connecting two terminals directly to the insulation, and accounting for the voltage drop against the cable insulation. This procedure would not even require transformers, voltage divider circuits, etc. because the Hall voltage is already of a suitable magnitude. A wealth of devices can be designed for M&C purposes as well as for other wide range of unexplored applications, by employing the phenomenon described in this work.

Conflict of interest

The author has no conflict of interest, financial or otherwise.

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Figures

a)

b)

Figure 1

a) Two-cable transmission line, and b) Equivalent two-port network

Figure 2

Electron in cable-1 under the influence of magnetic field due to cable-2

Figure 3

Depiction of transverse voltage generated in cable-1 with the electric field pointing left causing a sideward concentration of electrons.