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Highly Confined Mid-infrared Plasmonics in Graphene-plasma Cylindrical Structures

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Abstract: A new theoretical model is suggested for gyro-electric cylindrical waveguides incorporating graphene layers in this article. To validate the model, the analytical result of FOM is compared to simulation one prepared by COMSOL for a graphene nano-wire. The full agreement between them is seen, which confirmed the high accuracy of our model. As a special case of the general waveguide, a novel gyro-electric-based waveguide with double-layer graphene, constituting graphene-InSb-graphene-SiO₂-Si layers, is introduced and investigated. It is shown that the FOM of the designed waveguide could be altered via the chemical potential and the magnetic field.

Key-words: Multi-layer structure, graphene layer, gyro-electric medium, analytical model

1. Introduction

The emergence of graphene has suggested a new platform in “Plasmonics” science due to its notable features [1-4]. Nowadays, scientists have reported many novel properties for graphene in the THz region, where one of them is certainly the conductivity of the graphene. Based on this tunable parameter, many innovative devices have been designed and fabricated in plasmonics such as sensors [5, 6], couplers [7-9], filters [10-12], resonators [13-15], Radar Cross-Section (RCS) reduction-based devices [16-18] and circulators [19-22]. Among these devices, graphene-based waveguides play a remarkable role in graphene plasmonics, which are divided into various platforms such as planar [23-35], cylindrical [36-40], and elliptical structures [41-44]. It should be noted that graphene plasmonics is mostly used in the mid-infrared region whereas metal-based plasmonics is a promising candidate in near-infrared frequencies for various applications [45-49].

In graphene plasmonics, one of the famous ways to efficiently increase the performance of the designed structure is the integration of the graphene with other tunable materials such as chiral materials [50-57], and non-linear materials [58-68]. Cylindrical graphene structures, which are investigated in some articles [36, 37, 69-72], are one of the interesting platforms due to their potential applications. To the best of the author’s knowledge, no publication is investigated the plasma multi-layer graphene waveguides. This paper proposes a novel analytical model for these structures, which considers all propagating SPP waves and derives closed-form complicated relations for the field contributions of SPP waves. The proposed, general structure in this paper is constructed of gyro-electric multi-layer layers, where each graphene layer is located between two different gyro-electric materials.

The remainder of the article is organized as follows. In section 2, after introducing the general waveguide, we will propose a mathematical model for it. Then, a special exemplary structure will be studied in section 3. We will show that the propagating properties of the designed waveguide are tunable by the chemical potential and the DC magnetic bias. Finally, the article is concluded in section 4.

2. The Proposed General Structure and its Analytical Model

The configuration of the general structure is illustrated in Fig. 1. The general waveguide is formed of various gyro-electric layers, where each gyro-electric layer is surrounded by two different graphene layers. A perpendicular magnetic bias is applied in the z-direction.

The conductivity of the graphene in the N -th layer can be modeled by following relation [73]:

$$\sigma_N(\omega, \mu_{c,N}, \Gamma_N, T) = \frac{-je^2}{4\pi\hbar} \text{Ln} \left[\frac{2|\mu_{c,N}| - (\omega - j2\Gamma_N)\hbar}{2|\mu_{c,N}| + (\omega - j2\Gamma_N)\hbar} \right] + \frac{-je^2 K_B T}{\pi\hbar^2 (\omega - j2\Gamma_N)} \left[\frac{\mu_{c,N}}{K_B T} + 2 \text{Ln} \left(1 + e^{-\mu_{c,N}/K_B T} \right) \right] \quad (1)$$

In (1), Γ_N is the scattering rate, T is the temperature, and $\mu_{c,N}$ is the chemical potential for the N -th layer [73]. The permittivity tensor of each gyro-electric medium in the N -layer can be defined as [74]:

$$\bar{\bar{\epsilon}}_N = \epsilon_0 \begin{pmatrix} \epsilon_N & j\epsilon_{a,N} & 0 \\ -j\epsilon_{a,N} & \epsilon_N & 0 \\ 0 & 0 & \epsilon_{\square,N} \end{pmatrix} \quad (2)$$

with the following elements [75]:

$$\epsilon_N = \epsilon_{\infty,N} \left(1 - \frac{\omega_{p,N}^2 (\omega + j\nu_N)}{\omega [(\omega + j\nu_N)^2 - \omega_{c,N}^2]} \right) \quad (3)$$

$$\epsilon_{a,N} = \epsilon_{\infty,N} \left(\frac{\omega_{p,N}^2 \omega_c}{\omega [(\omega + j\nu_N)^2 - \omega_c^2]} \right) \quad (4)$$

$$\epsilon_{\square,N} = \epsilon_{\infty,N} \left(1 - \frac{\omega_{p,N}^2}{\omega (\omega + j\nu_N)} \right) \quad (5)$$

In (3)-(5), ν_N is the collision rate and $\epsilon_{\infty,N}$ is the background permittivity. The plasma and the cyclotron frequency are [75]:

$$\omega_{p,N} = \sqrt{\frac{n_s e^2}{\epsilon_0 \epsilon_{\infty,N} m^*}} \quad (6)$$

$$\omega_c = \frac{e B_0}{m^*} \quad (7)$$

Fig. 1. The cross-section configuration of the graphene-based cylindrical structures with gyro-electric layers.

By writing Maxwell's equations inside the N -th gyro-electric layer (suppose $e^{i\omega t}$) [74]:

$$\nabla \times \mathbf{E} = -j\omega \mu_N \mathbf{H} \quad (8)$$

$$\nabla \times \mathbf{H} = j\omega \bar{\bar{\epsilon}}_N \cdot \mathbf{E} \quad (9)$$

the z-component of the electromagnetic fields satisfy the following equations [74]:

$$\left(\nabla_{\perp}^2 + \frac{\epsilon_{\square,N}}{\epsilon_N} \frac{\partial^2}{\partial z^2} + (k_0^2 \epsilon_{\square,N} \mu_N) \right) E_z + k_0 \mu_N \frac{\epsilon_{a,N}}{\epsilon_N} \frac{\partial H_z}{\partial z} = 0 \quad (10)$$

$$\left(\nabla_{\perp}^2 + \frac{\partial^2}{\partial z^2} + (k_0^2 \epsilon_{\perp,N} \mu_N) \right) H_z - \frac{k_0 \epsilon_{\square,N} \epsilon_{a,N}}{\epsilon_N} \frac{\partial E_z}{\partial z} = 0 \quad (11)$$

where

$$\nabla_{\perp}^2 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \quad (12)$$

$$\epsilon_{\perp,N} = \epsilon_N - \frac{\epsilon_{a,N}^2}{\epsilon_N} \quad (13)$$

The z-components of electromagnetic fields inside the gyro-electric layer can be written as follows (m is an integer):

$$H_{z,N}(r, \varphi, z) = \int \sum_{m=-\infty}^{+\infty} e^{jk_z z} \exp(-jm\varphi) H_{z,N}^m(r) dk_z \quad (14)$$

$$E_{z,N}(r, \varphi, z) = \int \sum_{m=-\infty}^{+\infty} e^{jk_z z} \exp(-jm\varphi) E_{z,N}^m(r) dk_z \quad (15)$$

In (14)-(15) the propagation constant is indicated by k_z . By substituting (14)-(15) into (10)-(11), we achieve:

$$\left(\nabla_{\perp}^2 + k_0^2 \epsilon_{\square,N} \mu_N - \frac{\epsilon_{\square,N}}{\epsilon_N} k_z^2 \right) E_{z,N}^m - jk_0 k_z \mu_N \frac{\epsilon_{a,N}}{\epsilon_N} H_{z,N}^m = 0 \quad (16)$$

$$\left(\nabla_{\perp}^2 + k_0^2 \epsilon_{\perp,N} \mu_N - k_z^2 \right) H_{z,N}^m + jk_0 k_z \epsilon_{\square,N} \frac{\epsilon_{a,N}}{\epsilon_N} E_{z,N}^m = 0 \quad (17)$$

By combining (16) and (17), the following equation is derived:

$$\left(\nabla_{\perp}^2 + k_{r,2N-1}^2 \right) \left(\nabla_{\perp}^2 + k_{r,2N}^2 \right) H_{z,N}^m = 0 \quad (18)$$

where

$$A_{1,N} = -k_0^2 \mu_N \left(\epsilon_{\square,N} + \epsilon_{\perp,N} \right) + \left(\frac{\epsilon_{\square,N}}{\epsilon_N} \right) k_z^2 \quad (19)$$

$$A_{2,N} = \left(k_0^2 \epsilon_{\square,N} \mu_N - \frac{\epsilon_{\square,N}}{\epsilon_N} k_z^2 \right) \left(k_0^2 \epsilon_{\perp,N} \mu_N - k_z^2 \right) - k_0^2 k_z^2 \mu_N \epsilon_{\square,N} \left(\frac{\epsilon_{a,N}}{\epsilon_N} \right)^2 \quad (20)$$

Now, the characteristics equation can be expressed as:

$$s^4 + A_{1,N} s^2 + A_{2,N} = 0 \quad (21)$$

with the following roots:

$$k_{r,2N-1} = \sqrt{\frac{-A_{1,N} + \sqrt{A_{1,N}^2 - 4A_{2,N}}}{2}} \quad (22)$$

$$k_{r,2N} = \sqrt{\frac{-A_{1,N} - \sqrt{A_{1,N}^2 - 4A_{2,N}}}{2}} \quad (23)$$

For various regions, the roots can be considered as:

$$k_r = \begin{cases} k_{r,1}, k_{r,2} & i = 1, 2 \quad ; \text{ for layer } N = 1 \\ \dots & \dots \\ k_{r,2N-1}, k_{r,2N} & i = 2N-1, 2N \quad ; \text{ for layer } N \\ k_{r,2N+1}, k_{r,2N+2} & i = 2N+1, 2N+2 \quad ; \text{ for layer } N+1 \end{cases} \quad (24)$$

In (24), N is the number of the layer and i shows the index of the roots. Now, we write the z-component of the electromagnetic fields in various regions of Fig. 1:

$$H_z^m(\mathbf{r}) = \begin{cases} A_{m,1,1}J_m(k_{r,1}r) + A_{m,2,1}J_m(k_{r,2}r) & r < a_1 \\ A_{m,3,2}J_m(k_{r,3}r) + \\ B_{m,3,2}H_m^{(2)}(k_{r,3}r) + \\ A_{m,4,2}J_m(k_{r,4}r) + \\ B_{m,4,2}H_m^{(2)}(k_{r,4}r) & a_1 < r < a_2 \\ \dots \\ B_{m,2N+1,N+1}H_m^{(2)}(k_{r,2N+1}r) + \\ B_{m,2N+2,N+1}H_m^{(2)}(k_{r,2N+2}r) & r > a_N \end{cases} \quad (25)$$

$$E_z^m(\mathbf{r}) = \begin{cases} A_{m,1,1}T_{1,1}J_m(k_{r,1}r) + A_{m,2,1}T_{2,1}J_m(k_{r,2}r) & r < a_1 \\ T_{3,2} \left(A_{m,3,2}J_m(k_{r,3}r) + B_{m,3,2}H_m^{(2)}(k_{r,3}r) \right) + \\ T_{4,2} \left(A_{m,4,2}J_m(k_{r,4}r) + B_{m,4,2}H_m^{(2)}(k_{r,4}r) \right) & a_1 < r < a_2 \\ \dots \\ T_{2N+1,N+1}B_{m,2N+1,N+1}H_m^{(2)}(k_{r,2N+1}r) + \\ T_{2N+2,N+1}B_{m,2N+2,N+1}H_m^{(2)}(k_{r,2N+2}r) & r > a_N \end{cases} \quad (26)$$

Where

$$T_{i,N} = \frac{1}{-jk_0k_z\epsilon_{\square,N}\left(\frac{\epsilon_{a,N}}{\epsilon_N}\right)} \left(k_{r,i}^2 - k_0^2\epsilon_{\perp,N}\mu_N + k_z^2 \right) \quad i = 2N-1, 2N \quad ; N = 1, 2, \dots \quad (27)$$

The transverse components of electromagnetic fields are obtained as:

$$\begin{pmatrix} E_{r,N} \\ H_{r,N} \end{pmatrix} = \bar{\bar{\mathbf{Q}}}_N^{Pos} \frac{\partial}{\partial r} \begin{pmatrix} E_{z,N} \\ H_{z,N} \end{pmatrix} + \frac{m}{r} \bar{\bar{\mathbf{Q}}}_N^{Neg} \begin{pmatrix} E_{z,N} \\ H_{z,N} \end{pmatrix} \quad (28)$$

$$j \begin{pmatrix} E_{\varphi,N} \\ H_{\varphi,N} \end{pmatrix} = \bar{\bar{\mathbf{Q}}}_N^{Neg} \frac{\partial}{\partial r} \begin{pmatrix} E_{z,N} \\ H_{z,N} \end{pmatrix} + \frac{m}{r} \bar{\bar{\mathbf{Q}}}_N^{Pos} \begin{pmatrix} E_{z,N} \\ H_{z,N} \end{pmatrix} \quad (29)$$

In (28)-(29), the following matrices have been utilized:

$$\bar{\bar{\mathbf{Q}}}_N^{Pos} = \frac{1}{2} \left[\frac{1}{-k_z^2 + k_0^2 \varepsilon_{+,N} \mu_N} \begin{pmatrix} jk_z & -\omega \mu_0 \mu_N \\ \omega \varepsilon_0 \varepsilon_{+,N} & jk_z \end{pmatrix} + \frac{1}{-k_z^2 + k_0^2 \varepsilon_{-,N} \mu_N} \begin{pmatrix} jk_z & \omega \mu_0 \mu_N \\ -\omega \varepsilon_0 \varepsilon_{-,N} & jk_z \end{pmatrix} \right] \quad (30)$$

$$\bar{\bar{\mathbf{Q}}}_N^{Neg} = \frac{1}{2} \left[\frac{1}{-k_z^2 + k_0^2 \varepsilon_{+,N} \mu_N} \begin{pmatrix} jk_z & -\omega \mu_0 \mu_N \\ \omega \varepsilon_0 \varepsilon_{+,N} & jk_z \end{pmatrix} - \frac{1}{-k_z^2 + k_0^2 \varepsilon_{-,N} \mu_N} \begin{pmatrix} jk_z & \omega \mu_0 \mu_N \\ -\omega \varepsilon_0 \varepsilon_{-,N} & jk_z \end{pmatrix} \right] \quad (31)$$

Moreover,

$$\varepsilon_{\pm,N} = \varepsilon_N \pm \varepsilon_{a,N} \quad (32)$$

By applying the boundary conditions,

$$E_{z,N} = E_{z,N+1}, E_{\varphi,N} = E_{\varphi,N+1} \quad (33)$$

$$H_{z,N+1} - H_{z,N} = -\sigma E_{\varphi,N}, H_{\varphi,N+1} - H_{\varphi,N} = \sigma E_{z,N} \quad (34)$$

The final matrix is obtained,

$$\bar{\bar{S}}_{4N,4N} = \begin{pmatrix} A_{m,1,1} \\ A_{m,2,1} \\ \dots \\ B_{m,2N+1,N+1} \\ B_{m,2N+2,N+1} \end{pmatrix}_{4N,1} = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{pmatrix}_{4N,1} \quad (35)$$

In (35), the matrix $\bar{\bar{S}}$ is:

$$\bar{\bar{S}}_{4N,4N} = \begin{pmatrix} T_{1,1} J_m(k_{r,1} a_1) & \dots & \dots & 0 & 0 \\ R_{1,1,1}^J(a_1) & \dots & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & -R_{2N,N+1,2}^H(a_N) \end{pmatrix} \quad (36)$$

In (36), the following relations have been utilized,

$$\begin{pmatrix} R_{i,N,1}^J(\mathbf{r}) \\ R_{i,N,2}^J(\mathbf{r}) \end{pmatrix} = -j \left[\bar{\bar{\mathbf{Q}}}_N^{Neg} k_{r,i} J_m'(k_{r,i} r) + \frac{m}{r} \bar{\bar{\mathbf{Q}}}_N^{Pos} J_m(k_{r,i} r) \right] \begin{pmatrix} T_{i,N} \\ 1 \end{pmatrix} \quad i = 2N-1, 2N; N = 1, 2, \dots \quad (37)$$

$$\begin{pmatrix} R_{i,N,1}^H(\mathbf{r}) \\ R_{i,N,2}^H(\mathbf{r}) \end{pmatrix} = -j \left[\bar{\bar{\mathbf{Q}}}_N^{Neg} k_{r,i} H_m^{(2)}(k_{r,i} r) + \frac{m}{r} \bar{\bar{\mathbf{Q}}}_N^{Pos} H_m^{(2)}(k_{r,i} r) \right] \begin{pmatrix} T_{i,N} \\ 1 \end{pmatrix} \quad i = 2N-1, 2N; N \geq 2 \quad (38)$$

By setting $\det(\tilde{S}) = 0$, the propagation constant and thus other plasmonic features such as the Figure of Merit (FOM) (defined $FOM = Re(k_z)/2\pi Im(k_z)$ [76]) can be achieved.

3. Results and Discussions

In this section, first, we show the validity of our analytical model, and second, we investigate the performance of a new graphene-based cylindrical waveguide. In all reported results in this section, we assume that the thickness of graphene is $\Delta = 0.5 \text{ nm}$, its relaxation time is $\tau = 0.45 \text{ ps}$, and the temperature is $T = 300 \text{ K}$.

Fig. 2. Comparison of simulation (prepared by COMSOL) and analytical results (prepared by our analytical model) for FOM of a graphene-coated nano-wire as a function of frequency. The results have been depicted for the first two modes. The radius of the SiO_2 layer is 90 nm with a permittivity of 2.09 . The chemical potential is supposed to be 0.45 eV . The relaxation time of the graphene layer is 0.45 ps .

Before embarking on the study of the graphene-based cylindrical waveguide containing a gyro-electric layer, we investigate the accuracy of the model. Consider Fig. 2, where a graphene layer is located on a SiO_2 wire, with the permittivity of $\epsilon_{\text{SiO}_2} = 2.09$ and the radius of $R_{\text{SiO}_2} = 90 \text{ nm}$. FOM is one of the most important factors in studying the performance of plasmonic structures. Fig. 2 illustrates the simulation and analytical results of FOM of a graphene-coated nano-wire as a function of frequency. The results have been depicted for the fundamental mode ($m = 0$) and the first mode ($m = 1$). The analytical (dashed lines) and simulation results (solid lines) are prepared by the proposed analytical model and COMSOL software, respectively. In this figure, the SPP wave is a TM mode, because TE plasmonic waves cannot be propagated in this frequency range. FOM increases for the first mode with the increment of frequency but it decreases for the fundamental model. There is an excellent agreement between the analytical and simulation results, which validates the proposed analytical model and shows its high accuracy. Hence, in what follows, we only focus and report the analytical results of the suggested model.

Fig. 3. The cross-section configuration of a cylindrical gyro-electric waveguide with double-layer graphene.

Fig. 3 shows the cross-section of a new cylindrical gyro-electric waveguide with double-layer graphene, constituting graphene-InSb-graphene-SiO₂-Si layers. The permittivity of SiO₂ and Si layers are $\epsilon_{Si} = 11.9$, $\epsilon_{SiO_2} = 2.09$. The chemical potential is $\mu_c = 0.7 \text{ eV}$ unless otherwise stated. The plasma layer is the n-type InSb with the following parameters: $\epsilon_\infty = 15.68$, $n_s = 1.07 \times 10^{17} / \text{cm}^3$, $\nu = 0.314 \times 10^{13} \text{ s}^{-1}$, $m^* = 0.022m_e$ and m_e is the electron's mass. The geometrical parameters in our results are $t_{InSb} = R_{InSb} - R_{SiO_2} = 5 \text{ nm}$, $R_{Si} = 30 \text{ nm}$, $t_{SiO_2} = R_{SiO_2} - R_{Si} = 3 \text{ nm}$, unless otherwise stated.

Fig. 4. FOM versus frequency for various magnetic biases ($B_0 = 1, 2 \text{ T}$) for the first two modes.

As mentioned before, FOM is a key parameter for considering the proficiency of the nanostructures. Fig. 4 shows the analytical results of FOM for the cylindrical gyro-electric waveguide with double-layer graphene for various magnetic biases ($B_0 = 1, 2 \text{ T}$). The results have been depicted for the fundamental mode ($m = 0$) and the first mode ($m = 1$). It should be emphasized that the SPP waves are hybrid TE-TM modes due to the existence of the gyro-electric layer. The fundamental mode ($m = 0$) is cut-off free while the first mode ($m = 1$) has a cut-off frequency (about 31.5 THz). For the fundamental mode ($m = 0$), the FOM for the magnetic bias $B_0 = 1 \text{ T}$ is higher than the FOM for the magnetic bias $B_0 = 2 \text{ T}$ in the frequency range of $f < 25 \text{ THz}$. While the fundamental mode has a higher FOM as the external magnetic bias increases for $f > 25 \text{ THz}$. Furthermore, as seen in Fig. 4 (a), there is a maximum point for the FOM diagram for the frequency range of $f > 25 \text{ THz}$. For instance, the maximum of FOM for the magnetic bias of $B_0 = 2 \text{ T}$ occurs at $f = 33 \text{ THz}$ (FOM reaches 51). It is clear from Fig. 4 (b) that the FOM of the first mode ($m = 1$) does not depend strongly on the magnitude of the external magnetic bias.

Fig. 5 demonstrates the FOM as a function of the Si radius (R_{Si}) for two modes. In this figure, the magnetic bias is 1 T and the frequency is 35 THz. Other geometrical parameters remained fixed ($t_{SiO_2} = 3 \text{ nm}$, $t_{InSb} = R_{InSb} - R_{SiO_2} = 5 \text{ nm}$). As seen in this figure, the first mode has a cut-off radius (18 nm). Therefore, the structure must be designed for $R_{Si} < 18 \text{ nm}$ to operate as the single-mode.

In Fig. 6, we have depicted the FOM as a function of the SiO₂ thickness ($t_{SiO_2} = R_{SiO_2} - R_{Si}$) for the first two modes. Similar to Fig. 5, the applied bias is 1 T and the operation frequency is supposed to be 35 THz in this figure. Moreover, other geometrical parameters are $R_{Si} = 30 \text{ nm}$, $t_{InSb} = R_{InSb} - R_{SiO_2} = 5 \text{ nm}$. One can observe that FOM is very low for $t_{SiO_2} \rightarrow 0$. Indeed, for thin thicknesses of SiO₂, the propagation length is very low and thus the

SPP wave cannot propagate. It is seen from this figure that the fundamental mode has a higher FOM compared to the first mode.

As a final point, the FOM variations have been represented as a function of the chemical potential in Fig. 7. Similar to Fig. 5, 6, the magnetic bias is 1 T and the operation frequency is 35 THz. The geometrical parameters in this figure are $R_{Insb} = 38 \text{ nm}$, $R_{SiO_2} = 33 \text{ nm}$, $R_{Si} = 30 \text{ nm}$. As observed in this figure, the FOM for the fundamental mode increases with chemical potential increment. However, for the first mode, there is a maximum point that occurs in the vicinity of 0.7 eV in which the FOM reaches 28 at this point.

Fig. 5. Dependence of FOM on the Si radius (R_{Si}). The frequency is 35 THz.

Fig. 6. Dependence of FOM on the SiO₂ thickness ($t_{SiO_2} = R_{SiO_2} - R_{Si}$). The frequency is 35 THz.

Fig. 7. FOM variations versus the chemical potential (μ_c). The frequency is 35 THz.

4. Conclusion

In this paper, we analytically studied Surface Plasmon Polaritons (SPPs) in graphene-based cylindrical structures with gyroelectric layers. In the general waveguide, each graphene layer was surrounded by two various gyro-electric materials. New closed-form relations were derived for the field contributions of SPP waves. As a special case, a gyro-electric cylindrical waveguide with double-layer graphene was considered in this article. For the designed structure, the figure of merit (FOM) of 51 was reported for $B = 2 \text{ T}$ and $\mu_c = 0.7 \text{ eV}$, at the frequency of 33 THz. It was demonstrated that the FOM of the studied waveguide could be varied by tuning the chemical potential and the

magnetic bias. Harnessing the gyro-electric media together with graphene layers will open a new promising platform for the design and fabrication of novel plasmonic devices in the mid-infrared region.

Declarations

Ethics Approval: Not Applicable.

Consent to Participate: Not Applicable.

Consent to Publish: Not Applicable.

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