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Numerical Approach towards Gyrotactic Microorganisms hybrid nanoliquid flow with the Hall current and Magnetic field over a Spinning disk

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Abstract:

The article explores the upshot of Hall current, thermal radiation and magnetic field on hybrid nanoliquid flow over the surface of a spinning disk. The motive of the present effort to upgrade the heat transmission rate for engineering and industrial purposes. The hybrid nanoliquids as comparative to the conventional fluids have higher thermal properties. A special class of nanoparticles known as carbon nanotubes (CNTs) and iron ferrite Fe_3O_4 are added to the base fluid. The system of modeled equations is depleted into dimensionless differential equations through similarity transformation. The transform equations are further solved through Parametric Continuation method (PCM). For parametric study the embedding flow factors on velocity, energy, mass transmission and motile microorganism's concentration profiles have been sketched. The obtained results are compared with the existing literature, which shows best settlement. It concluded that the heat transmission rate reduces for Hall current and rises with radiative factor. The results perceived that the addition on CNTs in carrier fluid is more efficacious than any other types of nanoparticles, due to its C-C bond. CNTs nanoliquid can be more functionalized for the desired achievement, which can be utilize for a variety of applications by functionalization of non-covalent and covalent modification.

Keywords: Motile microorganism, MHD, Hall current, thermal radiation, PCM, spinning disk.

1. Introduction

The heat and mass transmission with nanoliquid flow runs over revolving disk has wide range implementation in the heat exchanger and electronic devices¹. The applications of such type problems are in computer hardware for storage purpose, thermal energy generating system, electronic instruments, gas turbine, spinning devices, chemical processes, geothermal industry, various types of medical instruments etc. The suction influence act on the fluid flow over a revolving disk is determined by Stuart². Turkyilmazoglu³ scrutinized the impact of stretching surface of the disk. Recently Tabassum and Mustafa⁴ studied non-Newtonian Reiner-Rivlin fluid past over a revolving disk. The non-Newtonian fluid flow model over the surface of revolving disk, under the magnetic strength was scrutinized by Khan et al.⁵ In MHD (magneto hydrodynamics) flow, the entropy production due to porosity of spinning disk deliberated by Rashidi et al.⁶. Hayat et al.⁷ highlighted the heat transmission in Darcy-Forchemmier flows of Copper *Cu* and silver *Ag* nanoliquid between the gap of two spinning stretchable disks. Khan et al.⁸ examined the unsteady thermal conductivity in Maxwell liquid over a revolving disk. Shuaib et al.⁹ illustrate the frictional nature of viscous fluid flow over a flexible surface of a rotating disk with heat transport characteristics. The Cattaneo-Christov model specially designed for heat flux, and axisymmetric Maxwell fluid flow between two revolving coaxially disks is studied by Ahmed et al.¹⁰. The significant outputs revealed that the velocity of fluid is decreases with Deborah number, and with thermal relaxation time the fluid temperature decline.

Hannes Alfven¹¹ was pioneered the field of MHD, for this work he won Nobel Prize (1970) in the field of Physics. The basic principle of MHD is to regulate fluid flow. Some applications of MHD mostly used in malignant tumour, arthritis, blood pressure and brain therapy. Siddiqui et al.¹² investigated the MHD movement of liquid flow in a porous medium with application throughout the respiratory track to monitor diseases. The MHD rotating boundary layer flow, over shrinking permeable surface was solved by numerical procedure (Keller-box method) in Ref.¹³. Neeraja et al.¹⁴ addressed MHD Casson liquid flow with convective boundary conditions and viscous dissipation over a deformable channel. The steady three-dimensional MHD Casson nanoliquid flow between two spinning plates has been scrutinized by Refs.¹⁵. Maryam et al.¹⁶ highlighted the unsteady MHD flow over a rotating porous surface of hybrid liquid. Lokesh et al.¹⁷ illustrated numerically the chemical reaction of the Casson nanoliquid over an expanding surface with heat

and mass transport. An unsteady three-dimensional MHD flow of nanoliquid is investigated by Rauf et al.¹⁸ as a result of rotation of infinite disc with periodic oscillation dependent on time. A numerical evaluation of the MHD Casson liquid over a deformable substrate with slip conditions is studied by Murthy¹⁹. Oyelakin et al.²⁰ revealed the upshot of the velocity slip in a tangent hyperbolic nanoliquid on the flow and heat transfer features. The key focus of the work in Ref.²¹ is to investigate the flow and heat transmission characteristics of copper and aluminum oxide hybrid nanoliquid over a radially shrinking surface with the MHD and Joule heating effect. Tlili et al.²² scrutinized an MHD flow of hybrid nanoliquid through a non-uniform stretched thick plane with slip effects.

The heat transfer in carbon-nanofluids has got an extensive attention among the researchers in different sectors of technologies. CNTs are the simple chemical structure along with the composition of carbon atoms, rolled in cylindrical form. CNTs have extraordinary thermophysical, chemical, electrical and mechanical features, can be utilized easily as a nanoparticle in base fluid. They have unique advantages on account of little size tube structure, such as large surface area, tube shape, configuration, chemical stability, hardness and their smallest dimension over other nanoparticles. CNTs depend on the number of graphene layers, which subdivided it into single walled & multi walled carbon nanotubes, abbreviated as SWCNTs and MWCNTs respectively. Khan et al.²³ has highlighted the physical aspects of entropy optimization within a rotating frame of carbon nanotubes (CNTs) in convective MHD effective flow. Anuar et al.²⁴ has evaluated the upshot of MHD on steady, two-dimensional induced flow of carbon nanotubes via the nonlinear surface. The main intention of Ref.²⁵ is to examined the electrical MHD spinning flow of single and multi-walled nanotubes based on several types of carrier fluid. Mahanthesh²⁶ studied the carbon nanoliquids flow passing via a revolving disk. The consequences of MHD on stagnation point flow having carbon nanotubes along stretch / shrink layer are studied by Anuar et al²⁷. The Entropy production of nanoliquid between the gap of two spinning discs under the magnetic and thermal radiation effect is investigated by Ref²⁸. Ahmed et al.²⁹ presents a novel model of the unstable MHD heat transmission flow over a shrinking surface in carbon nanotubes with variable viscosity. The MHD radiative and incompressible steady flow of Carreau nanofluid explored with carbon nanotubes is investigated by Nagalakshm & Vijaya³⁰. Tulu and Ibrahim³¹ scrutinized the carbon nanoliquid flow with the result of Cattaneo – Christov heat flux model due to stretchable rotating disks. Both SWCNTs and MWCNTs are known as the base fluid of ethylene glycol. The physical

prospects of hybrid nanoliquid under the impact of thermal radiation and slip was presented by Ghadikolaei and Gholinia³².

The motive of current work is to explore the upshot of Hall current, carbon nanotubes and iron ferrite nanoliquid flow over a spinning disk under the effect of thermal radiation and magnetic field. The second priority is to extend the idea of Ref.³³ and enhance the thermo physical properties of carrier fluids. As compared to conventional fluids the study of hybrid nanoliquids provides extraordinary enhancement in heat and mass transmission and thermal conductivity. Therefore, we are taking hybrid nanoliquid model, which are assemble of CNTs and Fe_3O_4 with base fluid water. The system of modeled equations is renovated into dimensionless differential equations through Karman's approach. Which are further tacked through Parametric Continuation method.

Fig.1: Spinning disk geometry.

2. Mathematical Formulation

This segment highlights the physical background and mathematical terminology of the present hybrid nanoliquid problem.

A. Physical description

In this study, we presume the steady hybrid nanoliquid flow passes over a spinning disk. The insulated spinning disk is placed at $z=0$, moving along z -axis. Magnetic field B_0 is uniformly

applied perpendicular to the disk surface. The disk is revolving with Ω (angular velocity). T and p correspond to temperature and pressure of nanofluid. Hall current has been signified by $m = \tau_e \omega_e$, here ω_e specify frequency of electron and τ_e specify collusion of electron. Where T_w and T_∞ are assumed as the temperature of the disk surface and away from the surface respectively. Figure 1 displays the geometry of the flow and cylindrical coordinate (r, ϕ, z) system.

B. Equation of Motion

On the basis of above assumption, the flow equations can be defined as³³⁻³⁵:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho_{hmf} \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right) + \frac{\partial p}{\partial r} = \mu_{hmf} \left(\frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma_{hmf} B_0^2}{(1+m^2)} (u - mv), \quad (2)$$

$$\rho_{hmf} \left(u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} - \frac{uv}{r} \right) = \mu_{hmf} \left(\frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma_{hmf} B_0^2}{(1+m^2)} (u - mv), \quad (3)$$

$$\rho_{hmf} \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) + \frac{\partial p}{\partial z} = \mu_{hmf} \left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right), \quad (4)$$

$$(\rho C p)_{hmf} \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = k_{hmf} \left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \frac{\partial q_r}{\partial z}, \quad (5)$$

$$\left(u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} \right) = D_{hmf} \frac{\partial^2 C}{\partial z^2}, \quad (6)$$

$$\left(w \frac{\partial \tilde{N}}{\partial z} + \tilde{w} \frac{\partial \tilde{N}}{\partial z} + \tilde{N} \frac{\partial \tilde{w}}{\partial z} \right) = D_n \frac{\partial^2 \tilde{N}}{\partial z^2}. \quad (7)$$

Here (u, v, w) demonstrate the velocity component and μ_{hmf} , α_{hmf} , σ_{hmf} and ρ_{hmf} demonstrates the dynamic, kinematic, thermal diffusivity, electrical conductivity and density of hybrid nanofluid. $\tilde{N}$ is the Density motile of microorganisms, D_n is the microorganism diffusion and Wc shows the

swimming speed of maximum cell respectively. The thermal conductivity and volumetric heat capacity are represented through k_{hnf} and $(\rho C_p)_{hnf}$ of the hybrid nanofluid respectively. Where qr is the radiative heat flux and can be simply express as³⁶.

$$- \quad qr = -\frac{4\sigma^*}{3k^*} \cdot \frac{\partial T^4}{\partial z}, \quad (8)$$

Here, σ^* and k^* is the Stefan Boltzmann and mean absorption coefficient respectively.

C. Boundary conditions

The boundary conditions are:

$$\begin{aligned} u=0, v=\Omega r, T=T_w, \tilde{N}=\tilde{N}_w, C=C_w, \quad \text{at } z=0, \\ u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, P=P_\infty, \tilde{N} \rightarrow \tilde{N}_\infty, C \rightarrow C_\infty \quad \text{at } z \rightarrow \infty. \end{aligned} \quad (9)$$

D. Similarity conversion

In order to lessned the system of PDEs, we defined the following variables as³⁷

$$\begin{aligned} v=r\Omega g, \quad u=r\Omega f', \quad w=-\sqrt{r\Omega v_f} f, \quad \Theta(\eta)=\frac{T-T_\infty}{T-T_w}, \\ P=P_\infty+2\Omega v_f P(\eta), \quad \eta=\sqrt{\frac{2\Omega}{v_f}} z, \quad \Phi=\frac{C-C_\infty}{C_w-C_\infty}, \quad h=\frac{n-n_\infty}{n_w-n_\infty}. \end{aligned} \quad (10)$$

Now, by using equation (8) in equations (1)-(7), we receive

$$2f''' + \frac{\mathbb{C}_1}{\mathbb{C}_4} (2ff'' - f'^2 + g^2) - \frac{\mathbb{C}_5}{\mathbb{C}_4} \frac{M}{(1+m^2)} (f' - mg) = 0, \quad (11)$$

$$2g'' + \frac{\mathbb{C}_1}{\mathbb{C}_4} (2fg' - 2f'g) - \frac{\mathbb{C}_5}{\mathbb{C}_4} \frac{M}{(1+m^2)} (g + mf') = 0, \quad (12)$$

$$\frac{1}{\text{Pr}} \Theta'' \left(1 + \frac{4N}{3} \right) + \frac{\mathbb{C}_2}{\mathbb{C}_3} f \Theta' = 0, \quad (13)$$

$$\Phi'' - Sc(f\Phi') = 0. \quad (14)$$

$$h'' + \text{Re}(2Scfh' + Pe(h'\Phi' - h\Phi'')) = 0. \quad (15)$$

Where

$$\mathbb{C}_1 = \frac{\rho_{hnf}}{\rho_f}, \mathbb{C}_2 = \frac{(\rho C_p)_{hnf}}{(\rho C_p)_f}, \mathbb{C}_3 = \frac{k_{hnf}}{k_f}, \mathbb{C}_4 = \frac{\mu_{hnf}}{\mu_f}, \mathbb{C}_5 = \frac{\sigma_{hnf}}{\sigma_f}.$$

Here, $\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3, \mathbb{C}_4, \mathbb{C}_5$ are the dimensionless constant. Similarly.

The boundary conditions also transform as:

$$\begin{aligned} f' = f = 0, \quad g = \Phi = \Theta = 1, \quad h = 1 \quad \text{at } \eta = 0, \\ f' \rightarrow 0, \Theta \rightarrow 0, \Phi \rightarrow 0, g \rightarrow 0, h = 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (16)$$

E. Thermo-physical properties

The thermal properties of hybrid nanofluid are expressed as³⁸:

$$\begin{aligned} \nu_{hnf} = \frac{\mu_{hnf}}{\rho_{hnf}}, \mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{5/2}(1-\phi_2)^{5/2}}, \frac{(\rho)_{hnf}}{(\rho)_f} = (1-\phi_2) \left\{ 1 - \left(1 - \frac{(\rho)Ms}{(\rho)_f} \right) \phi_1 \right\} + \frac{(\rho)_{CNT}}{(\rho)_f} \phi_2, \\ \frac{(\rho C_p)_{hnf}}{(\rho C_p)_f} = (1-\phi_2) \left\{ 1 - \left(1 - \frac{(\rho C_p)Ms}{(\rho C_p)_f} \right) \phi_1 \right\} + \frac{(\rho C_p)_{CNT}}{(\rho C_p)_f} \phi_2, \\ \frac{k_{hnf}}{k_{bf}} = \frac{1-\phi_2 + 2\phi_2 \frac{k_{CNT}}{(k_{CNT} - k_{bf})} - \ln \frac{k_{CNT} + k_{bf}}{2k_{bf}}}{1-\phi_2 + 2\phi_2 \frac{k_{bf}}{(k_{CNT} - k_{bf})} - \ln \frac{k_{CNT} + k_{bf}}{2k_{bf}}}, \quad \frac{k_{bf}}{k_f} = \frac{k_{MS} + (m-1)k_f - (m-1)\phi_1(k_f - k_{MS})}{k_{MS} + (m-1)k_f - \phi_1(k_f - k_{MS})}, \end{aligned} \quad (17)$$

Here, $(C_p)_{MS}$, ρ_{MS} and ρ_{CNT} specified specific heat capacities and densities of Fe_3O_4 and CNTs respectively.

Table. 1: The numerical properties of water, CNTs and Fe_3O_4 .

$\rho(\text{kg/m}^3)$	$C_p(\text{J/kgK})$	$k(\text{W/mK})$

Pure water	997.1	4179	0.613
SWCNTs	2600	425.0	6600
MWCNTs	1600	796.0	300.0
Fe_3O_4	5200	670.0	6.00

The constant terms of our calculation are radiation parameter, Prandtl number, Lewis number, Hartmann number, Schmidt number and Peclet number:

$$= N = \frac{4\sigma^* T_\infty^3}{\kappa^* \kappa_f}, \text{Pr} = \frac{\mu_f (\rho C_p)_f}{\rho_f \kappa_f}, Le = \frac{\nu_{hnf}}{D_B}, M^2 = \frac{\sigma_f B_0^2}{\rho_f \Omega}, Sc = \frac{\nu_f}{D_f}, Pe = \frac{bW_c}{D_n}. \quad (18)$$

The Skin friction, Sherwood numbers and Nusselt Number are rebound as³⁹:

$$C_f = \frac{\sqrt{\tau_{wr}^2 + \tau_{w\phi}^2}}{\rho_f (\Omega r)^2}, Sh = \frac{rj_w}{D_f (C_w - C_\infty)}, Nu = \frac{rq_w}{k_f (T_w - T_\infty)}. \quad (19)$$

τ_{wr} , $\tau_{w\phi}$, q_w and j_w is stand for radial stress, transverse shear stress, heat flux at and mass flux at the surface of the disk respectively.

$$\tau_{wr} = \left[\mu_{hnf} \left(\frac{du}{dz} + \frac{dw}{d\phi} \right) \right]_{z=0}, \tau_{w\phi} = \left[\mu_{hnf} \left(\frac{dv}{dz} + \frac{1}{r} \frac{dw}{d\phi} \right) \right]_{z=0}, q_w = -\frac{k_{hnf}}{k_f} \left(\frac{dT}{dz} \right)_{z=0}, j_w = -D_{hnf} \left(\frac{dC}{dz} \right)_{z=0}. \quad (20)$$

The drag force and heat transmission rate in dimensionless form is stated as:

$$\text{Re}^{\frac{1}{2}} C_f = \frac{\sqrt{(G'(0))^2 + (F'(0))^2}}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5}}, \text{Re}^{\frac{-1}{2}} Nu = -\frac{k_{hnf}}{k_f} \Theta'(0), \text{Re}^{\frac{-1}{2}} Sh = -\Phi'(0), \text{Re} = \frac{\Omega r^2}{\nu_f}. \quad (21)$$

3. Problem Solution:

For the results, the system of equations (10-13) are depleted to the first order by the following procedure, which are further solved through Parametric Continuation method (PCM):

$$\left. \begin{aligned} \eta &= \zeta_1, f = \zeta_2, f' = \zeta_3, f'' = \zeta_4, g = \zeta_5, g' = \zeta_6, \\ \Theta &= \zeta_7, \Theta' = \zeta_8, \Phi = \zeta_9, \Phi' = \zeta_{10}, h = \zeta_{11}, h' = \zeta_{12}, \end{aligned} \right\} \quad (22)$$

$$\left\{ \begin{array}{l} \zeta'_1 = 1, \zeta'_2 = \zeta_3, \zeta'_3 = \zeta_4, \\ \zeta'_4 = \frac{\mathbb{C}_5}{2\mathbb{C}_4} \frac{M}{(1+m^2)} (\zeta_3 - m\zeta_5) \frac{\mathbb{C}_1}{2\mathbb{C}_4} (2\zeta_2\zeta_4 - \zeta_3^2 + \zeta_5^2), \\ \zeta'_5 = \zeta_6, \zeta'_6 = \frac{\mathbb{C}_5}{2\mathbb{C}_4} \frac{M}{(1+m^2)} (\zeta_5 - m\zeta_3) \frac{\mathbb{C}_1}{\mathbb{C}_4} (\zeta_2\zeta_6 - \zeta_3\zeta_5), \\ \zeta'_7 = \zeta_8, \zeta'_8 = -\Pr \left(\frac{\mathbb{C}_3}{\mathbb{C}_4} \zeta_2\zeta_8 \right) / 1 + \frac{4N}{3}, \\ \zeta'_9 = \zeta_{10}, \zeta'_{10} = Sc(\zeta_2\zeta_{10}), \zeta'_{11} = \zeta_{12}, \\ \zeta'_{12} = -\text{Re} (2Sc\zeta_2\zeta_{12} + \text{Pe}(\zeta_{12}\zeta_{10} - \zeta_{12}\zeta'_{10})). \end{array} \right. \quad (23)$$

$$\left. \begin{array}{l} \zeta_2 = \zeta_4 = 0, \zeta_5 = 1, \zeta_7 = \zeta_9 = 1, \zeta_{10} = 1 \text{ at } \eta = 0, \\ \zeta_3 \rightarrow 0, \zeta_5 \rightarrow 0, \zeta_7 \rightarrow 0, \zeta_9 \rightarrow 0, \zeta_{10} \rightarrow 0 \text{ as } \eta \rightarrow \infty. \end{array} \right\} \quad (24)$$

4. Results and Discussion:

The discussion section scrutinizes the behaviour of velocity, temperature and motile microorganism's concentration distribution against the variation of several physical constraints for hybrid nanoliquid consist of CNTS and magnetic ferrite nanoparticles. Their out-turns are revealed through the comparative Figures (2-7) and Tables. The thermo physical characteristics are given in Table 1. To validate and ensure our results, we have plotted Table 2 for the numerical outcomes of $f''(0)$, $-g'(0)$, $-\Theta'(0)$ and $-\Phi'(0)$ to compare it with published work (Refs. 33, 38, 39), which show best agreement. The influence of magnetic parameter M , volume fraction ϕ_2 and rotation parameter Ω versus shear stresses are scrutinized in Table 3 & 4. It has been observed that with increment in magnetic parameter M , volume fraction ϕ_2 and rotation parameter Ω , the positive changes occurred in $f''(0)$ and $-g'(0)$, consequently the drag forces enhances. The effect of Ω , ϕ_1 , ϕ_2 , Pr and N against heat transfer rate is drawn via Table 5. The improving credit of volume friction parameters (ϕ_1, ϕ_2) decline the Nusselt number, which also result in enhances the fluid temperature of both Iron oxide and CNTs nanoliquid. The numerical outcomes for Sherwood number $-\Phi'(0)$ against volume friction parameters (ϕ_1, ϕ_2) and Schmidt number are discussed in Table 6.

The flow mechanism and coordinate geometry are exhibit in Figure 1. From Figure 2(a)-(c), we perceive that the increasing effects of Hall current parameter m enlarge the axial velocity $f'(\eta)$. Physically, $(\sigma_{mf}/1+m^2)$ electrical conductivity enhances with rising credit of m , which declines the damping impact of M on axial velocity $f'(\eta)$. Thus, slight away from the disk surface ($\eta \approx 5.0$), the both fluids achieved its peak velocity Figure 2(a). While an opposite trend has been observed in Figure 2(b)-(c). Because radial velocity $g(\eta)$ and temperature profiles $\Theta(\eta)$ reduces with the variation of Hall current parameter m . The axial velocity $f'(\eta)$ and radial velocity $g(\eta)$ profiles of hybrid nanoliquid decline against the growing effects of magnetic strength M , due to the resistive effect created by magnetic strength shown in Figure 3(a)-(b). While this opposing force also produce some heat energy, which enhances fluid temperature $\Theta(\eta)$ illustrated through Figure 3(c).

Figure 4(a)-(c) & 5(a)-(c) are sketched, for the purpose to display the upshot of volume fraction parameters (ϕ_1, ϕ_2) , where ϕ_1 express single and multi-wall carbon nanotubes quantities and ϕ_2 express iron ferrite Fe_3O_4 quantities, versus axial velocity $f'(\eta)$, radial and temperature profiles respectively. It can be concluded the rising credit of ϕ_1 and ϕ_2 reduces specific heat capacity of base fluid, while on other hand the variation of volume friction parameters increases the thermal expansion rate, as a result the velocity and temperature of fluid improves respectively.

Figure 6(a)-(b) revealed the influence of thermal radiation parameter N and Prandtl number Pr on temperature profile $\Theta(\eta)$ respectively. The fluid temperature enhances with the variation of thermal radiation, while declines with Prandtl effects. The heat energy radiated from the disk surface enhances the kinetic energy of fluid, which causes the rises in velocity as well as temperature of fluid. On the other hand, high Prandtl fluid has always greater specific heat capacity and kinematic viscosity, which effects fluid temperature to reduce. The influence of Lewis number Le and Peclet number Pe versus motile microorganism's concentration profile $h(\eta)$ has been shown through Figure 6(c)-(d). The rising credit of Lewis number reduces molecular diffusion rate, which causes increases in motile microorganism's concentration profile $h(\eta)$ Figure 6(c).

Similarly, microorganism diffusion also reduces versus growing values of Peclet number Figure 6(d).

From Figure 7, we perceive that mass transfer rate declines with positive increments of Schmidt number Sc . Physically, $Sc = \nu_f / D_f$ the rising trend of Schmidt number improves the kinematic viscosity of carrier fluid, which decrease the mass transfer rate.

Figure 2: Hall current parameter m effect on (a) axial velocity $f'(\eta)$ (b) radial velocity $g(\eta)$ (c) temperature $\Theta(\eta)$. When $\phi_1 = 0.01, \phi_2 = 0.2, Sc = 2.0, Pr = 6.2, M = 1.0, N = 0.4$.

Figure 3: Magnetic parameter M effect on (a) axial velocity $f'(\eta)$ (b) radial velocity $g(\eta)$ (c) temperature $\Theta(\eta)$. When $\phi_1 = 0.01, \phi_2 = 0.2, Sc = 2.0, Pr = 6.2, M = 1.0, N = 0.5$.

Figure 4: Volume friction parameter ϕ_1 (CNTs) effect on (a) axial velocity $f'(\eta)$ (b) radial velocity $g(\eta)$ (c) temperature $\Theta(\eta)$. When $\phi_1 = 0.01, \phi_2 = 0.2, Sc = 2.0, Pr = 6.2, M = 1.0, N = 0.5$.

Figure 5: Volume friction parameter $\phi_2(Fe_3O_4)$ effect on (a) axial velocity $f'(\eta)$ (b) radial velocity $g(\eta)$ (c) temperature $\Theta(\eta)$. When $\phi_1 = 0.01, \phi_2 = 0.2, Sc = 2.0, Pr = 6.2, M = 1.0, N = 0.5$.

Figure 6: (a) Radiation parameter N (b) Prandtl number Pr (c) Lewis number Le (d) Peclet number Pe effect on temperature $\Theta(\eta)$ and Motile microorganisms' concentration $h(\eta)$ profiles respectively. When $\phi_1 = 0.01, \phi_2 = 0.2, Sc = 2.0, Pr = 6.2, M = 1.0, N = 0.5$.

Figure 7: (a) Schmidt number Sc effects on concentration profile $\Phi(\eta)$ (b) volume friction parameter effects concentration profile $\Phi(\eta)$. When $\phi_1 = 0.01, Pr = 6.2, M=1.0, N=0.5$.

Table 2: The numerical of $f''(0)$, $-g'(0)$, $-\Theta'(0)$, and $-\Phi'(0)$, when $\phi_1 = \phi_2 = 0.5, Pr = 6.2, N=0.5$ and $M=2.0$.

	Yin et al. ³⁸	Acharya et al. ³³	Refs. ³⁹	Present work
$f''(0)$	0.51022941	0.5102295		0.5102297
$-g'(0)$	0.61591990	0.6159197		0.615199
$-\Theta'(0)$	0.93387285	0.9338728		0.9338731
$-\Phi'(0)$			-0.78555	-0.78555

Table 3: The numerical outcomes for skin friction $(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}f''(0)$ when $\phi_1 = \phi_2 = 0.5, Pr = 6.2, N=0.5$ and $M=2.0$.

$\frac{f''(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} \quad \frac{f''(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} \quad \frac{f''(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$					
Ω	M	ϕ_2	Fe_3O_4	$SWCNTs$	$MWCNTs$

2	2	0.2	0.473164	0.385087	0.371700
3			0.512794	0.477838	0.455406
4			0.548806	0.477798	0.496049
	3		0.563014	0.482898	0.496116
	4		0.564584	0.484453	0.497016
		0.3	0.578366	0.491306	0.497911
		0.4	0.591749	0.497773	0.498412

Table 4: The numerical outcomes for skin friction $\frac{g'(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$ when $\phi_1 = \phi_2 = 0.5, Pr = 6.2,$

$N=0.5$ and $M=2.0$.

$\frac{g'(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} \quad \frac{g'(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^2} \quad \frac{g'(0)}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$					
Ω	M	ϕ_2	Fe_3O_4	SWCNTs	MWCNTs
2	2	0.2	-0.63129	-0.48356	-0.43156
3			-0.68588	-0.58340	-0.58522
4			-1.71301	-0.64021	-1.64321
	3		-0.71862	-0.73153	-0.69853
	4		-0.76631	-0.81881	-0.76681
		0.3	-0.77488	-0.83194	-0.87494
		0.4	-1.78431	-0.88149	-0.88432

Table 5: The outcomes for Nusselt number $-\left(\frac{k_{mf}}{k_{nf}}\right)\Theta'(0)$ when $\phi_1 = \phi_2 = 0.5, Pr = 6.2, N=0.5$ and

$M=2.0$.

$-\left(\frac{k_{mf}}{k_{nf}}\right)\Theta'(0) \quad -\left(\frac{k_{mf}}{k_{nf}}\right)\Theta'(0) \quad -\left(\frac{k_{mf}}{k_{nf}}\right)\Theta'(0)$							
Ω	ϕ_1	ϕ_2	Pr	N	Fe_3O_4	$SWCNTs$	$MWCNTs$
2	0.02	0.2	6.2	0.2	-0.470880	-0.357508	-0.358063
3					-0.281709	-0.271003	-0.272758
4					-0.161703	-0.123278	-0.128023
	0.03				-0.139423	-0.017846	-0.020162
	0.04				-0.299640	-0.011775	-0.016102
		0.3			-0.140306	-0.325440	-0.305060
		0.4			-0.610370	-0.176231	-0.149620
			6.6		-1.802850	-0.508730	-2.036413
			7.0		-0.685082	-0.631178	-2.129232
				0.3	-0.232681	-0.216310	-1.109990
				0.4	-0.221650	-0.110080	-1.074766

Table 6: The numerical outcomes for Sherwood number $-\Phi'(0)$.

$-\Phi'(0) \quad -\Phi'(0) \quad -\Phi'(0)$					
ϕ_1	ϕ_2	Sc	Fe_3O_4	$SWCNTs$	$MWCNTs$
0.02	0.2	0.3	-0.615955	-0.621530	-0.621800
0.03			-0.643492	-0.649438	-0.649887
0.04			-1.673735	-0.674350	-0.674620
	0.3		-0.694710	-0.695040	-0.696122
	0.4		-0.712402	-0.717622	-0.718961
		0.4	-0.728808	-0.725706	-0.7375141
		0.5	-1.784112	-0.731965	-0.770660

5. Conclusion

The intention of present work to investigate the upshot of Hall current on *CNTs* and iron ferrite hybrid nanoliquid flow over a spinning disk under the influence of thermal radiation and magnetic field. To improve the heat transmission rate for engineering and industrial purposes, is the motive of the present work. Therefore, the present problem is modelled in form of PDEs, which are further depleted through similarity transformation. The transform equations are solved through Parametric Continuation method (PCM) for the numerical results. The key points are rebound as:

- The inclusion of (CNTs) and Fe_3O_4 nanoparticles in base fluid positively affect the heat and mass transmission.
- The opposing effect generated due to the Lorentz force is responsible for the decrease of axial $f'(\eta)$ and radial velocity $g(\eta)$ profile.
- While an opposite trend has been observed between temperature and magnetic strength, because the same retarding effect produces heat due to friction forces, which enhances the fluid temperature.
- The influence of Lewis number Le and Peclet number Pe both enhances the motile microorganism's concentration profile $h(\eta)$. Because, the rising credit of Lewis number reduces molecular diffusion rate, which increases motile microorganism's concentration profile $h(\eta)$.
- The positive effects of Hall current parameter m enlarge the axial velocity $f'(\eta)$. Physically, $\sigma_{mf}/1+m^2$ effective conductivity reduces with rising credit of m , which declines the damping impact of M on axial velocity $f'(\eta)$. Thus, slight away from the disk surface ($\eta \approx 5.0$), the both type of nanoliquid achieved its peak velocity

Competing interests: The authors declare no competing interests.

Data availability statement:

The data that support the findings of this study are available from the corresponding author upon reasonable request.

References

- ¹Y. Ma, R. Mohebbi, M. Rashidi, and Z. Yang, “Study of nanofluid forced convection heat transfer in a bent channel by means of lattice Boltzmann method,” [Physics of Fluids](#), vol. **30**, no. 3, p. 032001 (2018).
- ²J. Stuart, “On the effects of uniform suction on the steady flow due to a rotating disk,” [The Quarterly Journal of Mechanics and Applied Mathematics](#), vol. **7**, no. 4, pp. 446–457 (1954).
- ³M. Turkyilmazoglu, “Mhd fluid flow and heat transfer due to a stretching rotating disk,” [International Journal of Thermal Sciences](#), vol. **51**, pp. 195–201 (2012).
- ⁴M. Tabassum and M. Mustafa, “A numerical treatment for partial slip flow and heat transfer of non-newtonian reiner-rivlin fluid due to rotating disk,” [International Journal of Heat and Mass Transfer](#), vol. **123**, pp. 979–987(2018).
- ⁵N. A. Khan, S. Aziz and N. A.Khan. MHD flow of Powell–Eyring fluid over a rotating disk. [Journal of the Taiwan Institute of Chemical Engineers](#), **45**(6), pp.2859-2867 (2014).
- ⁶M.M. Rashidi, N. Kavyani and S. Abelman. Investigation of entropy generation in MHD and slip flow over a rotating porous disk with variable properties. [International Journal of Heat and Mass Transfer](#), **70**, pp.892-917 (2014).
- ⁷T. Hayat, H. Nazar, M. Imtiaz and A. Alsaedi. Darcy-Forchheimer flows of copper and silver water nanofluids between two rotating stretchable disks. [Applied Mathematics and Mechanics](#), **38**(12), pp.1663-1678 (2017).
- ⁸M. Khan, J. Ahmed and L. Ahmad. Application of modified Fourier law in von Kármán swirling flow of Maxwell fluid with chemically reactive species. [Journal of the Brazilian Society of Mechanical Sciences and Engineering](#), **40**(12), p.573 (2018).
- ⁹M. Shuaib, M. Bilal, MA. Khan and SJ. Malebary. Fractional Analysis of Viscous Fluid Flow with Heat and Mass Transfer over a Flexible Rotating Disk. “[Computer Modeling in Engineering & Sciences](#)”. Apr 1; **123**(1):377-400 (2020).
- ¹⁰J. Ahmed, M. Khan, & L. Ahmad. Effectiveness of homogeneous–heterogeneous reactions in Maxwell fluid flow between two spiraling disks with improved heat conduction features. [Journal of Thermal Analysis and Calorimetry](#), **139**(5), 3185-3195 (2020).
- ¹¹H. Alfvén. Existence of electromagnetic-hydrodynamic waves. [Nature](#), **150**(3805), 405-406 (1942).

- ¹²A. Siddiqui, N. Manzoor, K. Maqbool, A. B. Mann and S. Shaheen. Magnetohydrodynamic flow induced by ciliary movement: An application to lower respiratory track diseases. [Journal of Magnetism and Magnetic Materials](#), **480**, 164-170 (2019).
- ¹³M. E. H. Hafidzuddin, R. Nazar and N. Arifin. Application of the Keller-box Method to Magnetohydrodynamic Rotating Flow over a Permeable Shrinking Surface. [Embracing Mathematical Diversity](#), **36** (2019).
- ¹⁴A. Neeraja, R R. Devi, B. Devika, V. N. Radhika and M. K. Murthy. Effects of viscous dissipation and convective boundary conditions on magnetohydrodynamics flow of casson liquid over a deformable porous channel. [Results in Engineering](#), **4**, 100040 (2019).
- ¹⁵M. Zubair, Z. Shah, A. Dawar, S. Islam, P. Kumam and A. Khan. Entropy generation optimization in squeezing magnetohydrodynamics flow of casson nanofluid with viscous dissipation and joule heating effect. [Entropy](#), **21**(8), 747 (2019).
- ¹⁶M. Subhani and S. Nadeem. Numerical investigation into unsteady magnetohydrodynamics flow of micropolar hybrid nanofluid in porous medium. [Physica Scripta](#), **94**(10), 105220 (2019).
- ¹⁷H. J. Lokesh, B. J. Gireesha and K.G. Kumar. Characterization of chemical reaction on magnetohydrodynamics flow and nonlinear radiative heat transfer of Casson nanoparticles over an exponentially sheet. [Journal of Nanofluids](#), **8**(6), 1260-1266 (2019).
- ¹⁸A. Rauf, Z. Abbas and S.A. Shehzad. Interactions of active and passive control of nanoparticles on radiative magnetohydrodynamics flow of nanofluid over oscillatory rotating disk in porous medium. [Journal of Nanofluids](#), **8**(7), 1385-1396 (2019).
- ¹⁹M. K. Murthy. Numerical investigation on magnetohydrodynamics flow of Casson fluid over a deformable porous layer with slip conditions. [Indian Journal of Physics](#), 1-10 (2020).
- ²⁰I. S. Oyelakin, P. C. Lalramneihmawii, S. Mondal, S. K. Nandy and P. Sibanda. Thermophysical analysis of three-dimensional magnetohydrodynamic flow of a tangent hyperbolic nanofluid. [Engineering Reports](#), (2020).
- ²¹N. S. Khashi'ie, N. M. Arifin, R. Nazar, E. H. Hafidzuddin, N. Wahi & I. Pop. Magnetohydrodynamics (MHD) axisymmetric flow and heat transfer of a hybrid nanofluid past a radially permeable stretching/shrinking sheet with joule heating. [Chinese Journal of Physics](#), **64**, 251-263 (2020).

- ²²I. Tlili, H. A. Nabwey, G. P. Ashwinkumar & N. Sandeep. 3-D magnetohydrodynamic AA7072-AA7075/methanol hybrid nanofluid flow above an uneven thickness surface with slip effect. [Scientific Reports](#), **10**(1), 1-13 (2020).
- ²³S. A. Khan, M. I. Khan, T. Hayat & A. Alsaedi. Physical aspects of entropy optimization in mixed convective MHD flow of carbon nanotubes (CNTs) in a rotating frame. [Physica Scripta](#), **94**(12), 125009 (2019).
- ²⁴N. S. Anuar, N. Bachok, M. Turkyilmazoglu, N. M. Arifin & H. Rosali. Analytical and stability analysis of MHD flow past a nonlinearly deforming vertical surface in Carbon Nanotubes. [Alexandria Engineering Journal](#), (2020).
- ²⁵Z. Shah, E. Bonyah, S. Islam & T. Gul. Impact of thermal radiation on electrical mhd rotating flow of carbon nanotubes over a stretching sheet. [AIP Advances](#), **9**(1), 015115 (2019).
- ²⁶B. Mahanthesh, B. J. Gireesha, I.L. Animasaun, T. Muhammad & N. S. Shashikumar. MHD flow of SWCNT and MWCNT nanoliquids past a rotating stretchable disk with thermal and exponential space dependent heat source. [Physica Scripta](#), **94**(8), 085214 (2019).
- ²⁷N. S. Anuar, N. Bachok, N. M. Arifin & H. Rosali. MHD Flow past a Nonlinear Stretching/Shrinking Sheet in Carbon Nanotubes: Stability Analysis. [Chinese Journal of Physics](#), (2020).
- ²⁸K. Hosseinzadeh, A. Asadi, A. R. Mogharrebi, J. Khalesi, S. Mousavisani & D. D. Ganji. Entropy generation analysis of (CH₂OH)₂ containing CNTs nanofluid flow under effect of MHD and thermal radiation. [Case Studies in Thermal Engineering](#), **14**, 100482 (2019).
- ²⁹Z. Ahmed, Z., Nadeem, S., Saleem, S., & Ellahi, R. (2019). Numerical study of unsteady flow and heat transfer CNT-based MHD nanofluid with variable viscosity over a permeable shrinking surface. *International Journal of Numerical Methods for Heat & Fluid Flow*.
- ³⁰P. S. S. Nagalakshmi & N. Vijaya. MHD flow of Carreau Nanofluid Explored using CNT Over a Nonlinear Stretching sheet. [Frontiers in Heat and Mass Transfer \(FHMT\)](#), **14** (2020).
- ³¹A. Tulu & W. Ibrahim. MHD Slip Flow of CNT-Ethylene Glycol Nanofluid due to a Stretchable Rotating Disk with Cattaneo–Christov Heat Flux Model. [Mathematical Problems in Engineering](#), 2020 (2020).
- ³²S. S. Ghadikolaei & M. Gholinia. 3D mixed convection MHD flow of GO-MoS₂ hybrid nanoparticles in H₂O–(CH₂OH)₂ hybrid base fluid under the effect of H₂ bond. [International Communications in Heat](#), (2020).

- ³³N. Acharya, R. Bag & P. K. Kundu. Influence of Hall current on radiative nanofluid flow over a spinning disk: a hybrid approach. [Physica E: Low-dimensional Systems and Nanostructures](#), **111**, 103-112 (2019).
- ³⁴M. Veerakrishna, A.J. Chamkha, Hall effects on unsteady MHD flow of second grade fluid through porous medium with ramped wall temperature and ramped surface concentration, *Phys. Fluids* **30** (2018),
- ³⁵F.M. Abbasi, M. Gul, S.A. Shehzad, Hall effects on peristalsis of boron nitrideethylene glycol nanofluid with temperature dependent thermal conductivity, [Phys. E Low-dimens. Syst. Nanostruct.](#) **99**, 275–284 (2018).
- ³⁶F. Mabood, S.M. Ibrahim, M.M. Rashidi, M.S. Shadloo, G. Lorenzini, Non-uniform heat source/sink and Soret effects on MHD non-Darcian convective flow past a stretching sheet in a micropolar fluid with radiation, [Int. J. Heat Mass Transf.](#) **93**, 674–682 (2016).
- ³⁷F. Saba, N. Ahmed, U. Khan, ST. Mohyud-Din. A novel coupling of (CNT-Fe₃O₄/H₂O) hybrid nanofluid for improvements in heat transfer for flow in an asymmetric channel with dilating/squeezing walls. [International Journal of Heat and Mass Transfer](#). Jun 1; **136**:186-95 (2019).
- ³⁸C. Yin, L. Zheng, C. Zhang, X. Zhang, Flow and heat transfer of nanofluids over a rotating disk with uniform stretching rate in radial direction, [Propuls. Power Res.](#) **6** (1), 25–30 (2017).
- ³⁹ Tassaddiq, A., Khan, S., Bilal, M., Gul, T., Mukhtar, S., Shah, Z., & Bonyah, E. (2020). Heat and mass transfer together with hybrid nanofluid flow over a rotating disk. *AIP Advances*, *10*(5), 055317.

Author contributions

M. B and M.A.K.. wrote the original manuscript and performed the numerical simulations. M. S. reviewed the mathematical results and restructured the manuscript. All authors are agreed on the final draft of the submission file.

Figures

Figure 1

Spinning disk geometry.

Figure 2

"See the Supplemental Files section for the complete figure caption".

Figure 3

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Figure 4

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Figure 5

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Figure 6

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