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AmirAbbas Eslami Shafigh (✉ amir.shafig.86@gmail.com)

Riaam

Pante'a Davoudifar

Riaam

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Imitating Hadron Production with PHIT

AmirAbbas EslamiShafiq, Pante'a Davoudifar.

Abstract

We announce PHIT as a numerical model for simulating of hadroproduction and compare our results with other models and experimental data. Our code, although very simple, imitates the expected results acceptably compared to other more detailed physical models. Moreover, PHIT is fast and easily executable on an ordinary PC. These advantages make PHIT an ideal choice for practical applications of an event generator.

Introduction

The energy of a primary cosmic ray which begins the cascade of secondary particles, might exceed 10^{11} GeV. Recently, the Large Hadron Collider (LHC) at the CERN Laboratory has allowed us to access, for the first time, the energy region above 10^8 GeV. Hence, the range of energy in which the essence of extensive showers are developed, is mostly beyond the access of current technology and due to experimental shortcomings, particle physicists have failed to obtain a reliable theory of high energy particles interaction and they have to refer to air shower data to modulate their models and test their theorems in this field. Extensive air showers serve particle physics as a natural high energy laboratory.

Modeling of hadronic particle production is subject to large theoretical uncertainties [1]. None of the current models are able to give a fully consistent picture of the different observables within a given experiment [2]. As a result, any simulations of the extensive air showers based on current hadronic

interaction models will suffer from lack of certainty and full consistence with empirical data. Also, the interpretation of extensive air shower measurements, which relies strongly on the models suffers from these uncertainties. A review of common hadronic interaction models, occupied for simulating extended air showers, can also be found in [3].

It is obvious that in practical applications and researches which focus on the macroscopic manifestation of air cascades and how they might affect the other related phenomena or technologies potentially influenced by energetic particles, the diversity of models and their uncertainties might be a source of confusion. In this sort of researches, an ideal simulation is supposed to generate realistic and accurate output and the user is not interested in microscopic physics and the theoretical debates going on there.

In this work, we aim to develop a code which simulates the properties of generated hadrons based on the random choice of values from the empirical fits to experimental data. Produced Hadrons Imitation Tool, **PHIT**, is a

code, written in *FORTTRAN90*, and is desired to be independent from any particle interactions model and theoretical assumptions. Since some fit functions are based on specific models and theorems, this target cannot be totally fulfilled; but at least, we can assure that the used relations and distributions have the highest conformity with measurements.

Here we limit our research to the case of $p(\bar{p})p$ collisions; which are known to be the most elementary interactions and form the basis of our theories regarding the nature of high energy collisions. Also, all the interactions are non-diffractive in order to be comparable with non-diffractive collider data. The code can be extended to handle hadron-nucleus collisions in later steps. Also, in this version of PHIT, there is a tendency to keep the results near the average by not allowing fluctuation of individual multiplicities and forcing the mean transverse momentum of each event to follow the experimental average. In this paper, we describe the algorithm of PHIT and then we compare our results with other models and empirical data. Finally, as a pure output of our simulation, we study the dependence of inelasticity to collision energy.

I. The Algorithm

The main input of PHIT in this limited case of study is the collision energy. Almost all other parameters are determined based on the center of momentum energy, $\sqrt{s}$, and the type of the projectile and target particles which are antiproton and proton. PHIT begins by determining the charge multiplicity. Then, based on the multiplicity, the number of produced hadrons of each type is chosen. In the next step, transverse momentum and pseudorapidity of secondaries is specified and

finally, the properties of the leading particles are calculated and some corrections are made to secondaries to satisfy the conservation laws. We divide this procedure to seven main steps and discuss them in detail.

I-I Total Charge Multiplicity

The charge multiplicity, n_{ch} describes the number of produced charged particles per event. In PHIT, there are two available options for random choice of n_{ch} . One is the well-known negative binomial distribution (NBD). The measurements of charged particle multiplicity distribution have been known to be well described by NBD for more than three decades [4]. The other option is the recently suggested Shifted Gompertz Distribution (SGD) which has been extensively used in diffusion theory, social networks, and forecasting and now, first attempt to analyze the multiplicity data with SGD has produced good results [5]. Both distributions have two energy dependent parameters, however in very different forms.

NBD probability function can be written as:

$$P_{(n_{ch}),k}^{NBD}(n_{ch}) = \binom{n_{ch} + k - 1}{n_{ch}} \left(\frac{\langle n_{ch} \rangle / k}{1 + \langle n_{ch} \rangle / k} \right)^n \frac{1}{(1 + \langle n_{ch} \rangle / k)^k} \quad (1)$$

Based on empirical data in full rapidity range, dependence of average charge multiplicity, $\langle n_{ch} \rangle$, on $\sqrt{s}$ can be described reasonably well by a variety of expressions [6] [7] [8]:

$$\langle n_{ch} \rangle = 0.57 + 0.584 \ln(\sqrt{s}) + 0.127 \ln^2(\sqrt{s}) \quad (2)$$

$$\langle n_{ch} \rangle = 16.65 - 3.147 \ln(\sqrt{s}) + 0.334 \ln^2(\sqrt{s}) \quad (3)$$

$$\langle n_{ch} \rangle = 1.3 + 0.62 \ln(\sqrt{s}) + 0.59 \ln^2(\sqrt{s}) \quad (4)$$

Also, at higher energies, some researchers suggest a power law fit to the measured data [6] [8] [9] [10]:

$$\langle n_{ch} \rangle = 3.4 \sqrt{s}^{0.34} \quad (5)$$

$$\langle n_{ch} \rangle = -10.5 + 9.9 \sqrt{s}^{0.22} \quad (6)$$

$$\langle n_{ch} \rangle = 3.102 \sqrt{s}^{0.356} \quad (7)$$

$$\langle n_{ch} \rangle = 2.328 \sqrt{s}^{0.402} \quad (8)$$

We use a combination of (2) for $\sqrt{s} < 9.34 \text{ GeV}$ and (8) for higher energies. There are different fits for the other NBD parameter, k ; For instance, [11]. PHIT calculates it using this relation [8]:

$$\frac{1}{k} = -0.11 + 0.06 \ln(\sqrt{s}) \quad (9)$$

SGD probability function is parametrized as [5]:

$$P_{b,\eta}^{SGD}(n_{ch}) = b e^{-bn_{ch}} e^{-\eta e^{-bn_{ch}}} (1 + \eta(1 - e^{-bn_{ch}})) \quad (10)$$

$$b = 1.514 \sqrt{s}^{-0.459} \quad (11)$$

$$\eta = 357.7 \sqrt{s}^{-0.524} \quad (12)$$

The neutral particles multiplicity, n_N , has rarely been studied directly; because of the difficulty of detecting chargeless particles. One case, for example, is [12]:

$$\langle n_N \rangle \approx 1 + 0.52 \langle n_{ch} \rangle \quad (13)$$

In PHIT, we prefer not to force a total neutral multiplicity. Instead, the number of each neutral hadron is calculated using a relation with

the charge multiplicity of the same type. These relations are discussed in the next section.

Both distributions are very successful; except for somewhere between the collision energy of 540 GeV and 900 GeV where some deviations between the multiplicity data and these fits emerge. This shoulder structure in the multiplicity distribution has been known, since the results from UA5 collaboration were published [4]. It has been shown [13] that the violations can be interpreted as the weighted superposition of two sort of events, soft and semi-hard. The multiplicity distribution of each component can still be fitted by either of introduced formulae. This idea has been successfully implemented in several analyses at high energies with (at least) five free parameters [14] [15] [5]. Unfortunately, the energy dependence of these parameters in a wide range of rapidity has not been studied yet. This imperfection, of course, influences our results at collision energies about and above 1 TeV and limits the validity of our extrapolations. In the next versions, we will try to improve the code with two component multiplicity fits.

I-2 Multiplicity of Each Type

The procedure of determining the number of each type of produced hadrons begins with calculating the average multiplicity of each race. Then, based on the $n_{ch}/\langle n_{ch} \rangle$ ratio, the quantity of each sort of particles is selected. At last, the actual multiplicity of all hadrons in each event must be picked using this average value from a Poisson distribution [12] but we haven't applied this step to the current version of PHIT and the average values are used to reach a better estimation of inelasticity, the main pure output of the code. Here, we describe the detailed algorithm of average multiplicity calculations.

Charged Pion multiplicity can be related to the number of Kaons and Protons by defining a ratio coefficient:

$$\begin{aligned} n_{Ch} &= n_{p^\pm} + n_{k^\pm} + n_{\pi^\pm} \\ &= n_{\pi^\pm} (R_{p^\pm/\pi^\pm} + R_{k^\pm/\pi^\pm} + 1) \end{aligned} \quad (14)$$

$$\Rightarrow n_{\pi^\pm} = \frac{n_{Ch}}{(R_{p^\pm/\pi^\pm} + R_{k^\pm/\pi^\pm} + 1)} \quad (15)$$

Having measured the values of $R_{p^\pm/\pi^\pm}$ and $R_{k^\pm/\pi^\pm}$, we could determine the coefficient of proportionality of $n_{\pi^\pm}$ to n_{Ch} . Following the recent studies on these ratios [16] [17], we use these fits to empirical data:

$$R_{p^\pm/\pi^\pm} = 0.06 e^{-3.78/s^{0.25}} \quad (16)$$

$$R_{k^\pm/\pi^\pm} = 0.13 e^{-2.30/s^{0.25}} \quad (17)$$

In a similar way, we define n_{π^-} as a function of $n_{\pi^\pm}$:

$$n_{\pi^\pm} = n_{\pi^+} + n_{\pi^-} = n_{\pi^-} (R_{\pi^+/\pi^-} + 1) \quad (18)$$

$$\Rightarrow n_{\pi^-} = \frac{n_{\pi^\pm}}{R_{\pi^+/\pi^-} + 1} \quad (19)$$

At lower energies, this ratio can be determined directly from fits to empirical data [18]:

$$R_{\pi^+/\pi^-} = \frac{-1.55 + 0.82 \log(s) + 0.79/\sqrt{s}}{-2.98 + 0.94 \log(s) + 3.31/\sqrt{s}} \quad (20)$$

At higher energies, having the Kaon to Pion ratios, we can calculate R_{π^+/π^-} using R_{k^+/π^-} :

$$\begin{aligned} R_{\pi^+/\pi^-} &= R_{\pi^+/\pi^-} \left(\frac{R_{k^+/\pi^-}}{R_{k^+/\pi^-}} \right) = \left(\frac{R_{\pi^+/\pi^-}}{R_{k^+/\pi^-}} \right) R_{k^+/\pi^-} \\ &= \left(\frac{\pi^+/\pi^-}{k^+/\pi^-} \right) R_{k^+/\pi^-} \\ &= \left(\frac{k^-/\pi^-}{k^+/\pi^+} \right) R_{k^+/\pi^+} \end{aligned} \quad (21)$$

$$\Rightarrow R_{\pi^+/\pi^-} = \frac{R_{k^-/\pi^-}}{R_{k^+/\pi^+}} R_{k^+/\pi^+} \quad (22)$$

Inserting (16) and (17):

$$\begin{aligned} R_{\pi^+/\pi^-} &= \frac{0.13 e^{-2.15/s^{0.25}}}{0.11 e^{-2.05/s^{0.25}}} R_{k^+/\pi^+} \\ &= (1.18 e^{-0.10/s^{0.25}}) R_{k^+/\pi^+} \end{aligned} \quad (23)$$

This equation is solved using the fit obtained from [19]:

$$R_{k^+/\pi^+} = 0.79 e^{3.43/s^{0.25}} \quad (24)$$

Charged Kaon multiplicity can be calculated from the relations already described. The total number of charged Kaons already is in hand from (17) and the contribution of positives and negatives is given by (24), considering:

$$n_{k^-} = \frac{n_{k^\pm}}{R_{k^+/\pi^+} + 1} \quad (25)$$

Proton multiplicity is also already attained, knowing the multiplicity of the two other types and the sum of all charged particles numbers. The portion of positives and negatives is computed based on these relations [20]:

$$n_{p^-} = \frac{n_{p^\pm}}{R_{p^+/\pi^+} + 1} \quad (26)$$

$$R_{p^+/\pi^+} = R_{k^+/\pi^+}^{4.7619} \quad (27)$$

Neutral Pion Multiplicity cannot be measured directly because of the very short life

time of π^0 . Once they are produced, almost all of neutral pions decay to two gamma photons. So, the average number of π^0 should be around half the number of detected photons, n_γ :

$$n_{\pi^0} \approx \frac{1}{2} n_\gamma \quad (28)$$

However, At high energies, mainly due to η meson production with subsequent decay into pions and photons, a larger fraction of photons than expected by $n_{\pi^0} \approx \frac{1}{2} n_\gamma$

(28) are observed. So, with need to subtract the number of η -originated photons, $(n_\gamma)_\eta$, from the total n_γ to estimate the number of generated neutral pions. $(N_\gamma)_\eta$ is dependent on n_η , which fortunately has already been empirically studied [21] along with the decay modes and the branching ratios of η mesons.

The η mesons mainly decay to two photons or, by three-body decays, to pions, occasionally accompanied by gamma radiation. Considering all decay modes, listed in Table I, and their fraction [22], one can estimate the average number of emitted photons in an η decay:

$$(n_\gamma)_\eta \approx 3.33 n_\eta \quad (29)$$

Hence, the corrected n_{π^0} would be:

$$n_{\pi^0} \approx \frac{1}{2} (n_\gamma - (n_\gamma)_\eta) \Rightarrow n_{\pi^0} \approx 0.5 n_\gamma - 1.67 n_\eta \quad (30)$$

From this, we can deduce:

$$n_{\pi^0} \approx \frac{n_\gamma}{2 + 3.3 R_{\eta/\pi^0}} \quad (31)$$

Now, by defining the dependence of n_{π^0} on n_γ and R_{η/π^0} , we are able to insert experimental fits below to our relation and calculate n_{π^0} [23], [24]:

$$n_\gamma = \begin{cases} -1.27 + 0.52 \ln(s) + 0.148 \ln^2(s), & \sqrt{s} \leq 103 \text{ GeV} \\ -18.7 + 11.55 s^{0.1195}, & \sqrt{s} > 103 \text{ GeV} \end{cases} \quad (32)$$

$$R_{\eta/\pi^0} = 0.06 + 0.006 \ln(s) + 0.0011 \ln^2(s) \quad (33)$$

η DECAY MODES Fraction (Γ_i/Γ)	
Neutral modes	
neutral modes	(72.12 ± 0.34) %
2 γ	(39.41 ± 0.20) %
3 π^0	(32.68 ± 0.23) %
$\pi^0 2\gamma$	(2.56 ± 0.22) × 10 ⁻⁴
2 $\pi^0 2\gamma$	< 1.2 × 10 ⁻³
4 γ	< 2.8 × 10 ⁻⁴
invisible	< 1.0 × 10 ⁻⁴
Charged modes	
charged modes	(28.10 ± 0.34) %
$\pi^+ \pi^- \pi^0$	(22.92 ± 0.28) %
$\pi^+ \pi^- \gamma$	(4.22 ± 0.08) %
$e^+ e^- \gamma$	(6.9 ± 0.4) × 10 ⁻³
$\mu^+ \mu^- \gamma$	(3.1 ± 0.4) × 10 ⁻⁴
$\pi^+ \pi^- 2\gamma$	< 2.1 × 10 ⁻³
$\pi^+ \pi^- \pi^0 \gamma$	< 5 × 10 ⁻⁴
$\pi^0 \mu^+ \mu^- \gamma$	< 3 × 10 ⁻⁶

Table I Decay modes and branching ratios for mesons. [22]

At last stage, the **multiplicity of Neutral Kaons and Neutrons and their antiparticles**, is computed from these power law fits to experimental data [25] [24]:

$$n_{k^0} = 0.13 \sqrt{s}^{0.45} \quad (34)$$

$$n_n = 0.08 \sqrt{s}^{0.46} \quad (35)$$

In PHIT, no distinction between Neutrons and Antineutrons is defined. Also, K_S^0 and K_L^0 are assumed to be generated with the same probability.

I-3 Inelasticity and Leading Particle

After choosing the multiplicity of each type of particles and calculating the equivalent energy to provide their mass, if any energy is left, we randomly determine how much of the remained energy is going to be consumed to give momentum to the produced particles. Obviously, inelasticity, the fraction of total energy in center of momentum frame which is used in production of secondaries, is related to multiplicity of the event. A very naïve theoretical estimation of inelasticity that can be used to guide the code in the selection of inelasticity value, can be [26] [27] [7]:

$$\frac{K}{\langle K \rangle} = \frac{n}{\langle n \rangle} \quad (36)$$

K , is defined as the inelasticity of an event in which n secondaries emerge, while $\langle n \rangle$ and $\langle K \rangle$ represent the average number of secondaries and inelasticity at the same energy. The distribution of inelasticity in different energies and dependence of $\langle K \rangle$ on $\sqrt{s}$ has also been studied under different approaches [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38] [39]. In PHIT, we have not employed any of these theoretical or experimental approaches; simply because inelasticity could be, and is, a main output of our code. Moreover, inelasticity studies suffer from huge uncertainties and different researches have led to even contradictory results. For example, there is no agreement even on how the average inelasticity varies by rising of the collision energy. The decrease in inelasticity with energy is advocated by some authors, while others believe that the inelasticity is an increasing function of energy and the problem remains unsolved.

In our algorithm, we select a random value for inelasticity, between 0 and 1, without caring about any other factors, like on $\sqrt{s}$ or on

n_{ch} . Moreover, before each try to solve the collision, K_0 is randomly divided to two values, $(K_0)_n$ and $(K_0)_c$, which determine the supposed portion of energy that is consumed for generation of neutral and charged secondaries. Then, PHIT tries to reach a random selection of dynamical properties for all secondary particles. Although this procedure is dependent on the initial, randomly determined inelasticity, $K_0 = (K_0)_n + (K_0)_c$, the final value, K , is not necessarily or forcedly equal to the pre-selected inelasticity.

A huge fraction of the left energy, the elastic portion, is passed to the leading hadron. As it is well known, the leading particles, roughly defined as the energetic remnants of the two primary incident particles, may be of the same type as the incoming ones, or may undergo charge exchange, or may be excited to a resonance state. We only consider the charge exchange probability in our code. The possible charge exchange which are taken into account are listed in Table 2. Basically, after charge exchange, the number of positive, negative, and neutral pions should be adjusted to keep the total charge involved in the collision conserved. We skip this correction in PHIT assuming that total charge is conserved by other types of particles that are not considered in our code.

leader pion	leader	other pion
$p + \pi^0 \longrightarrow$	n	$+$ π^+
$n + \pi^+ \longrightarrow$	p	$+$ π^0
$\bar{p} + \pi^0 \longrightarrow$	$\bar{n}$	$+$ π^-
$\bar{n} + \pi^- \longrightarrow$	$\bar{p}$	$+$ π^0

Table 2 Charge exchange of the leading particles [40].

The probability for charge exchange varies with energy as [40]:

$$p = \begin{cases} 0.10 & \sqrt{s} \leq 19.4 \\ 0.10 + 0.345 \ln\left(\frac{E_t}{200}\right) & 19.4 < \sqrt{s} \leq 105. \\ 0.45 - 0.537 \ln\left(\frac{E_t}{200}\right) & 105. < \sqrt{s} \leq 969. \\ 0.03 & 969. < \sqrt{s} \end{cases} \quad (37)$$

E_l is the kinetic energy of the projectile particle in lab frame.

I-4 Transverse Momentum

The distribution of the transverse momentum of produced particles, P_T , has been the subject of many studies. While first investigations suggested a Boltzmannian distribution [41], today there is a widespread agreement among researchers, that the Tsallis distribution [42], in terms of only two parameters, fits to P_T spectrum better than all other suggestions [43] [44] [45] [46] [47]. Tsallis distribution also coincides with the Hagedorn formula [48] [49] which gives equally good fit to the hadron spectra [50]. Tsallis distribution has been expressed in slightly different forms. In this work, we have used a combination of the forms and the fit parameters suggested in [51] and [52]. The probability density function for generation of a hadron with transverse momentum magnitude of P_T , is given as:

$$P_{q,T}^{Tsallis}(P_T) = C \frac{P_T}{T} \left(1 + (q-1) \frac{P_T}{T}\right)^{-\frac{q}{q-1}} \quad (38)$$

The normalization factor, C , is calculated numerically in our code. This formula works in the whole range of P_T and it is not derived from a particular theory. It is only a generalization of the regular statistical mechanics and just offers the existence of some kind of equilibrium described by two parameters, T and q . The temperature T characterize its mean properties and the parameter q , known as the non-extensivity parameter, represents effects believed to be caused by fluctuations [53]. The dependence of these parameters to $\sqrt{s}$, as fitted in [51] to experimental data follows power law:

$$q(s) = 1.027\sqrt{s}^{0.01326} \quad (39)$$

$$T(s) = 1.1014\sqrt{s}^{-0.03262} \quad (40)$$

The course of action for choosing the random values of P_T begins with estimating a maximum limit by finding the point where the integral of distributions reaches unity. This is done numerically by defining a step for calculation of integral. Although the selected P_{Tmax} would basically be step dependent, choosing a sufficiently small step leads to an ignorable dependency. The default size for P_T in PHIT is $1. \text{keV}/c$. Afterwards, this confined range is divided into n sections with equal integral of probability density function. As it was defined, n represents the total number of secondaries. Each of these sections are going to be attributed to a produced hadron.

In assigning the P_T sections to particles, we should note that the average transverse momentum for pions, kaons, and protons is not the same and heavier hadrons tend to have larger P_T . While the difference is small in lower energies, for instance, $\langle P_T \rangle_{p^\pm}$ exceeds twice the value of the $\langle P_T \rangle_{\pi^\pm}$ when $\sqrt{s}$ reaches $1. \text{TeV}$ and higher [47]. To apply these considerations to our code, we utilize the exclusive type-based distribution and fit parameters in [52]:

$$P_{n,T}^{Tsallis}(E_T) = C E_T \left(1 + \frac{E_T}{nT}\right)^{-n} \quad (41)$$

$$E_T = \sqrt{P_T^2 + m^2} \quad (42)$$

$$T(s) = 0.082 + \frac{0.151}{\sqrt{s}} \quad (43)$$

$$n^{\pi^\pm}(s) = 6.41 + \frac{53.07}{\sqrt{s}} \quad (44)$$

$$n^{\pi^0}(s) = 7.23 + \frac{43.22}{\sqrt{s}} \quad (45)$$

$$n^k(s) = 6.72 + \frac{58.56}{\sqrt{s}} \quad (46)$$

$$n^{p^\pm, n}(s) = 8.76 + \frac{23.61}{\sqrt{s}} \quad (47)$$

We have assumed that $n^{k^0} \approx n^{k^\pm}$ and $n^n \approx n^{p^\pm}$ due to the lack of experimental data. The analysis for identified particles are done in a much smaller range of P_T limited by measured data. So, we have not based all the P_T determination algorithm on them. For deciding which section should be assigned to the current particle, PHIT picks a random section and calculates the integral of probability density function in this interval using the type dependent values. Then a random test is run to see if the section is selected. If yes, the attribution is done; else, another random section would be picked and the process would be repeated. This method leads to a random separation of particles and shifting the average P_T of each type to different values, as it is supposed to be. After attributing all sections to secondaries, a random value within each section is selected as the P_T of the ascribed particle.

The probability density distribution of transverse momentum decreases very sharply with P_T and as a result, our section-based approximation will fail to simulate the high P_T range and the mean transverse momentum would suffer large deviations from the experimental average. To solve this problem, the selected P_T values are corrected by multiplying them with $R_{\langle P_T \rangle}$:

$$R_{\langle P_T \rangle} = \frac{\langle |P_T| \rangle}{\langle |P_T| \rangle_0} \quad (48)$$

$\langle |P_T| \rangle_0$ is the average of the generated $|P_T|$ values, while $\langle |P_T| \rangle$ represent the empirically estimated mean transverse momentum magnitude as a function of $\sqrt{s}$ [54]

$$\langle |P_T| \rangle_0 = 0.413 - 0.0171 \ln(s) + 0.00143 \ln^2(s) \quad (49)$$

Another possible fit is [55]:

$$\langle |P_T| \rangle_0 = 0.0608 \log_{10}(\sqrt{s}) + 0.283 \quad (50)$$

Since the collision occurs in longitudinal dimension, there is no preferred

transverse direction and P_T of all particles has an equal chance of being positive or negative. Hence, the sign of P_T is selected randomly. If one calculates the sum of all P_T values generated in a large number of events, it would be almost zero; however, sum of secondary P_T values is not necessarily zero in each event. We don't force conservation of transverse momentum on secondary hadrons, assuming this non-zero sum balanced with the total momentum of all particles other than secondary hadrons. When P_T of all particles is calculated, PHIT attempts to find pairs of secondaries that mutual sign exchange within them will reduce the deviation of P_T sum from zero.

There are other available formulations of transverse momentum distributions, like [55] [56] [57] [58], but We have preferred the current approach in PHIT to keep the procedure and ideas simple and practical. We might add another option for P_T spectra based on these works in the next versions.

I-5 Pseudorapidity

Our encounter with the issue of distributing the longitudinal momentum, P_L is based on the practically popular quantity of pseudorapidity, defined as:

$$\eta = \frac{1}{2} \ln \left(\frac{|P| + P_L}{|P| - P_L} \right) \quad (51)$$

As it is obvious from this definition, the only quantity to be measured for the determination of pseudorapidity of a particle, independent of any identification mechanism, is its polar angle. The procedure used for choosing the η values is basically same as the case of transverse momentum. But unlike P_T , the suggested probability density functions for pseudorapidity are varied.

If one prefers to work directly with the $|P|$, instead of η , this parametrization of Tsallis distribution is available [59]:

$$P_{P_0,q}^{Tsallis}(|P|) = C_{P_0,q} \left(\frac{|P|}{P_0} \right)^2 \left(1 + \frac{q}{n} \left(\frac{|P|}{P_0} \right)^2 \right)^{-n} \quad (52)$$

That is the normalized momentum distribution of produced particles in the rest frames of respective emitting centers, where the normalization constant $C_{P_0,q}$ and n are:

$$C = (q-2)(q-3)/2P_0 \quad (53)$$

$$n = q/(q-1) \quad (54)$$

The empirical energy dependence of the parameters is:

$$P_0 = \begin{cases} 0.122 & \sqrt{s} < 382 \\ 0.0163 \log_{10}(\sqrt{s}) + 0.0799 & \sqrt{s} \geq 382 \end{cases} \quad (55)$$

$$q = 0.0295 \log_{10}(\sqrt{s}) + 1.020 \quad (56)$$

These relations are based on a previous work [60] on e^+e^- collisions and one should refer to discussions in these two papers for details of employing these fits for the case of pp collisions.

A recent parametrization of pseudorapidity distribution is introduced in [61]. Inclusive η distributions, measured by different collaborations are considered and the shapes of the pseudorapidity distributions agree with data up to a center of mass energy of $7. TeV$. While we plan to use the relations in the mentioned paper in PHIT for next versions, current code uses a simpler formulation [8]:

$$P_{C,\eta_0,\delta}^{Fermi}(\eta) = A \frac{C + \eta}{1 + \exp\left(\frac{\eta - \eta_0}{\delta}\right)} \quad (57)$$

This is a generalization of Fermi distribution, first put to use in [62]. The term A is the normalization coefficient calculated numerically in the code. C is a fit parameter which describes the magnitude of the distribution and the dip at $\eta = 0$. Both C and η_0 are found to increase with increasing $\sqrt{s}$, it seems δ is approximately independent of energy within errors. However, we have used a slightly rising fit for this parameter:

$$C = 3.7 + 1.15Ln(\sqrt{s}) + 0.25Ln^2(\sqrt{s}) \quad (58)$$

$$\eta_0 = 3.1 + 0.4Ln(\sqrt{s}) \quad (59)$$

$$\delta = 1.16 \sqrt{s}^{0.008} \quad (60)$$

Again, choosing the random values is essentially done with estimating a maximum η , dividing the range to sections with equal probability and assigning each of them to a particle. Nevertheless, the details are different. $(\eta_{max})_0$ is defined by considering that the Feynman scaling variable, x_F , does not exceed unity and from this we deduce:

$$(\eta_{max})_0 = \sinh^{-1} \left(\frac{\sqrt{s}}{|P_T|_{min}} \right) \quad (61)$$

Moreover, $(\eta_{max})_0$, the maximum pseudorapidity that a particle can obtain in this specific collision, cannot exceed the point where the integral of distribution reaches unity. The actual maximum pseudorapidity, η_{max} , is chosen randomly in this range:

$$0 < \eta_{max} < (\eta_{max})_0 \quad (62)$$

The process of allocating the pseudorapidity sections is a little tricky. First of all, the procedure is done separately but similarly for charged particles and neutrals; because, the experimental data is only available for charges, not for all the particles together. To solve this problem, we define a virtual energy for charges and neutrals, based on $(K_0)_c$ and $(K_0)_n$:

$$\sqrt{s}_c = (K_0)_c \sqrt{s} \quad (63)$$

$$\sqrt{s}_n = (K_0)_n \sqrt{s} \quad (64)$$

If the current set of pseudorapidity values fails to be approved, a new choice of $(K_0)_c$ and $(K_0)_n$ would be employed which obviously lead to a different set of distribution parameters and distinct sections. The next problem is about assigning the η values to particles which already have defined type and transverse momentum. A pure random allocation, or attribution of smaller P_T values to larger η , or vice versa does not lead to acceptable realistic results. In our code, particles are picked one by one and their relevant section is selected by two simple rules. For baryons, a randomly chosen section is allocated. In the case of mesons, the at collision energies below 2 TeV smallest available section of pseudorapidity is attributed to the particle. This rule is gradually reversed at higher energies and mesons take priority in occupying higher pseudorapidities, Physical constraints like the available energy for the particle or Feynman scaling variable are checked to not be violated by assigning the section to the current particle. This routine is repeated until all secondaries are matched to a specific section. Then a random pseudorapidity is defined for each particle from its relevant section. Finally, the sign of pseudorapidity values, which determines the forward or backward propagation of the particle, is picked randomly.

I-6 Colliding Particles

After determining the properties of all secondaries, the remnant energy, E_R , has to be divided within, roughly speaking, the two primary colliding ones. Since our main focus in PHIT was on producing realistic secondaries, we have developed a very simple procedure to deal with incident particles. First, the forward or backward direction of secondaries is determined randomly. This leads to a certain value of a total longitudinal momentum for the produced

particles, $(P_L)_s$, while we already know the total transverse momentum of secondaries, $(P_T)_s$. Hence, we have a system of equations here, based on conservation of momentum and energy, which must be solved:

$$E_p + E_t + E_s - E_0 = 0 \quad (65)$$

Here, P_L , P_T and E represent longitudinal momentum, transverse momentum and total energy and p , t and s refer to projectile and target particles and secondaries. Now, by choosing a random value for E_p , E_t would be determined immediately:

$$E_t = E_R - E_p \quad (66)$$

E_0 is the total available energy for all particles and E_R is the remained portion for colliding particles:

$$E_R = E_0 - E_s \quad (67)$$

By eliminating the equation of energy, the system reduces to:

$$\begin{cases} (P_L)_p^2 + (P_T)_p^2 = P_p^2 \\ (P_L)_t^2 + (P_T)_t^2 = P_t^2 \\ (P_L)_p + (P_L)_t = -(P_L)_s \\ (P_T)_p + (P_T)_t = -(P_T)_s \end{cases} \quad (68)$$

Where $P_p^2 = E_p^2 - m_p^2$ and $P_t^2 = E_t^2 - m_t^2$. There exists a set of real answers for this system on condition that:

$$P_s^2 = (P_L)_s^2 + (P_T)_s^2 \quad (69)$$

It is obvious that not every random choice of E_p satisfies the above condition. So, when there exists no answer, a new E_p would be picked randomly, until we reach an acceptable answer. Also, in similarity with the case of secondary particles, during each try, a random

maximum pseudorapidity is selected; to which, the pseudorapidity of both incident particles is limited. Moreover, we assume the pseudorapidity of the colliding particles must be larger than the smallest value within secondaries.

The mentioned assumptions guide the results toward a more acceptable distribution of pseudorapidity. It turns out that the allowed number of tries for finding a valid set of properties for the incident particles, which we call I_{try} , affects the results intensely. In fact, this is the only free parameter in our model and needs to be adjusted. By a trial and error approach, we are convinced that using the below function, the results are well-tuned over a large range of energy:

$$I_{try} = 50 \left(1 - \frac{1}{\sqrt{1 + \sqrt{s}/100}} \right) \quad (70)$$

2. The Results

The most important parameters of hadron production, particularly for the aim of simulating air shower development, are the multiplicity of secondary particles, the production ratio of neutral to charged particles and the inelasticity. These parameters are not well constrained by particle production measurements at accelerators and, moreover, the predictions of hadronic interaction models are very different [3]. In PHIT, multiplicity and ratio of neutrals to charges are basically considered as input parameters. The pure and significant outputs of our model are the inelasticity and the distribution of Feynman scaling function. Here, we first discuss the distribution of multiplicity and dynamical properties of generated particles by PHIT to show that they are compatible with experimental data and comparable with other model results.

Then, we focus on the inelasticity and its dependence on energy of collision as the main result of our study.

We have based the comparison of our results with other models on the work done by CORSIKA team [63] which compares several interaction models applied in the analysis of cosmic ray data in a systematic way. They present results of calculations with the CORSIKA air shower simulation program [40] employing five well-known models for the high energy hadronic interactions. The models are HDPM [64], VENUS [65], QGSJET [66], [67], SIBYLL [68] and DPMJET [69]. Our results, following their work, is based on 10000 simulated $\bar{p}p$ event at each specific energy. Although this study might seem outdated to be used for comparison, it is still the most comprehensive systematic study in its own type and is capable of serving us as a criterion to evaluate PHIT.

2-1 Multiplicity

The multiplicity is one of the key observables to describe the global properties of the interactions and has been the subject of long-standing experimental investigations. As mentioned before, we don't expect the distribution of multiplicity in PHIT to be satisfying when we reach the collision energies about 1 TeV . Nevertheless, the average multiplicities are not supposed to deviate too much from the experimental values, even above 1 TeV .

In **Table 3** measured multiplicities at collider energies [24] are compared with PHIT and other codes results. We have mentioned the minimum and maximum average charge multiplicity generated by other codes here; more details are found in the original research.

CORSIKA team has increased $\langle n_{ch} \rangle$ by 2, and $\langle n_p + n_{\bar{p}} \rangle$ by 1.5, and $\langle n_n + n_{\bar{n}} \rangle$ by 0.5 to account to the leading particles which systematically escape detection in experiments. There is no model describing the data clearly better than the other ones and PHIT outputs are not less satisfactory in comparison with other results.

Multiplicity distributions of charged particles in collisions at 540 GeV are plotted in **Figure I**. While all models seem to have fewer reactions at small multiplicities compared to experimental data, PHIT provides obviously more consistent results.

2-2 P_T Distribution

Transverse momentum of hadrons is among the basic physical observables in high-energy collisions that can be used to test the event generators. It is known that measurements of baryon and notably proton production is not reproduced well by the existing models [47]. PHIT, while having its own shortcomings, especially in the case of kaons, simulates protons better. The average of the measured transverse momenta for hadrons is compared with the same average of values generated by PHIT in **Table 4**. Data is taken from [70] [71] [72] [73].

The transverse momentum distributions of produced charged particles are shown in **Figure 2**. Below $p_T \approx 1 \text{ GeV}/c$ the distributions agree within about 50%. They differ most for large values of p_T which, represent only a very small fraction of all secondaries.

2-3 Pseudorapidity Distribution

Pseudorapidity distributions of charged particles at four different collision energies are

illustrated in **Figure 3-6**. Data is obtained from [74], [75], [76] and plotted by CORSIKA team. Obviously, all models have an acceptable degree of agreement with the data at lower energies. However, as collision energy grows, most of them tend to values higher than experimental results at large pseudorapidities; and those which not, fail to simulate the distribution at lower range. It seems that PHIT provides the overall best simulation of the experimental data.

2-4 Feynman x distribution

Feynman scaling variable or function is defined as the fraction of maximum longitudinal momentum that is carried by a secondary particle in the center of momentum frame.

$$x_F = \frac{2p_L}{\sqrt{s}} \quad (71)$$

The distributions of x_F for baryons and charged mesons are plotted separately in **Figure 7,8**. There is roughly a maximum difference of an order of magnitude in the results of different models. It should be reminded that the distinction between distribution of x_F for baryons and mesons is partially dependent on two simple rules as mentioned while we were explaining the method of attributing pseudorapidity sections.

$\sqrt{s}$ = 53.0 GeV	Experimental	Models		PHIT	$\sqrt{s}$ = 200. GeV	Experimental	Models		PHIT
		Min	Max				Min	Max	
$\langle n_{Ch} \rangle$	12.5	13.6	15.7	13.7	$\langle n_{Ch} \rangle$	22.4	21.1	24.9	20.9
$\langle n_{p^\pm} \rangle$	1.80	1.53	1.96	2.02	$\langle n_{p^\pm} \rangle$	2.00	1.87	2.53	2.37
$\langle n_n \rangle$	0.80	0.97	1.30	0.89	$\langle n_n \rangle$	1.50	1.30	1.93	1.34
$\langle n_{k^\pm} \rangle$	9.20	10.6	12.6	10.7	$\langle n_{k^\pm} \rangle$	17.9	17.3	20.4	16.7
$\langle n_{k^0} \rangle$	0.74	0.81	1.15	1.00	$\langle n_{k^0} \rangle$	1.50	1.58	2.21	1.82
$\langle n_{\pi^\pm} \rangle$	0.74	0.78	1.09	0.80	$\langle n_{\pi^\pm} \rangle$	1.50	1.36	2.08	1.38
$\langle n_\gamma \rangle$	11.2	12.0	15.3	13.1	$\langle n_\gamma \rangle$	22.2	22.3	24.6	22.7

$\sqrt{s}$ = 546. GeV	Experimental	Models		PHIT	$\sqrt{s}$ = 900. GeV	Experimental	Models		PHIT
		Min	Max				Min	Max	
$\langle n_{Ch} \rangle$	30.4	28.9	33.9	30.0	$\langle n_{Ch} \rangle$	36.6	33.8	39.6	35.8
$\langle n_{p^\pm} \rangle$	2.95	2.12	3.44	3.08	$\langle n_{p^\pm} \rangle$	2.50	2.33	4.05	3.54
$\langle n_n \rangle$	1.95	1.56	2.55	1.67	$\langle n_n \rangle$	1.50	1.72	2.96	1.93
$\langle n_{k^\pm} \rangle$	23.9	23.9	28.0	24.1	$\langle n_{k^\pm} \rangle$	29.9	28.1	32.7	28.7
$\langle n_{k^0} \rangle$	2.24	2.46	3.16	2.87	$\langle n_{k^0} \rangle$	2.74	3.02	3.79	3.56
$\langle n_{\pi^\pm} \rangle$	2.24	2.39	3.03	2.18	$\langle n_{\pi^\pm} \rangle$	2.74	2.96	3.65	2.72
$\langle n_\gamma \rangle$	33.0	31.1	34.8	35.1	$\langle n_\gamma \rangle$	41.4	36.5	41.5	43.2

Table 3 Average Multiplicity of secondary particles as measured in several collider energies, compared with the simulated results of PHIT and the minimum and maximum averages calculated using other models. $\langle n_n \rangle$ counts both Neutrons and Antineutrons

$\sqrt{s}$	62.4 GeV (pp)		200. GeV (pp)		540. GeV ($\bar{p}p$)	
	Data	PHIT	Data	PHIT	Data	PHIT
$\langle p_{T\pi^\pm} \rangle$	0.37 ± 0.01	0.36 ± 0.002	0.38 ± 0.02	0.38 ± 0.001	0.37 ± 0.01	0.40 ± 0.001
$\langle p_{Tk^\pm} \rangle$	0.55 ± 0.01	0.49 ± 0.006	0.57 ± 0.01	0.53 ± 0.005	0.48 ± 0.01	0.59 ± 0.005
$\langle p_{Tp^\pm} \rangle$	0.71 ± 0.03	0.51 ± 0.011	0.70 ± 0.02	0.56 ± 0.009	0.61 ± 0.03	0.62 ± 0.008

$\sqrt{s}$	900. GeV (pp)		1.0 TeV ($\bar{p}p$)		1.8 TeV ($\bar{p}p$)	
	Data	PHIT	Data	PHIT	Data	PHIT
$\langle p_{T\pi^\pm} \rangle$	0.40 ± 0.02	0.42 ± 0.001	0.37 ± 0.01	0.42 ± 0.001	0.39 ± 0.01	0.44 ± 0.001
$\langle p_{Tk^\pm} \rangle$	0.65 ± 0.06	0.62 ± 0.005	0.53 ± 0.012	0.63 ± 0.005	0.55 ± 0.01	0.67 ± 0.004
$\langle p_{Tp^\pm} \rangle$	0.76 ± 0.07	0.66 ± 0.007	0.67 ± 0.03	0.66 ± 0.007	0.72 ± 0.01	0.72 ± 0.007

Table 4 Comparison of the center-of-mass energy dependence of the average transverse momentum.

Figure 1 Multiplicity distributions of charged particles from non-diffractive collisions with $\sqrt{s} = 540 \text{ GeV}$. The gray solid curve shows the PHIT results and the triangles represent UA5 data [11].

Figure 2 Transverse momentum distributions of secondary charged particles at $\sqrt{s} = 540 \text{ GeV}$. The gray solid curve indicates the PHIT results

Figure 3 Pseudorapidity distributions of charged particles from $\bar{p}p$ collisions at $\sqrt{s} = 53 \text{ GeV}$. The gray solid curve represents PHIT results.

Figure 4 Pseudorapidity distributions of charged particles from $\bar{p}p$ collisions at $\sqrt{s} = 540 \text{ GeV}$. The gray solid curve represents PHIT results.

Figure 5 Pseudorapidity distributions of charged particles from $\bar{p}p$ collisions at $\sqrt{s} = 900 \text{ GeV}$. The gray solid curve represents PHIT results.

Figure 6 Pseudorapidity distributions of charged particles from $\bar{p}p$ collisions at $\sqrt{s} = 1800 \text{ GeV}$. The gray solid curve represents PHIT results.

Figure 7 Feynman function distributions for baryon and charged mesons from collisions of 10^{14} eV $\bar{p}$ on p ($\sqrt{s} \approx 433 \text{ GeV}$). The gray solid curve represents PHIT results.

Figure 8 Feynman function distributions for baryon and charged mesons from collisions of $10^{16}eV$ $\bar{p}$ on p ($\sqrt{s} \approx 4332.GeV$). The gray solid curve represents PHIT results.

2-5 Inelasticity

It is a well-known fact that, in $p(\bar{p})$ on p collisions, a large fraction of initial energy is carried away by two leading baryons and is not available for particle production. Inelasticity can be defined as the portion of energy that is consumed in particle generation:

$$K = 1 - \frac{(E_{l1} + E_{l2})}{\sqrt{s}} \quad (72)$$

$l1, l2$ represent the leading particles, by descending order of energy. Elasticity, on the other hand, is obviously defined as $1 - K$. However, some researchers are interested in the fraction of energy delivered by fastest baryon alone, not by the pair of leading particles. In this work, we follow the basic definition of K while we refer to inelasticity, but elasticity is defined as $E_{l1}/\sqrt{s}$. We aim to use PHIT for a more detailed study on inelasticity of hadronic collisions later. Here, we are content with remarking some key results.

While some investigations indicated that inelasticity is almost constant in a wide range of energy others claim it's an energy-dependent quantity. different approaches to this problem is already addressed. Our results indicate that inelasticity generally decreases with growth of collision energy, at least above energies about 10 GeV . $\langle K \rangle$ is plotted in **Figure 9** borrowed from [77]. For details about the data representation in this plot, please refer to the source and references mentioned there. PHIT outcomes are in acceptable agreement with the data compared to results of other models. Variation of elasticity with energy is shown in **Figure 10** and compared with model results studied by CORSIKA team. The solid curve indicates the average fraction of energy transported by the most energetic baryon; while the dashed line represents $1 - \langle K \rangle$. Despite the mess at lower energies, it seems that most of the models, including PHIT, tend to converge at about 1000 GeV . The similar behavior of models and comparative consistency of $\langle K \rangle$ with observations can be regarded as a sign of reliability of PHIT results.

Figure 9 Variation of average inelasticity with $\sqrt{s}$. The gray solid curve represents PHIT results. For details about data refer to [77].

Figure 10 Variation of average elasticity with $\sqrt{s}$. The solid curve indicates the average fraction of energy transported by the most energetic baryon; while the dashed line represents $1 - \langle K \rangle$. For more details refer to the definition of Elasticity in the text.

3. Conclusion

We introduced our numerical model for simulating $p(\bar{p})p$ interaction, which is capable of generalization to hadron-nucleus collisions, and analyzed its outputs briefly. The results are comparable with other theoretically rich physical models and moreover, in some aspects, our simulations are more consistent with experimental data. On the other hand, PHIT is a fast code. It can process a single collision typically by 10^8 calculations or, in other words, 100 million CPU cycles per a run. This means a regular CPU on a PC can namely process a collision in less than tenth of a second. Hence, being adequately fast and accurate, PHIT provides a suitable algorithm for practical applications like investigating the development of extended air showers, which is our ultimate goal.

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Figures

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