Observer based versus Non-Observer based Adaptive Control of Electrically Driven Cooperative Manipulators using q-analogue of Bernstein-Schurer-Stancu operator as Uncertainty approximator

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Observer based versus Non-Observer based Adaptive Control of Electrically Driven Cooperative Manipulators using q-analogue of Bernstein-Schurer-Stancu operator as Uncertainty approximator

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Abstract: In this paper, q -analogue of Bernstein-Schurer-Stancu operator is proposed for uncertainty estimation in adaptive control of cooperative electrically driven manipulators. The contribution of this paper is reducing the number of signals needed in construction of the regressor vector for uncertainty estimation due to the simplicity of q -analogue of Bernstein-Schurer-Stancu operator in comparison with other alternatives. It is verified that uniformly ultimately bounded stability of the tracking/approximation errors can be assured if the q -analogue of Bernstein-Schurer-Stancu operator is utilized as regressors. Moreover, it has been assumed that velocity signals are not available and an observer is needed to estimate these signals. Different trajectories including simple and complicated paths have been tested in simulation results. The results are also compared with the case in which velocity signals are in hand to show the accuracy and efficiency of the proposed observer-controller structure.

1. Introduction

We have recently witnessed an increasing research interest towards cooperative robot manipulators in the control community due to interesting applications in transportation, reconnaissance, search and exploration of the environment. In comparison to single manipulators, more reliability is obtained in cooperative systems thanks to the benefits of teamwork. Moreover, industry is faced by many applications of cooperative manipulators which are out of reach of the working capability of a single manipulator. However, the challenges of controlling a single and coordinated manipulators are completely different, since cooperative robots require more variables to be considered in a cohesive manner.

Due to the uncertainties involved in cooperative robots, many adaptive or robust control strategies have been presented. Model reference adaptive control (MRAC)

is a popular strategy. In [1], it has been applied to control of cooperative robots. The influence of Markovian switched couplings in cooperative manipulators has been compensated in [2] using an adaptive controller. In order to guarantee asymptotic convergence of the load position and boundedness of the internal forces in cooperative robots, an adaptive controller has been designed in [3]. The problem of networked control of cooperative manipulators has been studied in [4] with the assumption that the dynamical parameters of the arms are not known and thus, the controller is designed in an adaptive manner. For constrained cooperative robot manipulators, a model reference adaptive controller has been presented in [5]. In [6], adaptive control ideas have been utilized to estimate the external disturbances and uncertainties of the agents' and the object's dynamic parameters. It should be noted that in most of these controllers, the actuator dynamics have been excluded and the controller calculates the torque signal that should be applied on the joints.

In the field of robust control, many control algorithms have been presented for cooperative robots. Among robust nonlinear controllers, sliding mode control is very simple and efficient and thus, many different versions of sliding mode controllers have been presented in the literature. In [7], a second order sliding mode controller has been proposed. Sliding mode approach can be easily combined by other controllers and mathematical tools. For example, in [8-10], fuzzy sliding mode controllers for cooperative robots have been introduced. A sliding mode controller based on low-pass filter has been presented in [11]. The performance of integral sliding mode on cooperative robot manipulators has been investigated in [12]. Also, dynamic surface control and supervisory sliding mode control have been presented for cooperative manipulators [13-14]. Combination of

adaptive and robust control is very interesting and various valuable approaches in this field have been presented for cooperative robots [15-18]. However, as mentioned above, in these robust controllers also the actuator dynamics has been ignored and in practical implementations, this question arises that how the desired torque should be provided.

Fuzzy systems and neural networks have also been frequently applied to robust or adaptive control of cooperative robots. In [19], a fuzzy backstepping controller for dual-arm robotic systems has been presented. In [20], an adaptive fuzzy controller with the capability of force tracking has been introduced. In [21], an adaptive cooperative controller using fuzzy rules and dynamic action surface has been presented and tested experimentally. Impedance control for cooperative manipulators using neural networks as been studied in [22]. Recurrent neural networks have been used in impedance control cooperative manipulators [23]. Combination of neural networks and sliding mode has been proposed in [24]. Universal approximation property is the main core of these applications of fuzzy systems and neural networks. However, we are confronted with large number of tuning parameters in the controllers designed based on fuzzy systems and neural networks [25]. Moreover, the number of required signals for building the regressor vector in uncertainty estimation using fuzzy systems and neural networks is considerable [26]. To solve these shortcomings of fuzzy systems and neural networks, function approximation-based controllers using the Fourier series expansion [27], differential equations [28] and Bernstein polynomials [29] are suitable alternatives. In this paper, q -analogue of Bernstein-Schurer-Stancu operator is introduced as uncertainty estimator in adaptive control of cooperative robots. Moreover, actuator dynamics are also considered and consequently, the

controller calculates the voltages applied to motors in cooperative control of robot manipulators. In high-speed applications, the advantage of designing controller by considering actuator dynamics is more tangible [30].

Many industrial robots do not provide the velocity and force feedbacks and therefore, various observer-based controllers have been proposed in the literature. In [31], using a linear observer, a robust decentralized controller for cooperative robots has been presented. Disturbance observer has also been utilized for control of multiple robots [32-33]. Some observers based on sliding mode strategy have been presented in [34-35] with the aim of force estimation. In [36], an observer-based controller for cooperative robots has been presented. However, the information of the inertia is required in this controller. In [37], a robust control scheme without velocity measurement has been presented that needs the inertia matrix to construct the observer dynamics. In [38], a model-free observer for estimation of velocity signals has been presented. However, the proposed controller in [38] is a simple model-based controller using feedback linearization technique. It must be emphasized that the aforementioned observer-controller structures are based on the torque control strategy in which the actuator dynamics is excluded.

In this paper, a model-free observer-controller structure is proposed for cooperative robots. Uncertainties are estimated using q -analogue of Bernstein-Schurer-Stancu operator which is simpler and less computational in comparison with fuzzy systems and neural networks. The unknown parameters of the estimator will be regulated online using adaptation law. Also, the proposed observer for velocity estimation is model-free. Thus, the proposed controller is superior than most of previous related works due to its model-free property in

observer and controller design. Moreover, the uncertainty estimator in the proposed method is novel and more efficient in comparison with neuro/fuzzy estimators. Also, from practical point of view, the proposed controller is more suitable due to its voltage-based structure that include the actuator dynamics in modelling and controller design. In order to highlight the satisfactory performance of the observer, a comparison with non-observer-based controller has been presented. Simulation results have been illustrated using interesting and complicated trajectories to show high capabilities of the proposed scheme in performing difficult tasks. Indeed, in order to make the influence of q -analogue of Bernstein-Schurer-Stancu operator more dominant, it has been removed from the control law and the results are illustrated that shows undeniable effect of this estimator in enhancing the controller performance.

The rest of this paper is organized as follows. Section 2, introduces q -analogue of Bernstein-Schurer-Stancu operator. Section 3, presents the dynamical model of cooperative manipulators considering permanent magnet DC motors as actuators. Section 4 explains controller design using full state feedback. Section 5 describes controller design using linear observer accompanied by stability analysis and performance evaluation. Section 6 presents simulation results and section 7 concludes the paper. Matrices and vectors are exhibited in bold form. It should also mention that in this paper $\bar{\lambda}(\bullet)$ and $\underline{\lambda}(\bullet)$ are maximum singular value and its minimum, respectively. Norm of a vector like $\mathbf{y} \in \Re^n$ is presented as $\|\mathbf{y}\| = \sqrt{\mathbf{y}^T \mathbf{y}}$ and for a matrix is addressed as $\|\mathbf{E}(\mathbf{y})\| = \sqrt{\bar{\lambda}(\mathbf{E}(\mathbf{y})^T \mathbf{E}(\mathbf{y}))}$.

2. Function Approximation using the q-analogue of Bernstein-Schurer-Stancu Operators

Let $\alpha, \beta, p \in \mathbb{N}^0$ (the set of all non-negative integers) be such that $0 \leq \alpha \leq \beta$, then for any $f \in C[0, 1+p]$, $q \in (0, 1)$, the q -Bernstein-Schurer-Stancu Operators are defined by [39].

$$S_{N,p}^{\alpha,\beta}(f, q, t) = \sum_{k=0}^{N+p} f\left(\frac{[k]_q + \alpha}{[N]_q + \beta}\right) \begin{bmatrix} N+p \\ k \end{bmatrix}_q t^k (1-t)_q^{N+p-k}, \quad \forall t \in [0, 1] \quad (1)$$

where $\begin{bmatrix} N \\ k \end{bmatrix}_q$ represents the q -binomial coefficient. In other words,

$$\begin{bmatrix} N \\ k \end{bmatrix}_q = \frac{[N]_q!}{[N-k]_q! [k]_q!} \quad (2)$$

where N and k are any integers that satisfy $N \geq k \geq 0$. The q -factorial is represented by $[k]_q!$ and is described as follows.

$$[k]_q! = \begin{cases} [k]_q [k-1]_q \dots 1 & k \geq 1 \\ 1 & k = 0 \end{cases} \quad (3)$$

where

$$[k]_q = \begin{cases} \frac{(1-q^k)}{(1-q)}, & q \neq 1 \\ k & q = 1 \end{cases} \quad (4)$$

denotes the q -integer of the number $k \in \mathbb{N}$. According to Theorem 1 of [39], regarding any function $f \in C[0, 1+p]$ and $0 < q < 1$ it is proved that the sequence $S_{N,p}^{\alpha,\beta}(f, q, t)$ converges uniformly to $f(t)$ on $[0, 1]$. It can be shown that

$$S_{N,p}^{\alpha,\beta}(f, q, t) = \Lambda_f^T \xi_f \quad (5)$$

where

$$\Lambda_f = \left[f\left(\frac{\alpha}{[N]_q + \beta}\right) \quad [N+p]_q f\left(\frac{1+\alpha}{[N]_q + \beta}\right) \quad \dots \quad f\left(\frac{[N+p]_q + \alpha}{[N]_q + \beta}\right) \right]^T \in \Re^{N+p+1} \quad (6)$$

is a vector consisting of tunable parameters and

$$\xi_f = \left[(1-t)_q^{N+p} \quad t(1-t)_q^{N+p-1} \quad \dots \quad t^{N+p} \right]^T \in \Re^{N+p+1} \quad (7)$$

is the vector of basis functions. Equation (1) can be shown in the linear form (5), which is a common form in adaptive control.

2. Dynamical Modeling

2.1. System Overview and Assumptions

Assume that an object must be controlled in the workspace by some manipulators. Suppose that m manipulators with n degree-of-freedom are involved. The first step to derive the dynamical model of this system is to establish separate coordinate frames as presented in Fig. 1. The purpose of the control system is to regulate the end-effector and object positions and orientations in the frame $OXYZ$ which is the base frame. As shown in Fig. 1, $O_oX_oY_oZ_o$ is the coordinate frame of the object. Suppose that this frame is placed at center of mass (COM) of the object. In this figure, $O_{ei}X_{ei}Y_{ei}Z_{ei}$ is the coordinate frame of the i th manipulator which is placed at its grasp point. Now, consider the following assumptions [40-41].

Assumption 1: For each robot, the dimensions of the workspace and DOF are equal. In other words, manipulators are non-redundant.

Fig 1. The Coordinate robotic system.

Assumption 2: The relative movements between all end-effectors and the object is zero due to the rigid attachments.

Assumption 3: For each robot, there is no uncertainty in its kinematics. Also, the Jacobian matrix is full-rank and known for all manipulators. In other words, we are not faced with kinematics singularities while moving.

Assumption 4: Applying forces from manipulators to the object does not result in distortion of the object. In other words, a rigid object is studied.

2.2. Actuator Model

According to [42], the rotational behavior of n permanent magnet DC motors driving a robot manipulator is given by

$$\mathbf{J}_{ai} \ddot{\boldsymbol{\theta}}_i + (\mathbf{b}_{ai} + \mathbf{k}_{ai} \mathbf{k}_{bi} \mathbf{R}_i^{-1}) \dot{\boldsymbol{\theta}}_i = \mathbf{k}_{ai} \mathbf{R}_i^{-1} \mathbf{v}_i(t) - \mathbf{r}_i \boldsymbol{\tau}_{li}, \quad i=1,2,\dots,m \quad (8)$$

where $\boldsymbol{\theta}_i \in \mathbb{R}^n$, $\mathbf{J}_{ai} \in \mathbb{R}^{n \times n} [kg.m^2]$, $\mathbf{b}_{ai} \in \mathbb{R}^{n \times n} [N.m.s / rad]$,

$\mathbf{k}_{ai} \in \mathbb{R}^{n \times n} [N.m / A]$, $\mathbf{k}_{bi} \in \mathbb{R}^{n \times n} [N.m / A]$, $\mathbf{R}_i \in \mathbb{R}^{n \times n} [\Omega]$, $\mathbf{v}_i(t) \in \mathbb{R}^n [v]$ and

$\boldsymbol{\tau}_{li} \in \mathbb{R}^n [N.m]$ are the vector of motor angular positions, lumped actuator rotor

inertias along with the motor side elements of the transmission, the lumped actuator damping coefficients and motor side elements of the transmission, motor-torque constants, back EMF constants, armature resistances, armature voltages, and load torques, respectively. Also, we have

$$\dot{\mathbf{q}}_i = \mathbf{r}_i \dot{\boldsymbol{\theta}}_i, \quad i=1,2,\dots,m \quad (9)$$

where $\mathbf{r}_i$ is an $n \times n$ gear-ratio matrix.

Remark 1: There is an assumption in equation (8) which consider the mechanical time constant to be more than it's electrical one.

2.3. Manipulators Dynamic Model

According to [43-44], the i th manipulator has the following motion equation:

$$\mathbf{D}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{G}_i(\mathbf{q}_i) = \boldsymbol{\tau}_{li} - \mathbf{J}_{ei}^T(\mathbf{q}_i)\mathbf{F}_{ei}, \quad i=1,2,\dots,m \quad (10)$$

where the vector $\mathbf{q}_i \in \mathbb{R}^n$ indicates the joint variables. $\mathbf{D}_i(\mathbf{q}_i) \in \mathbb{R}^{n \times n}$, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i \in \mathbb{R}^{n \times 1}$ and $\mathbf{G}_i(\mathbf{q}_i) \in \mathbb{R}^n$ are the inertia matrix, centrifugal-Coriolis and gravitational forces and the Jacobian matrix is $\mathbf{J}_{ei}(\mathbf{q}_i) \in \mathbb{R}^{n \times n}$. Also, $\mathbf{F}_{ei} \in \mathbb{R}^n$ is the force that the end-effector of the i th manipulator exerts on the common object at the grasp point.

Unified modelling of m manipulators driven by permanent magnet DC motors (8) is

$$\bar{\mathbf{D}}(\mathbf{q})\ddot{\mathbf{q}} + \bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{r}^{-1}\mathbf{k}_a\mathbf{R}^{-1}\mathbf{v} - \mathbf{J}_e^T(\mathbf{q})\mathbf{F}_e \quad (11)$$

that

$$\begin{aligned}
\mathbf{r}^{-1}\mathbf{k}_a\mathbf{R}^{-1} &= \text{blockdiag}\left[\mathbf{r}_1^{-1}\mathbf{k}_{a1}\mathbf{R}_1^{-1} \quad \dots \quad \mathbf{r}_m^{-1}\mathbf{k}_{am}\mathbf{R}_m^{-1}\right] \in \Re^{mn \times mn} \\
\mathbf{q} &= \begin{bmatrix} \mathbf{q}_1^T & \mathbf{q}_2^T & \dots & \mathbf{q}_m^T \end{bmatrix}^T \in \Re^{mn} \\
\mathbf{v} &= \begin{bmatrix} \mathbf{v}_1^T & \mathbf{v}_2^T & \dots & \mathbf{v}_m^T \end{bmatrix}^T \in \Re^{mn} \\
\bar{\mathbf{D}}(\mathbf{q}) &= \text{blockdiag}\left[\mathbf{D}_1(\mathbf{q}_1) + \mathbf{r}_1^{-1}\mathbf{J}_{a1}\mathbf{r}_1^{-1} \quad \mathbf{D}_2(\mathbf{q}_2) + \mathbf{r}_2^{-1}\mathbf{J}_{a2}\mathbf{r}_2^{-1} \quad \dots \quad \mathbf{D}_m(\mathbf{q}_m) + \mathbf{r}_m^{-1}\mathbf{J}_{am}\mathbf{r}_m^{-1}\right] \in \Re^{mn \times mn} \\
\mathbf{J}_e(\mathbf{q}) &= \text{blockdiag}\left[\mathbf{J}_{e1}^T(\mathbf{q}_1) \quad \dots \quad \mathbf{J}_{em}^T(\mathbf{q}_m)\right] \in \Re^{mn \times mn} \\
\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) &= \text{blockdiag}\left[\mathbf{C}_1(\mathbf{q}_1, \dot{\mathbf{q}}_1) + \mathbf{r}_1^{-1}(\mathbf{b}_{a1} + \mathbf{k}_{a1}\mathbf{k}_{b1}\mathbf{R}_1^{-1})\mathbf{r}_1^{-1} \quad \dots \right. \\
&\quad \left. \mathbf{C}_m(\mathbf{q}_m, \dot{\mathbf{q}}_m) + \mathbf{r}_m^{-1}(\mathbf{b}_{am} + \mathbf{k}_{am}\mathbf{k}_{bm}\mathbf{R}_m^{-1})\mathbf{r}_m^{-1}\right] \in \Re^{mn \times mn} \\
\mathbf{G}(\mathbf{q}) &= \begin{bmatrix} \mathbf{G}_1(\mathbf{q}_1) & \dots & \mathbf{G}_m(\mathbf{q}_m) \end{bmatrix} \in \Re^{mn} \\
\mathbf{F}_e &= \begin{bmatrix} \mathbf{F}_{e1}^T & \mathbf{F}_{e2}^T & \dots & \mathbf{F}_{em}^T \end{bmatrix}^T \in \Re^{mn}
\end{aligned} \tag{12}$$

2.4. Dynamic Model of the Object

According to [40,45], the motion of the object is described as:

$$\mathbf{D}_o(\mathbf{x}_o)\ddot{\mathbf{x}}_o + \mathbf{C}_o(\mathbf{x}_o, \dot{\mathbf{x}}_o)\dot{\mathbf{x}}_o + \mathbf{G}_o(\mathbf{x}_o) = \mathbf{F}_o \tag{13}$$

where $\mathbf{x}_o \in \Re^n$ denotes the object position and its orientation relative to the base frame (OXYZ) and (O_oX_oY_oZ_o) is the object frame. The object inertial matrix $\mathbf{D}_o(\mathbf{x}_o) \in \Re^{n \times n}$ is a positive definite symmetric matrix. $\mathbf{G}_o(\mathbf{x}_o) \in \Re^n$ shows the gravitational vector. $\mathbf{C}_o(\mathbf{x}_o, \dot{\mathbf{x}}_o) \in \Re^{n \times n}$ is the centrifugal and Coriolis matrix. $\mathbf{F}_o \in \Re^n$ denotes the force exerted on the center of mass of the object. The matrix $\mathbf{J}_o(\mathbf{x}_o) \in \Re^{mn \times n}$ is in the form of

$$\mathbf{J}_o(\mathbf{x}_o) = \begin{bmatrix} \mathbf{J}_{o1}^T(\mathbf{x}_o) & \dots & \mathbf{J}_{om}^T(\mathbf{x}_o) \end{bmatrix}^T \tag{14}$$

where $\mathbf{J}_{oi}(\mathbf{x}_o) \in \Re^{n \times n}$ is the object frame (O_oX_oY_oZ_o) Jacobian matrix relative to the i th robot end-effector frame (O_{ei}X_{ei}Y_{ei}Z_{ei}). Therefore, $\mathbf{F}_o$ is obtained

$$\mathbf{F}_o = \mathbf{J}_o^T(\mathbf{x}_o)\mathbf{F}_e \quad (15)$$

If $\mathbf{F}_o$ is known, then $\mathbf{F}_e$, can be divided into two orthogonal parts corresponding to the motion of the object and the internal force production. In other words, $\mathbf{F}_e$ is given by [40-41]:

$$\mathbf{F}_e = (\mathbf{J}_o^T(\mathbf{x}_o))^+ \mathbf{F}_o + \mathbf{F}_I \quad (16)$$

where $(\mathbf{J}_o^T(\mathbf{x}_o))^+ \in \mathfrak{R}^{mn \times n}$ is the pseudo-inverse given by $\mathbf{J}_o(\mathbf{x}_o)(\mathbf{J}_o^T(\mathbf{x}_o)\mathbf{J}_o(\mathbf{x}_o))^{-1}$ and $\mathbf{F}_I \in \mathfrak{R}^{mn}$ is the internal force vector and must be in the null space of $\mathbf{J}_o^T(\mathbf{x}_o)$.

In other words, we have:

$$\mathbf{J}_o^T(\mathbf{x}_o)\mathbf{F}_I = 0 \quad (17)$$

Using (13) and (16), we can write

$$\mathbf{F}_e = (\mathbf{J}_o^T)^+ (\mathbf{G}_o + \mathbf{C}_o \dot{\mathbf{x}}_o + \mathbf{D}_o \ddot{\mathbf{x}}_o) + \mathbf{F}_I \quad (18)$$

2. 5. The Unified Dynamics of Cooperative Robots

Based on (11) and (13), it is concluded that the total number of position variables is $(m+1)n$; n of which are independent, since if the object trajectory is specified, based on Assumptions 1 and 2, the joint trajectory for each manipulator is uniquely obtained. Therefore, due to dependency in position variables, (11) and (13) are not suitable for analysing system dynamics. Thus, the object position $\mathbf{x}_o$ can be used to describe the behavior of the overall system due to its independence. Consequently, (11) can be modified based on $\mathbf{x}_o$.

Let $\mathbf{x}_{ei} \in \mathfrak{R}^n$ be the position and orientation of the i th end-effector. Therefore, $\dot{\mathbf{x}}_{ei}$ is in relation with $\dot{\mathbf{q}}_i$ by the Jacobian matrix $\mathbf{J}_{ei}(\mathbf{q}_i)$ as

$$\dot{\mathbf{x}}_{ei} = \mathbf{J}_{ei}(\mathbf{q}_i)\dot{\mathbf{q}}_i \quad (19)$$

and the relationship between $\dot{\mathbf{x}}_{ei}$ and $\dot{\mathbf{x}}_o$ is given by

$$\dot{\mathbf{x}}_{ei} = \mathbf{J}_{oi}(\mathbf{x}_o)\dot{\mathbf{x}}_o \quad (20)$$

By combining (19) and (20), we can write

$$\mathbf{J}_{ei}(\mathbf{q}_i)\dot{\mathbf{q}}_i = \mathbf{J}_{oi}(\mathbf{x}_o)\dot{\mathbf{x}}_o \quad (21)$$

Since it is supposed that the manipulators operate in a non-singular space, the inverse of the Jacobian matrix $\mathbf{J}_{ei}(\mathbf{q}_i)$ is computable. Considering all the manipulators acting on the object at the same time, yields

$$\dot{\mathbf{q}} = \mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o)\dot{\mathbf{x}}_o \quad (22)$$

Taking time derivative of (22) results in

$$\ddot{\mathbf{q}} = \mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o)\ddot{\mathbf{x}}_o + \frac{d}{dt}(\mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o))\dot{\mathbf{x}}_o \quad (23)$$

Using (18), (22) and (23), the dynamic equation (11) can be written as

$$\begin{aligned} & \bar{\mathbf{D}}\mathbf{J}_e^{-1}\mathbf{J}_o\ddot{\mathbf{x}}_o + \bar{\mathbf{D}}\frac{d}{dt}(\mathbf{J}_e^{-1}\mathbf{J}_o)\dot{\mathbf{x}}_o + \bar{\mathbf{C}}\mathbf{J}_e^{-1}\mathbf{J}_o\dot{\mathbf{x}}_o + \mathbf{G} + \mathbf{J}_e^T\mathbf{F}_I \\ & + \mathbf{J}_e^T(\mathbf{J}_o^T)^+(\mathbf{D}_o\ddot{\mathbf{x}}_o + \mathbf{C}_o\dot{\mathbf{x}}_o + \mathbf{G}_o) = \mathbf{r}^{-1}\mathbf{k}_a\mathbf{R}^{-1}\mathbf{v} \end{aligned} \quad (24)$$

Premultiplying both sides of (24) by $\mathbf{J}_o^T\mathbf{J}_e^{-T}$, and using the fact that $\mathbf{J}_o^T\mathbf{F}_I = 0$, the comprehensive dynamic model is given by

$$\mathbf{M}_c(\mathbf{x}_o)\ddot{\mathbf{x}}_o + \mathbf{C}_c(\mathbf{x}_o, \dot{\mathbf{x}}_o)\dot{\mathbf{x}}_o + \mathbf{G}_c(\mathbf{x}_o) + \delta(\mathbf{x}_o, \mathbf{v}) = \mathbf{u} \quad (25)$$

where

$$\begin{aligned} \mathbf{M}_c(\mathbf{x}_o) &= \mathbf{J}_o^T(\mathbf{x}_o)\mathbf{J}_e^{-T}(\mathbf{q})\bar{\mathbf{D}}(\mathbf{q})\mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o) + \mathbf{D}_o(\mathbf{x}_o) \in \Re^{n \times n} \\ \mathbf{C}_c(\mathbf{x}_o, \dot{\mathbf{x}}_o) &= \mathbf{J}_o^T(\mathbf{x}_o)\mathbf{J}_e^{-T}(\mathbf{q})\left(\bar{\mathbf{D}}(\mathbf{q})\frac{d}{dt}(\mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o)) + \bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{J}_e^{-1}(\mathbf{q})\mathbf{J}_o(\mathbf{x}_o)\right) \\ &+ \mathbf{C}_o(\mathbf{x}_o, \dot{\mathbf{x}}_o) \in \Re^{n \times n} \\ \mathbf{G}_c &= \mathbf{J}_o^T\mathbf{J}_e^{-T}\mathbf{G}(\mathbf{q}) + \mathbf{G}_o \in \Re^n \\ \mathbf{u} &= \mathbf{J}_o^T\mathbf{J}_e^{-T}\mathbf{v} \in \Re^n \\ \delta(\mathbf{x}_o, \mathbf{v}) &= \mathbf{J}_o^T\mathbf{J}_e^{-T}(\mathbf{I}_{mn} - \mathbf{r}^{-1}\mathbf{k}_a\mathbf{R}^{-1})\mathbf{v} \in \Re^n \end{aligned}$$

where $\mathbf{I}_{mn}$ is a block diagonal matrix consisted of m identity matrix with the dimension n .

3. Full State Feedback Design

According to (25), the mathematical model of cooperative robots contains complicated matrices that will impose difficulties on the control system and reduce its capabilities. In other words, due to complexity or uncertainties in the matrices and vectors of (25), the controller design for such system is a challenging task. Moreover, increasing the number of cooperative manipulators and its relevant DOF, make the above difficulty more hypersensitive. The control objective is hence to regulate the center of mass of the object to track a predefined trajectory in the task space while the controller is faced with unknown uncertainties and external disturbances in the dynamics of manipulator. To this mean, function approximation technique is developed in this section to get a model free control scheme. Rewrite (25) in the form of:

$$\ddot{\mathbf{x}}_o = \mathbf{u} + \boldsymbol{\sigma}(t) \quad (26)$$

where

$$\boldsymbol{\sigma}(t) = (\mathbf{M}_c^{-1}(\mathbf{x}_o) - \mathbf{I}_n)\mathbf{u} - \mathbf{M}_c^{-1}(\mathbf{C}_c \dot{\mathbf{x}}_o + \mathbf{G}_c + \boldsymbol{\delta}) \quad (27)$$

shows a vector with n -dimension containing uncertainties with $\mathbf{I}_n$ which identifies an $n \times n$ identity matrix. The assumption below is prescribed prior to the controller derivation specifications

Assumption 5: Assume that $\boldsymbol{\sigma}(t)$ nonlinear function to be bounded, unknown and time varying. Furthermore, the variation bound has been presumed unavailable. Consequently, administering conventional robust or adaptive control methods is not at hand [46]. Attend the controller as below

$$\mathbf{u} = \ddot{\mathbf{x}}_{od} - \mathbf{K}_v \dot{\mathbf{e}}_o - \mathbf{K}_p \mathbf{e}_o - \hat{\boldsymbol{\sigma}}(t) \quad (28)$$

where $\mathbf{K}_p \in \mathbb{R}^{n \times n}$ and $0 < \mathbf{K}_v \in \mathbb{R}^{n \times n}$ are constant gain matrices, $\hat{\boldsymbol{\sigma}}(t)$ is approximation of $\boldsymbol{\sigma}(t)$ and $\mathbf{e}_o = \mathbf{x}_o - \mathbf{x}_{od}$ declares tracking error for the object position/orientation.

Assumption 6: The desired trajectory is smooth, in the sense that $\mathbf{x}_{od} \in \mathbb{R}^n$ is bounded and uniformly continuous. Moreover, up to a necessary order, its derivatives are bounded and uniformly continuous.

Replacing $\mathbf{u}$ in (26) by (28) gives

$$\ddot{\mathbf{e}}_o + \mathbf{K}_v \dot{\mathbf{e}}_o + \mathbf{K}_p \mathbf{e}_o = \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t) \quad (29)$$

Define

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_n & \mathbf{I}_n \\ -\mathbf{K}_p & -\mathbf{K}_v \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_n \\ \mathbf{I}_n \end{bmatrix} \in \mathbb{R}^{2n \times n}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{e}_o \\ \dot{\mathbf{e}}_o \end{bmatrix} \in \mathbb{R}^{2n}$$

Thus, Eq. (29) is given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}(\boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t)) \quad (30)$$

Converging $\mathbf{x}(t)$ to zero can result from (30), if an adaptation rule is designed so that $\hat{\boldsymbol{\sigma}}(t) \rightarrow \boldsymbol{\sigma}(t)$. With this in mind, the q-analogue of Bernstein-Schurer-Stancu operator is used to represent $\boldsymbol{\sigma}(t)$ as linear mixture of basis functions as

$$\boldsymbol{\sigma}(t) = \mathbf{W}_\sigma^T \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma \quad (31)$$

where β_1 is the basis function number, $\mathbf{W}_\sigma \in \mathbb{R}^{n\beta_1 \times n}$ is q-analogue of Bernstein-Schurer-Stancu operator coefficient, $\mathbf{Z}_\sigma \in \mathbb{R}^{n\beta_1}$ is the regressor vector, and $\boldsymbol{\varepsilon}_\sigma$ is the reconstruction error vector of $\boldsymbol{\sigma}(t)$. Obviously, based on the q-analogue of Bernstein-Schurer-Stancu operator, the corresponding estimates can be given by the equation below

$$\hat{\boldsymbol{\sigma}}(t) = \hat{\mathbf{W}}_\sigma^T \mathbf{Z}_\sigma \quad (32)$$

where $\hat{\mathbf{W}}_\sigma \in \mathfrak{R}^{n\beta_1 \times n}$ is the estimation of $\mathbf{W}_\sigma$. Now, by replacing (31) and (32) into (30), we have

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}(\tilde{\mathbf{W}}_\sigma^T \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma) \quad (33)$$

in which $\tilde{\mathbf{W}}_\sigma = \mathbf{W}_\sigma - \hat{\mathbf{W}}_\sigma$ is parametric error the q-analogue of Bernstein-Schurer-Stancu operator coefficients.

Remark 2: It is important to note that $\mathbf{v}$ is the controller output that is going to be assigned to the robotic system. Consequently, the control law that is responsible for regulation of the position is defined as

$$\mathbf{v} = \mathbf{J}_e^T(\mathbf{q})\mathbf{F}_{lc} + \mathbf{J}_e^T(\mathbf{q})(\mathbf{J}_o^T(\mathbf{x}_o))^+ \mathbf{u} \quad (34)$$

where the internal force control term $\mathbf{F}_{lc} \in \mathfrak{R}^{mn}$ is assigned as

$$\mathbf{F}_{lc} = \mathbf{K}_I (\mathbf{F}_{ld} - \mathbf{F}_I) + \mathbf{F}_{ld} \quad (35)$$

with $\mathbf{K}_I \in \mathfrak{R}^{mn \times mn}$ representing a positive diagonal gain matrix. It is important to mention that $\mathbf{F}_{lc}$ is in the null space of $\mathbf{J}_o^T(\mathbf{x}_o)$, since $\mathbf{F}_{ld}$ has that property.

Here is the theorem that aggregates the main outcomes of suggested method

Theorem 1: Consider the dynamic system in (26) and Assumption 5. Assume the described controller in (28) in which the lumped uncertainty is approximated by (32). Using σ -modification, we have

$$\dot{\hat{\mathbf{W}}}_\sigma = \mathbf{Q}_\sigma^{-1}(\mathbf{Z}_\sigma \mathbf{x}^T \boldsymbol{\Xi} \mathbf{B} - \kappa_\sigma \hat{\mathbf{W}}_\sigma) \quad (36)$$

where κ_σ is a positive scalar and $\mathbf{Q}_\sigma \in \mathfrak{R}^{n\beta_1 \times n\beta_1} > 0$ shows its learning rate. The vectors $\mathbf{x}(t)$ and $\tilde{\mathbf{W}}_\sigma$ are uniformly ultimately bounded, if the number of regressor terms, control gains and other design parameters are selected properly.

Proof: Consider the following function

$$V(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) = \mathbf{x}^T \boldsymbol{\Xi} \mathbf{x} + Tr(\tilde{\mathbf{W}}_\sigma^T \mathbf{Q}_\sigma \tilde{\mathbf{W}}_\sigma) \quad (37)$$

where $Tr(\bullet)$ is the trace operator. The time derivative of (37) accompanying (33) and (36) leads to

$$\dot{V}(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) = \mathbf{x}^T (\mathbf{A}^T \Xi + \Xi \mathbf{A}) \mathbf{x} + 2\mathbf{x}^T \Xi \mathbf{B} \boldsymbol{\varepsilon}_\sigma + 2\kappa_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \hat{\mathbf{W}}_\sigma) \quad (38)$$

Since $\mathbf{A}$ is Hurwitz; it is concluded that

$$\mathbf{A}^T \Xi + \Xi \mathbf{A} + \mathbf{Q} = \mathbf{0} \quad (39)$$

where $\Xi \in \Re^{2n \times 2n}$ and $\mathbf{Q} \in \Re^{2n \times 2n}$ are symmetric positive-definite matrices.

Therefore, (38) is clarified and simplified to

$$\dot{V}(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) = -\mathbf{x}^T \mathbf{Q} \mathbf{x} + 2\mathbf{x}^T \Xi \mathbf{B} \boldsymbol{\varepsilon}_\sigma + 2\kappa_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \hat{\mathbf{W}}_\sigma) \quad (40)$$

Result 1: Suppose that the required number of regressor terms of q- Bernstein-Schurer-Stancu operators has been selected and $\boldsymbol{\varepsilon}_\sigma$ is negligible. Then, the σ -modification terms in (36) is not needed. Therefore, (40) is converted to

$$\dot{V}(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) \leq -\mathbf{x}^T \mathbf{Q} \mathbf{x}$$

In order to verify the asymptotic convergence of $\mathbf{x}$, the Barbalat's Lemma can be used.

Result 2: Suppose that $\boldsymbol{\varepsilon}_\sigma$ is large. It is easy to prove the following inequalities hold

$$-\mathbf{x}^T \mathbf{Q} \mathbf{x} + 2\mathbf{x}^T \Xi \mathbf{B} \boldsymbol{\varepsilon}_\sigma \leq -\frac{1}{2} \underline{\lambda}(\mathbf{Q}) \|\mathbf{x}\|^2 + \frac{2\bar{\sigma}^2(\Xi \mathbf{B})}{\underline{\lambda}(\mathbf{Q})} \|\boldsymbol{\varepsilon}_\sigma\|^2 \quad (41)$$

$$2Tr(\tilde{\mathbf{W}}_\sigma^T \hat{\mathbf{W}}_\sigma) \leq Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) - Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \quad (42)$$

where $\bar{\sigma}(\Xi \mathbf{B})$ represent the maximum singular value of $\Xi \mathbf{B}$. Together with these relationships

$$V(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) \leq \bar{\lambda}(\Xi) \|\mathbf{x}\|^2 + \bar{\lambda}(\mathbf{Q}_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \quad (43)$$

One can write (40) in the form of

$$\begin{aligned}\dot{V} = & -\mu V + (\mu\bar{\lambda}(\Xi) - \frac{1}{2}\underline{\lambda}(\mathbf{Q}))\|\mathbf{x}\|^2 \\ & + (\mu\bar{\lambda}(\mathbf{Q}_\sigma) - \kappa_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) + \frac{2\bar{\sigma}^2(\Xi\mathbf{B})}{\underline{\lambda}(\mathbf{Q})}\|\boldsymbol{\varepsilon}_\sigma\|^2 + \kappa_\sigma Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma)\end{aligned}\quad (44)$$

By selecting

$$\mu \leq \min \left\{ \frac{\underline{\lambda}(\mathbf{Q})}{2\bar{\lambda}(\Xi)}, \frac{\kappa_\sigma}{\bar{\lambda}(\mathbf{Q}_\sigma)} \right\} \quad (45)$$

Equation (44) can be further derived as

$$\dot{V}(\mathbf{x}, \tilde{\mathbf{W}}_\sigma) \leq -\mu V + \frac{2\bar{\sigma}^2(\Xi\mathbf{B})}{\underline{\lambda}(\mathbf{Q})}\|\boldsymbol{\varepsilon}_\sigma\|^2 + \kappa_\sigma Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \quad (46)$$

As a result $\dot{V} < 0$, whenever

$$V > \frac{2\bar{\sigma}^2(\Xi\mathbf{B})}{\mu\underline{\lambda}(\mathbf{Q})} \sup_{t_0 < \tau} \|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2 + \frac{\kappa_\sigma}{\mu} Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \quad (47)$$

Hence, we have proved that $\mathbf{x}$ and $\tilde{\mathbf{W}}_\sigma$ are uniformly ultimately bounded. ■

In order to verify the boundedness of the internal force errors, consider (48) which is obtained by (25), (28) and (34).

$$(\mathbf{F}_{ld} - \mathbf{F}_l) = (\mathbf{I}_n + \mathbf{K}_l)^{-1} \left\{ (\mathbf{J}_o^T)^+ (\boldsymbol{\delta} - \mathbf{u} + \mathbf{M}_c \ddot{\mathbf{x}}_o + \mathbf{C}_c \dot{\mathbf{x}}_o + \mathbf{G}_c) \right\} \quad (48)$$

Obviously, the force error is reversely related to $(\mathbf{I}_n + \mathbf{K}_l)$ and bounded. It can be suppressed to a desired value by choosing an enough big value for $\mathbf{K}_l$ gain.

3.1. Performance Evaluation

In the previous subsection, the boundedness of the closed loop signals was verified. It is to be noted that considering practical issues in implantations, transient performance fulfilment is also a tremendous key issue. (46) provides

$$V(t) \leq V(t_0)e^{-\mu(t-t_0)} + \frac{2\bar{\sigma}^2(\Xi\mathbf{B})}{\mu\underline{\lambda}(\mathbf{Q})} \sup_{t_0 < \tau < t} \|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2 + \frac{\kappa_\sigma}{\mu} Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \quad (49)$$

Using (37), we have

$$\|\mathbf{x}\| \leq \sqrt{\frac{V}{\underline{\lambda}(\Xi)}} \quad (50)$$

This leads to

$$\begin{aligned} \|\mathbf{x}\| \leq & \sqrt{\frac{V(t_0)}{\underline{\lambda}(\Xi)}} e^{-0.5\mu(t-t_0)} + \sqrt{\frac{2}{\mu\underline{\lambda}(\Xi)\underline{\lambda}(\mathbf{Q})}} \bar{\sigma}(\Xi\mathbf{B}) \sup_{t_0 < \tau < t} \|\boldsymbol{\varepsilon}_\sigma(\tau)\| \\ & + \sqrt{\frac{\kappa_\sigma}{\mu\underline{\lambda}(\Xi)} \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma)} \end{aligned} \quad (51)$$

that guarantees $\|\mathbf{x}\|$ is bounded. Its bound is given by (51). This also shows that by tuning some gains, one can enhance the reduction rate of $\|\mathbf{x}\|$. Therefore,

$$\lim_{t \rightarrow \infty} \|\mathbf{x}(t)\| \leq \sqrt{\frac{2}{\mu\underline{\lambda}(\Xi)\underline{\lambda}(\mathbf{Q})}} \bar{\sigma}(\Xi\mathbf{B}) \sup_{t_0 < \tau} \|\boldsymbol{\varepsilon}_\sigma(\tau)\| + \sqrt{\frac{\kappa_\sigma}{\mu\underline{\lambda}(\Xi)} \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma)} \quad (52)$$

The result is fulfilling the analysis of performance transient.

4. Controller Design Using Linear State Estimated Feedback

As mentioned before, in many real-world examples, velocity signals are not measured and usually only the position signal is in hand. Besides, speed measurement increases costs and reduces the bandwidth of the system. The reason is that velocity signals are more contaminated by noise [47]. A common approach to address this issue is computing the velocity by differentiating the measured position. This may lead to the emergence of the chattering phenomenon in the system signals. To handle these challenges imposed by the velocity state, an observer-based form of the controller shown in previous section is presented. The proposed control input and observer dynamics are as follows

$$\mathbf{u} = \ddot{\mathbf{x}}_{\text{od}} - \mathbf{K}_v \dot{\hat{\mathbf{e}}}_o - \mathbf{K}_p \hat{\mathbf{e}}_o - \hat{\boldsymbol{\sigma}}(t) \quad (53)$$

$$\begin{cases} \dot{\hat{\mathbf{e}}}_o = \boldsymbol{\rho} + \mathbf{L}_v(\mathbf{e}_o - \hat{\mathbf{e}}_o) \\ \dot{\boldsymbol{\rho}} = \mathbf{L}_p(\mathbf{e}_o - \hat{\mathbf{e}}_o) \end{cases} \quad (54)$$

where $0 < \mathbf{K}_v \in \mathfrak{R}^{n \times n}$ and $0 < \mathbf{K}_p \in \mathfrak{R}^{n \times n}$ are gain matrices; $\hat{\mathbf{e}}_o = \hat{\mathbf{x}}_o - \mathbf{x}_{\text{od}}$ is the estimation of $\mathbf{e}_o(t)$; $\mathbf{L}_v \in \mathfrak{R}^{n \times n}$ and $\mathbf{L}_p \in \mathfrak{R}^{n \times n}$ are diagonal and positive observer gain matrices and $\hat{\boldsymbol{\sigma}}(t)$ is defined as before. Let us make the following assumption on the structure of $\mathbf{K}_p$, $\mathbf{K}_v$, $\mathbf{L}_v$ and $\mathbf{L}_p$.

Assumption 7: $\mathbf{K}_p$, $\mathbf{K}_v$, $\mathbf{L}_v$ and $\mathbf{L}_p$ satisfy the following equations.

$$\mathbf{K}_p = \mu \mathbf{K}_v \quad (55)$$

$$\mathbf{K}_v = (k_v + \gamma) \mathbf{I}_n \quad (56)$$

$$\mathbf{L}_p = \mu l_v \mathbf{I}_n \quad (57)$$

$$\mathbf{L}_v = (l_v + \mu) \mathbf{I}_n \quad (58)$$

where μ , k_v , γ and l_v are positive constants and $k_v > \mu$. Substituting (53) into (26), and using definition (55) yields

$$\ddot{\mathbf{e}}_o + \mathbf{K}_v \dot{\hat{\mathbf{e}}}_o + \mu \mathbf{K}_v \hat{\mathbf{e}}_o = \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t) \quad (59)$$

Define

$$\mathbf{s}_1 = \dot{\mathbf{e}}_o + \mu \mathbf{e}_o = \dot{\mathbf{x}}_o - \dot{\mathbf{x}}_{or} \quad (60)$$

$$\mathbf{s}_2 = \ddot{\mathbf{x}}_o + \mu \tilde{\mathbf{x}}_o = \ddot{\mathbf{x}}_o - \ddot{\mathbf{x}}_{o\ell} \quad (61)$$

where

$$\tilde{\mathbf{x}}_o = \mathbf{e}_o - \hat{\mathbf{e}}_o = \mathbf{x}_o - \hat{\mathbf{x}}_o \quad (62)$$

denotes the task-space position estimation error. Then, the closed-loop error

dynamics are given by

$$\ddot{\mathbf{e}}_o = -\mathbf{K}_v \mathbf{s}_1 + \mathbf{K}_v \mathbf{s}_2 + \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t) \quad (63)$$

$$\dot{\mathbf{s}}_2 + (l_v \mathbf{I}_n - \mathbf{K}_v) \mathbf{s}_2 = -\mathbf{K}_v \mathbf{s}_1 + \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t) \quad (64)$$

where we have utilized (61) and the following equality

$$\ddot{\mathbf{x}}_{o\ell}(t) - \ddot{\mathbf{x}}_{od}(t) = l_v \mathbf{s}_2(t) \quad (65)$$

Utilizing the procedure as same as those done in previous section, we present $\hat{\boldsymbol{\sigma}}(t)$ and $\boldsymbol{\sigma}(t)$ by (32) and (31), respectively, with the same parameters definitions and dimensions. Substituting (31) and (32) into (63) and (64) yields the output tracking loop dynamics as follows

$$\ddot{\mathbf{e}}_o = -\mathbf{K}_v \mathbf{s}_1 + \mathbf{K}_v \mathbf{s}_2 + \tilde{\mathbf{W}}_\sigma \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma \quad (66)$$

$$\dot{\mathbf{s}}_2 + (l_v \mathbf{I}_n - \mathbf{K}_v) \mathbf{s}_2 = -\mathbf{K}_v \mathbf{s}_1 + \tilde{\mathbf{W}}_\sigma \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma \quad (67)$$

where $\tilde{\mathbf{W}}_\sigma = \mathbf{W}_\sigma - \hat{\mathbf{W}}_\sigma$ is the parametric error related to the q -analogue of Bernstein-Schurer-Stancu operator's coefficients.

4.1. Stability Analysis

To investigate the system stability, a Lyapunov candidate function is introduced as follows

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) = \frac{1}{2} \mathbf{y}^T \mathbf{P} \mathbf{y} + \frac{1}{2} \text{Tr}(\tilde{\mathbf{W}}_\sigma^T \boldsymbol{\Gamma}_\sigma \tilde{\mathbf{W}}_\sigma) \quad (68)$$

where

$$\mathbf{y} = \begin{bmatrix} \dot{\mathbf{e}}_o^T & (\mu \mathbf{e}_o)^T & \dot{\mathbf{x}}_o^T & (\mu \tilde{\mathbf{x}}_o)^T \end{bmatrix}^T \in \Re^{4n} \quad (69)$$

$$\mathbf{P} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{I}_n & 2\mu^{-1}k_v\mathbf{I}_n \end{bmatrix} & \mathbf{0}_{2n} \\ \mathbf{0}_{2n} & \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{I}_n & 2\mu^{-1}k_v\mathbf{I}_n + \mathbf{I}_n \end{bmatrix} \end{bmatrix} \in \Re^{4n \times 4n} \quad (70)$$

with $\mathbf{0}_{2n}$ denoting the null matrix of dimension $2n$ and $\Gamma_\sigma \in \Re^{n\beta_1 \times n\beta_1}$ is a positive-definite matrix.

Lemma 1: The upper and lower bounds of (68) are as follows,

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq \frac{1}{2} \bar{\lambda}(\mathbf{P}) \|\mathbf{y}\|^2 + \frac{1}{2} \bar{\lambda}(\Gamma_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \quad (71)$$

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \geq \frac{1}{2} \underline{\lambda}(\mathbf{P}) \|\mathbf{y}\|^2 + \frac{1}{2} \underline{\lambda}(\Gamma_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \quad (72)$$

with $\bar{\lambda}(\mathbf{P})$ and $\underline{\lambda}(\mathbf{P})$ defined as

$$\bar{\lambda}(\mathbf{P}) = 6\mu^{-1}k_v, \quad \underline{\lambda}(\mathbf{P}) = \frac{1}{3} \quad (73)$$

Proof: Introducing

$$\mathbf{z}^T = \begin{bmatrix} \mathbf{s}_1^T & (\mu \mathbf{e}_o)^T & \mathbf{s}_2^T & (\mu \tilde{\mathbf{x}}_o)^T \end{bmatrix} \quad (74)$$

Function (68) could be rewritten as

$$V(\mathbf{z}, \tilde{\mathbf{W}}_\sigma) = \frac{1}{2} \mathbf{z}^T \Lambda \mathbf{z} + \frac{1}{2} Tr(\tilde{\mathbf{W}}_\sigma^T \Gamma_\sigma \tilde{\mathbf{W}}_\sigma) \quad (75)$$

where

$$\Lambda = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & 2\mu^{-1}k_v\mathbf{I}_n - \mathbf{I}_n \end{bmatrix} & \mathbf{0}_{2n} \\ \mathbf{0}_{2n} & \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & 2\mu^{-1}k_v\mathbf{I}_n \end{bmatrix} \end{bmatrix} \in \Re^{4n \times 4n} \quad (76)$$

It is not difficult to demonstrate that

$$\begin{aligned} \frac{1}{2}(\underline{\lambda}(\Lambda)\|\mathbf{z}\|^2 + \underline{\lambda}(\Gamma_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma)) \leq \\ V(\mathbf{z}, \tilde{\mathbf{W}}_\sigma) \leq \frac{1}{2}(\bar{\lambda}(\Lambda)\|\mathbf{z}\|^2 + \bar{\lambda}(\Gamma_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma)) \end{aligned} \quad (77)$$

where

$$\underline{\lambda}(\Lambda) = 1, \quad \bar{\lambda}(\Lambda) = 2\mu^{-1}k_v \quad (78)$$

By definition

$$\mathbf{z} = \mathbf{T}\mathbf{y} \quad (79)$$

using (69) and

$$\mathbf{T} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{0}_n & \mathbf{I}_n \end{bmatrix} & \mathbf{0}_{2n} \\ \mathbf{0}_{2n} & \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{0}_n & \mathbf{I}_n \end{bmatrix} \end{bmatrix} \in \Re^{4n \times 4n} \quad (80)$$

It can be easily shown that

$$\frac{1}{3}\|\mathbf{y}\|^2 \leq \|\mathbf{z}\|^2 \leq 3\|\mathbf{y}\|^2 \quad (81)$$

Together with (77) and (78) it implies (71)-(73)■

Differentiating (68) with respect to time; using (56), (66), (67), rearranging with some mathematical simplification, we have

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) = \underbrace{-\mathbf{s}_1^T k_v \mathbf{I}_n \mathbf{s}_1 + \dot{\mathbf{e}}_o^T \mu \mathbf{I}_n \dot{\mathbf{e}}_o - \mathbf{s}_2^T k_v \mathbf{I}_n \mathbf{s}_2 + 2\mathbf{e}_o^T \mu k_v \mathbf{I}_n \dot{\mathbf{e}}_o + 2\tilde{\mathbf{x}}_o^T \mu k_v \mathbf{I}_n \dot{\tilde{\mathbf{x}}}_o}_{\wp} \\ - \mathbf{s}_1^T \gamma \mathbf{I}_n \mathbf{s}_1 - \mathbf{s}_2^T \gamma \mathbf{I}_n \mathbf{s}_2 - \mathbf{s}_2^T (l_d \mathbf{I}_n - 2\mathbf{K}_v) \mathbf{s}_2 + (\mathbf{s}_1^T + \mathbf{s}_2^T) \boldsymbol{\varepsilon}_\sigma \\ + (\mathbf{s}_1^T + \mathbf{s}_2^T) \tilde{\mathbf{W}}_\sigma^T \mathbf{Z}_\sigma - Tr(\tilde{\mathbf{W}}_\sigma^T \Gamma_\sigma \dot{\tilde{\mathbf{W}}}_\sigma) \end{aligned} \quad (82)$$

It can be easily shown that the five first term of Eq. (82) can be simplified as

$$\wp = -\dot{\mathbf{e}}_o^T k_v \mathbf{I}_n \dot{\mathbf{e}}_o - \mathbf{e}_o^T \mu k_v \mathbf{I}_n \mu \dot{\mathbf{e}}_o + \dot{\mathbf{e}}_o^T \mu \mathbf{I}_n \dot{\mathbf{e}}_o - \dot{\tilde{\mathbf{x}}}_o^T k_v \mathbf{I}_n \dot{\tilde{\mathbf{x}}}_o - \tilde{\mathbf{x}}_o^T \mu k_v \mathbf{I}_n \mu \dot{\tilde{\mathbf{x}}}_o. \quad \text{Thus, Eq.}$$

(82) is written as

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) = & -\mathbf{y}^T \mathbf{Q}_o \mathbf{y} - \mathbf{s}_1^T \gamma \mathbf{I}_n \mathbf{s}_1 - \mathbf{s}_2^T \gamma \mathbf{I}_n \mathbf{s}_2 - \mathbf{s}_2^T (l_v \mathbf{I}_n - 2\mathbf{K}_v) \mathbf{s}_2 \\ & + (\mathbf{s}_1^T + \mathbf{s}_2^T) \boldsymbol{\varepsilon}_\sigma - Tr(\tilde{\mathbf{W}}_\sigma^T (\Gamma_\sigma \dot{\tilde{\mathbf{W}}}_\sigma - \mathbf{Z}_\sigma (\mathbf{s}_1^T + \mathbf{s}_2^T))) \end{aligned} \quad (83)$$

where

$$\mathbf{Q}_o = \text{blockdiag}[(k_v - \mu)\mathbf{I}_n, k_v \mathbf{I}_n, k_v \mathbf{I}_n, k_v \mathbf{I}_n] \in \Re^{4n \times 4n} \quad (84)$$

By selecting the update laws as

$$\begin{aligned} \dot{\hat{\mathbf{W}}}_\sigma &= \Gamma_\sigma^{-1} \mathbf{Z}_\sigma(t) \aleph^T(t) + \Delta(t) \\ \dot{\Delta}(t) &= -\Gamma_\sigma^{-1} \dot{\mathbf{Z}}_\sigma(t) \aleph^T(t) + \mu \Gamma_\sigma^{-1} \mathbf{Z}_\sigma(t) \aleph^T(t) - \bar{\kappa}_\sigma \Gamma_\sigma^{-1} \hat{\mathbf{W}}_\sigma(t) \end{aligned} \quad (85)$$

in which upper dot represents derivative with respect to time,

$$\aleph(t) = \mathbf{e}_o(t) + \tilde{\mathbf{x}}_o(t) \quad (86)$$

and $\bar{\kappa}_\sigma$ is positive number; (83) is simplified as

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq & -\underline{\lambda}(\mathbf{Q}_o) \|\mathbf{y}\|^2 - \gamma \|\mathbf{s}_1\|^2 - \gamma \|\mathbf{s}_2\|^2 + \|\mathbf{s}_1\| \|\boldsymbol{\varepsilon}_\sigma\| + \|\mathbf{s}_2\| \|\boldsymbol{\varepsilon}_\sigma\| \\ & + \bar{\kappa}_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \end{aligned} \quad (87)$$

where the inequality $l_v \mathbf{I}_n - 2\mathbf{K}_v > 0$ has been used. Using the inequality

$\|\mathbf{s}_1\|, \|\mathbf{s}_2\| \leq \|\mathbf{z}\| \leq \sqrt{3} \|\mathbf{y}\|$, (87) is given by

$$\dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq -\underline{\lambda}(\mathbf{Q}_o) \|\mathbf{y}\|^2 + 2\sqrt{3} \|\mathbf{y}\| \|\boldsymbol{\varepsilon}_\sigma\| + \bar{\kappa}_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \quad (88)$$

Result 3: If enough basis functions are utilized in the regressor vector such that the estimation error is very small. In other words, the term σ -modification can be deleted in (85). Therefore,

$$\dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq -\underline{\lambda}(\mathbf{Q}_o) \|\mathbf{y}\|^2 \quad (89)$$

Thus, $\mathbf{y}$ asymptotically converges to zero.

Result 4: Assume that $\boldsymbol{\varepsilon}_\sigma$ in (88) is large. Then, we can write:

$$-\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 + 2\sqrt{3}\|\mathbf{y}\|\|\boldsymbol{\varepsilon}_\sigma\| \leq -0.5\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 + \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 \quad (90)$$

Regarding this equation and using (42); (88) is rewritten as

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) &\leq -0.5\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 - 0.5\bar{\kappa}_\sigma \text{Tr}(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) + \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 \\ &\quad + 0.5\bar{\kappa}_\sigma \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \end{aligned} \quad (91)$$

Considering (71), one can easily relate (91) to V as follows

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) &\leq -\omega V + 0.5(\omega\bar{\lambda}(\mathbf{P}) - \underline{\lambda}(\mathbf{Q}_o))\|\mathbf{y}\|^2 + \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 \\ &\quad + 0.5(\omega\bar{\lambda}(\Gamma_\sigma) - \bar{\kappa}_\sigma)\text{Tr}(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) + 0.5\bar{\kappa}_\sigma \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \end{aligned} \quad (92)$$

Now, by choosing ω as

$$\omega \leq \min \left\{ \frac{\underline{\lambda}(\mathbf{Q}_o)}{\bar{\lambda}(\mathbf{P})}, \frac{\bar{\kappa}_\sigma}{\bar{\lambda}(\Gamma_\sigma)} \right\} \quad (93)$$

Equation (92) is simplified to

$$\dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq -\omega V + \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 + 0.5\bar{\kappa}_\sigma \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \quad (94)$$

Thus, if $(\mathbf{y}, \tilde{\mathbf{W}}_\sigma)$ belongs to the following set, $\dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma)$ is negative-definite.

$$(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \in \Omega \equiv \left\{ (\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \left| V > \frac{6}{\omega\underline{\lambda}(\mathbf{Q}_o)} \sup_{t_0 \leq \tau} \|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2 + \frac{\bar{\kappa}_\sigma}{2\omega} \text{Tr}(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \right. \right\} \quad (95)$$

Therefore, the uniform ultimate boundedness of $\mathbf{y}$ and $\tilde{\mathbf{W}}_\sigma$ is proven. Moreover, according to (95), by choosing proper values for ω , $\bar{\kappa}_\sigma$, and $\mathbf{Q}_o$, the observer error can become smaller. It should be considered that the internal force boundedness errors can be checked out just like before.

4.2 Performance Evaluation

To show the satisfactory performance of the proposed controller-observer

structure in the transient state due to the tracking errors originated from the initial conditions, (94) is solved to obtain:

$$V(t) \leq V(t_0)e^{-\omega(t-t_0)} + \frac{6}{\omega \underline{\lambda}(\mathbf{Q}_o)} \sup_{t_0 \leq \tau} \|\mathbf{\varepsilon}_\sigma(\tau)\|^2 + \frac{\bar{K}_\sigma}{2\omega} Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \quad (96)$$

Considering (72), which presents the lower bound of (68), one can conclude that,

$$\|\mathbf{y}\| \leq \sqrt{\frac{2V}{\underline{\lambda}(\mathbf{P})}} \quad (97)$$

That is simplified to (98) by employing (96)

$$\begin{aligned} \|\mathbf{y}\| \leq & \sqrt{\frac{2V(t_0)}{\underline{\lambda}(\mathbf{P})}} e^{-0.5\omega(t-t_0)} + \sqrt{\frac{12}{\omega \underline{\lambda}(\mathbf{P}) \underline{\lambda}(\mathbf{Q}_o)}} \sup_{t_0 \leq \tau} \|\mathbf{\varepsilon}_\sigma(\tau)\| \\ & + \sqrt{\frac{\bar{K}_\sigma}{\omega \underline{\lambda}(\mathbf{P})}} Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \end{aligned} \quad (98)$$

It is concluded from (98) that the amplitude of observer and controller errors is bounded and its bound is the summation of an exponential functions plus a constant. This completes the transient performance analysis. ■

5. Numerical Evaluation of Designed Controller

To test the proposed controller, consider Fig. 2. More details about Coriolis and centripetal matrix, Jacobian, the inertia matrix and the gravity terms is given in [48]. Assume that the reference frame is placed exactly in the point that its distance is the same from the of two manipulators. Suppose that $l_{1,1} = l_{2,1} = 2.05m$, $l_{1,2} = l_{2,2} = 2.05m$, and $l_{1,3} = l_{2,3} = 0.5m$ showing each link length. $m_{1,1} = m_{2,1} = 1kg$, $m_{1,2} = m_{2,2} = 1kg$, and $m_{1,3} = m_{2,3} = 0.3kg$ denote the link masses. $I_{1,1} = I_{2,1} = 0.7kgm^2$, $I_{1,2} = I_{2,2} = 0.5kgm^2$, and $I_{1,3} = I_{2,3} = 0.3kgm^2$ are the links moments of inertia. About motors, assume that $\mathbf{R}_1 = \mathbf{R}_2 = 2.1\mathbf{I}_3$,

$\mathbf{J}_{a1} = \mathbf{J}_{a2} = 2 \times 10^{-4} \mathbf{I}_3$, $\mathbf{r}_1 = \mathbf{r}_2 = 10^{-2} \mathbf{I}_3$, $\mathbf{k}_{a1} = \mathbf{k}_{a1} =$
 $\mathbf{k}_{b2} = \mathbf{k}_{b2} = \text{diag}(0.189, 0.219, 0.202)$ and
 $\mathbf{b}_{a1} = \mathbf{b}_{a2} = \text{diag}(1.48 \times 10^{-3}, 0.817 \times 10^{-3}, 1.38 \times 10^{-3})$. About the object, assume that

$$\mathbf{D}_o(\mathbf{x}_o) = \text{diag}(m_o, m_o, I_o), \quad \mathbf{G}_o(\mathbf{x}_o) = \begin{bmatrix} 0 \\ m_o g \\ 0 \end{bmatrix} \quad (99)$$

where I_o is the inertia of the object and m_o is its mass. The grasp matrices are:

$$\mathbf{J}_{o1}(\mathbf{x}_o) = \begin{bmatrix} 1 & 0 & l_{1l} \sin(x_{o3}) \\ 0 & 1 & -l_{1l} \cos(x_{o3}) \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{J}_{o2}(\mathbf{x}_o) = \begin{bmatrix} 1 & 0 & -l_{2l} \sin(x_{o3}) \\ 0 & 1 & l_{2l} \cos(x_{o3}) \\ 0 & 0 & 1 \end{bmatrix} \quad (100)$$

In (100), l_{il} is the distance between the centre of the mass of the object and the i th end-effector, which is half of the object length. In this simulation, the object mass m_o and the object inertia I_o are set to 0.2kg and 0.1kgm², respectively. The controller is model-free in the sense that $\mathbf{M}_c(\mathbf{x}_o)$, $\mathbf{C}_c(\mathbf{x}_o, \dot{\mathbf{x}}_o)$ and $\mathbf{G}_c(\mathbf{x}_o)$ are not available.

Fig 2. Two 3-DOFcooperative manipulators [49]

Consider

$$\boldsymbol{\tau}_{d,i}(t) = [0.4\cos(2t) \quad 0.1\sin(t) + \cos(2t) \quad 0.2\sin(t)]^T, \quad i=1,2 \quad (101)$$

as external disturbance which is imposed on both manipulators. To study the controller response, three different desired trajectories are defined in the following cases.

Case 1: Simple path and robustness analysis

Consider the following path:

$$\mathbf{x}_{od}(t) = \begin{bmatrix} 0.0625(\sin(2t) + \sin(8t)) \\ 0.0625\sin(8t + 1.6) + 2 + 0.0625\sin(2t + 1.6) \\ 0 \end{bmatrix} \quad (102)$$

The desired value of $\mathbf{F}_{ld}(t)$ is zero. The initial speeds of two 3-DOF manipulators are zero for both robots and initial values of $\mathbf{q}$ is $\mathbf{q}(0) = [2.5968 \quad -2.0519 \quad -0.5448 \quad 0.5448 \quad 2.0519 \quad 0.5448]^T$. The numerical values of the controller-observer structure are $l_v = 45, \gamma = 5, k_v = 15, \mu = 10$ and $\mathbf{K}_I = 100\mathbf{I}_6$. To estimate the function $\boldsymbol{\sigma}(t)$, the first 11 terms of the q -analogue of Bernstein-Schurer-Stancu operator are chosen. So, $\hat{\mathbf{W}}_\sigma \in \mathbb{R}^{33 \times 3}$. The initial values of the estimated parameters are set to zero. The matrix of learning rates is selected as $\boldsymbol{\Gamma}_\sigma = 10^{-5} \times \mathbf{I}_{33}$, where $\mathbf{I}_{33}$ is the 33×33 identity matrix. In first case, suppose that the estimation error is negligible. As a result, we can assume that $\bar{\kappa}_\sigma = 0$. For non-observer-based control strategy, all simulation settings are the same as observer-based one, except that velocity signal is accessible.

Under these settings, the desired path and controller response are plotted in Fig. 3.

As it can be seen in this figure, the controller is successful in tracking the reference trajectory in the presence of external disturbances and uncertainties. The tracking errors along the x and y axes and orientation are presented in Fig. 4. As it can be seen in this figure, these signals are acceptable. The angular positions for the first and second arms are presented in Fig. 5 and Fig. 6, respectively. Motor voltages as control efforts are given in Fig. 7 and Fig. 8. As shown in these figures, motor voltages are reasonable. For $\mathbf{F}_l$, the tracking performances are illustrated in Fig. 9 and Fig. 10, respectively. The observer errors in estimating position and velocity signals of the centre of the mass of the object are seen in Fig. 11 and Fig. 12, respectively. In Fig.13, the estimated lumped uncertainties have been plotted. Based on this figure, both strategies (observer-based controller and full-state feedback controller) are capable of uncertainty estimation with high accuracy. To emphasize the effect of $\hat{\mathbf{\sigma}}(t)$ (estimation of uncertainties) in (53), the outputs are also presented in the case that $\hat{\mathbf{\sigma}}(t)$ is excluded from the controller. This case is certainly the best way to test the uniformly ultimately bounded stability, since the reconstruction error $\mathbf{\epsilon}_\sigma$ is not small. In other words, the reconstruction error $\mathbf{\epsilon}_\sigma$, takes its maximum value in this situation. Fig. 14 shows the performance of trajectory tracking in this scenario.

Fig 3. The object trajectory tracking in the XY plane

Fig 4. Position/orientation tracking errors

Fig 5. The angles of the first manipulator joints

Fig 6. The angles of the second manipulator joints

Fig 7. Control signals of the first manipulator

Fig 8. Control signals of the second manipulator

Fig 9. The internal force for the first manipulator

Fig 10. The internal force for the second manipulator

Figure11. The position/orientation estimation error for the observer-based approach

Figure 12. The velocity estimation error for the observer-based approach

Figure 13. The estimation performance of $\sigma(t)$

Fig 14. Trajectory tracking in the absence of $\hat{\sigma}(t)$ in (53)

Case 2: Tracking a Complex Trajectory

Now consider the following path which is more complicated in comparison with **Case 1**.

$$\mathbf{x}_{od}(t) = \begin{bmatrix} \frac{0.5 \cos(t+4)}{1 + \sin(t)^2} & 1.65 + \frac{0.5 \sin(t) \cos(t)}{1 + \sin(t+4) \sin(t)} & 0 \end{bmatrix}^T \quad (103)$$

In this simulation, assume that we have $\mathbf{q}(0) = [-0.2272 \quad -2.2961 \quad 2.5233 \quad -2.5233 \quad 2.2961 \quad 3.3688]^T$. Other

controller parameters are the same as given in **Case 1**. Tracking performance of the object in the XY plane is given Fig. 15. The tracking errors along the x and y-coordinate and the orientation error are illustrated in Fig. 16. The angular positions for both arms are given in Fig. 17 and Fig. 18. Motor voltages for this case are presented in Fig. 19 and Fig. 20. As it can be seen in these figures, these signals are smooth and bounded. For the internal forces, the tracking performances are presented in Fig. 21 and Fig. 22. Based on the obtained results, both controllers (observer-based and full-state controllers) have provided good performances for the dual-arm robotic system. The observer error in calculation of the position and velocity of the COM of the object are shown in Fig. 23 and Fig. 24, respectively. The estimated uncertainties are given in Fig. 25. According to this figure, both controllers (observer-based and full-state controllers) are successful in uncertainty estimation. In order to highlight the role of uncertainty estimation (the term $\hat{\mathbf{g}}(t)$) in (53), $\hat{\mathbf{g}}(t)$ has been excluded from the controller and the results are compared in Fig. 26. As it can be seen in this figure, satisfactory performance of the controller is completely dependent to high accurate uncertainty estimation. In other words, the influence feedback and its gains in not

sufficient for acceptable performance of the controller.

Fig 15. The object trajectory tracking in the XY plane

Fig 16. Position/orientation tracking errors

Fig 17. The angles of the first manipulator joints regarding the complex trajectory

Fig 18. The angles of the second manipulator joints regarding the complex trajectory

Fig 19. Control signals of the first manipulator

Fig 20. Control signals of the second manipulator

Fig 21. The internal force for the first manipulator

Fig 22. The internal force for the second manipulator

Figure23. The position/orientation estimation error for the observer-based approach

Figure 24. The velocity estimation error for the observer-based approach

Figure 25. The estimation performance of $\sigma(t)$

Fig 26. Trajectory tracking in the absence of $\hat{\sigma}(t)$ in (53)

Case 3: Tracking a More Complicated Trajectory

Consider the following desired trajectory

$$\mathbf{x}_{od}(t) = \begin{bmatrix} 0.125\sin(11t) + 0.125\sin(2t) \\ 0.125\sin(11t + 1.6) + 2 + 0.125\sin(2t + 1.6) \\ 0 \end{bmatrix} \quad (104)$$

In this simulation, assume that we have $\mathbf{q}(0) = [2.5607 \quad -1.9798 \quad -0.5809 \quad 0.5809 \quad 1.9798 \quad 0.5809]^T$. In this trajectory fast and complicated reactions between various dynamics of the system inevitable. All the controller parameters are given in Case 1. The tracking performance of the object in the XY plane is given in Fig. 27. The tracking errors along the x and y-coordinate and the orientation error are illustrated in Fig. 28. By proper selection of control parameters these errors are reducible. The angular positions for both arms are given in Fig. 29 and Fig. 30. Motor voltages for this case are presented in Fig. 31 and Fig. 32. As it can be seen in these figures, these signals are smooth and bounded. For the internal forces, the tracking performances are presented in Fig. 33 and Fig. 34. Based on the obtained results, both controllers (observer-based and full-state controllers) have provided good performances for the dual-arm robotic system. The observer errors in estimating position and velocity signals of the centre of the mass of the object are seen in Fig. 35 and Fig. 36, respectively. The estimated uncertainties are given in Fig. 37. According to this figure, both controllers (observer-based and full-state controllers) are successful in uncertainty estimation. In order to highlight the role of uncertainty estimation (the term $\hat{\mathbf{\sigma}}(t)$) in (53), $\hat{\mathbf{\sigma}}(t)$ has been excluded from the controller and the results are compared in Fig. 38. It can be concluded from this figure that the q -analogue of Bernstein-Schurer-Stancu operator has a

considerable effect in reducing the tracking error.

Fig 27. The object trajectory tracking in the XY plane

Fig 28. Position/orientation tracking errors

Fig 29. The angles of the first manipulator joints regarding more complicated trajectory

Fig 30. The angles of the second manipulator joints regarding more complicated trajectory

Fig 31. Control signals of the first manipulator

Fig 32. Control signals of the second manipulator

Fig 33. The internal force for the first manipulator

Fig 34. The internal force for the second manipulator

Figure 35. The position/orientation estimation error for the observer-based approach

Figure 36. The velocity estimation error for the observer-based approach

Figure 37. The estimation performance of $\sigma(t)$

Fig 38. Trajectory tracking in the absence of $\hat{\sigma}(t)$ in (53)

6. Conclusion

In this paper, an observer-controller approach has been introduced for cooperative robots using q-analogue of Bernstein-Schurer-Stancu operator as an alternative for fuzzy systems and neural networks. Both the controller and observer are model-free. Also, the actuator dynamics has been included in modelling and controller design. In order to highlight the satisfactory performance of the observer, a comparison with non-observer-based controller was presented. Stability analysis and performance evaluation for both cases were presented. Simulation results were illustrated using simple and complicated trajectories to show high capabilities of the proposed scheme in performing difficult tasks. Indeed, simulation results showed the considerable influence of this estimator in enhancing the controller performance.

Data Availability Statements

The data that support the findings of this study are available from the corresponding author upon reasonable request.

COMPLIANCE WITH ETHICAL STANDARDS

The authors declare that this paper has never been submitted to other journals for simultaneous review processes and it has not been published before (partially or completely). Moreover, this paper has not been divided into different sections in order to have more submissions. The data has not been fabricated or changed in favor of the conclusions. In addition, no theory, data, or text belonging to other authors and publications was included as if it was our own and proper acknowledgement to other works has been provided.

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Research involving Human Participants and/or Animals

The authors declare that no human participant or animal was included in this work.

The authors state that this submission does not require any informed consent due to the fact that the results were acquired by computer simulation.

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