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Local Calibration of a Weak Stiffness On-Machine Measuring Device with a New 3-D Calibrator

Helin Li¹, Bin Lin^{1,*}, Tianyi Sui¹, Tian Huang¹, Haitao Liu¹, Fukan Yu¹, Pengcheng Zhao¹, Yueting Jia¹, Fuan Zhu¹

Abstract

This paper presents a new local calibration method for a weak stiffness on-machine measuring (OMM) device. First, the original volumetric error models' union is established, and an equivalent dimensionality reduction algorithm is proposed to reduce the model's dimensionality, thus, the novel equivalent volumetric error models' union (EVEMU) is derived, which is more suitable for limited calibration data. For better flexibility and efficiency of error measurements, a new 3-D ball array calibrator is proposed, and its local calibration procedures can be executed directly in the 3-D space. Then, the smoothly clipped absolute deviation (SCAD) regularization algorithm with *oracle* property is introduced to identify equivalent coefficients as well as select the optimal volumetric error sub-model of EVEMU, which can closely represent the real error state of the weak stiffness OMM device. Experiments demonstrated that methods proposed in this paper are effective, and provide new insights for the error compensation of similar devices.

Keywords: On-machine measurement; Weak stiffness; Volumetric error; Calibration; 3-D calibrator; Smoothly clipped absolute deviation

1 Introduction

The manufacturing and measurement technologies of free-form surface parts have been rapidly developing with the increasing demand for such parts in aeronautical engineering. the use of offline measurement instruments typically requires the removal of the workpiece from the machine tool, which can introduce systematic errors, on-machine measurement can overcome this disadvantage and has shown remarkable achievements regarding high precision[1]. The accuracy is one of the most important indicators of measuring devices, however, the hardware accuracy of a mechanical structure is limited,

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especially with the increasing demands for lightweight structures, some new weak stiffness measuring devices appear, which brings new challenges to the improvement of their accuracy. In the past few decades, the software-based error compensation method has been proven to be a cost-effective method that can improve the geometric accuracy of measuring devices[2]. and it can be achieved by following three steps [3]: (1) develop an error model for the machine; (2) use a measuring instrument to measure the errors; and (3) conduct error compensation using the error model.

As for the error modeling, Many effective error modeling methods have been proposed such as the homogeneous transformation matrix (HTM) [4], multi-body system (MBS) [5], screw theory [6], differential motion matrix method (DMM) [7] and elasto-geometrical error modeling method [8], etc. For the measurement of the error, the approaches can be distinguished into “direct” and “indirect” methods [9]. The “direct” method measures directly one or several geometric error items for one measurement setup. The ISO 230-1 [10] describes the main direct measurement methods that require precise measurement instruments, such as a laser interferometer with different combinations of optics kit. Nevertheless, most direct measurement methods require skilled operators, careful setting, and long measurement times, while for volumetric error compensation, the efficiency of the direct measurement is an issue [11]. On the other hand, the indirect method has become one of the main research fields due to its efficiency advantages. In particular, indirect measurement focuses on the tooltip location as the superposition of single geometric errors [11] and relies on multiple measurements in the machine work volume. In indirect methods, reference artifacts and laser tracker[12] are mainly utilized. In the past few decades, several reference artifacts have been developed, which have been presented in ISO 10360-2:2009 [13]. They can be classified into three categories: one-dimensional (1-D) artifacts that include step gauges, ball bars, and 1-D ball arrays; two-dimensional (2-D) artifacts that include ball plates and hole plates; and three-dimensional (3-D) artifacts that include 3-D grid artifacts and 3-D ball plates. In particular, Zhang et al. [14] proposed a geometric calibration method for a 3 linear axis coordinate measuring machine (CMM) based on a 1-D ball array, and several measurements were required to identify 21 geometric errors. Pahk et al. [15] developed a geometric error identification method based on ball bar measurements. A volumetric error model was constructed based on the geometric errors which were modeled as polynomial, and the ordinary least square (OLS) method was utilized to identify the coefficients of the respective geometric errors. Kunzmann et al. [16] proposed a ball plate-based calibration method, where the ball plate was fixed at different measurement positions, and then, the identified errors were separated through geometric relations. Trapet et al. [17] presented a calibration procedure using a hole plate, where the hole plate was measured in 4 positions in the working space, and 21 errors could be calculated separately. Liebrich et al. [18] proposed a 3-D ball plate for detecting machine tool errors. The ball plate could detect deviations of the X, Y, and Z coordinates at several points, which were used to identify a single geometric error with

equations, or were used as direct input of a spatial compensation grid [19]. Viprey et al. [20] designed a multifunctional bar and developed a program for identifying 21 geometric errors of the linear axes. In addition, several new methods have also been adopted for the indirect measurement of 5-axis geometric errors, such as ball bars [21], R-test [22] and others[23]. For large machines, multilateration methods with the laser tracker, such as the 12-line method [24], are commonly used. As for error compensation, several numerical compensation strategies have been proposed. The final result compensation is a feasible method for a measuring device[25], which compensates the measured values by adding the compensation vectors to the values.

The above literature review indicates that a great deal of work has been conducted for error modeling, measurement and compensation, and has made remarkable achievements. However, these methods are mainly for machine tools or CMMs with relatively high stiffness, and relatively few investigations have been conducted for the accuracy improvement of weak stiffness measuring devices. In terms of error modeling, when there are many axes, the error model may have redundancy, methods to deal with the redundancy still deserve further development. Concerning the error measurements, most of the direct and indirect measurement methods with laser trackers rely on expensive optical measurement instruments and time-consuming operations. As for artifact-based indirect measurement methods, the existing 1- and 2-D artifacts have limited measurable dimensions, most of which need to be parallel to some axis or plane in order to measure explicit single geometric errors, thus, many measurements are required for measuring all geometric errors of a machine, so, these methods are relatively time-consuming, too. Some 3-D artifacts allow the measurement of volumetric errors, which can be used to identify single geometric errors or as direct input of the spatial compensation grid [19], however, these artifacts often have a large volume and lack flexibility. Therefore, the calibrator with better flexibility and efficiency is worth studying.

The prominent feature of the weak stiffness measuring device is “deformations” and the resulting “interactions” in this paper, as shown in Fig. 1, it means that the device may have large deformations at limited positions, which will affect the absolute positioning accuracy of the device, and as a result, their geometric errors may not be independent and perform like interacting with each other. Therefore, measuring explicit geometric errors of every axis and directly taking them into the volumetric error model may not be effective enough, the indirect method is more appropriate. Thus, we are no longer dedicated to identifying the explicit geometric errors in this paper, but to seeking a further equivalent solution in the intermediate solution space. First, we establish the original volumetric models union based on MBS [26], DMM [7], and response surface method [27], then, a new equivalent dimensionality reduction method is proposed, to construct the equivalent volumetric models union (EVEMU), which is more suitable for small sample measuring data. To improve the measurement flexibility and efficiency of artifacts as much as possible, a new 3-D ball array calibrator is proposed, it has 3

measurable dimensions, a more simple structure with a small volume compared to conventional 2-D or 3-D calibrators. furthermore, the calibration procedures are presented, and the concept of “local calibration” is introduced, which aims at improving the accuracy of specific local volumes, instead of calibrating the entire machine working volume. To weaken “interactions”, we transformed the physical “interactions” problem into the multicollinearity problem of linear algebra, and the smoothly clipped absolute deviation (SCAD) algorithm with *oracle* property is introduced to deal with the multicollinearity and to select the optimal volumetric error sub-model from EVEMU at the same time. Local calibration experiments on a ZYX OMM device with weak stiffness are performed. The correctness and feasibility of the methods proposed in this paper are verified experimentally.

The rest of this paper is organized as follows: Section 2 introduces the EVEMU method; Section 3 proposes the 3-D ball array calibrator and its local calibration procedures; Section 4 introduces the SCAD-based identification and volumetric error compensation; Section 5 verifies experimentally the methods proposed in this paper, and Section 6 are conclusions.

2 EVEMU

To demonstrate the theory proposed in this paper, a 3-axis weak stiffness OMM measuring device is introduced (Fig. 1). The OMM device is designed to measure the internal and external freeform surfaces of thin-wall workpieces for several machine tools in production lines, so, it has a long cantilever structure and is movable that can move to different machine tools for on-machine measurement, however, the movement of the OMM device may lead to changes in accuracy, consequently, a relatively effectively instant calibration method is required in production scene. The OMM device is composed of a tactile probe and three orthogonal linear axes, i.e., Z-axis, X-axis, and Y-axis. In addition, there is an external rotary A-axis of another machine tool, and the workpiece is fixed on the A-axis. During the inspection, the tactile probe inspects along the intersection line of the workpiece over the X-Y plane (Fig. 1). Then, the A-axis rotates at a certain angle, the OMM device inspects again along the intersection line and repeats the above steps until the required measured data are obtained. The regions formed by these intersecting lines are always within the measuring volume shown in Fig. 1, which is a local volume within the entire working volume of the OMM device, and will be studied in this paper. Since the A-axis is calibrated beforehand, this paper only focuses on the local calibration of the three-axis OMM device. Due to the long cantilever structure and lightweight requirements, the OMM device shows relatively weak stiffness property especially at limited positions of the device, which causes the measured values to deviate from the theoretical values and is shown in Fig.8, and we define a weak stiffness structure if its volumetric error exceeds 1mm in every movement of 300mm. Therefore, the instantly online error calibration or accuracy maintenance of the OMM device are important for its industrial application, and this paper focuses on solving this problem.

Figure 1. Schematic illustration of the ZXY-type OMM device, it is designed to measure the internal and external freeform surfaces of thin-wall workpieces for several machine tools in production lines, due to its long cantilever structure and lightweight requirements, the OMM device shows the relatively weak stiffness property especially at limit positions of the device, and cause the measured values to deviate from the theoretical values, we define a weak stiffness structure if its volumetric error exceeds 1mm in every movement of 300mm. Therefore, the instantly online error calibration or accuracy maintenance of the OMM device are important for its industrial application; and the detailed cutaway view shows the measuring volume of local calibration, and this paper focuses on solving this problem

2.1 Original volumetric error models' union

For each linear Axis, there are 6 position-dependent geometric errors (PDGEs) according to ISO230-1 [10]. If the component moves along the X-axis by x , the motion produces 6 geometric errors: one position error $\delta_x(x)$, two straightness errors $\delta_y(x)$ and $\delta_z(x)$, and three angle errors $\varepsilon_x(x)$, $\varepsilon_y(x)$, and $\varepsilon_z(x)$, and they can form an error vector $\Delta \mathbf{X}$ as shown in Eq. (1). Similarly, the PDGEs of the Y-axis are defined as $\Delta \mathbf{Y}$, and those of the Z-axis are defined as $\Delta \mathbf{Z}$. In addition, there are three position-independent geometric errors (PIGEs) between axes, namely straightness error vector $\Delta \mathbf{S}$. And they can form a geometric error column matrix ξ as shown in Eq. (1)

$$\begin{aligned}
 \Delta \mathbf{X} &= [\delta_x(x), \delta_y(x), \delta_z(x), \varepsilon_x(x), \varepsilon_y(x), \varepsilon_z(x)]^T \\
 \Delta \mathbf{Y} &= [\delta_x(y), \delta_y(y), \delta_z(y), \varepsilon_x(y), \varepsilon_y(y), \varepsilon_z(y)]^T \\
 \Delta \mathbf{Z} &= [\delta_x(z), \delta_y(z), \delta_z(z), \varepsilon_x(z), \varepsilon_y(z), \varepsilon_z(z)]^T \\
 \Delta \mathbf{S} &= [\alpha_{xy}, \alpha_{zx}, \alpha_{zy}]^T \\
 \xi &= [\Delta \mathbf{X} \quad \Delta \mathbf{Y} \quad \Delta \mathbf{Z} \quad \Delta \mathbf{S}]^T
 \end{aligned} \tag{1}$$

The ideal homogeneous coordinate transformation matrix from the tactile probe coordinate system to the machine's

base coordinate system $\mathbf{T}_{wt_ideal} = f_{ideal}(x, y, z)$ can be derived via MBS theory[26], but due to PDGEs and PIGEs, the actual transformation matrix $\mathbf{T}_{wt_actual}$ is not equal to $\mathbf{T}_{wt_ideal}$, $\mathbf{T}_{wt_actual} = f_{actual}(x, y, z, \Delta\mathbf{X}, \Delta\mathbf{Y}, \Delta\mathbf{Z}, \Delta\mathbf{S})$ can be established via DMM[7]. Then, the volumetric error model of the OMM device can be derived by neglecting the higher-order error terms (above the 2nd order) and errors of the tactile probe based on $\mathbf{T}_{wt_actual}$ and $\mathbf{T}_{wt_ideal}$, as shown in Eq. (2)

$$\mathbf{E}(x, y, z) = \mathbf{P}_{ideal} - \mathbf{P}_{actual} = (\mathbf{T}_{wt_ideal} - \mathbf{T}_{wt_actual})\mathbf{P}_t = \mathbf{J}\xi \quad (2)$$

where $\mathbf{P}_t = [0, 0, 0, 1]^T$ is a vector to filter volumetric error, $\mathbf{P}_{ideal} = [x_{ideal}, y_{ideal}, z_{ideal}]^T$ represents the ideal coordinates of the measured points, $\mathbf{P}_{actual} = [x, y, z]^T$ represents the actual coordinates of measured points by the OMM device, the difference between them is the volumetric error $\mathbf{E}(x, y, z)$, and $\mathbf{J}$ is the Jacobian matrix of $\mathbf{E}$ about ξ .

For PDGEs in $\Delta\mathbf{X}$, $\Delta\mathbf{Y}$ and $\Delta\mathbf{Z}$, such as $\delta_x(x)$, the real analytic function between $\delta_x(x)$ and x is unknown, or cannot be explicitly expressed, the unary polynomial model $\delta'_x(x)$ concerning x is applied to fit the real $\delta_x(x)$ based on the response surface methodology [27], it is an averaged fitting model on y and z about x . For PIGEs, $\Delta\mathbf{S}$ is position-independent, so it is treated as a constant vector. therefore, the ξ can be fitted as Eq. (3).

$$\xi = \mathbf{G}\mathbf{K}$$

$$\text{where } \mathbf{G} = \begin{bmatrix} \mathbf{G}_x & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_y & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_z & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_\alpha \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \mathbf{K}_{dxx} \\ \mathbf{K}_{dyx} \\ \mathbf{M} \\ \mathbf{K}_{ezz} \\ \mathbf{M} \\ \Delta\mathbf{S} \end{bmatrix} \quad (3)$$

where $\mathbf{G}$ is the fitting variable matrix of $\mathbf{K}$, $\mathbf{G}_x$, $\mathbf{G}_y$ and $\mathbf{G}_z$ are row matrices of each polynomial's variable, $\mathbf{G}_x = [x^{np} \ x^{np-1} \ L \ x^0]$, $\mathbf{G}_y = [y^{np} \ y^{np-1} \ L \ y^0]$, $\mathbf{G}_z = [z^{np} \ z^{np-1} \ L \ z^0]$, the “np” is the highest order of each polynomial, and should be set in advance, $\mathbf{G}_\alpha = [1 \ 1 \ 1]$ because the PIGEs are constants. $\mathbf{K}$ is the polynomial's coefficient matrix, $\mathbf{K}_{dxx} = [k_{dxx(np)} \ k_{dxx(np-1)} \ L \ k_{dxx0}]^T$, $\mathbf{K}_{dyx} = [k_{dyx(np)} \ k_{dyx(np-1)} \ L \ k_{dyx0}]^T$, etc. “ $\mathbf{K}_{ABC}$ ” like column matrices are coefficient matrices of each polynomial related to ξ , and the “ k_{ABCi} ” like expressions represent a polynomial coefficient, while the first letter of the subscript represents the error type, in particular, “d” represents a position error and “e” represents an angle error. The second letter of the subscript represents the direction of the affected axis, the third one represents the axis that moves, and the last one represents the corresponding variable's order. By Substituting Eq. (3) into Eq. (2), Eq. (4) is derived.

$$\mathbf{E} = \mathbf{J}\mathbf{G}\mathbf{K} \quad (4)$$

By merging $\mathbf{JG}$ to $\mathbf{A}$, as shown in Eq. (5), and Eq. (6) is obtained

$$\mathbf{A} = \mathbf{JG} \quad (5)$$

where $\mathbf{A}$ is a new Jacobian matrix about $\mathbf{K}$, and $\mathbf{E}$ becomes a generalized linear model.

$$\mathbf{E} = \mathbf{AK} \quad (6)$$

In Eq. (3) to Eq. (6), the selection of “np” is important for the bias-variance performance of a model, inappropriate choice of it may cause overfitting or underfitting problems. Aguado et al. [28] observed that the polynomial order is limited by the physical behavior of the geometric errors, and a third-order polynomial is a good approximation choice. Fu et al. [29] demonstrated that, sometimes, second- or first-order models may be preferable than third-order ones for PDGEs fitting. Thus, orders not greater than 3 are considered as comprehensive choices, so, “np” is set as 3. It should be noted that “np” in this paper just represents the highest order of each polynomial and the optimal order will be automatically selected by the measured data and identification algorithm, so, each polynomial of PDGEs is actually a union of its sub-polynomials whose orders are not greater than “np”. As a result, Eq. (6) is actually a union of its possible volumetric error sub-models, also called the original volumetric error models’ union in this paper.

2.2 EVEMU based on equivalently dimensionality reduction

When calibration, $\mathbf{A}$ in Eq. (6) will be constructed from series of measured data, and $\mathbf{K}$ is the parameter vector to be identified. Before identifying $\mathbf{K}$, it is necessary to check the redundancy of Eq. (6) [30], because if Jacobian matrix $\mathbf{A}$ is not column full rank, $\mathbf{K}$ has no unique solution. When using the iterative method to solve $\mathbf{K}$, different initial values may produce different solutions. Although some analytical methods can solve its general solution [31], if there is a method which can equivalently reduce the dimensionality of Eq. (6), then, the equivalent $\mathbf{A}$ is column full rank, the corresponding equivalent $\mathbf{K}$ ’s solution will be unique, for the same amount of observed data, the dimensionality reduced equivalent model will relatively improve the data utilization and the confidence of the solution [32], especially when the dimensionality is significantly reduced with limited measured data. Assuming that there is a dimensionality reduction matrix $\mathbf{W}_{r \times h}$ and its Moore-Penrose inverse matrix $\mathbf{W}_{n \times r}^\dagger$, and they satisfy the relationship shown in Eq. (7). Then, the dimensionality reduction can be realized.

$$\mathbf{E} = \mathbf{A}_{m \times h} \left(\mathbf{W}_{h \times r}^\dagger \mathbf{W}_{r \times h} \right) \mathbf{K}_{h \times 1} \quad (7)$$

Where $\mathbf{A}_{m \times h}$ and $\mathbf{K}_{h \times 1}$ are same with those in Eq. (6), their subscripts represent the number of rows and columns, “m” equals 3, “h” represents the highest dimensionality of the model, and equals $18 \times (\text{np} + 1) + 3$ because of 18 PDGEs and 3 PIGEs, the “r” in $\mathbf{W}_{r \times h}$ and $\mathbf{W}_{h \times r}^\dagger$ is the column rank of $\mathbf{A}_{m \times h}$. Then, Multiplying $\mathbf{A}_{m \times h}$ by $\mathbf{W}_{h \times r}^\dagger$, the

equivalent $\mathbf{X}_{m \times r}$ of $\mathbf{A}_{m \times h}$ is obtained, the equivalent $\boldsymbol{\beta}_{r \times l}$ of $\mathbf{K}_{h \times l}$ can be derived through Multiplying $\mathbf{W}_{r \times h}$ by $\mathbf{K}_{h \times l}$, as shown in Eq. (8).

$$\begin{aligned}\mathbf{X}_{m \times r} &= \mathbf{A}_{m \times h} \mathbf{W}_{r \times h}^\dagger \\ \boldsymbol{\beta}_{r \times l} &= \mathbf{W}_{r \times h} \mathbf{K}_{h \times l}\end{aligned}\tag{8}$$

Based on Eq. (7) and Eq. (8), the $\mathbf{E}$ can also be represented by the new $\mathbf{X}_{m \times r}$ and $\boldsymbol{\beta}_{r \times l}$ in Eq. (9). Comparing Eq. (9) with Eq. (7), the column dimensionality of $\mathbf{A}_{m \times h}$ and the row dimensionality $\mathbf{K}_{h \times l}$ is equivalently reduced from “h” to “r”, and thus Eq. (9) is called the equivalent volumetric error models’ union (EVEMU) in this paper.

$$\mathbf{E} = \mathbf{X}_{m \times r} \boldsymbol{\beta}_{r \times l}\tag{9}$$

To construct the appropriate $\mathbf{W}_{r \times h}$, the singular value decomposition (SVD) is one of the possible methods [33], it is a widely used dimensionality reduction algorithm in machine learning, but it is mainly based on the eigenvalues and eigenvectors of $\mathbf{A}_{m \times h} (\mathbf{A}_{m \times h})^T$ and $(\mathbf{A}_{m \times h})^T \mathbf{A}_{m \times h}$, which may lead to each term in the $\boldsymbol{\beta}_{r \times l}$ contains a lot of polynomial coefficients k s and introduce new problems. To include each k in the terms of $\boldsymbol{\beta}_{r \times l}$ as independently as possible, a new construction algorithm of $\mathbf{W}_{r \times h}$ is proposed according to the characteristics of $\mathbf{A}_{m \times h}$, and the key idea is similar to the combining like terms operation and matrix construction operation in polynomial algebra, as shown in Algorithm 1.

Algorithm 1. Computational procedure for the proposed construction of $\mathbf{W}_{r \times h}$

```

input:  $\mathbf{A}$ , polynomial matrix of size  $m \times h$  and its column rank is  $r$ 
input:  $\mathbf{K}$ , coefficient matrix of size  $h \times 1$ 
output:  $\mathbf{W}$ , dimensionality reduction matrix of  $\mathbf{A}$  and its size is  $r \times h$ 

1  $\mathbf{W} = \text{Matrix}()$ 
2 for  $i=1$  to  $\mathbf{A}.\text{nrows}$  do
3   for  $j=1$  to  $\mathbf{A}.\text{ncols}$  do
4     if  $\mathbf{A}[i, j] \neq 0$  then
5        $\text{coef}_{ij} = \mathbf{A}[i, j].\text{coef}$ 
6        $\text{vars}_{ij} = \mathbf{A}[i, j] / \text{coef}_{ij}$ 
7        $\text{temp\_coefs}_{ij} = \mathbf{A}[i, :] / \text{vars}_{ij}$ 
8        $\text{temp\_mask}_{ij} = \text{Zeros}[1, \mathbf{A}.\text{nrows}]$ 
9       for  $k=1$  to  $\mathbf{A}.\text{ncols}$  do
10        if  $\text{temp\_coef}_{ij}[1, k].\text{is\_number}$  and  $\text{temp\_coef}_{ij}[1, k] \neq 0$  then
11           $\text{temp\_mask}_{ij}[1, k] = 1$ 
12        end
13      end for
14       $\text{temp\_W}_{ij} = \text{temp\_coefs}_{ij} \text{ o } \text{temp\_mask}_{ij}$ 
15       $\text{is\_in\_w} = 0$ 
16      if  $\mathbf{W}.\text{nrows} == 0$  then
17         $\mathbf{W}[\mathbf{W}.\text{nrows} + 1, :] = \text{temp\_W}$ 
18      else
19         $\text{temp\_}\beta = \mathbf{W} \cdot \mathbf{K}$ 
20         $\text{temp\_}\beta_{ij} = \text{temp\_W}_{ij} \cdot \mathbf{K}$ 
21        for  $t = 1$  to  $\text{temp\_}\beta.\text{ncols}$  do
22          if  $\text{abs}(\text{temp\_}\beta[t, :]) == \text{abs}(\text{temp\_}\beta_{ij})$  then
23             $\text{is\_in\_w} = 1$ 
24            break
25          end
26        end
27        if  $\text{is\_in\_w} == 0$  then
28           $\mathbf{W}[\mathbf{W}.\text{nrows} + 1, :] = \text{temp\_W}$ 
29        end
30      end
31    end for
32  end for
33 return  $\mathbf{W}$ 

```

where the input is the same $\mathbf{A}_{m \times h}$ and $\mathbf{K}_{h \times 1}$ in Eq. (7), the dimensionality reduction matrix $\mathbf{W}$ is initialized as an empty matrix. $\mathbf{A}.\text{nrows}$ operator represents to get the row's amount of $\mathbf{A}$, $\mathbf{A}.\text{ncols}$ is to get the column's amount of $\mathbf{A}$, $\mathbf{A}[i, j].\text{coef}$ is to get the constant coefficient of the term $\mathbf{A}[i, j]$ without any variables, e.g. $(-x^2y).\text{coef} = -1$. The operator "o" is Hadamard product [34]. The algorithm scans matrix row by row and column by column, and performs the following process: if $\mathbf{A}[i, j]$ not zero, then, to get its coefficient coef_{ij} and calculate the vars_{ij} of $\mathbf{A}[i, j]$, Divide the row vector $\mathbf{A}[i, :]$ by vars_{ij} to get the temporary coefficient row vector temp_coefs_{ij} . With the help of mask row matrix temp_mask_{ij} and Hadamard product, to retain constant term in temp_coefs_{ij} , and set the non-constant term to 0, then produce temp_W_{ij} . If the absolute value of $\text{temp_W}_{ij} \cdot \mathbf{K}$ does not exist in any row of current absolute $\mathbf{W} \cdot \mathbf{K}$, temp_W_{ij} will be appended to the end of $\mathbf{W}$ and become a new row. Until all elements of being processed, the generated $\mathbf{W}$ with size $r \times h$ will be returned. Then, $\mathbf{W}_{h \times r}^\dagger$ of Eq. (9) can be solved by SVD using $\mathbf{W}_{h \times r}^\dagger$ [31].

For the OMM device in this paper, based on Eq. (6) and "np" = 3, $\mathbf{A}_{3 \times 75}$ is derived, and the dimensionality reduction

matrix $\mathbf{W}_{45 \times 75}$ is constructed by Algorithm 1. The $\mathbf{W}_{75 \times 45}^\dagger$ is calculated by SVD. Multiply $\mathbf{A}_{3 \times 75}$ by $\mathbf{W}_{75 \times 45}^\dagger$ to get the $\mathbf{X}_{3 \times 45}$, which shows that the column dimensionality of the variable matrix $\mathbf{A}_{3 \times 75}$ is equivalently reduced from 75 to 45, $\mathbf{W}_{45 \times 75}$ times $\mathbf{K}_{75 \times 1}$ to get $\boldsymbol{\beta}_{45 \times 1}$, which means that the dimensionality coefficient vector $\mathbf{K}_{75 \times 1}$ is reduced from 75 to 45. Some of the k s in $\mathbf{K}_{75 \times 1}$ are combined to construct new equivalent k_{eq} s in $\boldsymbol{\beta}_{45 \times 1}$. Their relationship is shown in Eq. (9) and the detailed results are shown in the appendix.

$$\begin{aligned} \mathbf{E} = \mathbf{A}_{3 \times 75} \mathbf{K}_{75 \times 1} &\rightarrow \mathbf{E} = \mathbf{A}_{3 \times 75} \left(\mathbf{W}_{75 \times 45}^\dagger \mathbf{W}_{45 \times 75} \right) \mathbf{K}_{75 \times 1} \rightarrow \mathbf{E} = \mathbf{X}_{3 \times 45} \boldsymbol{\beta}_{45 \times 1} \\ \mathbf{A}_{3 \times 75} &\rightarrow \mathbf{A}_{45 \times 75} \mathbf{W}_{75 \times 45}^\dagger \rightarrow \mathbf{X}_{3 \times 45} \\ \mathbf{K}_{75 \times 1} &\rightarrow \mathbf{W}_{45 \times 75} \mathbf{K}_{75 \times 1} \rightarrow \boldsymbol{\beta}_{45 \times 1} \end{aligned} \quad (10)$$

Compared with the original volumetric error models' union ($\mathbf{A}_{3 \times 75} \mathbf{K}_{75 \times 1}$), which has 75 coefficients, the new EVEMU ($\mathbf{X}_{3 \times 45} \boldsymbol{\beta}_{45 \times 1}$) contains only 45 coefficients; the highest dimension of the volumetric error model is equivalently reduced from 75 to 45, and the amount of corresponding possible sub-models is reduced from about 2^{75} to 2^{45} . Such a dimensionality reduction operation reduces the search space of solutions, and, for the same amount of data, the solution efficiency and data utilization based on Eq. (9) will be improved, thus, if the measuring data is limited, the EVEMU is more suitable for calibration.

3 3-D ball array calibrator and its local calibration procedures

In Eq. (9), a series of (E_x, E_y, E_z) data is required. Artifact-based calibrators are often used to acquire such observations in calibration. Among them, standard balls are commonly used since their centers can act as spatial references [14, 35]. In this paper, to improve the local measuring efficiency and flexibility, a new 3-D ball array calibrator and its local calibration procedures are proposed (Fig. 2). the new calibrator is mainly inspired by the 1-D ball array [14, 35] and ball bar[21], and its overall layout is similar to the ball bar and machine checking gauge system of *Renishaw*[36] to some extent.

3.1 3-D ball array calibrator

The 3-D ball array calibrator is composed of two 1-D standard ball arrays [14, 35] with an L-type structure. According to data statistics, when measuring the volume, the bending shape of most intersecting lines can be generally approximated as the shape of the letter “L”. Therefore, the L-type structure is closer to the workpiece surface, and does not increase the complexity of the calibrator's structure significantly. There are 9 standard balls (Ball1–Ball9; Fig. 2) and 2 standard blocks (Block1 and Block2; Fig.2) fixed on linear tubes. Their detailed dimensions and technical requirements are optimized to ensure that the change of the center position of each ball is within 3 μm under a trigger force of 1 N exerted by the tactile probe at different spatial positions under an ambient temperature of $20 \pm 5^\circ\text{C}$.

Figure 2. Schematic illustration of the 3-D ball array calibrator and its center fixture

Before using in calibration, the center coordinates of the standard balls and the normal vectors of the standard blocks of the calibrator should be calibrated in advance by another instrument with higher accuracy and precision. In this paper, this process is called “pre-calibration”, and the calibrated values are treated as the ideal values corresponding to $\mathbf{P}_{ideal}$ in Eq. (2). Subsequently, the same center coordinates of the standard balls and normal vectors of the standard blocks can be measured by the OMM device, and the measured values are treated as the actual values corresponding to $\mathbf{P}_{actual}$ in Eq. (2). Since the standard balls and blocks are fixed on the calibrator, the corresponding center coordinates and normal vectors are theoretically constant relative to the calibrator itself, irrespective of being measured at different positions or by different instruments, which is the theoretical basis of local calibration. However, due to various error sources, in practice, there will be deviations $\mathbf{E}(E_x, E_y, E_z)$ between ideal and actual values. A method for establishing a uniform coordinate system (CS) is defined to compare the errors. The method is based on the following steps, which is also indicated in Fig. 3:

- 1) Position of Origin: Coincide with the center coordinate of Ball1 $\mathbf{P}_1$
- 2) Orientation of Z-axis $\mathbf{V}_z$: same as the normal vector of Block1 $\mathbf{V}'_z$
- 3) Orientation of X-axis $\mathbf{V}_x$: same as $\mathbf{V}'_x$, where $\mathbf{V}'_x = \mathbf{V}'_z \times \mathbf{V}'_y$, and $\mathbf{V}'_y$ indicates the normal vector of Block2
- 4) Orientation of Y-axis $\mathbf{V}_y$: $\mathbf{V}_y = \mathbf{V}_z \times \mathbf{V}_x$

As it can be seen in the above steps, the origin position refers to Ball1; thus, Ball1 is called the “position tracing ball”, which establishes a 3-degree-of-freedom (DOF) position reference. The orientation of the uniform CS refers to Block1 and Block2; thus, the two blocks are called “orientation tracing blocks”, which establish another 3-DOF orientation reference. In general, the uniform CS establishes a 6-DOF reference with regard to Ball1, Block1, and Block2. Consequently, the origin point error becomes zero relative to itself. For pre-calibration or local calibration, the OMM device and the pre-calibration instruments measure coordinate values of balls and blocks on the same calibrator in their own machine CS. After performing the above-mentioned procedure for establishing the same uniform CS, the coordinate values measured by different machines are represented in the same uniform CS. Then, the deviation between their values can be quantitatively compared. Consequently, the procedure for establishing a uniform CS is also a procedure that transforms the coordinate values from the measuring machine CS into the uniform CS. As it can be seen in Fig. 3, if there is a ball center $\mathbf{P}_i$ on the calibrator, its cartesian coordinates are measured through pre-calibration and represented in the pre-calibration CS as $\mathbf{P}_{i_ideal_pre}$. The same $\mathbf{P}_i$ is measured by the OMM device and is represented in the calibration CS as $\mathbf{P}_{i_actual_self}$. By establishing uniform CS, the homogeneous transformation matrices $\mathbf{T}_{uni_pre}$ and $\mathbf{T}_{uni_self}$ can be defined as Eq. (11):

$$\begin{aligned}\mathbf{T}_{uni_pre} &= \begin{bmatrix} \mathbf{V}_{x_pre} & \mathbf{V}_{y_pre} & \mathbf{V}_{z_pre} & \mathbf{P}_{1_pre} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\ \mathbf{T}_{uni_self} &= \begin{bmatrix} \mathbf{V}_{x_self} & \mathbf{V}_{y_self} & \mathbf{V}_{z_self} & \mathbf{P}_{1_self} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\ \mathbf{T}_{self_uni} &= \mathbf{T}_{uni_self}^{-1}\end{aligned}\tag{11}$$

where $\mathbf{P}_{1_pre}, \mathbf{V}_{x_pre}, \mathbf{V}_{y_pre}, \mathbf{V}_{z_pre}$ in $\mathbf{T}_{uni_pre}$ are the $\mathbf{P}_1, \mathbf{V}_x, \mathbf{V}_y, \mathbf{V}_z$ values in the pre-calibration CS, and $\mathbf{P}_{1_self}, \mathbf{V}_{x_self}, \mathbf{V}_{y_self}, \mathbf{V}_{z_self}$ in $\mathbf{T}_{uni_self}$ are the $\mathbf{P}_1, \mathbf{V}_x, \mathbf{V}_y, \mathbf{V}_z$ values in the calibration CS. $\mathbf{P}_{i_ideal_pre}$ can be transformed into the uniform CS as $\mathbf{P}_{i_ideal_uni}$, and $\mathbf{P}_{i_actual_self}$ as $\mathbf{P}_{i_actual_uni}$. Their error in the uniform CS is $\mathbf{E}_{i_uni}$, which can also be transformed into the calibration CS and become one of the volumetric errors $\mathbf{E}_{i_self}$ for the calibration application through Eq. (12). Fig. 3 illustrates the relationships among these factors. The calculation chain of error from up to down indicates how $\mathbf{E}_{i_self}$ is calculated for the same $\mathbf{P}_i$, and the simplified traceability chain from left to right side indicates how the standard of length metrology is traced. Moreover, the pre-calibration instrument should be also calibrated in advance, to ensure that its measured values can trace to the international standard of length metrology of the International System of Units (SI); thus, the local calibration could finally trace to the SI.

$$\mathbf{E}_{i_self} = \mathbf{P}_{i_ideal_self} - \mathbf{P}_{i_actual_self}$$

where

$$\mathbf{P}_{i_ideal_self} = \mathbf{T}_{self_uni} \mathbf{T}_{uni_pre} \mathbf{P}_{i_ideal_pre}$$

(12)

Figure 3. Volumetric error calculation and traceability chains of the 3-D ball array calibrator

In order to capture the volumetric error of the entire measuring volume, the 3-D ball array calibrator should be placed at several different local calibration positions and measured by the OMM device. Each local calibration position produces a group of volumetric error values concerning the center coordinates of the balls, and has a 6-DOF traceable reference at Ball1. Each error of Ball1 is zero since it coincides with the origin point of the uniform CS mentioned in Section 3.1. If the center coordinates of Ball1 are different, there will be more than one zero-error points in the same measuring volume in the calibration CS, which would make the traceability chain contradictory. However, if the different local calibration positions have the same center coordinate of Ball1, there will be only one zero-error point in the measuring volume, which will produce a stable traceability chain to SI units. To this end, a center fixture was designed to keep the center coordinate of

Ball1 the same when the right side of the calibrator moves at different positions, the overall layout is similar to the machine checking gauge system of *Renishaw*[36]. As it can be observed in Fig. 2, Ball1 is in contact with three circumferentially-arranged and uniformly-distributed ruby balls in front of the fixture. One end of a high-strength cable is consolidated with Ball1, while the other end is tightened by an extension spring. The tension of the extension spring can be adjusted by the tension adjusting nut, to keep Ball1 in contact with the ruby balls all the time. When the right side of the calibrator is placed at a different position, the position of the left-side center fixture is fixed all the time, and the center coordinates of Ball1 can remain unchanged, since the center of Ball1 is restrained by the three ruby balls. The fixture limits the three translational DOFs of the calibrator, but provides a stable traceability chain.

3.2 Local calibration procedures

For a certain measuring volume of a local calibration task, as shown in Figs. 4(a) and (b) corresponding to Fig. 1, several local calibration positions, such as position1 and position_{*i*}, are required to traverse the internal space and surround the outer profile of the measuring volume. All positions have the same center coordinate of Ball1, and the bounding polygon composed of the outer ball centers should cover the measuring volume. An insufficient number of positions may lead to poor calibration performance, since limited volumetric error data cannot reflect the error features of the entire measuring volume. An excessive number of positions will result in more accurate calibration results, but requires more measuring time. The efficiency and accuracy of local calibration with the proposed calibrator can be controlled by the density of positions, and the trade-off between accuracy and efficiency needs to be considered under each application scene. Through preliminary experiments, it was found that eight positions can meet the requirements of this study; there are many different eight-position layouts, and one of the available layouts is depicted in Figs. 4(c) and (d). The local calibration procedure with eight positions is as follows:

- 1) Position order: It is set according to the distance between the calibrator and OMM device from near to far; thus, the calibrator is fixed at positions 4, 8, 3, 7, 2, 6, 1, 5 in turn. At each position, the OMM device measures following the measuring order.
- 2) Measuring order: It is set according to the distance between Ball1 and other blocks or balls from near to far; thus, the measuring order is Ball1, Block1, Block2, Ball2, to Ball9 in turn.
- 3) Ball measurement: To measure multiple discrete points on a spherical surface, the points should cover as much of the sphere surface as possible. Then, the center coordinates of the balls can be fitted by the least square method according to ISO 10360-6:2001 [37].

- 4) Block measurement: To measure multiple points scattered on the upper surface of the block, these points should cover as much area as possible. Then, the normal vectors of the blocks can be fitted by the least square method [37].

Figure 4. Several local calibration positions with the 3-D ball array calibrator: (a) Side view of several local calibration positions, several local calibration positions, such as position1 and position_i, are required to traverse the internal space and surround the outer profile of the measuring volume. All positions have the same center coordinate of Ball1, and the bounding polygon composed of the outer ball centers should contain the measuring volume; (b) Front view of several local calibration positions; (c) Eight local calibration positions, where the bounding polygon contains the measuring volume; (d) Eight local calibration positions and their relative position with the OMM device

At the eight local calibration positions, each center coordinate of the balls $\mathbf{P}_{i_actual_self}(x_{i_self}, y_{i_self}, z_{i_self})$ produces its corresponding volumetric error. Since in the following sections, the results are mainly calculated in the calibration CS,

simplified expressions such as (x_i, y_i, z_i) , (E_{xi}, E_{yi}, E_{zi}) without subscript “_self”, “_uni”, “_pre” will be all represented in the calibration CS as default. There are 72 (x_i, y_i, z_i) and (E_{xi}, E_{yi}, E_{zi}) for the eight local calibration positions, since at each position, there is the same calibrator with nine balls.

Compared to conventional calibrators, such as 1-D ball arrays [14], 2-D hole plates [17], or 3-D ball plates [18], the proposed calibrator has three measurable dimensions, a relatively simple structure, and a small volume at the same time. There is no need to be fixed strictly along with some axes or planes by several times, the proposed calibration procedures are executed directly in the 3-D space, and the efficiency and accuracy can be flexibly controlled by the density of the calibration positions. It does not rely on expensive optical instruments such as laser tracker in use, most parts of the calibrator could be mature industrial products; therefore, its cost is relatively low and has the potential for large-scale application.

4 SCAD-based identification and compensation

As mentioned in Section 2, there are 2^{45} possible sub-models of the EVEMU, the sub-model with optimal statistical significance is the optimal estimation of the actual model, which, from the statistical perspective, is treated as the actual sub-model of the OMM device in this paper. thus, the identification is required to select the optimal sub-model. The identification objective is to solve 45 ks coefficients of $\beta_{45 \times 1}$ in Eq. (9) by several (E_{xi}, E_{yi}, E_{zi}) measured from the local calibration procedures, as described in Section 3. When ks is determined, the actual sub-model is simultaneously selected. If some ks values are identified as zeros, the highest dimension of the volumetric error model union may be reduced; then, the sub-model becomes a sparse sub-model [38]. With measured volumetric error data, Eq. (9) can also be expressed as Eq. (13):

$$\mathbf{E} = \mathbf{X}\boldsymbol{\beta}$$

where

$$\mathbf{E} = \begin{bmatrix} E_{x1} & E_{y1} & E_{z1} & K & E_{xi} & E_{yi} & E_{zi} & K & E_{xn} & E_{yn} & E_{zn} \end{bmatrix}^T \quad (13)$$

$$\mathbf{X} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & K & y_1^2 & y_1^3 & z_1^3 & K & 0 & 0 & 0 \\ M & M & M & M & M & M & M & M & M & M & M \\ 0 & 0 & 0 & K & 0 & 0 & 0 & K & 0 & 0 & 0 \\ M & M & M & M & M & M & M & M & M & M & M \\ 0 & 0 & 0 & K & 0 & 0 & 0 & K & -x_N z_N^3 & -x_N z_N^2 & -x_N z_N \end{bmatrix}$$

$$\mathbf{E} \in \mathbf{R}^N, \mathbf{X} \in \mathbf{R}^{N \times Q}, \boldsymbol{\beta} \in \mathbf{R}^Q, N = 3n$$

where $\mathbf{E}$ is an N-dimension vector, which is a flattened sequence of (E_{xi}, E_{yi}, E_{zi}) . Thus, N is equal to $3n$, where n represents the total number of balls to be calibrated. If a local calibration procedure for eight positions is adopted, then,

$n=72$ and $N=216$. β is a Q -dimension vector of ks , Q is 45, $\mathbf{X}$ is a $N \times Q$ matrix, whose terms are obtained with respect to β . In actual measurements, apart from systematic component errors, there is also noise. Eq. (13) can be further expressed as Eq. (14):

$$\mathbf{E} = \mathbf{X}\hat{\beta} + \epsilon \quad (14)$$

$$\hat{\mathbf{E}} = \mathbf{X}\hat{\beta} \quad (15)$$

where $\hat{\beta}$ is the estimation of the coefficients and the systematic components of ks as well, the parameter ϵ is an N -dimension residual vector which represents bias between actual and estimated values, and represents the random deviation identified by algorithms. The systematic components of volumetric error $\mathbf{E}$ can be estimated by $\hat{\mathbf{E}}$, as defined by Eq. (15).

OLS regression is commonly used to identify the parameters $\hat{\beta}$ for linear equations [39]. OLS can obtain an unbiased estimation of $\hat{\beta}$ by minimizing the sum of squares of the difference between the observed and estimated values of the sample. Its objective function is given as Eq. (16):

$$\hat{\beta} = \arg \min_{\beta \in R^P} \|\mathbf{E} - \mathbf{X}\beta\|_2^2 \quad (16)$$

Nevertheless, OLS may overfit data when it handles a model with high dimension and limited $\mathbf{X}$ data, since it cannot perform well in multicollinearity problems occurring in $\mathbf{X}$. Even after the dimension reduction performed in Section 2, the number of dimensions is as high as 45. The multicollinearity problems are caused by the interaction among geometric errors of structures with weak stiffness. Increasing the number of measurements can overcome multicollinearity to some extent; however, additional measurements require additional local calibration time.

In the past few decades, several methods have been proposed to solve multicollinearity problems. Among them, several regularization algorithms, also called penalized least-squares regression algorithms, have been well developed[40]. Such algorithms retain the items that are most relevant to the coefficients and reduce or even eliminate the effects of miscellaneous coefficients. Breiman [41] proposed Ridge regressions with L2 penalty; Tibshirani [42] proposed the least absolute shrinkage and selection operator (LASSO) with L1 penalty, and Fan [43] proposed the SCAD method. Among them, LASSO and SCAD belong to regularized sparse algorithms, which eliminate some redundant coefficients and keep the most relevant ones. Ridge and LASSO models are biased regression methods, while SCAD is approximately unbiased [43]. SCAD also has demonstrated the so-called *oracle* property [43], which means that it performs well in identifying parameters and selecting the sub-model as if it knows the actual sub-model ahead in an asymptotic sense [44]. Thus, SCAD

is a theoretically ideal method for model identification of the weak stiffness OMM device. OLS is also an unbiased method and has no sparsity ability, therefore, its model structure is fixed. However, SCAD can select the sub-model among 2^{45} candidates, and has more possibilities. The objective function of SCAD is given as Eq. (17):

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= \arg \min_{\boldsymbol{\beta} \in R^P} \frac{1}{2} \|\mathbf{E} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \sum_{p=1}^P \varphi_{\lambda, \gamma}(\beta_p) \\ \text{where} \quad \varphi_{\lambda, \gamma}(\beta_p) &= \begin{cases} \lambda |\beta_p|, & 0 \leq |\beta_p| \leq \lambda \\ -\frac{(\beta_p^2 - 2\gamma\lambda |\beta_p| + \lambda^2)^2}{2(\gamma-1)}, & \lambda \leq |\beta_p| \leq \gamma\lambda \\ \frac{(\gamma+1)\lambda^2}{2}, & |\beta_p| \geq \gamma\lambda \end{cases} \quad (17) \\ p &\in \{1, K, P\}, \gamma > 2, \lambda \geq 0 \end{aligned}$$

where λ, γ are tuning parameters which affect the identification performance; $\varphi_{\lambda, \gamma}(\beta_p)$ is the nonconvex penalty function, and β_p is a coefficient of $\boldsymbol{\beta}$. To find the optimal λ and γ , a hybrid approach combining BIC, CV, and convexity diagnostics are introduced [44], which is implemented as a free available R package called *ncvreg* [44]. When there is a certain value in γ , the *ncvreg* solver selects the optimal value of λ based on the smallest averaged cross-validation error (CVE) from a sequence of λ values, and fits the SCAD at the same time by a coordinate descent algorithm. In order to automatically select γ , the 2-D *GridSearchCV* is applied [45], which combines a 2-D grid search and the *ncvreg* solver. The steps are as follows:

- (a) A 2-D candidate parameter grid is set in advance, which includes $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \mathbf{K}, \lambda_j, \lambda_{j+1}, \mathbf{K}, \lambda_{s-1}, \lambda_s]$ and $\boldsymbol{\gamma} = [\gamma_0, \gamma_1, \mathbf{K}, \gamma_j, \gamma_{j+1}, \mathbf{K}, \gamma_{s-1}, \gamma_s]$;
- (b) Every γ_i value in $\boldsymbol{\gamma}$ and the complete $\boldsymbol{\lambda}$ are substituted into the *ncvreg* solver in a loop. The CVE for every γ_i and λ_i can be obtained through the *ncvreg* solver. The optimal γ_i and λ_i values are the ones with the smallest CVE value.

When searching the optimal γ_i and λ_i , the optimal $\hat{\boldsymbol{\beta}}$ can be obtained at the same time; thus, the optimal equivalent volumetric error sub-model is also acquired. It is worth noting that, since the penalty function $\varphi_{\lambda, \gamma}(\beta_p)$ is non-convex, the optimal solution obtained following the above method may still be a local optimal solution rather than the global one. Nevertheless, as long as the CVE is small enough to meet the industrial demands, the CVE performance is feasible for industrial applications.

After identification, the optimal estimation $\hat{\mathbf{b}}$ is obtained. The calibrated OMM device can be applied to measure other $\mathbf{P}_i'(x_i', y_i', z_i')$ points, apart from the local calibration points within the measuring volume. The coordinates of these measurement points $\mathbf{P}_i'$ produce corresponding $\mathbf{X}_i'$ values according to Eq. (18). The corresponding optimal estimation of the volumetric error $\mathbf{E}_i'$ is $\hat{\mathbf{E}}_i'$, as given by Eq. (17). By adding the $\hat{\mathbf{E}}_i'$ to $\mathbf{P}_i'$, the compensated $\mathbf{P}_i'$, i.e., $\hat{\mathbf{P}}_i'$, is derived, as shown in Eq. (19).

$$\hat{\mathbf{E}}_i' = \mathbf{X}_i' \hat{\mathbf{b}} \quad (18)$$

$$\hat{\mathbf{P}}_i' = \mathbf{P}_i' + \hat{\mathbf{E}}_i' \quad (19)$$

5 Experimental case study

5.1 Experimental setup

In this paper, a custom-made OMM device is introduced, which has been built for medium accuracy (Figs. 1, 7, and 8). Its main specifications are listed in Table 1. The motion of the OMM device is controlled by a fully closed-loop structure with three linear encoders, which are used to measure the displacement in the X, Y, and Z directions. A tactile probe (*Renishaw OMP40-2*) is mounted at the end of the OMM device, and a precision ruby ball with a diameter of 3 mm is placed at the end of the stylus. During measurement, the probe moves towards the measuring point at a slow speed; when the probe ball is in contact with the object surface and just exceeds the triggering threshold of the tactile probe, the OMM device immediately stops moving, and the coordinate values of the contact point are captured by locking the linear encoder measured values at the same time.

The main experiments are illustrated in Figs. 6–8. More specifically, Fig. 5 shows the pre-calibration of the 3-D ball array calibrator by a *Hexagon Classic* CMM, which acquires the ideal values of the calibrator corresponding to Fig. 3. The specifications of the CMM are listed in Table 2. Several pre-calibration experiments indicated that the standard deviation of the ball center coordinates measured at different time points and positions in the same uniform CS was less than 0.002 mm. This demonstrated that the calibrator is relatively stable and meets the medium precision application requirement. Subsequently, a local calibration procedure at 8 positions was performed on the OMM device using the 3-D ball array calibrator. The local calibration at Position3 is as shown in Fig. 6, while the entire procedure is actually the same as that shown in Figs. 4(c) and (d), where the actual values of the calibrator corresponding to Fig. 3 were acquired. In order to verify the local calibration performance (Fig. 7), a *Leica AT901-LR* laser tracker with a maximum observed deviation of 0.005 mm was utilized. A spherical target reflector was mounted at the end of the OMM device and close to the tactile probe

to reflect the laser emitted by the laser tracker; thus, the spatial coordinates of the target reflector could be measured. In any of the above experiments, the environmental temperature was controlled at $20\pm 3^{\circ}\text{C}$, and the machine state and other ambient conditions were also maintained as similar as possible.

Figure 5. Pre-calibration of the 3-D ball array calibrator by the *Hexagon Classic* CMM

Figure 6. Local calibration at 8 positions using the 3-D ball array calibrator and the center fixture

Figure 7. Local calibration verification using the *Leica AT901-LR* laser tracker

Table 1. OMM device's Specifications for local calibration

Item	Value
Scale	X: 0 – 1000 mm, Y: 0 – 550 mm; Z: 0 – 300 mm
Drivers	X, Y, and Z: rotary motor with ball screw
Linear encoders	X, Y, and Z accuracy: $\pm 3 \mu\text{m/m}$
Tactile probe	Sense direction: Omni-directional $\pm X, \pm Y, \pm Z$ Repeatability: $1 \mu\text{m}$ with 2σ Stylus length: 50 mm Probe ball: $\varnothing 3$ ruby ball

Table 2. Specifications of the *Hexagon Classic* CMM used for pre-calibration

Item	Value
Scale	X: 0 – 1000 mm, Y: 0 – 900 mm; Z: 0 – 600 mm
<i>MPE</i>	$0.6\mu\text{m} \pm (2.5\mu\text{m} + 10^{-3}L / 750)$

5.2 Local calibration results

After pre-calibration, local calibration at 8 positions, and data preprocessing using Eq. (16), all ball center coordinates (x_i, y_i, z_i) and the corresponding volumetric errors (E_{xi}, E_{yi}, E_{zi}) are transformed into the calibration CS. Fig. 8(a) demonstrates the volumetric error vectors (30X magnification) on each ball center of the 8 local calibration positions, which correspond to Figs. 4(c) and (d). It can be observed that, the farther away from Ball1, the larger the magnitude of the volumetric error vectors, while the volumetric error at Ball1 is zero. Figs. 8 (b), (c), and (d) depict the volumetric error components in the X, Y, and Z directions, respectively. As it can be seen in Fig. 8(b), for a certain position, E_x generally increases with increasing X, and Ball9 had the largest E_x . As it can be seen in Fig. 8(c), E_y can be divided into two groups: one group contains Position1–Position4 with values lower than zero; the other group contains Position5–Position8

with values are greater than zero; the two groups are divided by Ball1. In general, the absolute values of E_y also increase with increasing distance from Ball1. As it can be observed in Fig. 8(d), E_z can also be divided into two groups similar to E_y ; these two groups are also divided by Ball1. The reason behind the existence of two groups is that, since their reference origin is the center of Ball1, the continuously changing volumetric error will produce positive and negative values relative to the middle zero point. In fact, even for an OMM device with medium or low accuracy, the volumetric error values are not small enough. If the OMM device was highly rigid, such high error would not occur theoretically. The reason is that the OMM device has a relatively weak stiffness, especially along the X and Z axes. On the other hand, repeated local calibration experiments revealed that the repeatability of the OMM device prototype is less than ± 0.008 mm, which demonstrated that error compensation is possible.

Figure 8. Volumetric error vectors of the OMM device in cartesian CS at 8 local calibration positions: (a) Volumetric error vectors (30X magnification); (b) X and E_x ; (c) Y and E_y ; (d) Z and E_z

5.3 SCAD-based identification and volumetric error compensation

As mentioned in Section 4, the 2-D *GridSearchCV* was employed to select the optimal values of λ and γ . A large-

and a small-scale 2-D grid were investigated, as shown in Fig. 9. For the large-scale 2-D grid (Figs. 9(a) and (b)), the dimension γ was set as a sequence from 2.0001 to 100, which was equally spaced into 50 values. More specifically, the minimum value of γ was set to 2.0001, since in SCAD, γ should be greater than 2 [44]. λ was set as a sequence of values from 1E-10 to 1E-1 with a length of 100, equally spaced on the log scale according to the default sequence of the *ncvreg* solver. For the small-scale 2-D grid (Figs. 9(c) and (d)), the γ was set as a sequence of values from 2.0001 to 10, with 50 equally spaced values as well. The parameter λ was set as a sequence of values from 1E-10 to 1E-1 with a length of 100, equally spaced on the log scale. Under the large-scale 2-D grid (Fig. 9(a)), the minimum CVE was equal to 0.005842233 and its corresponding selected γ_i and λ_i were 20.00081633 and 0.0000259, respectively. Under the small-scale 2-D grid (Fig. 9(c)), the minimum CVE was equal to 0.005865556 and its corresponding selected γ_i and λ_i were 5.592387755 and 0.000021, respectively. Among both the large- and small-scale 2-D grids, the lowest CVE value was 0.005842233 in the large-scale 2-D grid; thus, the optimal tuning parameters were $\gamma_i=20.00081633$ and $\lambda_i=0.0000259$.

Figure 9. Tuning parameter selection for the SCAD algorithm: (a) CVEs on the large-scale 2-D grid about γ and λ ; (b) selected λ on each γ with the minimum CVE in the large-scale 2-D grid; (c) CVEs on the small-scale 2-D grid about γ and λ ; (d) selected λ on each γ with the minimum CVE in the small-scale 2-D grid

After having determined the optimal γ_i and λ_i values, the identified $\hat{\mathbf{b}}$ of Eq. (13) is obtained (Table 3). There were 41 non-zero k s and 4 zero k s values. By substituting the k s values into Eq. (13), the optimal equivalent volumetric error sub-

model can be obtained (Eq. (14)), which is a sparse sub-model of the EVEMU Eq. (9).

Table 3. k_s values of $\hat{\beta}$ identified by SCAD using the optimal $\gamma_i=20.00081633$ and $\lambda_i=0.0000259$

Item	Value	Item	Value	Item	Value	Item	Value
k_{dxx1}	2.950490E-03	k_{dxx2}	3.813183E-06	k_{dxx3}	-2.775042E-09	k_{dxy2}	-1.785525E-05
k_{dxy3}	5.117030E-08	k_{dxz1}	1.669860E-02	k_{dxz2}	-3.462166E-04	k_{dxz3}	2.035311E-06
k_{dyx2}	1.167908E-06	k_{dyx3}	1.848295E-10	k_{dyy1}	2.017827E-04	k_{dyy2}	0.000000E+00
k_{dyy3}	-1.832424E-08	k_{dyz1}	5.660915E-02	k_{dyz2}	-1.673767E-03	k_{dyz3}	1.599664E-05
k_{dxx2}	-1.747859E-06	k_{dxx3}	-9.975762E-10	k_{dzy2}	1.548582E-05	k_{dzy3}	-7.038459E-08
k_{dzz1}	0.000000E+00	k_{dzz2}	-1.328144E-04	k_{dzz3}	-3.380137E-10	k_{eqx1}	4.290736E-04
k_{eqx2}	-4.037066E-02	k_{eqy1}	2.007601E-03	k_{eqy2}	-5.909335E-01	k_{eqz1}	2.516920E-03
k_{eqz2}	-1.512266E-03	k_{eqz3}	7.274573E-01	k_{exx1}	6.939124E-06	k_{exx2}	6.795187E-09
k_{exx3}	-2.011528E-11	k_{exz1}	-2.410024E-05	k_{exz2}	4.178833E-10	k_{exz3}	8.430376E-09
k_{eyz1}	-3.560507E-05	k_{eyz2}	0.000000E+00	k_{eyz3}	5.256019E-09	k_{ezx1}	2.169027E-06
k_{ezx2}	0.000000E+00	k_{ezx3}	-3.121463E-12	k_{ezz1}	-5.102954E-05	k_{ezz2}	1.537837E-07
k_{ezz3}	-3.872882E-09						

In order to verify the error compensation vectors obtained based on the method proposed in this paper, discrete points in the measuring volume were selected as the comparison object. Based on the size of the bounding polygon formed by the 3-D ball array calibrator of 8 positions (Section 3.2), the internal space was uniformly sampled, and the steps of each axis were set as follows: X with 100/3 mm, Y with 50/3 mm, and Z with 10/3 mm; thus, several discrete points located within the bounding polygon were obtained. The points between Ball1 and Ball2 are removed because of the narrow space, and 2924 available points left in total, the X, Y, and Z coordinates of which are substituted into Eq. (15), and the volumetric error vectors in 3-D space are estimated (Fig. 10). These are also volumetric error compensation vectors for the OMM device to be calibrated. the initial point of a vector represents the actual coordinates, the vector itself represents the volumetric error compensation, and its terminal point represents the compensated coordinates; thus, the terminal points are the optimal estimation of the ideal coordinates.

Figure 10. Volumetric error compensation vectors (15X magnification)

5.4 Verification

In order to verify the accuracy compensation performance, a *Leica AT901* Absolute laser tracker was used (Fig. 7). In order to fully cover the same measuring volume with local calibration, a larger cubic volume was measured. Taking the measuring accuracy and efficiency into consideration, the volume was uniformly discretized into a series of points. The measuring scale and discrete step length were as follows: X: - 100 mm – 800 mm with a discrete step of 100 mm; Y: - 100 – 400 mm with a discrete step of 100 mm; Z: - 40 mm – 80 mm with a discrete step of 20 mm. A target reflector was fixed at the end of the OMM device as close to the tactile probe as possible, and the end of the OMM device carried the target reflector to locate the discretized points one by one. Every time the OMM device located a measuring point, it remained still for a few seconds to filter out the possible deviation induced by mechanical vibration, and then, the laser tracker measured the spatial coordinate values of the target reflector. At each measuring point, the OMM device measured a coordinate value, and the laser tracker measure a spatial coordinate value as well. Similar to local calibration, the former is treated as the actual value and the latter as the ideal value. All values are measured in their own coordinate systems. Through coordinate transformation and data post-processing, the data measured by the laser tracker can be converted into the OMM device CS, and then, the values measured by the OMM device are subtracted to obtain the volumetric error. The points and corresponding volumetric error are illustrated in Fig. 11(a). To facilitate the comparison with the 2924 compensation vectors estimated by local calibration (Fig. 10), the volumetric error values measured by the laser tracker were linearly interpolated

to the same 2924 points. The error reference origin was relocated to a position coincident with the center of Ball1, as shown in Fig. 9(b). This way, the volumetric error estimated by local calibration can be qualitatively and quantitatively compared with that measured by the tracker.

Figure 11. Volumetric error measured by the laser tracker. (a) Volumetric error in a cubic volume; (b) Volumetric error measured by the laser tracker interpolated into a bounding polygon with the same 2924 points as in local calibration (Fig. 10)

In order to determine the difference of the volumetric error interpolated from the laser tracker data and the compensation vectors estimated by OLS and SCAD in local calibration, three planes across the 2924 points were selected (Fig. 12). Figs. 12(a), (d), and (g) depict an XY plane at $Z=33.333$ mm; Figs. 12(b), (e), and (h) depict a YZ plane at $X=366.667$ mm; Figs. 12(c), (f), and (i) depict an XZ plane at $Y=50$ mm. Figs. 12 (a), (b), and (c) show the volumetric error (30X magnification) interpolated by the laser tracker data; Figs. 12(d), (e), and (f) show the compensation vectors (30X magnification) estimated by SCAD; Figs. 12(g), (h), and (i) show the compensation vectors (30X magnification) estimated by OLS.

Figs. 12 (a)–(c) indicate that the directions and magnitudes of the volumetric error vectors change regularly along the X, Y, and Z directions, and the vector changes between adjacent points are gentle without mutation. On the XY plane, the volumetric error was mainly along the +X direction, and its magnitude increased with increasing X; on the YZ plane, the direction of the vectors was mainly along the +Z direction, and the magnitude increased gradually from the upper left to the lower right part of the plot; on the XZ plane, the direction of the vectors was mainly along the upper right direction in the plot, and the magnitude increased with the increase of X. By comparing Fig. 12(a) with Figs. 12(d) and (g), it can be seen that, similar to the data measured by the laser tracker, the magnitude of compensation vector estimated by OLS and SCAD also increased with increasing X. However, in both cases, a slight upward trend towards the upper right corner of the plot was observed, while the data of the laser tracker did not have such an obvious trend. In addition, the vector magnitude

estimated by OLS was quite different from that of the laser tracker, while that estimated by SCAD was more similar. By comparing Fig. 12(b) with Figs. 12(e) and (h), it can be deduced that the direction of the compensation vectors estimated by OLS and SCAD was generally close to the data of the laser tracker. However, in the middle area of Z, the vectors predicted by OLS and CAD pointed clearly towards the upper right direction. In particular, in the area where Z was around 50 mm, the vector direction estimated by OLS was opposite to that of the laser tracker, while this problem was not observed in the results estimated by SCAD. By comparing Fig. 12(c) with Figs. 12(f) and (i), it can be observed that the vectors estimated by SCAD were similar to the interpolated volumetric error of laser tracker; nevertheless, in the areas with large X and Z values, the direction of the compensation vectors estimated by OLS was opposite to that of the laser tracker.

Figure 12. Volumetric error interpolated by the laser tracker data and compensation vectors estimated by OLS and SCAD during local calibration (30X magnification); (a)–(c): Volumetric error interpolated from the laser tracker data; (d)–(f): Compensation vectors estimated by SCAD; (g)–(i): Compensation vectors estimated by OLS; (a, d, g) XY plane at $Z=33.333$ mm; (b, e, h) YZ plane at $X=366.667$ mm; (c, f, i) XZ plane at

$$Y=50 \text{ mm};$$

Based on the above qualitative analysis, it can be deduced that the compensation vectors estimated by OLS have defects in certain areas, while those estimated by SCAD are more similar to the laser tracker measurements. Still, the estimated values were not exactly the same with the values interpolated by the laser tracker (Fig. 13). More specifically, after adding the compensation vectors to the volumetric error vectors, there will always be a residual volumetric error. For a point $\mathbf{P}_{i_actual}$ in the measuring volume, the residual volumetric error after compensation is defined as $\mathbf{V}_{residual}$, which, according to Eq. (20), is equal to $\mathbf{V}_{las}$ minus $\mathbf{V}_{comp}$, where $\mathbf{V}_{las}$ is the volumetric error interpolated from the laser tracker data and $\mathbf{V}_{comp}$ is the compensation vector estimated by SCAD or OLS. If there is a method, which, for the same $\mathbf{V}_{las}$, can make the magnitude of $\mathbf{V}_{residual}$ smaller than that obtained by other competitive compensation methods, then, this method is considered to have better compensation performance. Based on $\mathbf{V}_{comp}$, the metric R_{resmag} is further introduced to quantitatively compare the compensation performance. According to Eq. (21), it is equal to the L_2 norm of $\mathbf{V}_{residual}$ divided by the L_2 norm of $\mathbf{V}_{las}$, i.e., it is the ratio of the magnitude of the volumetric error after compensation to that before compensation. If R_{resmag} is within 0–1, it means that, after compensation, the volumetric error has been reduced and the accuracy has been effectively improved. In general, the smaller the R_{resmag} , the better the performance of the method. If R_{resmag} is greater than 1, it means that the compensation does not improve the accuracy, but worsens the result.

$$\mathbf{V}_{residual} = \mathbf{V}_{las} - \mathbf{V}_{comp} \quad (20)$$

$$R_{resmag} = \frac{\|\mathbf{V}_{residual}\|_2}{\|\mathbf{V}_{las}\|_2} \quad (21)$$

Figure 13. Residual volumetric error after compensation

The R_{resmag} values of the 2924 points of the SCAD and OLS estimations were calculated. Their histogram is shown in Fig. 14, where Fig. 14(a) is the R_{resmag} comparison between SCAD and OLS, Fig. 14(b) is the detailed histogram of SCAD,

and Fig. 14(c) is the detailed histogram of OLS. In addition, some basic statistics of R_{resmag} are given in Table 4. It can be observed that, after SCAD compensation, the R_{resmag} of all points was less than 0.471, the average value was 0.260 with an Std of 0.070, and most of the data (within the 25th – 75th percentile interval) were concentrated between 0.214 – 0.301 (Table 4). This indicates that the accuracy was generally improved by about 3 – 5 times (within the 25th – 75th percentile interval) and all the R_{resmag} values were within the 0.104–0.471 range; that is, the accuracy was improved by 2–10 times (within the entire percentile interval), which proves that SCAD can significantly improve the measurement accuracy after compensation. Compared to SCAD, the average value of R_{resmag} after OLS compensation was about 1.737 times that of SCAD (~0.451) and the Std value was about 5.29 times that of SCAD, indicating that the compensation accuracy of the OLS method is relatively poor. Moreover, the large Std value suggests that OLS is not stable enough. To make matters worse, according to Figs. 14(a) and (c), there were 251 R_{resmag} values of OLS larger than 1, which accounts for about 8.584% of the total points; these points do not improve the accuracy, but deteriorate the compensation. The maximum R_{resmag} value of OLS was 2.634, which means the OLS increased the error of the original data by 2.634 times, which is difficult to be allowed in industrial applications. The above results indicate that, compared to SCAD, OLS is not a reliable method.

Table 4. Basic statistics of R_{resmag}

Method	Count	Mean	Std	Min	25th percentile	50th percentile	75th percentile	Max
SCAD	2924	0.260	0.070	0.104	0.214	0.252	0.301	0.471
OLS	2924	0.451	0.372	0.133	0.253	0.336	0.448	2.634

Figure 14. R_{resmag} histograms: (a) R_{resmag} histogram of SCAD and OLS; (b) Detailed R_{resmag} histogram of SCAD; (c) Detailed R_{resmag} histogram of OLS

The SCAD-based compensation is more effective than OLS for several reasons. First, the OMM device's volumetric error changes regularly in the measuring volume, and the regularity is the premise of using small samples to effectively estimate the population. Second, the EVEMU method offers a massive amount of possible volumetric error sub-models of

structures with weak stiffness, the EVEMU is basically derived from the physical structure of the OMM device and combined with the polynomial fitted PDGEs, incorporating both physical meaning and data correlation. Then, the 3-D ball array calibrator and its local calibration procedures effectively expose the volumetric error. Finally, the SCAD performs well in selecting the optimal volumetric error sub-model due to its approximately unbiased and *oracle* properties. It is worth noting that the experiments in Section 5 were repeated several times, and each time, a similar accuracy improvement performance was obtained, which supports the theory proposed in this study.

6 Conclusions

This paper primarily proposed an integrated local calibration method including error measurement, modeling, identification, and compensation for a weak stiffness measuring device. The EVEMU method was proposed for error modeling. A new 3-D ball array calibrator and its local calibration procedure were proposed for better error measurement flexibility and efficiency purposes, SCAD was introduced to identify and compensate the volumetric error for weakening the multicollinearity problem as well as dealing with the “interaction” problem. The verification experiment revealed that the SCAD-based local calibration approach is more effective than OLS. The main conclusions are as follows:

1. An equivalent dimensionality reduction algorithm is proposed for the construction of EVEMU, which equivalently reduced the highest dimension of the volumetric error model from 75 to 45, and the amount of corresponding possible sub-models is reduced from about 2^{75} to 2^{45} . the dimensionality reduced EVEMU is more suitable for limited calibration data. And due to the equivalent method, the error measurements can be performed directly in the 3-D space, and are not limited by identifying explicit geometric errors, but to seek the equivalent solution in the intermediate solution space. which relatively improves the calibration efficiency to a certain extent. The proposed method provides new insight and a theoretical tool for error modeling of multi-linear axes structures.
2. A new 3-D ball array calibrator and its local calibration procedure are proposed. Compared to conventional calibrators, the new calibrator has three measurable dimensions, a simple structure and small volume at the same time. All the local calibration procedures are executed directly in the 3-D space instead of combining many measurements strictly along with some axes or planes, and dose not rely on expensive optical instruments such as laser trackers in application. The efficiency and accuracy of calibration can be flexibly controlled by the density of the calibration positions.
3. the SCAD algorithm with approximately unbiased and *oracle* properties is introduced to simultaneously identify equivalent coefficients and select the optimal volumetric error sub-model for weak stiffness OMM device. Experiments demonstrated that SCAD-based identification and compensation can improve the accuracy of the measuring volume by

about 3–5 times (within the 25th – 75th percentile interval), and by 2–10 times (within the entire percentile interval). Compared to OLS, SCAD performs more accurately, which can more closely represent the real error state of the machine. OLS has an inferior performance, which can even worsen the error compensation.

In this paper, a novel relatively concise and effective error compensation solution for a weak stiffness OMM device. The 3-orthogonal linear axes structures, which are widely used in 3- or 5- axis machine tools and CMMs or OMMs, are almost the foundation of the worldwide industry. This paper provides new possible accuracy improvement support for the design and manufacture of such machines. On the other hand, the averaged accuracy improvement of the OMM device is limited (not more than one order of magnitude) in this paper, so, the accuracy and stability of the 3-D ball array calibrator are still worth further improvement in future studies.

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Authors' contributions Helin Li developed the idea and designed the study, performed the research, analyzed the data, and wrote the paper; Bin Lin, Tianyi Sui, Tian Huang, and Haitao Liu contributed to developing and refining the ideas and providing prominent technical guidance; Fukan Yu designed the on-machine measuring device; Pengcheng Zhao helped perform all the experiments and collect the corresponding data; Yueting Jia performed experiments of the laser tracker and collected the corresponding data; Yueting Jia performed experiments of the hexagon CMM and collected the corresponding data; all authors discussed the results and revised the manuscript.

Availability of data and material All data generated or analyzed during this study are included in the present article.

Code availability Not applicable

Declarations

Competing interests Not applicable

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Consent to participate Not applicable

Consent for publication Not applicable

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Appendix

To better show some sparse matrixes, the compressed sparse row format (CSR, CRS or Yale format)[1] is introduced.

$$\mathbf{E} = \mathbf{A}_{3 \times 75} \mathbf{K}_{75 \times 1} \rightarrow \mathbf{E} = \mathbf{A}_{3 \times 75} \left(\mathbf{W}_{75 \times 45}^\dagger \mathbf{W}_{45 \times 75} \right) \mathbf{K}_{75 \times 1} \rightarrow \mathbf{E} = \mathbf{X}_{3 \times 45} \boldsymbol{\beta}_{45 \times 1}$$

$$\mathbf{A}_{3 \times 75} \rightarrow \mathbf{A}_{45 \times 75} \mathbf{W}_{75 \times 45}^\dagger \rightarrow \mathbf{X}_{3 \times 45}$$

$$\mathbf{K}_{75 \times 1} \rightarrow \mathbf{W}_{45 \times 75} \mathbf{K}_{75 \times 1} \rightarrow \boldsymbol{\beta}_{45 \times 1}$$

$$\mathbf{A}_{3 \times 75} = \begin{bmatrix} \mathbf{A1} \\ \mathbf{A2} \\ \mathbf{A3} \end{bmatrix}$$

where

$$\mathbf{A1}_{1 \times 75} = \begin{bmatrix} x^3, x^2, x, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -x^3y, -x^2y, -xy, -y, \\ y^3, y^2, y, 1, 0, z^3, z^2, z, 1, 0, \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -yz^3, -yz^2, -yz, -y, -y, 0, 0 \end{bmatrix}$$

$$\mathbf{A2}_{1 \times 75} = \begin{bmatrix} 0, 0, 0, 0, x^3, x^2, x, 1, 0, y^3, \\ y^2, y, 1, 0, z^3, z^2, z, 1, 0, 0, 0, \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, xz^3, xz^2, xz, x, 0, 0, 0 \end{bmatrix}$$

$$\mathbf{A3}_{1 \times 75} = \begin{bmatrix} 0, 0, 0, 0, 0, 0, 0, 0, x^3, x^2, x, 1, x^3y, x^2y, xy, y, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\ 0, 0, 0, 0, 0, y^3, y^2, y, 1, 0, z^3, \\ z^2, z, 1, yz^3, yz^2, yz, y, -xz^3, -xz^2, -xz, -x, 0, 0, 0, 0, 0, 0, -x, y \end{bmatrix}$$

$$\begin{aligned} \mathbf{W}_{45 \times 75} = & \text{CSR}\left(\begin{bmatrix} 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 \\ 1, 1, 1, 1, 1, 1, 1, -1, -1, 1 \end{bmatrix}, \right. \\ & \begin{bmatrix} 0, 1, 2, 3, 27, 51, 20, 21, 22, 23, 26, 71, 72, 24, 25, 48, 49, 50, 68, 69, 70, 4, \\ 5, 6, 71, 7, 31, 55, 28, 29, 30, 52, 53, 54, 8, 9, 10, 67, 73, 11, 35, 59, 12, 13, \\ 14, 15, 34, 63, 74, 32, 33, 56, 57, 58, 60, 61, 62, 64, 65, 66 \end{bmatrix}, \\ & \begin{bmatrix} 0, 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 28, 29, 30, \\ 31, 32, 33, 34, 35, 36, 39, 42, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, \\ 58, 59, 60 \end{bmatrix}, \\ & [45, 75] \\ &) \end{aligned}$$

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$$\mathbf{X}_{3 \times 45} = \mathbf{A}_{45 \times 75} \mathbf{W}_{75 \times 45}^\dagger = \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \\ \mathbf{X3} \end{bmatrix}$$

where

$$\begin{aligned} \mathbf{X1} &= \begin{bmatrix} x^3, x^2, x, 1, -x^3y, -x^2y, -xy, y, y^3, y^2, z^3, z^2, z, -yz^3, -yz^2, -yz, \\ 0, 0 \end{bmatrix} \\ \mathbf{X2} &= \begin{bmatrix} 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, xz^3, xz^2, xz, x^3, x^2, x, 1, y^3, y^2, y, z^3, \\ z^2, z, 0 \end{bmatrix} \\ \mathbf{X3} &= \begin{bmatrix} 0, x^3, x^2, \\ x, 1, x^3y, x^2y, xy, y, y^3, y^2, z^3, z^2, z, yz^3, yz^2, yz, -xz^3, -xz^2, -xz \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \boldsymbol{\beta}_{45 \times 1} &= \mathbf{W}_{45 \times 75} \mathbf{K}_{75 \times 1} \\ &= \begin{bmatrix} k_{dxx3}, k_{dxx2}, k_{dxx1}, k_{dxx0} + k_{dxy0} + k_{dxz0}, k_{ezx3}, k_{ezx2}, k_{ezx1}, -\alpha_{xy} + k_{dxy1} - k_{ezx0} - k_{ezz0}, k_{dxy3}, k_{dxy2}, \\ k_{dxz3}, k_{dxz2}, k_{dxz1}, k_{ezz3}, k_{ezz2}, k_{ezz1}, k_{dyx3}, k_{dyx2}, k_{dyx1} + k_{ezz0}, k_{dyx0} + k_{dyy0} + k_{dyz0}, k_{dyy3}, k_{dyy2}, k_{dyy1}, \\ k_{dyz3}, k_{dyz2}, k_{dyz1}, k_{dxx3}, k_{dxx2}, -\alpha_{zx} + k_{dxx1} - k_{eyz0}, k_{dxx0} + k_{dzy0} + k_{dzz0}, k_{exx3}, k_{exx2}, k_{exx1}, \\ \alpha_{zy} + k_{dzy1} + k_{exx0} + k_{exz0}, k_{dzy3}, k_{dzy2}, k_{dzz3}, k_{dzz2}, k_{dzz1}, k_{exz3}, k_{exz2}, k_{exz1}, k_{eyz3}, k_{eyz2}, k_{eyz1} \end{bmatrix}^T \\ &= \begin{bmatrix} k_{dxx3}, k_{dxx2}, k_{dxx1}, k_{eqx2}, k_{ezx3}, k_{ezx2}, k_{ezx1}, k_{eqx1}, k_{dxy3}, k_{dxy2}, \\ k_{dxz3}, k_{dxz2}, k_{dxz1}, k_{ezz3}, k_{ezz2}, k_{ezz1}, k_{dyx3}, k_{dyx2}, k_{eqy1}, k_{eqy2}, k_{dyy3}, k_{dyy2}, k_{dyy1}, \\ k_{dyz3}, k_{dyz2}, k_{dyz1}, k_{dxx3}, k_{dxx2}, k_{eqz1}, k_{eqz3}, k_{exx3}, k_{exx2}, k_{exx1}, \\ k_{eqz2}, k_{dzy3}, k_{dzy2}, k_{dzz3}, k_{dzz2}, k_{dzz1}, k_{exz3}, k_{exz2}, k_{exz1}, k_{eyz3}, k_{eyz2}, k_{eyz1} \end{bmatrix}^T \end{aligned}$$

where

$$\begin{aligned} k_{eqx1} &= k_{dxy1} - k_{ezx0} - k_{ezz0} - \alpha_{xy}, k_{eqx2} = k_{dxx0} + k_{dxy0} + k_{dxz0}, \\ k_{eqy1} &= k_{dyx1} + k_{ezz0}, k_{eqy2} = k_{dyx0} + k_{dyy0} + k_{dyz0}, \\ k_{eqz1} &= k_{dxx1} - k_{eyz0} - \alpha_{zx}, k_{eqz2} = \alpha_{zy} + k_{dzy1} + k_{exx0} + k_{exz0}, \\ k_{eqz3} &= k_{dxx0} + k_{dzy0} + k_{dzz0} \end{aligned}$$

References of Appendix

[1] Eisenstat, Stanley C., et al. Yale sparse matrix package. i. the symmetric codes. YALE UNIV NEW HAVEN CT DEPT OF COMPUTER SCIENCE, 1977.