Fully Coupled Stochastic Geomechanical-Geochemical - Reservoir Modeling for Fracture Reservoirs

dulian Dushan zeqiraj (✉ d_zeqiraj@hotmail.com)

Faculty of Geology and Mining, Department of Energy Resources, Tirana, Albania

Research article

Keywords: Fully coupled modelling, hydro-geomechanical-geochemical processes, fracture reservoirs

Posted Date: December 23rd, 2020

DOI: <https://doi.org/10.21203/rs.3.rs-129792/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

FULLY COUPLED STOCHASTIC GEOMECHANICAL-GEOCHEMICAL - RESERVOIR MODELING FOR FRACTURE RESERVOIRS

¹ Dulian Zeqiraj

¹ Faculty of Geology and Mining, Department of Energy Resources, Tirana, Albania

Email: d_zeqiraj@hotmail.com

Abstract

Carbonate reservoirs contain a significant portion of the world's oil reserves. The uptake of oil from these reservoirs is difficult due to the complex hydro-thermo-geomechanical-geochemical processes. In recent years many efforts have been made to model these processes in fractured rocks. A few studies have been done on the stochastic nature of these processes. Geomechanical models need to take into account the reservoir rock deformations that are the result of the pores and fracture pressure, while geochemical models deal with the chemical reactions that take place into fracture and matrix with their principal processes such dissolution and precipitation. In this paper we will make the modeling from the stochastic point of view of these processes by making a model to deal with such parameters and coefficients that are stochastic, seeing them as random variables. In our knowledge this is the first work of this kind. From the geomechanical point of view we will use the stochastic nature of the Biot linear model with uncertain coefficients, while for the stochastic geochemical model we will use the Langrazhian formulation.

Key words: Fully coupled modelling; hydro-geomechanical-geochemical processes; fracture reservoirs.

Introduction

A solid-to-fluid coupling that occurs when change in applied stress produces a change in fluid pressure or fluid mass and vice-versa a fluid-to-solid coupling produces change to fluid pressure and mass. In other words dynamic porous media behavior is described by means of Biot theory of poroelasticity. The deterministic Biot model is a early and popular model that describes the coupled response of a linear elastic porous media and a fluid in it for small deformations and is based in moment conservation and mass conservation law. Many of works done for flow in fracture media uses Stokes flow in fracture and Biot poroelasticity model for fluids in the other zones of poroelasticity properties. After Biot model there has been a broad spectrum of work with Terzaghi etc. Then the Biot model has been coupled with Finite Element Method in past works for describing the fluid flow in a deformable porous media. But rarely has been proved the robustness and uncertainty quantification for these coupled models. Let start by considering two field Biot model from [1].

$$-\nabla \cdot \sigma = f \quad \text{in } D \quad (1)$$

$$-\nabla \cdot u - \lambda^{-1}(p_T - \alpha p_F) = 0 \quad \text{in } D \quad (2)$$

$$\lambda^{-1}(\alpha p_T - \alpha^2 p_F) - s_0 p_f + \nabla \cdot (k \nabla p_f) = g \quad \text{in } D \quad (3)$$

Where

the stress tensor is defined as $\sigma := 2\mu\epsilon(u) - p_T \mathbf{I}$ and:

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad s_0 = \frac{\alpha - \phi}{2\mu d^{-1} + \lambda} \quad x \times D, y \in \Gamma_y \quad (4)$$

In reality values of E, k, v, α and s_0 are not known with certainty, due to the measurement errors or in some cases they vary in large scale. Botti et al. [2] discussed the Biot stochastic model and found a numerical solution using polynomial chaos, but without a robust analysis. As we said before there has been a lot of work for the deterministic Biot model, but there has been little work for the robust stochastic Biot model. One of the methods to deal with the uncertainty is the Uncertainty Quantification (UQ) method. This method has been developed in the last decades to tackle the problem of uncertainty input of parameters. The principle of these methods is to decompose random quantities on suitable stochastic approximation bases.

To define the new stochastic model let express E and k as function of the following form according to **Arbaz Khan et al [6]**:

$$E(x, y) = e_0(x) + \sum_{k=1}^{M_1} e_k(x) y_k, \quad x \in D, y \in \Gamma_y := \Gamma_1 \times \dots \times \Gamma_{M_1}, \quad (6)$$

$$k(x, z) = k_0(x) + \sum_{k=1}^{M_1} K_k(x) z_k, \quad x \in D, z \in \Gamma_z := \Gamma_1 \times \dots \times \Gamma_{M_2}, \quad (7)$$

Where y and z are vector of parameters, $y = (y_1, \dots, y_{M_1})$ and $z = (z_1, \dots, z_{M_2})$ and random variables. Assuming that $y_k, z_k \in \Gamma_k = [-1, 1]$ for each k and $\Gamma := \Gamma_y \times \Gamma_z$ the problem now is: Find $u : D \times \Gamma \rightarrow \mathbb{R}^d$ and $p_T, p_F : D \times \Gamma \rightarrow \mathbb{R}$ such that:

$$-\nabla \cdot \sigma(x, Y) = f(x) \quad \text{in } D \times \Gamma \quad (8)$$

$$-\nabla \cdot u(x, Y) - \frac{(p_T(x, Y) - \alpha p_F(x, Y))}{\lambda(x, y)} = 0 \quad \text{in } D \times \Gamma \quad (9)$$

$$\frac{\alpha p_T(x, Y)}{\lambda(x, y)} - \left(\frac{\alpha^2}{\lambda(x, y)} + s_0(x, y) \right) p_F(x, Y) + \nabla \cdot (k(x, z) \nabla p_F(x, Y)) = g(x) \quad \text{in } D \times \Gamma. \quad (10)$$

From (6) and (7) we notice that they have the same structure as **Karhunen–Loève** expansions, that is:

$$\mu(x, y) = \frac{E(x, y)}{2(1 + v)}, \quad \lambda(x, y) = \frac{E(x, y)v}{(1 + v)(1 - 2v)}, \quad x \in D, y \in \Gamma_y. \quad (11)$$

Stochastic Galerkin Mixed Finite Element Method (SG-MFEM).

Following A. Khan, C.E. Powell [6] the system of equations implementing Stochastic Galerkin Mixed Finite Element Method (SG-MFEM) can be written in the following form:

$$-\nabla \cdot u(x, Y) = f(x) \quad \text{in } D \times \Gamma, \quad (12)$$

$$-\nabla \cdot u(x, Y) - \tilde{\lambda}^{-1} p_1(x, Y) = 0 \quad \text{in } D \times \Gamma, \quad (13)$$

$$\tilde{\lambda}^{-1} \alpha p_1(x, Y) - \tilde{s}_0 p_2(x, Y) + \nabla \cdot (K(x, z) \nabla p_F(x, Y)) = g(x) \quad \text{in } D \times \Gamma \quad (14)$$

$$-\tilde{\lambda}^{-1} (p_T(x, Y) - \alpha p_F(x, Y)) + \tilde{\lambda}^{-1} E(x, y) p_1(x, Y) = 0 \quad \text{in } D \times \Gamma, \quad (15)$$

$$-\tilde{s}_0 p_F(x, Y) + \tilde{s}_0 E(x, y) p_2(x, Y) = 0 \quad \text{in } D \times \Gamma. \quad (16)$$

Now we need to discretize them in a matrix form

With the assumed ordering of the degrees of freedom, the bloks of the coefficient matrix in are then given by:

$$\mathcal{A} := \begin{pmatrix} \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{11}^k & \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{21}^k & 0 & 0 \\ \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{12}^k & \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{22}^k & 0 & 0 \\ 0 & 0 & \tilde{\lambda} \sum_{k=0}^{M_1} G_k \otimes \tilde{C}_k & 0 \\ 0 & 0 & 0 & \tilde{s}_0 \sum_{k=0}^{M_1} G_k \otimes \tilde{C}_k \end{pmatrix} \quad (18)$$

$$B := \begin{pmatrix} 0 & 0 & \alpha \tilde{\lambda}^{-1} \otimes C_b & \tilde{s}_0 I \otimes C_b \\ I \otimes B_1 & I \otimes B_2 & -\tilde{\lambda}^{-1} I \otimes C & 0 \end{pmatrix}, \quad C := \begin{pmatrix} \sum_{k=0}^{M_2} \tilde{G}_k \otimes D_k & 0 \\ 0 & 0 \end{pmatrix},$$

Where $\tilde{G}_0 = G_0 = I$ (the $n_y \times n_y$ identity matrix) and $\tilde{G}_k = G_{k+M_1}$ for $k=1, \dots, M_2$.

The linear system in so-called kronecker form is as follow

$$\left(G_0 \otimes K_0 + \sum_{k=1}^{M_1} G_k \otimes K_k + \sum_{k=1}^{M_2} \tilde{G}_k \otimes \tilde{K}_k \right) x = z$$

Where the solution vector has form $x = (x_1^T, \dots, x_{n_y}^T)^T$, with

$$x_j^T := (u_{1,j}^T, u_{2,j}^T, p_{1,j}^T, p_{2,j}^T, p_{F,j}^T, p_{T,j}^T)^T, \quad j = 1, \dots, n_y.$$

$$\text{We have then } K_0 := \begin{pmatrix} \tilde{\mu} A_{11}^0 & \tilde{\mu} A_{21}^0 & 0 & 0 & 0 & B_1^T \\ \tilde{\mu} A_{12}^0 & \tilde{\mu} A_{22}^0 & 0 & 0 & 0 & B_2^T \\ 0 & 0 & \tilde{\lambda}^{-1} \tilde{C}_0 & 0 & \alpha \tilde{\lambda}^{-1} C_b^T & -\tilde{\lambda}^{-1} C \\ 0 & 0 & 0 & \tilde{s}_0 \tilde{C}_0 & \tilde{s}_0 C_b^T & 0 \\ 0 & 0 & \alpha \tilde{\lambda}^{-1} \tilde{C}_b & \tilde{s}_0 \tilde{C}_b & D_0 & 0 \\ B_1 & B_2 & -\tilde{\lambda}^{-1} C & 0 & 0 & 0 \end{pmatrix} z = g_0 \otimes \begin{pmatrix} f_1 \\ f_2 \\ 0 \\ 0 \\ g \\ 0 \end{pmatrix} \quad (20)$$

and for $k = 1, \dots, M_1$ and $\ell = 1, \dots, M_2$; we have:

$$K_k := \left(\begin{array}{cccc|cc} \tilde{\mu}A_{11}^k & \tilde{\mu}A_{21}^k & 0 & 0 & 0 & 0 \\ \tilde{\mu}A_{12}^k & \tilde{\mu}A_{22}^k & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\lambda}^{-1}\tilde{C}_k & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{s}_0\tilde{C}_k & 0 & 0 \\ 0 & 0 & 0 & 0 & D_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right), \quad \tilde{K}_\ell := \left(\begin{array}{cccc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_\ell & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$K_0 X + \sum_{k=1}^{M_1} K_k X G_k + \sum_{\ell=1}^{M_2} \tilde{K}_\ell X \tilde{G}_\ell = Z \quad (21)$$

This the final matrix form for stochastic (uncertain) Biot poroelasticity coefficients

Simulating Complex Reactions on Particles

Lagrange methods have the advantage that they can take into account non-Fickian transport even in multiporous systems. These methods discretize a finite number of particles where each represents a part of the total mass and then cause these particles to move in accordance with simple mechanisms of: advection, dispersion, diffusion. In this way they avoid the usual Eulerian reactions, more precisely numerical dispersions and oscillations. But what happen when it comes to chemical kinetic reactions that can occur between these particles? These methods then take into account the case when the absorption reactions are of a slow nature and in this way can be treated with transition probabilities. In the case when we have nonlinear chemical reactions involving more than one chemical species, these methods fail because in this case the concentrations must be re-evaluated at every step of the time. This is a disadvantage because most of the common processes in geochemistry are complex, nonlinear, there is more than one species in their composition, and rock water reactions are quite substantial. A good approach would be a Euler-Lagrange combination where the coefficients of reactions are determined by concentrations, but these processes are time-consuming. Parkhurst and Wissmeier, 2015, developed a new module, Phreeqcrm, a special module of Phreeqc, which is based on highly sophisticated algorithms, but which are also those of a deterministic nature. In principle it seems a good idea a Lagrangian-Phreeqcrm combination. In this combination each particle can be seen as a bin containing a defined volume of water and thus reactions occur within the particles in accordance with their concentrations. However, we must be realistic because we need to constantly inject empty particles.

We introduce here briefly the algorithm that is based on random walk particle tracking method. It calculates the reaction probability by combining the probability that two particles will merge and the thermodynamic probability that two particles will react, given the collocation

$$P(\text{react}) = P(\text{react}|\text{colocation}) \times P(\text{colocation})$$

The latter colocation probability depends on time interval and separation:

$$P(s|\Delta t) = \int_{\Delta s} v(s|\Delta t) ds \approx v(s|\Delta t) \Delta s \quad (2)$$

In the above formula s is the distance between two particles, v the density function, t is the time. The colocation density function is:

$$v_{i,j}(s|\Delta t) = \frac{\exp\left[-\frac{1}{4\Delta t} s^T (\mathbf{D}_i + \mathbf{D}_j)^{-1} s\right]}{\sqrt{(4\pi\Delta t)^d \det[\mathbf{D}_i + \mathbf{D}_j]}} \quad (3)$$

For the particles that are close to each other in such way that they can exchange mass in proportion with the velocity we can write the following equation:

$$m_j(t + \Delta t) - m_j(t) = \frac{1}{2} \sum_{i \neq j} (m_i - m_j)^* v_{i,j}(s|\Delta t) \Delta s \quad (4)$$

According to [6], **Benson and Bolster, 2016**, masses may update according by the following equation

$$\begin{aligned} & \left(1 + \frac{1}{2} \sum_{i \neq j} v_{i,j}(s|\Delta t) \Delta s\right) m_j(t + \Delta t) - \\ & \frac{1}{2} \sum_{i \neq j} v_{i,j}(s|\Delta t) \Delta s m_i(t + \Delta t) = m_j(t) \end{aligned} \quad (5)$$

Implementation of CPR (Complex Particle Reactions) (MATLAB code, [2], Nicholas. B)

DEFINE THE REACTION HERE

Reaction: $\text{CaCO}_3(s) + 2\text{H}^+(aq) \rightarrow \text{Ca}^{2+}(aq) + \text{CO}_2(g) + \text{H}_2\text{O}$;-----Kinetic
 Reaction: $\text{CaCO}_3 \rightleftharpoons \text{Ca}^{2+} + \text{CO}_3^{2-}$ -----Equilibrium
 kf=1.0; -----Kinetic rate constant-----lam=1e-3; -----Decay rate
 A0=1.0; -----B0=1.15; ----- E0=1.25; Initial concentration

Figure 1. Initial Concentration of particles. Number of particles = 2500. (From MATLAB code, [2], Nicholas. B)

Figure 2. Concentration of particles after t=10-time unity towards positions. (From MATLAB code, [2], Nicholas. B)

Figure 3. Concentration of particles after t=10-time unity. (From MATLAB code, [2], Nicholas. B)

Fully coupled stochastic geomechanical-geochemical - reservoir modeling for fracture reservoirs

Our model now incorporates even the reservoir model that is a black oil model. We have used the MRST-SINTEF software to incorporate all the three parts of the fully coupled geomechanical-

geochemical - reservoir modeling for fracture reservoirs. Our is a 2D model that has 1 injection well in the origin (0,0) and producer well in the upper right corner, so the wells are settled in a diagonal manner. The algorithm of the proposed model is given bellow:

Algorithm of the proposed model

1. Initialize the black oil reservoir model at $t = 1s$
 - 1.a Initialize the stochastic geomechanical model at $t + \Delta t$
 - 1.b Solve the stochastic geomechanical model at $t + \Delta t$
 - 1.c Update the Biot stochastic coefficient of the photoelasticity for the next step
2. Initialize the stochastic geochemical Lagrangian model at $t + \Delta t$
 - 2.a Solve the stochastic geomechanical model at $t + \Delta t$
 - 2.b Update the chemical coefficients at large Darcy scale for the next step
 - 2.c Update the relative permeability in base of calcite dissolution
3. Return to step 1 solve the reservoir model at time $t + \Delta t$
4. Repeat the procedure until to the final time of simulation

We have used here mrst-SINEF an open-source software in MATLAB and further developing with our algorithm.

Figure 4. Pressure and saturation for the deterministic case left and the stochastic geochemical geomechanical model right for three phase black oil model with one injection well in origin and one production well in upper right corner, see [appendix](#) for video simulation of the above (MRST-SINTEF, D. Zeqiraj)

Conclusions

To the best of our knowledge, we have for the first time been able to create a complex model that assembles the stochastic features of the Biot poroelastic linear model with the Lagrangian geochemical stochastic model based on the software of the American Geological Survey Phreeqcrm. The problem is quite complex and in the future the possibility to implement it in the three dimensional case should be considered, although the difficulties on the horizon seem great not for building the model but for the time it takes on the computer, so models need to be found and algorithms to be developed for the more intelligent ways to "mitigate" this problem as delicate as it is challenging

Appendix

(supplementary material---

<https://drive.google.com/file/d/1AMhAI4az6LjdoR5HMmjovUvG7Lcw7nzz/view?usp=sharing>)

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

The author declares He have no competing interests

Funding

Author of manuscript declares that the data for research are theoretical data, but in case of disponible of real data the program in MATLAB would have real value.

Authors' contributions

I am the only corresponding author of this manuscript

Acknowledgements

The author of this work acknowledgement the akademik and personnel assistance staff of Department of Energy Resorces, Faculty of Geology and Minning, Tirana, Albania for their distinguished support for realizing this work.

References

- [1] Jeonghun J Lee, Kent-Andre Mardal, and Ragnar Winther. (Parameter-robust discretization and preconditioning of Biot's consolidation model). SIAM Journal on Scienti_c Computing, 39(1): A1{A24, 2017
- [2] Nicholas B. Engdahl , David A. Benson , and Diogo Bolster:"Lagrangian simulation of mixing and reactions in complex geochemical systems", 2017
- [3] Michele Botti, Daniele A Di Pietro, Olivier Le Matre, and Pierre Sochala. Numerical approximation of poroelasticity with random coefficients using polynomial chaos and hybrid high-order methods. arXiv preprint arXiv:1903.11885, 2019.

- [4] Jeonghun J Lee, Kent-Andre Mardal, and Ragnar Winther. Parameter-robust discretization and preconditioning of Biot's consolidation model. *SIAM Journal on Scientific Computing*, 39(1):A1{A24, 2017.
- [5] Ricardo Oyarzua and Ricardo Ruiz-Baier. Locking-free finite element methods for poroelasticity. *SIAM Journal on Numerical Analysis*, 54(5):2951{2973, 2016.
- [6] Arbaz Khan, Catherine E Powell, and David J Silvester. Robust preconditioning for stochastic Galerkin formulations of parameter-dependent nearly incompressible elasticity equations. *SIAM Journal on Scientific Computing*, 41(1):A402{A421, 2019
- [7] Arbaz Khan, Catherine E. Powell. Parameter-robust stochastic Galerkin mixed approximation for linear poroelasticity with uncertain inputs
- [8] Paster, A., T. Aquino, and D. Bolster (2015), Incomplete mixing and reactions in laminar shear flow, *Physical Review E*, 92 (1), 012,922, doi:10.1103/PhysRevE.92.012922.
- [9] Pedretti, D., A. Lassin, and R. D. Beckie (2015), Applied Geochemistry Analysis of the potential impact of capillarity on long-term geochemical processes in sulphidic waste- rock dumps, *Applied Geochemistry*, 62, 75{83, doi:10.1016/j.apgeochem.2015.03.017.
- [10] Steefel, C., et al. (2015), Reactive transport codes for subsurface environmental simulation, *Computational Geosciences*, 19 (3), 445{478, doi:10.1007/s10596-014-9443-x.
- [11] Tompson, A. F. B., and D. Dougherty (1992), Particle-grid methods for reacting flows in porous-media with application to Fisher equation, *Appl. Math. Model.* 16 (7), 374{383.

Figures

Figure 1

Initial Concentration of particles. Number of particles = 2500. (From MATLAB code, [2], Nicholas. B)

Figure 2

Concentration of particles after t=10-time unity towards positions. (From MATLAB code, [2], Nicholas. B)

Figure 3

Concentration of particles after $t=10$ -time unity. (From MATLAB code, [2], Nicholas. B)

Figure 4

Pressure and saturation for the deterministic case left and the stochastic geochemical geomechanical model right for three phase black oil model with one injection well in origin and one production well in upper right corner, see appendix for video simulation of the above (MRST-SINTEF, D. Zeqiraj)