A simple method of searching for a fixed solution in a coordinate domain

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Research Article

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A simple method of searching for a fixed solution in a coordinate domain.

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Abstract Recently, more and more Global Navigation Satellite Systems satellites are available for observation. Apart from obvious advantages, this brings new challenges in developing efficient computational methods of processing signals obtained from more satellites than to date. From the viewpoint of computation process efficiency, the most critical step is ambiguity resolution. Because of the discrete character of ambiguities, the search procedure is employed to perform this task. The search space dimension significantly impacts a search procedure computational load. The time needed for obtaining a solution raises considerably when the search space dimension is greater. Therefore, the improved version of the well-known concept of searching for a fixed solution in a three-dimensional coordinate domain was proposed and tested. The math model of the proposed approach is presented together with the algorithm of the search procedure. The numerical experiments were designed for simulated and real data. The simulated data has been prepared based on the concept proposed by the authors. This data simulation method is described in detail. The test results confirmed the reduction of the time needed to obtain the results when the proposed method is applied. This advantage over traditional methods unveils in the case of many satellites. Moreover, the real-data test pointed out that using the new approach is beneficial in reducing the computation time also in the case of a few satellites if long sessions are processed.

Keywords GNSS data processing · Ambiguity Function · integer least-squares · Voronoi cells · precise satellite positioning

Introduction

The Global Navigation Satellite Systems (GNSS) growth brings both new opportunities and challenges in signal processing methods. The apparent advantage of the increasing number of

satellites possible to observe is the rise in the number of observations collected by the GNSS receiver. On the other hand, besides new signals, each new satellite provides new unknown values, i.e., ambiguities to be estimated. The rise in amounts of parameters substantially impacts the computational load. Even the most popular and efficient methods of optimizing the computational process in the classic approach, for example, in the LAMBDA method (Teunissen, 1995, 1999; de Jonge and Tiberius, 1996; Liu et al., 1999; Chang et al., 2005; Zhou 2011; Xu, 2012) are sensitive to the number of ambiguities. Xu proved that if observed satellites exceed a specific number, even the most efficient decorrelation techniques do not improve the search procedure efficiency (Xu, 2001). If there is more than a specific number of satellites, the decorrelation procedure takes more time than the search procedure performed without such a transformation. Thus, the cost of the enhancement technique outnumbers the benefits that are got from it! Therefore, it is justified to reconsider the concept of searching for integer ambiguities in a coordinate domain. The search space dimension is constant and amounts to three in such a case. Thus, the search space dimension does not depend on the number of satellites. Consequently, the computational load of a search procedure does not rise with increasing the number of satellites. It can even decrease if a search region is set correctly. Usually, the search region is formed based on the float position covariance matrix. In that case, this region shrinks with an increasing number of satellites because this typically improves satellites' configuration geometry and float solution precision. Then, fewer candidates are needed for carrying out the search procedure. The concept of searching for a fixed solution in the coordinate domain dates back to the early years of satellite navigation system development. Its classic form utilizes the Ambiguity Function (AF) method in the computation process. The AF method of processing the GPS carrier phase observations was the first time proposed by Counselman and Gourevitch (1981) and then by Remondi (1984). A substantial improvement of the AF method has been proposed by Cellmer et al. (2010). They developed the Modified Ambiguity Function Approach (MAFA). A description of the MAFA method can be found in, for example, (Cellmer 2012 2013 2015; Cellmer et al. 2010 2017 2018 2021; Kwasniak et al. 2016 2017; Baselga 2010; Nowel et al. 2018). Some researchers use the MAFA method foundations in recent research in airplane attitude determination (Wang et al. 2007; Wang et al. 2019; Wu et al. 2019, Zhao et al., 2022). Nevertheless, we do not use the functional model of the MAFA method in the present contribution. Unlike the MAFA, all formulas used in the search procedure proposed in this article are based solely on the float solution results. In contrast, the MAFA method requires the whole carrier phase data set for forming the observation equations when testing the candidates. Still, however, we take advantage of the other solutions used in the MAFA method: creating the search region in the coordinate domain and setting an optimal search step (Cellmer et al. 2018, 2021). The search region is in the form

of an error ellipsoid of the approximate position, and the optimal search step's length derivation is based on a concept of the Voronoi cells (VC) and an actual satellites' configuration. According to Hassibi and Boyd (1998), the Voronoi cell of the point X_0 in a lattice is the set of points in space closer to X_0 than to any other point in the lattice. Xu (2006) used the VC to analyze an ambiguity resolution problem. In Teunissen's integer equivariant estimation theory, the VC is called an "in-pull region," where distance accounts for the non-Euclidean metric (Teunissen, 1999, 2003). Cellmer et al. (2021) used the VC in the meaning of a region containing candidates that give the same solution from the methods based on searching for a fixed solution in a coordinate domain. This meaning of the VC is assumed in our work. The search step's length resulting from the method proposed by Cellmer et al. (2021) is the longest distance between neighboring candidates that one at the same time guarantees we do not skip any VC included in the search region. In contrast to the MAFA method, the search procedure proposed in this contribution does not need the whole data set to test successive candidates. The search procedure is conducted based on approximate position coordinates and their covariance matrix. Both magnitudes are obtained from a float solution.

In the next section, the classic form of the mixed integer-real least squares estimation math model is introduced to familiarize readers with the notation used in this work. The third section is devoted to the main topic of this work, i.e., the search procedure in a coordinate domain (SinCD). Sole results from a float solution are used in this procedure as input. Two types of computational experiments are described in the fourth section. They are based on real and simulated data. The results of those experiments illustrate the effects of the proposed method. The conclusions are presented in the last section.

The classic approach of precise positioning

The math model of precise satellite positioning is (Hoffman-Wellenhof et al., 2008; Leick et al., 2015):

$$\mathbf{y} = \mathbf{A}\mathbf{a}+\mathbf{B}\mathbf{b}+\mathbf{e} \tag{1}$$

where **y** is the data vector, **a** is the integer ambiguity vector, **b** is the real parameter vector, **A** and **B** are the corresponding design matrices, and **e** is the noise vector. The **y** vector consists of double differenced (DD) carrier phase observations and/or code observations. The **b** vector consists of the baseline components. Sometimes it can also contain other parameters, e.g., atmospheric corrections. The classic approach of precise satellite positioning consists of the three stages:

1) The least-squares (LS) solution in which the integer nature of ambiguities is discarded. This solution is called the "float solution."

2) The integer ambiguity resolution. This stage also includes a validation of the results.

3) The solution with integer ambiguities. This solution is called the "fixed solution."

The LS estimates of the **a** and **b** vectors, in the first stage, are obtained from:

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{\hat{a}} & \mathbf{Q}_{\hat{a}\hat{b}} \\ \mathbf{Q}_{\hat{b}\hat{a}} & \mathbf{Q}_{\hat{b}} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{y} \\ \mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{y} \end{bmatrix} \tag{2}$$

where: $\begin{bmatrix} \mathbf{Q}_{\hat{a}} & \mathbf{Q}_{\hat{a}\hat{b}} \\ \mathbf{Q}_{\hat{b}\hat{a}} & \mathbf{Q}_{\hat{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{A} & \mathbf{A}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{B} \\ \mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{A} & \mathbf{B}^{\mathrm{T}}\mathbf{P}_{\mathrm{y}}\mathbf{B} \end{bmatrix}^{-1}$ and $\mathbf{P_y}$ is the observation weight matrix.

The LS estimator of the integer ambiguity vector is defined as:

$$\breve{\mathbf{a}} = \arg\min_{\mathbf{z}\in Z^n} \left\| \hat{\mathbf{a}} - \mathbf{z} \right\|^2_{\mathbf{Q}_{\hat{a}}} \tag{3}$$

The expression at the arg min(.) operator denotes a weighted norm of the $(\hat{\mathbf{a}} - \mathbf{z})$ vector:

$$\left\| \hat{\mathbf{a}} - \mathbf{z} \right\|^2_{\mathbf{Q}_{\hat{a}}} = \left(\hat{\mathbf{a}} - \mathbf{z}\right)^{\mathrm{T}} \mathbf{Q}^{-1}_{\hat{a}} \left(\hat{\mathbf{a}} - \mathbf{z}\right) \tag{4}$$

It is necessary to employ the search procedure since the solution domain of formula (3) has a discrete character. The ambiguity resolution step is always associated with the validation of the results. The last step in precise positioning is a "fixed solution," which incorporates the integer ambiguities obtained in the previous step:

$$\breve{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{b}\hat{a}}\mathbf{Q}^{-1}_{\hat{a}} \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right) \tag{5}$$

The critical part of this three-step approach of precise positioning is the step of ambiguity resolution. To date, many improvements to the search procedure have been developed and successfully applied in practice. However, still, the search procedure is conducted in *n*-dim ambiguity space. The concept of moving the search procedure from the *n*-dim ambiguity space to the 3-dim coordinate space is presented in the next section.

The search procedure in a coordinate domain

The conditional solution (5) derived from model (2) describes the change of the **b** vector resulting from the modification of the **a** vector. The method proposed in this work utilizes the relationship of type (5) but with swapped the **a** and **b** vectors. This modification also concerns corresponding cofactor matrices. The conditional solution resulting from the model (2) can be

alternatively formulated as the change of the **a** vector resulting from the change of the **b** vector. Such a formula can be derived using block matrix:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & -\mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1} \end{bmatrix} \tag{6}$$

where the first block **I** is an identity matrix. Premultiplying normal equation system (2) by the **T** matrix, we get the formula for the $\hat{\mathbf{a}}$ vector conditioned by the $\hat{\mathbf{b}}$ vector:

$$\hat{\mathbf{a}} = \left(\mathbf{Q}_{\hat{a}} - \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\mathbf{Q}_{\hat{b}\hat{a}}\right)\mathbf{A}^{\mathrm{T}}\mathbf{P}_{y}\mathbf{y} + \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\hat{\mathbf{b}} \tag{7}$$

Thus the change in the **b** vector, such as, e.g., $\breve{\mathbf{b}} = \hat{\mathbf{b}} + \Delta\mathbf{b}$ results in the modification of the **a** vector:

$$\breve{\mathbf{a}} = \left(\mathbf{Q}_{\hat{a}} - \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\mathbf{Q}_{\hat{b}\hat{a}}\right)\mathbf{A}^{\mathrm{T}}\mathbf{P}_{y}\mathbf{y} + \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\breve{\mathbf{b}} \tag{8}$$

The difference of the equations (7) and (8) leads to:

$$\breve{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\left(\hat{\mathbf{b}} - \breve{\mathbf{b}}\right) \tag{9}$$

The description of applying the above formula in the proposed method is presented in the further part of this section. This method is carried out according to the following steps: forming a search region in coordinate space, subsequently establishing a grid of candidates inside it, then testing each candidate and choosing this, which best satisfies the criterion assumed on the tests' stage. Although the main stages of the proposed approach are similar to those in the classic approach, the details differ significantly. The proposed method assumes that the float solution is obtained the same way as the traditional approach. Next, the search region, as the confidence region of the float solution, is established. In the 3-dim coordinate space, this region takes the form of an error ellipsoid of an approximate position from the float solution. The initial candidates (IC) grid is formed inside the search space. The arrangement of the IC set has to guarantee that no the VC of integer ambiguity vector located inside the search region is omitted. On the other hand, to satisfy the computational efficiency, the distance between neighboring IC should be as long as possible so that the number of IC is small. The rules of forming the search region and setting the grid of IC inside it, which account above conditions, are described in detail by Cellmer et al. (2021). Figure 1 shows some examples of an arrangement of IC inside the error ellipsoid. The examples are taken from the tests described in the last section of the article. The essence of the proposed method is included in the next step of the data processing. Let us name the ICs with $\mathbf{b}_{\mathrm{IC}}$. For *i*-th $\mathbf{b}_{\mathrm{IC}}$ we compute the corresponding ambiguity vector:

$$\mathbf{a}_{ICi} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\left(\hat{\mathbf{b}} - \mathbf{b}_{ICi}\right) \tag{10}$$

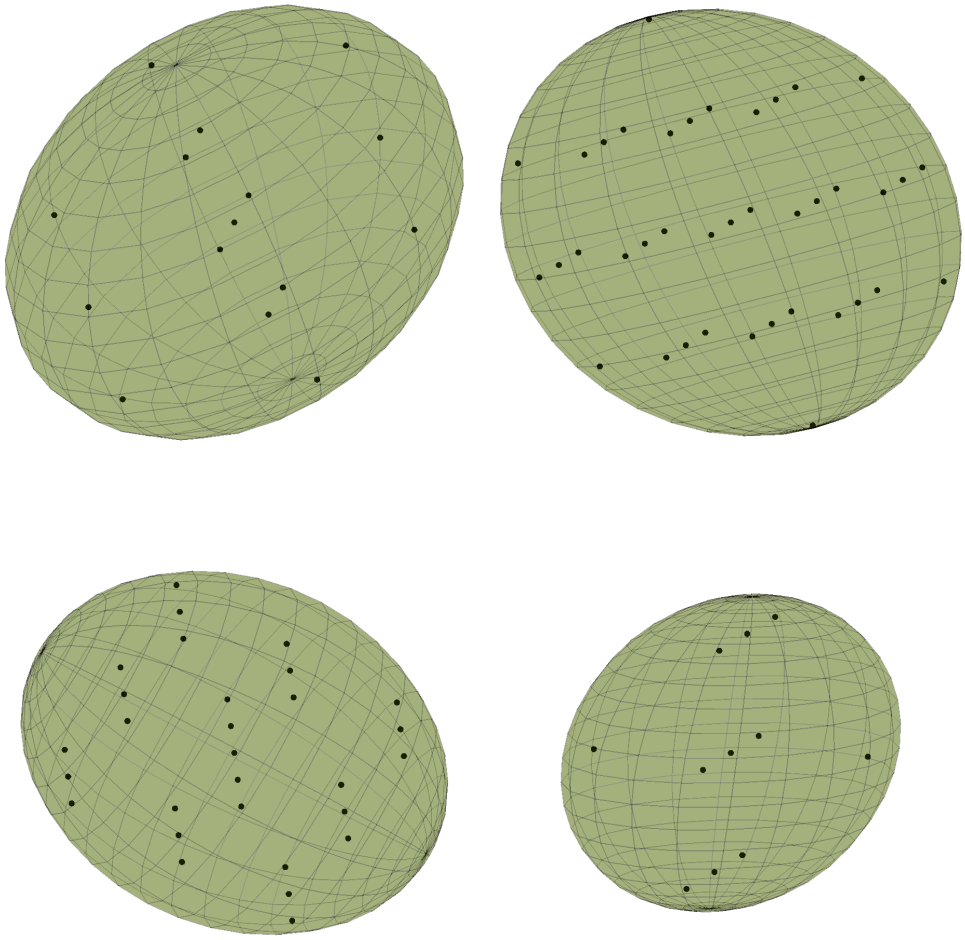


Fig. 1 Examples of the arrangements of the initial candidates inside the error ellipsoid.

Each ambiguity vector obtained by (10) is rounded to the nearest integer vector:

$$\mathbf{a}_{FCi} = round\left(\mathbf{a}_{ICi}\right) \tag{11}$$

The integer vector $\mathbf{a}_{\mathrm{FC}i}$ corresponds to the *i*-th final candidate. Each vector $\mathbf{a}_{\mathrm{FC}i}$ is tested to find that one which minimizes criterion:

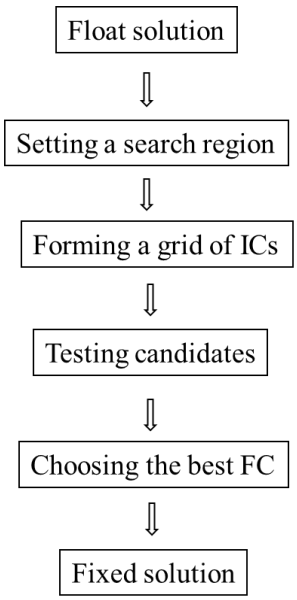
$$\mathbf{a}_{FC\min} = \arg\min_{\mathbf{z}\in Z^n} \left\|\hat{\mathbf{a}} - \mathbf{a}_{FCi}\right\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}}} \tag{12}$$

The $\mathbf{a}_{\mathrm{FC}\min}$ vector is employed to compute the final solution:

$$\mathbf{b}_{fix} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Q}^{-1}_{\hat{\mathbf{a}}}\left(\hat{\mathbf{a}} - \mathbf{a}_{FC\min}\right) \tag{13}$$

171 The successive steps of the computational process are depicted in Figure 2. The first three steps
172 of the computation process were discussed in (Cellmer et al., 2018; 2021). The primary issue
173 of the proposed method is included in the fourth step of the computational process: ICs testing.

174



176 **Fig. 2** Computational process of precise positioning based on search procedure in coordinate
177 domain

178 This step comprises operations (10) and (11), and also computing the norm on the right hand
179 of formula (12). Note the number of ICs can be greater than the number of FCs. The rules of
180 setting optimal search steps guarantee that no integer candidate is skipped. On the other hand,
181 a few ICs may fall into the same VC and consequently give the same FC. This is the cost of
182 reducing the dimension of a search space and has no significant impact on a computational load
183 if a search step length is set correctly.

184

185 **Numerical experiments**

186 *The real data test*

187 The first experiment is based on real data collected in two permanently operating EUREF
188 stations: WTZR and WTZZ (Germany), on 12, May, 2019, during a 24 h observation session.
189 The subject of the experiment is to test the performance of the new search procedure. The
190 analysis compares the new method with the classic one implemented in the LAMBDA method.
191 In the latter approach, a search for a fixed solution conducts in an ambiguity domain. The very
192 short baseline (1.62 m) was chosen for the tests to avoid any impact of atmospheric delays as
193 extraneous factors irrelevant in comparative analysis. Such a short baseline lets to take the

assumption that all atmospheric delays are eliminated by forming double differences of observations. Only the single-frequency GPS signals were processed in the experiment. The data used in this test was already utilized in another experiment, described by Cellmer et al. (2021). The interval between consecutive epochs was 30 seconds. The WTZR was set as a reference station and the WTZZ as a rover. The precise position of the rover was computed based on the data collected in a 24h session. This position was used as a reference for further comparisons. Next, the whole observation set was divided into short observation sessions. The 2000 subsequent epochs at 30-second intervals from the beginning of the 24h observation session were adopted as the starting epochs for the successive short sessions. The tests were carried out for sessions of various lengths: from a single-epoch solution to 30 minutes sessions. There were 2000 separate sessions tested for each session length. There were 7 satellites in most sessions. Only in some cases were observations collected from 8 satellites. The approximate position in each session was obtained using the DD code and phase observations discarding the integer nature of ambiguities (the float solution). The error ellipsoid of the approximate position was formed using the covariance matrix from a float solution and setting the confidence level to 0.999. The final precise position was estimated using two approaches. The first way is the method proposed in the article and the second way is the LAMBDA method. For simplicity, the first approach is denoted as #1 and the second as #2. The results were compared. In the second approach, the computations were conducted using the Matlab function *LAMBDA_06.m* retrieved from the TU Delft website: https://www.tudelft.nl/citg/over-faculteit/afdelingen/geoscience-remote-sensing/research/lambda/lambda (LAMBDA method, 2021). Table 1 lists the solutions' success rates (SR) in both approaches. The SR values were computed separately for each session length. The ambiguities obtained from 24h session were assumed to be actual.

Table 1 Success rates as a percentage of correct solutions in the samples of 2000 observation sessions

	Session length [minutes]							
	Single-epoch	1	2	5	10	15	20	30
	Success rates [%]							
#1 (SinCD)	45.6	59.8	70.2	85.3	94.8	98.2	98.6	100.0
#2 (LAMBDA)	45.2	60.7	72.4	90.0	99.8	100.0	100.0	100.0

Thus, the tested short-session solutions containing the actual ambiguity values are considered the correct solutions. The SR values are expressed in the percentage of correct solutions among 2000 sessions with a given length. The differences in SR ranged from 0 (for 30 min sessions) to 5 (for 10 min sessions). The proposed method achieved higher SR for single-epoch solutions. In other cases, better SR was obtained by the LAMBDA method. Although the percentage of

correct solutions is not precisely the same in both approaches, the differences in their values are minor. The mean value of absolute differences in SR in both approaches is 2.0%. The core element of the analysis is a comparison of the time needed for finding the correct solution in both approaches. Figure 3 depicts the mean computation time computed from 2000 sessions separately for each session length. On the X-axis are session lengths expressed in minutes, whereas on the Y-axis is a computation time in seconds. The log scale has been applied on the Y-axis. The session length has no impact on computation time in the LAMBDA method. Its value amounts slightly over 10-3 seconds for all session lengths.

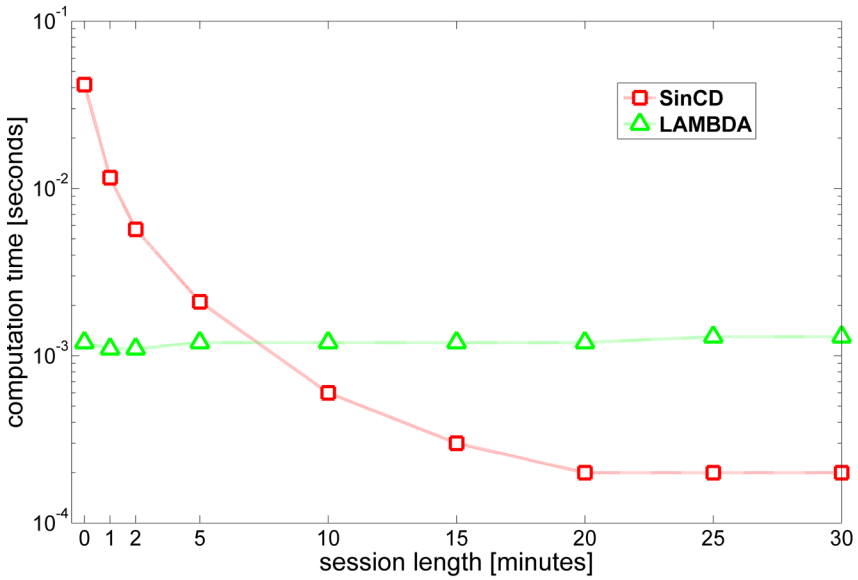


Fig. 3 Relationship between a session length and a computation time in the SinCD and the LAMBDA methods.

At the same time, the impact of session length on computation time is significant in the SinCD method. Its value ranges from 0.2*10-3 (for the longest sessions) to 41.8*10-3 in the case of single-epoch positioning. Shorter computation time in the SinCD method than in the LAMBDA is for 10-minute and longer sessions. In the present case study, the number of satellites amounts to 7 or, in some sparse sessions, 8. A relationship between the number of satellites and a computation time in both approaches can be evaluated based on simulated data. The following experiment has been designed for this purpose.

The test based on simulated data

It has been assumed that the positioning was conducted based on single-epoch data. The computations were repeated 2000 times for each assumed number of satellites. The satellite distribution was generated randomly in each epoch.

The simulated code-observations set can be modeled using simulated observation residuals:

$$\nabla\Delta\mathbf{P} = \nabla\Delta\boldsymbol{\rho}^{(\text{actual})} + \nabla\Delta\mathbf{v}_c \tag{14}$$

or using simulated shift between approximated position and actual position

$$\nabla\Delta\mathbf{P} = \nabla\Delta\boldsymbol{\rho}^{(\text{actual})} + \delta\boldsymbol{\rho} \tag{15}$$

where: $\nabla\Delta\mathbf{P}$ is double differenced (DD) code-observations vector, $\nabla\Delta\mathbf{v}_c$ is DD code-observation residuals vector, $\nabla\Delta\boldsymbol{\rho}^{(\text{actual})}$ is DD actual geometric distances vector, $\delta\boldsymbol{\rho}$ is a part of a computed DD geometric distance, resulting from a shift of approximated position regard to the actual one. The last term in (11) can be expressed in linear form as:

$$\delta\boldsymbol{\rho} = \mathbf{A}\mathbf{dx} \tag{16}$$

where the **A** matrix is a design matrix of DD code-observations.

Subtracting (15) from (14) and inserting (16) we get observation equations for simulated code-data:

$$\nabla\Delta\mathbf{v}_c = \mathbf{A}\mathbf{dx} \tag{17}$$

in which the $\nabla\Delta\boldsymbol{\rho}^{(\text{actual})}$ and the $\nabla\Delta\mathbf{P}$ vectors vanish. All terms of (17) are prepared in the simulation procedure. Each entry of the $\nabla\Delta\mathbf{v}_c$ vector on the left side of equation (17) is formed by generating four residuals with assumed normal distribution $N[0,\sigma_c]$ where σ_c is the standard deviation of code-observations and then forming single and double differences. Four residuals have to be generated because this amount of pseudo-ranges is needed to create a single DD observation. The entries of the **dx** vector are values generated from the range of plus/minus a few meters. Preparing a design matrix **A** for simulated data set does not require knowledge about exact satellite positions. The *jth* row of the undifferenced design matrix $\mathbf{A_u}$ consist of the ratios of components of satellite-rover distance and this distance:

$$\mathbf{A_u}(j,:) = \left[-\frac{\Delta x_{rov}^j}{\rho_{rov}^j}, -\frac{\Delta y_{rov}^j}{\rho_{rov}^j}, -\frac{\Delta z_{rov}^j}{\rho_{rov}^j} \right] \tag{18}$$

That vector can be alternatively expressed by azimuth Az_j and elevation angle El_j in a local, horizontal coordinate system:

$$\mathbf{A_u}(j,:) = \left[-\cos(El_j)\cos(Az_j), -\cos(El_j)\sin(Az_j), -\sin(El_j) \right] \tag{19}$$

Thus, it is enough to generate satellites' azimuths and elevation angles to form a design matrix for undifferenced observations. The **A** matrix for DD observations is created by subtracting the row related to the reference satellite from the other rows of the $\mathbf{A_u}$ matrix:

$$\mathbf{A}(j,:) = \mathbf{A_u}(j,:) - \mathbf{A_u}(ref,:), \text{ for all } j \neq ref. \tag{20}$$

Thus, a design matrix for simulated data is formed using satellites' azimuths and elevations generated in ranges (0;2π) and (0;π/2) respectively. The observation equations for the simulated carrier phase observations can be derived analogously. They take the form similar to those of the equation (17):

$$\nabla\Delta\mathbf{v}_{ph} = \tfrac{1}{\lambda}\mathbf{A}\mathbf{d}\mathbf{x} + \mathbf{a} \tag{21}$$

The only difference is an additional term of the ambiguity vector **a**, which must be generated as a random integer number. The DD carrier phase residuals on the left hand are expressed in cycles, so the first term on the right hand is divided by carrier phase wavelength λ. There were considered the various number of satellites in a single-epoch positioning. The computations were repeated 2000 times for each number of satellites. Data were simulated and processed separately in each epoch using two approaches: the SinCD and the LAMBDA methods. The single-epoch positioning was conducted in a two-stage process: In the first stage, the observation set consisted of both types of code- and phase observations. The float solution results, not satisfying the integer nature of ambiguities, were the same in the two approaches. The precise position (fixed solution) was computed using the SinCD and LAMBDA methods. The SRs calculated from 2000 repetitions achieve almost the same values in both approaches for all tested numbers of satellites. The only difference in those values occurs in the case of 9 satellites and amounts to 0.1% (99.9% in the SinCD method and 100% in the LAMBDA method). Table 2 lists the SR values for both methods with respect to the number of satellites. Since the SR for the number of satellites in the range of 10-30 achieves the same values amounting to 100.0% they have been placed in the single column.

Table 2 Success rates as a percentage of correct solutions in 2000 repetitions of single-epoch positioning for various numbers of satellites

	Number of satellites				
	6	7	8	9	10-30
	Success rates [%]				
#1 (SinCD)	37.4	84.4	99.0	99.9	100.0
#2 (LAMBDA)	37.4	84.4	99.0	100.0	100.0

Figure 4 depicts the computation time (CT) needed to obtain a solution by two tested methods. The CT was computed as a mean value from 2000 repetitions for each of the assumed numbers

of satellites. The CT of the LAMBDA method for the number of satellites fewer than 10 is stable and low. In contrast, the computational time of the SinCD method significantly decreases, in this range, when the number of satellites increases. The opposite situation starts when the number of satellites amounts to 20 or more. The computational time of the LAMBDA method starts to increase considerably, whereas the computational time of the SinCD method decreases gradually. Starting from 25 satellites providing the observations for a single-epoch positioning, the computational time of processing data is shorter in the SinCD method than in the LAMBDA method.

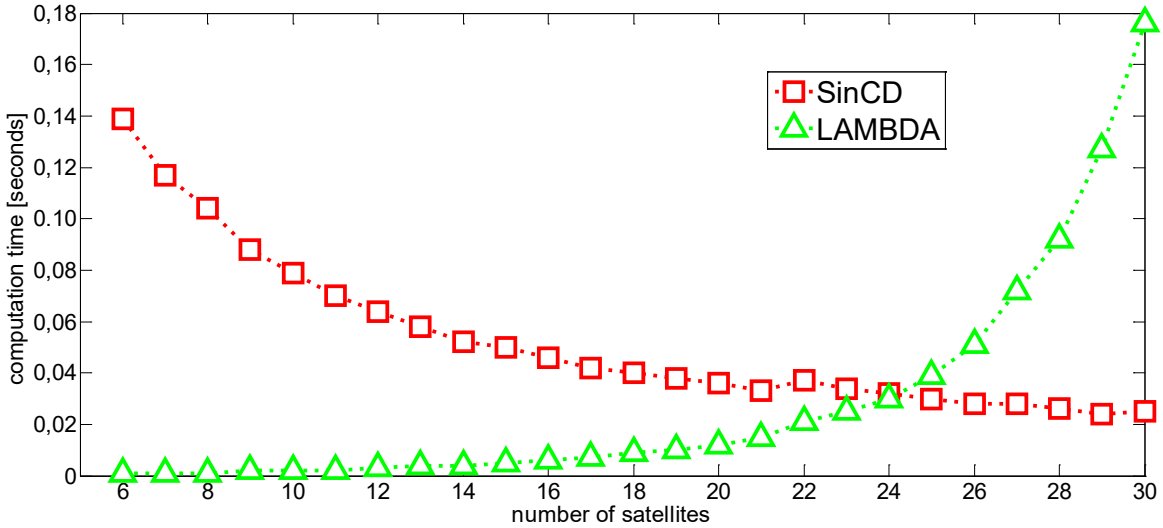


Fig. 4 The computation time of the SinCD and the LAMBDA methods for a various number of satellites. The single-epoch positioning was performed based on simulated data.

The results of the tests confirmed the good performance of the proposed method. Unfortunately, based on the real data test, we should admit that the SR is slightly fewer than in the traditional method in the cases with a small number of satellites and short observation sessions. However, the difference is not significant. On the other hand, for sessions 10-minute and longer with 7 or 8 satellites providing observation set, the CT is shorter in the proposed method than in the traditional one. Based on the test with simulated data, one can conclude that in a case of a single-epoch positioning, the CT is shorter in the proposed method than in the traditional one if the number of satellites is greater than 24.

Conclusions

The motivation of the work was the apparent finding that the dimension of the search space is non-negligible for search procedure efficiency. The time needed for obtaining a solution raises considerably when the search space dimension is greater. Therefore, the concept of searching

for a fixed solution in a coordinate domain instead of in an ambiguity domain was investigated. As a result, the new method was developed and tested. A major advantage of the proposed method is moving a search procedure from multi-dimensional ambiguity space to only three-dimensional coordinate space. Such an operation allows shortening computation time, thereby increasing the efficiency of the precise positioning. This feature reveals oneself in the case of an observation set obtained from many satellites. It has great importance in the prospect of increasing the number of satellites resulting from developing modern satellite navigation systems. The test results confirmed shortening computation time when using the proposed method and that its advantage over traditional methods unveils in the case of many satellites. The real data test pointed out that applying the new approach is beneficial in the case of long sessions, even if we deal with a small number of satellites.

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