Sampled-Data Exponential Synchronization of Markovian Jump Chaotic Lur'e Systems with Multiple Time Delays

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Sampled-Data Exponential Synchronization of Markovian Jump Chaotic Lur'e Systems with Multiple Time Delays

Te Yang* Zhen Wang* Xia Huang[†] Jianwei Xia[‡]

Abstract

This paper studies the mean square exponential synchronization control of the Markovian jump chaotic Lur'e systems (MJCLSs) with multiple time delays via aperiodic sampling. Firstly, by introducing the mode-dependent single/double integral terms and considering the system state at the sampling instant, a new mode-dependent Lyapunov functional based on the looped-functional is constructed. On the basis of the constructed functional and by estimating the expectation of the solution of MJCLSs with multiple time delays, an exponential synchronization criterion in the mean square sense is presented. Then, based on the obtained stability conditions, a design method for aperiodic sampled-data controller (ASDC) to ensure the exponential synchronization is proposed. Finally, the effectiveness of the developed method is demonstrated by the Chua's circuit model and a neural network model. Compared with the existing literature, an admissible larger upper bound of sampling period can be obtained.

Keywords: Markovian jump chaotic Lur'e systems; multiple time delays; aperiodic sampled-data controller; looped-functional; exponential stability

1 Introduction

The Lur'e system is a special kind of nonlinear systems proposed by Lur'e in the 1940s when studying the automatic pilot control of aircraft. It consists of a linear term and a sector-bounded nonlinear term. In [1–4], a large number of actual nonlinear systems such as positive rolling attractors, hyperchaotic systems, Chua's circuits and neural networks can all be simulated as chaotic Lur'e systems. However, the structure of an actual control system is often changed due to sudden external environment disturbances, changes in the coupling part of the subsystem and the random component failures and maintenance. In this case, we can use

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the Markovian jump systems (MJSs) to simulate the dynamics. MJSs are widely used to simulate numerous actual engineering systems with unpredictable change in structure and parameters, such as aerospace systems, power systems and manufacturing systems. As we all know, the control problem of nonlinear MJSs has always been a hot spot in control field, especially the MJCLSs composed of linear and nonlinearities parts. Combining the random Lur'e-type Lyapunov functional and the implicit Markovian model method, sliding mode control problem was investigated for MJCLSs via event-triggered control in [5]. For the discrete-time MJCLSs with control saturation and exogenous L_2 type interference, the control synthesis problem of local stability and minimizing finite L_2 gain was solved in [6]. The model predictive control problem for a series of MJCLSs at the mercy of parameter uncertainties and input constraints was studied in [7]. The reduced-order and full-order observers for MJCLSs differential inclusion system were designed in [8], which makes the corresponding error system exponentially stable.

Due to the wide application of digital controller in computer control systems and embedded control systems, networked control systems have been developed rapidly. As the theoretical basis, the stability study of sampled-data control system is particularly important [9–14]. As an important behavior of complex dynamic networks, synchronization has potential applications in the field of image processing, chemical reactions, communication safety and processes for storing and retrieving information. Since the pioneering work of synchronization in [15], the research on sampled-data synchronization of chaotic Lur'e systems has aroused the interest of many scholars. Based on the input delay approach, linear matrix inequalities and free-weighting matrix method, synchronization conditions in the global sense for chaotic Lur'e systems were proposed in [16, 17]. A lower conservative design method for sampled-data controller and synchronization criteria were presented in [18] by constructing a Lyapunov functional that only requires the positiveness at sampling instants. Furthermore, most of the results consider the synchronization of chaotic delayed systems. Ref. [19, 20] used this type of functional proposed in Ref. [18] to consider the sampled-data synchronization problem of delayed chaotic Lur'e systems. A novel integral inequality approach and a novel Lyapunov functional was constructed by using more nonlinear information of the chaotic Lur'e delayed system, and less conservative stability conditions were presented in [21, 22]. H_∞ synchronization and output feedback synchronization were studied in [23, 24] by constructing Lyapunov functionals. Quasi-synchronization of discrete-time Lur'e-type switched systems with parameter mismatching and Lur'e networks with proportional delay were studied in [25, 26]. The synchronization problem of uncertain delayed Lur'e system with random sampling based on Bernoulli distribution was addressed through a new non-fragile stochastic control method in [27]. Some less conservative synchronization conditions were presented based on the quantized sampled-data controller in [28, 29]. Since neural networks and chaotic Lur'e systems have similar structures with nonlinear functions, much literature about sampled-data synchronization of Markovian jump neural networks (MJNNs) are of great significance to the research of sampled-data synchronization of MJCLSs. Considering coupled MJNNs with different forms of time delays, Ref. [30, 31] addressed the stability and synchronization prob-

lems. Considering the unknown transfer rate, and by constructing a mode-dependent Lyapunov functional, sampled-data synchronization of MJNNs was reported in [32, 33]. Based on constructing mode-dependent one-sided and two-sided looped-functionals, the random sampled-data synchronization problem of MJNNs was studied in [34] and Ref. [34] concluded that two-sided looped-functionals has lower conservative. The H_∞ synchronization problems of MJNNs were studied by constructing a composite disturbance rejection controller and a mode-dependent synchronous controller in [35, 36], respectively.

Unlike such a wealth of research on sampled-data synchronization of MJNNs, the sampled-data synchronization of MJCLSs is less concerned, therefore, it needs further Consideration. Considering the event-trigger mechanism and the quantization effect, Ref. [37] studied the asynchronous control problem of MJCLSs. By considering partial accessibility and using hidden Markov model, Ref. [38] discussed the problem of asynchronous control of MJCLSs with limited time interval. By fully considering the probability distribution characteristics of packet loss, the random synchronization problem of a semi-MJCLSs with packet loss and multi-period sampling was studied in [39]. Through the above discussion, we can find that most of the chaotic Lur'e systems studied in the existing published papers are with single time-delay, and rarely literature considers the situation of multiple time delays. In addition, for MJCLSs, compared with the mode-independent Lyapunov functional, the mode-dependent Lyapunov functional is less conservative in many cases. However, in most studies of MJCLSs under aperiodic sampling, only the mode-dependent of the partial Lyapunov matrices is considered. However, when the single integral and double integral terms are introduced to construct the functional, the introduction of the mode-dependent matrices will inevitably bring about less conservative results. Furthermore, if one considers the use of sampled states to construct functionals, is it possible to construct mode-dependent functionals as well? Considering the above problems, the ASDC is further designed to guarantee exponential synchronization of the error system by constructing a lower conservative functional.

The following contributions have been presented:

1. A lemma to estimate the expectation of the solution of the MJCLSs with multiple time delays is proposed.
2. By introducing mode-dependent single/double integral terms to the Lyapunov functional, and considering mode-dependent information of the sampled-state, a innovative Lyapunov functional is constructed.
3. On the basis of the constructed Lyapunov functional and the improved integral inequality, the exponential synchronization criterion in the mean square sense is presented, and the ASDC is designed.

Notations: The following are the significant symbols used in this article. $\mathbb{R}$: real numbers set; $\mathbb{R}^n$: n -dimensional Euclidean space; $\mathbb{R}^{m \times n}$: $m \times n$ real matrices set; $I(0)$: identity (zero) matrix; $H^{-1}(H^T)$: the inverse (transpose) of the matrix H ; $\text{diag}\{\dots\}$: block-diagonal matrix; $H > 0(\geq 0)$: real symmetric positive-definite (positive-semidefinite) matrix; $\mathbb{E}\{\cdot\}$: mathematical expectation; $\text{col}\{\zeta_1, \zeta_2\}$: the vector stacking ζ_1

on the top of ζ_2 ; $\|A\| : \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^T A)}$; $*$: symmetric components in a symmetric matrix; $\text{sym}\{Q\}$: $Q + Q^T$; $\lambda_{\min}(B)$ ($\lambda_{\max}(B)$): minimum (maximum) eigenvalues of matrix B; $\Pr\{N\}$: Probability of event N.

2 Problem Formulation

Consider the master system as following:

$$\mathcal{M} : \begin{cases} \dot{x}(t) = \mathcal{A}_{r(t)}x(t) + \sum_{v=1}^m \mathcal{B}_{v,r(t)}x(t - d_v) + \mathcal{W}_{r(t)}\eta(\mathcal{D}x(t)), \\ q(t) = \mathcal{C}_{r(t)}x(t), \end{cases} \quad (1)$$

and the slave system

$$\mathcal{N} : \begin{cases} \dot{y}(t) = \mathcal{A}_{r(t)}y(t) + \sum_{v=1}^m \mathcal{B}_{v,r(t)}y(t - d_v) + \mathcal{W}_{r(t)}\eta(\mathcal{D}y(t)) + u(t), \\ p(t) = \mathcal{C}_{r(t)}y(t), \end{cases} \quad (2)$$

where $u(t) = \mathcal{K}_{r(t)}(q(t_k) - p(t_k))$, $t_k \leq t < t_{k+1}$. $x(t)$, $y(t) \in \mathbb{R}^n$ are state vectors, $q(t)$, $p(t) \in \mathbb{R}^l$ are output vectors, $u(t) \in \mathbb{R}^n$ is control input, and $\mathcal{K}_{r(t)} \in \mathbb{R}^{n \times l}$ is control gain matrix. d_v is the constant time-delay for $v = 1, 2, \dots, m$. Assuming that $\eta(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a diagonal nonlinearity and $\eta_\sigma(\cdot)$ belongs to the sector $[0, s_\sigma]$ for $\sigma = 1, 2, \dots, n$. $\mathcal{A}_{r(t)}$, $\mathcal{B}_{v,r(t)}$, $\mathcal{W}_{r(t)}$, $\mathcal{D}$ and $\mathcal{C}_{r(t)}$ are known constant matrices. The sampling interval $h_k = t_{k+1} - t_k$, $k \geq 0$, the lower and upper bounds of the sampling interval is presented by h_m and h_M , i.e., $0 \leq h_m \leq h_k \leq h_M$ with $t_0 = 0$.

Assuming $r(t)$ is a continuous-time markovian chain and it takes values from $\mathcal{N} = \{1, 2, \dots, N\}$ with generator $\Gamma \triangleq [r_{ij}]_{N \times N}$ given by

$$\Pr\{r(t+l) = j \mid r(t) = i\} = \begin{cases} r_{ij}l + o(l), i \neq j \\ 1 + r_{ii}l + o(l), i = j \end{cases}$$

here, $l > 0$, $\lim_{l \rightarrow 0}(o(l)/l) = 0$. $r_{ij} \geq 0$ is transition rate from mode i at time t to mode j at time $t+l$ if $i \neq j$ and $r_{ii} = -\sum_{j=1, j \neq i}^N r_{ij}$. To simplify, we denote $r(t) = i \in \mathcal{N}$. Let $\varsigma(t) = x(t) - y(t)$ is the synchronization error, we can express the error system like that:

$$\dot{\varsigma}(t) = \mathcal{A}_i\varsigma(t) + \sum_{v=1}^m \mathcal{B}_{v,i}\varsigma(t - d_v) + \mathcal{W}_i\xi(\mathcal{D}\varsigma(t), y(t)) - \mathcal{K}_i\mathcal{C}_i\varsigma(t_k), \quad (3)$$

here $\xi(\mathcal{D}\varsigma(t), y(t)) = \eta(\mathcal{D}(\varsigma(t) + y(t))) - \eta(\mathcal{D}(y(t)))$. Let $\eta_\sigma(\cdot)$ pertains to sector $[0, s_\sigma]$, for any $\sigma = 1, 2, \dots, n$, it is easy to find like this

$$0 \leq \frac{\xi_\sigma(d_\sigma^T \varsigma, y)}{d_\sigma^T \varsigma} = \frac{\eta_\sigma[d_\sigma^T(\varsigma + y)] - \eta_\sigma(d_\sigma^T \varsigma)}{d_\sigma^T \varsigma} \leq s_\sigma, \quad (4)$$

where d_σ^T represents the σ th row vector of $\mathcal{D}$. From [19], for any matrices $\mathcal{V}_{\rho,i} = \text{diag}\{v_{\rho,1,i}, v_{\rho,2,i}, \dots, v_{\rho,n,i}\} \geq 0$, $\rho = 1, 2$, the inequalities can be presented like this:

$$\varsigma^T(t) \mathcal{D}^T \mathcal{S} \mathcal{V}_{\rho,i} \xi(\mathcal{D}\varsigma(t), y(t)) - \xi(\mathcal{D}\varsigma(t), y(t))^T \mathcal{V}_{\rho,i} \xi(\mathcal{D}\varsigma(t), y(t)) \geq 0, \quad (5)$$

where $\mathcal{S} = \text{diag}\{s_1, s_2, \dots, s_n\}$.

The objective of this research is to develop a new design method for ASDC via a novel Lyapunov functional to make the master system $\mathcal{M}$ and the slave system $\mathcal{N}$ mean square exponential synchronize. Before giving the results of stability analysis, we supply a stability definition and two useful lemmas.

Definition 1. [40] *The synchronization error system (3) is exponentially stable in the mean square sense, if there exists a decay coefficient $\beta > 0$ and a decay rate $\alpha > 0$ such that*

$$\mathbb{E}\{\|\varsigma(t)\|^2\} \leq \beta e^{-2\alpha t} \mathbb{E}\{\|\varsigma_0\|_c^2\}, \quad \forall t \geq 0, \quad (6)$$

where $\|\varsigma_0\|_c^2 = \sup_{-\chi \leq \theta \leq 0} \{\|\varsigma(\theta)\|^2 + \|\dot{\varsigma}(\theta)\|^2\}$, and $\chi = \max\{d_v, h_M\}$, $v = 1, 2, \dots, m$.

Lemma 1. *Concerning system (3), the following inequality holds:*

$$\mathbb{E}\{\|\varsigma(t)\|^2\} \leq \theta_1 \mathbb{E}\{\|\varsigma(t_k)\|^2\} + \mathbb{E}\left\{\sum_{v=1}^m \int_{t_k-d_v}^{t_k} \theta_2 \varepsilon \|\varsigma(s)\|^2 ds\right\}, \quad (7)$$

where

$$\begin{aligned} \theta_1 &= (5 + 5h_M^2 \|\mathcal{K}_i \mathcal{C}_i\|^2) \varepsilon, \\ \theta_2 &= 5h_M m \|\mathcal{B}_{v,i}\|^2, \\ \varepsilon &= e^{5h_M^2 \|\mathcal{A}_i\|^2 + 5h_M^2 \sum_{v=1}^m m \|\mathcal{B}_{v,i}\|^2 + 5h_M^2 \|\mathcal{W}_i\|^2 \|\mathcal{S} \mathcal{D}\|^2}. \end{aligned}$$

Proof. Integrating system (3) from t_k to t leads to

$$\begin{aligned} \|\varsigma(t)\| &\leq \|\varsigma(t_k)\| + \left\| \int_{t_k}^t \mathcal{A}_i \varsigma(s) ds \right\| + \left\| \int_{t_k}^t \mathcal{K}_i \mathcal{C}_i \varsigma(t_k) ds \right\| \\ &\quad + \left\| \int_{t_k}^t \sum_{v=1}^m \mathcal{B}_{v,i} \varsigma(s - d_v) ds \right\| + \left\| \int_{t_k}^t \mathcal{W}_i \xi(\mathcal{D}\varsigma(s), y(s)) ds \right\|. \end{aligned}$$

In addition, from Cauchy-Schwarz inequality, we can easily obtain that

$$\begin{aligned} \|\varsigma(t)\|^2 &\leq 5\|\varsigma(t_k)\|^2 + 5 \left\| \int_{t_k}^t \mathcal{A}_i \varsigma(s) ds \right\|^2 + 5 \left\| \int_{t_k}^t \sum_{v=1}^m \mathcal{B}_{v,i} \varsigma(s - d_v) ds \right\|^2 \\ &\quad + 5 \left\| \int_{t_k}^t \mathcal{W}_i \xi(\mathcal{D}\varsigma(s), y(s)) ds \right\|^2 + 5 \left\| \int_{t_k}^t \mathcal{K}_i \mathcal{C}_i \varsigma(t_k) ds \right\|^2, \end{aligned}$$

and

$$\mathbb{E}\{\|\varsigma(t)\|^2\} \leq 5\mathbb{E}\{\|\varsigma(t_k)\|^2\} + 5h_M \mathbb{E}\left\{\int_{t_k}^t \|\mathcal{A}_i \varsigma(s)\|^2 ds\right\} + 5h_M \mathbb{E}\left\{\int_{t_k}^t \left\| \sum_{v=1}^m \mathcal{B}_{v,i} \varsigma(s - d_v) \right\|^2 ds\right\}$$

$$+ 5h_M \mathbb{E} \left\{ \int_{t_k}^t \left\| \mathcal{W}_i \xi(\mathcal{D}\varsigma(s), y(s)) \right\|^2 ds \right\} + 5h_M \mathbb{E} \left\{ \int_{t_k}^t \left\| \mathcal{K}_i \mathcal{C}_i \varsigma(t_k) \right\|^2 ds \right\}.$$

It can be found from (4) that

$$\|\xi(\mathcal{D}\varsigma(t), y(t))\|^2 \leq \|\mathcal{S}\mathcal{D}\|^2 \|\varsigma(t)\|^2,$$

and

$$\begin{aligned} & 5h_M \mathbb{E} \left\{ \int_{t_k}^t \left\| \sum_{v=1}^m \mathcal{B}_{v,i} \varsigma(s - d_v) \right\|^2 ds \right\} \\ & \leq 5h_M \mathbb{E} \left\{ \sum_{v=1}^m \int_{t_k}^t m \left\| \mathcal{B}_{v,i} \varsigma(s - d_v) \right\|^2 ds \right\} \\ & \leq 5h_M m \mathbb{E} \left\{ \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \int_{t_k - d_v}^{t - d_v} \left\| \varsigma(s) \right\|^2 ds \right\} \\ & \leq 5h_M m \mathbb{E} \left\{ \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \int_{t_k - d_v}^{t_k} \left\| \varsigma(s) \right\|^2 ds \right\} + 5h_M m \mathbb{E} \left\{ \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \int_{t_k}^t \left\| \varsigma(s) \right\|^2 ds \right\}. \end{aligned}$$

Thence, we can obtain

$$\begin{aligned} \mathbb{E} \{ \|\varsigma(t)\|^2 \} & \leq 5\mathbb{E} \{ \|\varsigma(t_k)\|^2 \} + 5h_M \left\| \mathcal{A}_i \right\|^2 \mathbb{E} \left\{ \int_{t_k}^t \left\| \varsigma(s) \right\|^2 ds \right\} \\ & \quad + 5h_M m \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \mathbb{E} \left\{ \int_{t_k - d_v}^{t_k} \left\| \varsigma(s) \right\|^2 ds \right\} + 5h_M m \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \mathbb{E} \left\{ \int_{t_k}^t \left\| \varsigma(s) \right\|^2 ds \right\} \\ & \quad + 5h_M \left\| \mathcal{W}_i \right\|^2 \left\| \mathcal{S}\mathcal{D} \right\|^2 \mathbb{E} \left\{ \int_{t_k}^t \left\| \varsigma(s) \right\|^2 ds \right\} + 5h_M^2 \left\| \mathcal{K}_i \mathcal{C}_i \right\|^2 \mathbb{E} \{ \|\varsigma(t_k)\|^2 \} \\ & \leq (5 + 5h_M^2 \left\| \mathcal{K}_i \mathcal{C}_i \right\|^2) \mathbb{E} \{ \|\varsigma(t_k)\|^2 \} + (5h_M \left\| \mathcal{A}_i \right\|^2 + 5h_M m \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \\ & \quad + 5h_M \left\| \mathcal{W}_i \right\|^2 \left\| \mathcal{S}\mathcal{D} \right\|^2) \mathbb{E} \left\{ \int_{t_k}^t \left\| \varsigma(s) \right\|^2 ds \right\} + 5h_M m \sum_{v=1}^m \left\| \mathcal{B}_{v,i} \right\|^2 \mathbb{E} \left\{ \int_{t_k - d_v}^{t_k} \left\| \varsigma(s) \right\|^2 ds \right\}. \end{aligned}$$

From the Gronwall inequality, we can obtain (7). $\square$

Remark 1. For error system (3), the estimation of expectation of the system solution is presented by Lemma 1, which plays an important role in analysis of the mean square exponential stability.

Lemma 2. [41] For given symmetric matrices $R > 0$ and N_1, N_2, N_3 , the following inequality satisfies:

$$- \int_a^b \dot{\varsigma}^T(t) R \dot{\varsigma}(t) ds \leq \varpi^T \Omega \varpi, \quad (8)$$

where

$$\Omega = \tau(N_1 R^{-1} N_1^T + \frac{1}{3} N_2 R^{-1} N_2^T + \frac{1}{5} N_3 R^{-1} N_3^T) + \text{Sym}\{N_1 \Pi_1 + N_2 \Pi_2 + N_3 \Pi_3\},$$

$$\Pi_1 = \check{e}_1 - \check{e}_2, \quad \Pi_2 = \check{e}_1 + \check{e}_2 - 2\check{e}_3, \quad \Pi_3 = \check{e}_1 - \check{e}_2 - 6\check{e}_3 + 6\check{e}_4, \quad \tau = a - b,$$

$$\varpi = [x^T(b), \quad x^T(a), \quad \frac{1}{\tau} \int_a^b x^T(s) ds, \quad \frac{2}{\tau^2} \int_a^b \int_a^s x^T(u) du ds]^T$$

$$\check{e}_i = [0_{n \times (i-1)n}, \quad I_{n \times n}, \quad 0_{n \times (4-i)n}], \quad i = 1, 2, 3, 4.$$

3 Stability analysis and controller design

In this section, a mode-dependent two-sided looped-functional was constructed. Using the novel functional, we discuss the exponentially synchronization in the mean square sense for MJCLSs with multiple time delays. The ASDC design scheme for error system is proposed. For simplicity, we use nomenclature for vectors and matrices in the following.

$$\begin{aligned}
\omega_{1,v} &= \frac{1}{d_v} \int_{t-d_v}^t \varsigma(s) ds, & \omega_{2,v} &= \frac{2}{d_v^2} \int_{t-d_v}^t \int_{t-d_v}^s \varsigma(u) du ds, & \alpha_{1,v} &= \frac{e^{2\alpha d_v} - 1}{2\alpha}, \\
\alpha_{2,v} &= \frac{e^{2\alpha d_v} - 1 - 2\alpha d_v}{4\alpha^2}, & \alpha_{3,v} &= \frac{e^{2\alpha d_v} - 1 - 2\alpha d_v + 2\alpha^2 d_v^2}{8\alpha^3}, & \alpha_4 &= \frac{e^{-2\alpha h_M} - 1 + 2\alpha h_M}{4\alpha^2}, \\
\zeta^T(t) &= [\varsigma^T(t), \varsigma^T(t-d_1), \varsigma^T(t-d_2), \dots, \varsigma^T(t-d_m), \omega_{1,1}^T(t), \omega_{1,2}^T(t), \dots, \omega_{1,m}^T(t), \\
&\quad \omega_{2,1}^T(t), \omega_{2,2}^T(t), \dots, \omega_{2,m}^T(t), \varsigma^T(t_k), \varsigma^T(t_{k+1}), \varsigma^T(t), \xi^T(\mathcal{D}\varsigma(t), y(t))], \\
\zeta_1^T(t) &= [(t_{k+1} - t)\mu_1^T(t), (t - t_k)\mu_2^T(t)], \\
\zeta_2^T(t) &= [\mu_1^T(t), \mu_2^T(t)], \quad \zeta_3^T = [\varsigma^T(t_k), \varsigma^T(t_{k+1})], \\
\mu_1(t) &= \varsigma(t) - \varsigma(t_k), \quad \mu_2(t) = \varsigma(t) - \varsigma(t_{k+1}), \\
e_\iota &= [0_{n \times (\iota-1)n}, I_{n \times n}, 0_{n \times (3m+5-\iota)n}], \quad \iota = 1, 2, \dots, 3m+5.
\end{aligned}$$

Theorem 1. *Given scalars $\alpha > 0$, δ and γ , the slave system (2) is exponentially synchronization in the mean square sense to the master system (1) if there exist positive definite matrices P_i , $Q_{v,i}$, Q_v , $\mathcal{R}_{v,i}$, $\mathcal{R}_v$, U_i , U , Z_i , matrices X_i , Y_i , S_i , $M_{1,v,i}$, $M_{2,v,i}$, $M_{3,v,i}$, G_i , L_i , and diagonal matrices $\mathcal{V}_{1,i} > 0$, $\mathcal{V}_{2,i} > 0$ such that following conditions hold for $h_k \in \{h_m, h_M\}$, $\forall i \in \mathcal{N}$, $v = 1, 2, \dots, m$:*

$$\hat{\Xi}_1(h_k) = \begin{bmatrix} \Phi_1 + \Phi_2 + h_k \Phi_3 & \sqrt{d_1} \tilde{\Xi}_1 & \sqrt{d_2} \tilde{\Xi}_2 & \sqrt{d_2} \tilde{\Xi}_3 & \dots & \sqrt{d_m} \tilde{\Xi}_m \\ * & \tilde{R}_{1,i} & 0 & 0 & \dots & 0 \\ * & * & \tilde{R}_{2,i} & 0 & \dots & 0 \\ * & * & * & \tilde{R}_{3,i} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \dots & \tilde{R}_{m,i} \end{bmatrix} < 0, \quad (9)$$

$$\hat{\Xi}_2(h_k) = \begin{bmatrix} \Phi_1 + \Phi_4 + h_k \Phi_5 & \sqrt{d_1} \tilde{\Xi}_1 & \sqrt{d_2} \tilde{\Xi}_2 & \sqrt{d_2} \tilde{\Xi}_3 & \dots & \sqrt{d_m} \tilde{\Xi}_m \\ * & \tilde{R}_{1,i} & 0 & 0 & \dots & 0 \\ * & * & \tilde{R}_{2,i} & 0 & \dots & 0 \\ * & * & * & \tilde{R}_{3,i} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \dots & \tilde{R}_{m,i} \end{bmatrix} < 0, \quad (10)$$

$$e^{2\alpha d_v} \sum_{j=1}^N r_{ij} Q_{v,j} - Q_v < 0, \quad (11)$$

$$e^{2\alpha d_v} \sum_{j=1}^N r_{ij} R_{v,j} - R_v < 0, \quad (12)$$

$$\sum_{j=1}^N r_{ij} U_j - U < 0 \quad (13)$$

where

$$\begin{aligned} \pi_1 &= [e_{3m+2}^T - e_1^T, \quad e_1^T - e_{3m+3}^T]^T, \quad \pi_2 = [e_1^T - e_{3m+2}^T, \quad e_1^T - e_{3m+3}^T]^T, \quad \pi_3 = [e_{3m+2}^T, \quad e_{3m+3}^T]^T, \\ \pi_4 &= [e_{3m+4}^T, \quad e_{3m+4}^T]^T, \quad \pi_5 = [e_{3m+4}^T, \quad 0]^T, \quad \pi_6 = [0, \quad e_{3m+4}^T]^T, \quad \pi_{7,v} = e_1 - e_{v+1}, \\ \pi_{8,v} &= e_1 + e_{v+1} - 2e_{m+v+1}, \quad \pi_{9,v} = e_1 - e_{v+1} - 6e_{m+v+1} + 6e_{2m+v+1}, \quad \pi_{10} = [e_1^T - e_{3m+2}^T, \quad 0]^T, \\ \pi_{11} &= [0, \quad e_1^T - e_{3m+3}^T]^T, \quad \pi_{12,v} = [e_1^T, \quad e_{v+1}^T, \quad e_{m+v+1}^T, \quad e_{2m+v+1}^T]^T, \\ \pi_{13} &= [e_{3m+2}^T, \quad e_{3m+3}^T]^T, \quad \pi_{14} = e_1 - e_{3m+2}, \quad \pi_{15} = e_1 - e_{3m+3}, \\ \Phi_1 &= e_1^T \left[2\alpha P_i + \left(\sum_{j=1}^N r_{ij} \right) P_j + \sum_{v=1}^m e^{2\alpha d_v} Q_{v,i} + \sum_{v=1}^m \alpha_{1,v} Q_v \right] e_1 + e_{3m+4}^T (U_i + h_M U) e_{3m+4} \\ &\quad - \sum_{v=1}^m e_{v+1}^T Q_{v,i} e_{v+1} + e_{3m+4}^T \left(\sum_{v=1}^m \alpha_{1,v} \mathcal{R}_{v,i} + \sum_{v=1}^m \alpha_{2,v} \mathcal{R}_v \right) e_{3m+4} + \pi_{13}^T [2\alpha h_M^2 Z_i + h_M^2 \left(\sum_{j=1}^N r_{ij} Z_j \right)] \pi_{13} \\ &\quad + \text{sym} \left\{ e_1^T P_i e_{3m+4} + \pi_1^T (Y_i \pi_2 + S_i \pi_3) + \pi_{14}^T [2\alpha X_i + \left(\sum_{j=1}^N r_{ij} X_j \right)] \pi_{15} + e_{3m+4}^T X_i \pi_{15} + \pi_{14}^T X_i e_{3m+4} \right. \\ &\quad \left. + (\delta e_1^T G_i + e_{3m+4}^T G_i + \gamma e_{3m+2}^T G_i) \left(-e_{3m+4} + \mathcal{A}_i e_1 + \sum_{v=1}^m \mathcal{B}_{v,i} e_{v+1} + \mathcal{W}_i e_{3m+5} - \mathcal{K}_i \mathcal{C}_i e_{3m+2} \right) \right. \\ &\quad \left. + \sum_{v=1}^m \pi_{12,v}^T (M_{1,v,i} \pi_{7,v} + M_{2,v,i} \pi_{8,v} + M_{3,v,i} \pi_{9,v}) \right\}, \\ \Phi_2 &= \text{sym} \left\{ e_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{1,i} e_{3m+5} \right\} - 2e_{3m+5}^T \mathcal{V}_{1,i} e_{3m+5}, \\ \Phi_3 &= \text{sym} \left\{ (\pi_5^T + 2\alpha \pi_{10}^T) (Y_i \pi_2 + S_i \pi_3) + \pi_{10}^T Y_i \pi_4 + \pi_{10}^T \left[\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right] \right\} + \pi_{13}^T Z_i \pi_{13}, \\ \Phi_4 &= \text{sym} \left\{ e_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{2,i} e_{3m+5} \right\} - 2e_{3m+5}^T \mathcal{V}_{2,i} e_{3m+5}, \\ \Phi_5 &= \text{sym} \left\{ (\pi_6^T + 2\alpha \pi_{11}^T) (Y_i \pi_2 + S_i \pi_3) + \pi_{11}^T Y_i \pi_4 + \pi_{11}^T \left[\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right] \right\} - \pi_{13}^T Z_i \pi_{13}, \\ \tilde{\Xi}_v &= [\pi_{12,v}^T M_{1,v,i}, \quad \pi_{12,v}^T M_{2,v,i}, \quad \pi_{12,v}^T M_{3,v,i}], \\ \tilde{\mathcal{R}}_{v,i} &= \text{diag} \{ -\mathcal{R}_{v,i}, \quad -3\mathcal{R}_{v,i}, \quad -5\mathcal{R}_{v,i} \}. \end{aligned}$$

Furthermore, the ASDC gain matrix in (3) is

$$\mathcal{K}_i = G_i^{-1} L_i. \quad (14)$$

Proof. Constructing mode-dependent two-sided looped-functional for the system (3)

$$\mathcal{V}(\varsigma(t), i, t) = \sum_{\epsilon=1}^7 \mathcal{V}_\epsilon(\varsigma(t), i, t), \quad (15)$$

with

$$\begin{aligned}
\mathcal{V}_1(\varsigma(t), i, t) &= e^{2\alpha t} \varsigma^T(t) P_i \varsigma(t), \\
\mathcal{V}_2(\varsigma(t), i, t) &= \sum_{v=1}^m \int_{t-d_v}^t e^{2\alpha(s+d_v)} \varsigma^T(s) Q_{v,i} \varsigma(s) ds + \sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \varsigma^T(s) Q_v \varsigma(s) ds d\theta, \\
\mathcal{V}_3(\varsigma(t), i, t) &= \sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \dot{\varsigma}^T(s) \mathcal{R}_{v,i} \dot{\varsigma}(s) ds d\theta + \sum_{v=1}^m \int_{-d_v}^0 \int_{\theta}^0 \int_{t+\beta}^t e^{2\alpha(s-\beta)} \dot{\varsigma}^T(s) \mathcal{R}_v \dot{\varsigma}(s) ds d\beta d\theta, \\
\mathcal{V}_4(\varsigma(t), i, t) &= \int_{t_k}^t e^{2\alpha s} \dot{\varsigma}^T(s) U_i \dot{\varsigma}(s) ds + \int_{-h_M}^0 \int_{t+\theta}^t e^{2\alpha s} \dot{\varsigma}^T(s) U \dot{\varsigma}(s) ds d\theta, \\
\mathcal{V}_5(\varsigma(t), i, t) &= 2e^{2\alpha t} \mu_1^T(t) X_i \mu_2(t), \\
\mathcal{V}_6(\varsigma(t), i, t) &= (t_{k+1} - t)(t - t_k) e^{2\alpha t} \zeta_3^T Z_i \zeta_3, \\
\mathcal{V}_7(\varsigma(t), i, t) &= 2e^{2\alpha t} \zeta_1^T(t) [Y_i \zeta_2(t) + S_i \zeta_3].
\end{aligned}$$

Let $\mathcal{L}$ be the weak infinitesimal operator of the random process $\{\varsigma(t), i, t \geq 0\}$ acting on $\mathcal{V}(\varsigma(t), i, t)$. Then, for any $i \in \mathcal{N}$, calculating $\mathcal{L}\mathcal{V}(\varsigma(t), i, t)$ of (3) gives

$$\begin{aligned}
\mathcal{L}\mathcal{V}_1(\varsigma(t), i, t) &= 2\alpha e^{2\alpha t} \varsigma^T(t) P_i \varsigma(t) + 2e^{2\alpha t} \varsigma^T(t) P_i \dot{\varsigma}(t) + e^{2\alpha t} \varsigma^T(t) \left(\sum_{j=1}^N r_{ij} P_j \right) \varsigma(t) \\
&= e^{2\alpha t} \left\{ \varsigma^T(t) \left[e_1^T \left(2\alpha P_i + \left(\sum_{j=1}^N r_{ij} P_j \right) \right) e_1 + 2e_1^T P_i e_{3m+4} \right] \varsigma(t) \right\},
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{L}\mathcal{V}_2(\varsigma(t), i, t) &= e^{2\alpha t} \left\{ \varsigma^T(t) \left[\sum_{v=1}^m e^{2\alpha d_v} Q_{v,i} + \sum_{v=1}^m \alpha_{1,v} Q_v \right] \varsigma(t) - \sum_{v=1}^m \varsigma^T(t - d_v) Q_{v,i} \varsigma(t - d_v) \right\} \\
&\quad - e^{2\alpha t} \sum_{v=1}^m \int_{t-d_v}^t \varsigma^T(s) Q_v \varsigma(s) ds + \sum_{v=1}^m \int_{t-d_v}^t e^{2\alpha(s+d_v)} \varsigma^T(s) \left(\sum_{j=1}^N r_{ij} Q_{v,j} \right) \varsigma(s) ds.
\end{aligned}$$

It follows from (11) and $Q_v > 0$ that

$$\sum_{v=1}^m \int_{t-d_v}^t e^{2\alpha(s+d_v)} \varsigma^T(s) \left(\sum_{j=1}^N r_{ij} Q_{v,j} \right) \varsigma(s) ds \leq \sum_{v=1}^m \int_{t-d_v}^t e^{2\alpha s} \varsigma^T(s) Q_v \varsigma(s) ds \leq e^{2\alpha t} \sum_{v=1}^m \int_{t-d_v}^t \varsigma^T(s) Q_v \varsigma(s) ds,$$

that is, from (11), there is the following inequality

$$\sum_{v=1}^m \int_{t-d_v}^t e^{2\alpha(s+d_v)} \varsigma^T(s) \left(\sum_{j=1}^N r_{ij} Q_{v,j} \right) \varsigma(s) ds - e^{2\alpha t} \sum_{v=1}^m \int_{t-d_v}^t \varsigma^T(s) Q_v \varsigma(s) ds \leq 0. \quad (16)$$

Therefore,

$$\begin{aligned}
\mathcal{L}\mathcal{V}_2(\varsigma(t), i, t) &\leq e^{2\alpha t} \left\{ \varsigma^T(t) \left[\sum_{v=1}^m e^{2\alpha d_v} Q_{v,i} + \sum_{v=1}^m \alpha_{1,v} Q_v \right] \varsigma(t) - \sum_{v=1}^m \varsigma^T(t - d_v) Q_{v,i} \varsigma(t - d_v) \right\} \\
&= e^{2\alpha t} \varsigma^T(t) \left[e_1^T \left(\sum_{v=1}^m e^{2\alpha d_v} Q_{v,i} + \sum_{v=1}^m \alpha_{1,v} Q_v \right) e_1 - \sum_{v=1}^m e_{v+1}^T Q_{v,i} e_{v+1} \right] \varsigma(t).
\end{aligned}$$

$$\begin{aligned}
\mathcal{LV}_3(\zeta(t), i, t) &= e^{2\alpha t} \left[\sum_{v=1}^m \alpha_{1,v} \dot{\zeta}^T(t) \mathcal{R}_{v,i} \dot{\zeta}(t) + \sum_{v=1}^m \alpha_{2,v} \dot{\zeta}^T(t) \mathcal{R}_v \dot{\zeta}(t) - \sum_{v=1}^m \int_{t-d_v}^t \dot{\zeta}^T(s) \mathcal{R}_{v,i} \dot{\zeta}(s) ds \right] \\
&\quad + \sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \dot{\zeta}^T(s) \left(\sum_{j=1}^N r_{ij} \mathcal{R}_{v,j} \right) \dot{\zeta}(s) ds d\theta \\
&\quad - e^{2\alpha t} \sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t \dot{\zeta}^T(s) \mathcal{R}_v \dot{\zeta}(s) ds d\theta.
\end{aligned}$$

Similarly, the inequality (12) implies the following inequality:

$$\sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \dot{\zeta}^T(s) \left(\sum_{j=1}^N r_{ij} \mathcal{R}_{v,j} \right) \dot{\zeta}(s) ds d\theta - e^{2\alpha t} \sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t \dot{\zeta}^T(s) \mathcal{R}_v \dot{\zeta}(s) ds d\theta \leq 0. \quad (17)$$

We further apply Lemma 2 and get

$$\begin{aligned}
-\sum_{v=1}^m \int_{t-d_v}^t \dot{\zeta}^T(s) \mathcal{R}_{v,i} \dot{\zeta}(s) ds &\leq \sum_{v=1}^m \zeta^T(t) \left[d_v \sum_{\kappa=1}^3 \frac{1}{2\kappa-1} \pi_{12,v}^T M_{\kappa,v,i} \mathcal{R}_{v,i}^{-1} M_{\kappa,v,i}^T \pi_{12,v} \right. \\
&\quad \left. + \sum_{v=1}^m \text{sym}\{\pi_{12,v}^T (M_{1,v,i} \pi_{7,v} + M_{2,v,i} \pi_{8,v} + M_{3,v,i} \pi_{9,v})\} \right] \zeta(t),
\end{aligned}$$

that is,

$$\begin{aligned}
\mathcal{LV}_3(\zeta(t), i, t) &\leq e^{2\alpha t} \left[\sum_{v=1}^m \alpha_{1,v} \dot{\zeta}^T(t) \mathcal{R}_{v,i} \dot{\zeta}(t) + \sum_{v=1}^m \alpha_{2,v} \dot{\zeta}^T(t) \mathcal{R}_v \dot{\zeta}(t) - \sum_{v=1}^m \int_{t-d_v}^t \dot{\zeta}^T(s) \mathcal{R}_{v,i} \dot{\zeta}(s) ds \right] \\
&\leq e^{2\alpha t} \zeta^T(t) \left\{ \left[e_{3m+4}^T \left(\sum_{v=1}^m \alpha_{1,v} \mathcal{R}_{v,i} + \sum_{v=1}^m \alpha_{2,v} \mathcal{R}_v \right) e_{3m+4} \right] \right. \\
&\quad + \left[\sum_{v=1}^m d_v \sum_{\kappa=1}^3 \frac{1}{2\kappa-1} \pi_{12,v}^T M_{\kappa,v,i} \mathcal{R}_{v,i}^{-1} M_{\kappa,v,i}^T \pi_{12,v} \right. \\
&\quad \left. \left. + \sum_{v=1}^m \text{sym}\{\pi_{12,v}^T (M_{1,v,i} \pi_{7,v} + M_{2,v,i} \pi_{8,v} + M_{3,v,i} \pi_{9,v})\} \right] \right\} \zeta(t). \\
\mathcal{LV}_4(\zeta(t), i, t) &= e^{2\alpha t} \dot{\zeta}^T(t) (U_i + h_M U) \dot{\zeta}(t) - \int_{t-h_M}^t e^{2\alpha s} \dot{\zeta}^T(s) U \dot{\zeta}(s) ds + \int_{t_k}^t e^{2\alpha s} \dot{\zeta}^T(s) \left(\sum_{j=1}^N r_{ij} U_j \right) \dot{\zeta}(s) ds \\
&\leq e^{2\alpha t} \dot{\zeta}^T(t) (U_i + h_M U) \dot{\zeta}(t) - \int_{t_k}^t e^{2\alpha s} \dot{\zeta}^T(s) U \dot{\zeta}(s) ds + \int_{t_k}^t e^{2\alpha s} \dot{\zeta}^T(s) \left(\sum_{j=1}^N r_{ij} U_j \right) \dot{\zeta}(s) ds.
\end{aligned}$$

The inequality (13) implies the following inequality:

$$\int_{t_k}^t e^{2\alpha(s+t-t_k)} \dot{\zeta}^T(s) \left(\sum_{j=1}^N r_{ij} U_j \right) \dot{\zeta}(s) ds - \int_{t_k}^t e^{2\alpha(s+t-t_k)} \dot{\zeta}^T(s) U \dot{\zeta}(s) ds \leq 0, \quad (18)$$

so it is easy to get

$$\begin{aligned}
\mathcal{LV}_4(\zeta(t), i, t) &\leq e^{2\alpha t} \dot{\zeta}^T(t) (U_i + h_M U) \dot{\zeta}(t) \\
&= e^{2\alpha t} \zeta^T(t) e_{3m+4}^T (U_i + h_M U) e_{3m+4} \zeta(t). \\
\mathcal{LV}_5(\zeta(t), i, t) &= e^{2\alpha t} \left\{ [\zeta^T(t) - \zeta(t_k)^T] \left(4\alpha X_i + 2 \left(\sum_{j=1}^N r_{ij} X_j \right) \right) [\zeta(t) - \zeta(t_{k+1})] \right\}
\end{aligned}$$

$$\begin{aligned}
& + 2\varsigma^T(t)X_i[\varsigma(t) - \varsigma(t_k + 1)] + 2[\varsigma^T(t) - \varsigma(t_k)^T]X_i\dot{\varsigma}(t) \Big\} \\
& = 2e^{2\alpha t}\varsigma^T(t) \left[\pi_{14}^T \left(2\alpha X_i + \left(\sum_{j=1}^N r_{ij} X_j \right) \right) \pi_{15} + e_{3m+4}^T X_i \pi_{15} + \pi_{14}^T X_i e_{3m+4} \right] \varsigma(t). \\
\mathcal{LV}_6(\varsigma(t), i, t) & = e^{2\alpha t} \varsigma^T(t) \pi_{13}^T \left\{ 2\alpha(t_{k+1} - t)(t - t_k)Z_i + (t_{k+1} - t)Z_i - (t - t_k)Z_i \right. \\
& \quad \left. + (t_{k+1} - t)(t - t_k) \left(\sum_{j=1}^N r_{ij} Z_j \right) \right\} \pi_{13} \varsigma(t) \\
& \leq e^{2\alpha t} \varsigma^T(t) \pi_{13}^T \left\{ 2\alpha h_M^2 Z_i + [(t_{k+1} - t) - (t - t_k)]Z_i + h_M^2 \left(\sum_{j=1}^N r_{ij} Z_j \right) \right\} \pi_{13} \varsigma(t). \\
\mathcal{LV}_7(\varsigma(t), i, t) & = 2e^{2\alpha t} \varsigma^T(t) \left\{ \pi_1^T (Y_i \pi_2 + S_i \pi_3) + (t_{k+1} - t) [\pi_5^T (Y_i \pi_2 + S_i \pi_3) + \pi_{10}^T Y_i \pi_4] \right. \\
& \quad \left. + (t - t_k) [\pi_6^T (Y_i \pi_2 + S_i \pi_3) + \pi_{11}^T Y_i \pi_4] \right\} \varsigma(t) \\
& \quad + 4\alpha e^{2\alpha t} \varsigma^T(t) \left\{ (t_{k+1} - t) [\pi_{10}^T (Y_i \pi_2 + S_i \pi_3)] + (t - t_k) [\pi_{11}^T (Y_i \pi_2 + S_i \pi_3)] \right\} \varsigma(t) \\
& \quad + 2e^{2\alpha t} \varsigma^T(t) \left\{ (t_{k+1} - t) \left[\pi_{10}^T \left(\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right) \right] \right. \\
& \quad \left. + (t - t_k) \left[\pi_{11}^T \left(\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right) \right] \right\} \varsigma(t) \\
& = 2e^{2\alpha t} \varsigma^T(t) \left\{ \pi_1^T (Y_i \pi_2 + S_i \pi_3) + (t_{k+1} - t) \left[(\pi_5^T + 2\alpha \pi_{10}^T) (Y_i \pi_2 + S_i \pi_3) + \pi_{10}^T Y_i \pi_4 \right. \right. \\
& \quad \left. \left. + \pi_{10}^T \left(\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right) \right] + (t - t_k) \left[(\pi_6^T + 2\alpha \pi_{11}^T) (Y_i \pi_2 + S_i \pi_3) \right. \right. \\
& \quad \left. \left. + \pi_{11}^T Y_i \pi_4 + \pi_{11}^T \left(\left(\sum_{j=1}^N r_{ij} Y_j \right) \pi_2 + \left(\sum_{j=1}^N r_{ij} S_j \right) \pi_3 \right) \right] \right\} \varsigma(t).
\end{aligned}$$

In addition, for any a free weighting matrix G_i , and scalars γ and δ , we always have

$$\begin{aligned}
0 & = 2e^{2\alpha t} [\delta \varsigma^T(t) G_i + \dot{\varsigma}^T(t) G_i + \gamma \varsigma^T(t_k) G_i] \\
& \quad \times \left[-\dot{\varsigma}(t) + \mathcal{A}_i \varsigma(t) + \sum_{v=1}^m \mathcal{B}_{v,i} \varsigma(t - d_v) + \mathcal{W}_i \xi(\mathcal{D}\varsigma(t), y(t)) - \mathcal{K}_i \mathcal{C}_i \varsigma(t_k) \right] \\
& = 2e^{2\alpha t} \varsigma^T(t) \left[(\delta e_1^T G_i + e_{3m+4}^T G_i + \gamma e_{3m+2}^T G_i) \right. \\
& \quad \left. \times (-e_{3m+4} + \mathcal{A}_i e_1 + \sum_{v=1}^m \mathcal{B}_{v,i} e_{v+1} + \mathcal{W}_i e_{3m+5} - \mathcal{K}_i \mathcal{C}_i e_{3m+2}) \right] \varsigma(t). \tag{19}
\end{aligned}$$

Furthermore, on the basis of (5), the following inequality is satisfied:

$$\begin{aligned}
0 & \leq 2e^{2\alpha t} \frac{t_{k+1} - t}{h_k} [\varsigma(t)^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{1,i} \xi(\mathcal{D}\varsigma(t), y(t)) - \xi(\mathcal{D}\varsigma(t), y(t))^T \mathcal{V}_{1,i} \xi(\mathcal{D}\varsigma(t), y(t))] \\
& \quad + 2e^{2\alpha t} \frac{t - t_k}{h_k} [\varsigma(t)^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{2,i} \xi(\mathcal{D}\varsigma(t), y(t)) - \xi(\mathcal{D}\varsigma(t), y(t))^T \mathcal{V}_{2,i} \xi(\mathcal{D}\varsigma(t), y(t))], \\
& = 2e^{2\alpha t} \varsigma^T(t) \left[\frac{t_{k+1} - t}{h_k} (e_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{1,i} e_{3m+5} - e_{3m+5}^T \mathcal{V}_{1,i} e_{3m+5}) \right. \\
& \quad \left. + \frac{t - t_k}{h_k} (e_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_{2,i} e_{3m+5} - e_{3m+5}^T \mathcal{V}_{2,i} e_{3m+5}) \right] \varsigma(t). \tag{20}
\end{aligned}$$

We insert the right side of (19) and (20) to $\mathcal{LV}$ and let $L_i = G_i \mathcal{X}_i$, then $\mathcal{LV}(\varsigma(t), i, t)$ can be expressed as:

$$\mathcal{LV}(\varsigma(t), i, t) \leq e^{2\alpha t} \varsigma^T(t) \left[\frac{t_{k+1} - t}{h_k} \hat{\Xi}_1(h_k) + \frac{t - t_k}{h_k} \hat{\Xi}_2(h_k) \right] \varsigma(t), \quad (21)$$

where

$$\begin{aligned} \hat{\Xi}_1(h_k) &= \Phi_1 + \Phi_2 + h_k \Phi_3 + \sum_{v=1}^m \sum_{\kappa=1}^3 \frac{d_v}{2\kappa - 1} \pi_{12,v}^T M_{\kappa,v,i} \mathcal{R}_{v,i}^{-1} M_{\kappa,v,i}^T \pi_{12,v}, \\ \hat{\Xi}_2(h_k) &= \Phi_1 + \Phi_4 + h_k \Phi_5 + \sum_{v=1}^m \sum_{\kappa=1}^3 \frac{d_v}{2\kappa - 1} \pi_{12,v}^T M_{\kappa,v,i} \mathcal{R}_{v,i}^{-1} M_{\kappa,v,i}^T \pi_{12,v}. \end{aligned}$$

It is noted that

$$\begin{aligned} \hat{\Xi}_1(h_k) &= \frac{h_k}{h_M} \hat{\Xi}_1(h_M) + \frac{h_M - h_k}{h_M} \hat{\Xi}_1(h_m), \\ \hat{\Xi}_2(h_k) &= \frac{h_k}{h_M} \hat{\Xi}_2(h_M) + \frac{h_M - h_k}{h_M} \hat{\Xi}_2(h_m). \end{aligned}$$

From (9) and (13), we can find that

$$\hat{\Xi}_1(h_k) < 0, \quad \hat{\Xi}_2(h_k) < 0.$$

Thus, we can further get that

$$\mathcal{LV}(\varsigma(t), i, t) \leq 0, \quad t \in [t_k, t_{k+1}). \quad (22)$$

It suggests that, for $t \in [t_k, t_{k+1})$, one has

$$\mathbb{E}\{\mathcal{V}(t)\} \leq \mathbb{E}\{\mathcal{V}(t_k)\} \leq \mathbb{E}\{\mathcal{V}(t_{k-1})\} \leq \dots \leq \mathbb{E}\{\mathcal{V}(0)\}. \quad (23)$$

On the basis of Lemma 1 and (23), we can conclude that

$$\begin{aligned} \mathbb{E}\{\|\varsigma(t)\|^2\} &\leq \theta_1 \mathbb{E}\{\|\varsigma(t_k)\|^2\} + \mathbb{E}\left\{ \sum_{v=1}^m \int_{t_k - d_v}^{t_k} \theta_2 \varepsilon \|\varsigma(s)\|^2 ds \right\} \\ &= \sum_{v=1}^m \frac{\theta_1}{\lambda_{\min}(P_i) e^{2\alpha t_k}} e^{2\alpha t_k} \lambda_{\min}(P_i) \mathbb{E}\{\|\varsigma(t_k)\|^2\} \\ &\quad + \mathbb{E}\left\{ \sum_{v=1}^m \int_{t_k - d_v}^{t_k} \frac{\theta_2 \lambda_{\min}(Q_{v,i}) e^{2\alpha(t_k + d_v)} \varepsilon \|\varsigma(s)\|^2 ds}{\lambda_{\min}(Q_{v,i}) e^{2\alpha(t_k + d_v)}} \right\} \\ &\leq \sum_{v=1}^m \frac{\theta_1}{\lambda_{\min}(P_i) e^{2\alpha t_k}} \mathbb{E}\left\{ e^{2\alpha t_k} \varsigma^T(t_k) P_i z(t_k) \right\} \\ &\quad + \mathbb{E}\left\{ \sum_{v=1}^m \frac{\theta_2 \varepsilon \int_{t_k - d_v}^{t_k} e^{2\alpha(t_k + d_v)} \varsigma^T(s) (Q_{v,i}) \varsigma(s) ds}{\lambda_{\min}(Q_{v,i}) e^{2\alpha(t_k + d_v)}} \right\} \\ &\leq \sum_{v=1}^m \max \left\{ \frac{\theta_1}{\lambda_{\min}(P_i) e^{2\alpha t_k}}, \frac{\theta_2 \varepsilon}{\lambda_{\min}(Q_{v,i}) e^{2\alpha(t_k + d_v)}} \right\} \mathbb{E}\{\mathcal{V}_1(t_k) + \mathcal{V}_2(t_k)\} \\ &\leq \sum_{v=1}^m \max \left\{ \frac{\theta_1}{\lambda_{\min}(P_i) e^{2\alpha t_k}}, \frac{\theta_2 \varepsilon}{\lambda_{\min}(Q_{v,i}) e^{2\alpha(t_k + d_v)}} \right\} \mathbb{E}\{\mathcal{V}(t_k)\} \end{aligned}$$

$$\leq \sum_{v=1}^m e^{2\alpha h_M} \max \left\{ \frac{\theta_1}{\lambda_{\min}(P_i)}, \frac{\theta_2 \varepsilon}{\lambda_{\min}(Q_{v,i}) e^{2\alpha d_v}} \right\} e^{-2\alpha t} \mathbb{E} \{ \mathcal{V}(0) \}. \quad (24)$$

It can be computed that

$$\begin{aligned} \mathbb{E} \{ \mathcal{V}(0) \} &= \mathbb{E} \{ \varsigma^T(0) P_i \varsigma(0) \} + \mathbb{E} \left\{ \sum_{v=1}^m \int_{-d_v}^0 e^{2\alpha(s+d_v)} \varsigma^T(s) Q_{v,i} \varsigma(s) ds \right\} \\ &\quad + \mathbb{E} \left\{ \sum_{v=1}^m \int_{-d_v}^0 \int_{\theta}^0 e^{2\alpha(s-\theta)} \varsigma^T(s) Q_v \varsigma(s) ds d\theta \right\} \\ &\quad + \mathbb{E} \left\{ \sum_{v=1}^m \int_{-d_v}^0 \int_{\theta}^0 e^{2\alpha(s-\theta)} \dot{\varsigma}^T(s) \mathcal{R}_{v,i} \dot{\varsigma}(s) ds d\theta \right\} \\ &\quad + \mathbb{E} \left\{ \sum_{v=1}^m \int_{-d_v}^0 \int_{\theta}^0 \int_{\beta}^0 e^{2\alpha(s-\beta)} \dot{\varsigma}^T(s) \mathcal{R}_v \dot{\varsigma}(s) ds d\theta \right\} \\ &\quad + \mathbb{E} \left\{ \int_{-h_M}^0 \int_{\theta}^0 e^{2\alpha s} \dot{\varsigma}^T(s) U \dot{\varsigma}(s) ds d\theta \right\} \\ &\leq \mathbb{E} \{ \lambda_{\max}(P_i) \|\varsigma(0)\|^2 \} + \mathbb{E} \left\{ \sum_{v=1}^m \alpha_{1,v} \lambda_{\max}(Q_{v,i}) \sup_{-d_v \leq \theta \leq 0} \|\varsigma(\theta)\|^2 \right\} \\ &\quad + \mathbb{E} \left\{ \sum_{v=1}^m \alpha_{2,v} \lambda_{\max}(Q_v) \sup_{-d_v \leq \theta \leq 0} \|\varsigma(\theta)\|^2 \right\} + \mathbb{E} \left\{ \sum_{v=1}^m \alpha_{2,v} \lambda_{\max}(\mathcal{R}_{v,i}) \sup_{-d_v \leq \theta \leq 0} \|\dot{\varsigma}(\theta)\|^2 \right\} \\ &\quad + \mathbb{E} \left\{ \sum_{v=1}^m \alpha_{3,v} \lambda_{\max}(\mathcal{R}_v) \sup_{-d_v \leq \theta \leq 0} \|\dot{\varsigma}(\theta)\|^2 \right\} + \mathbb{E} \left\{ \alpha_4 \lambda_{\max}(U) \sup_{-h_M \leq \theta \leq 0} \|\dot{\varsigma}(\theta)\|^2 \right\} \\ &\leq (\varrho_1 + \varrho_2 + \varrho_3) \sup_{-d_v \leq \theta \leq 0} \mathbb{E} \{ \|\varsigma(\theta)\|^2 \} + (\varrho_4 + \varrho_5 + \varrho_6) \sup_{-\chi \leq \theta \leq 0} \mathbb{E} \{ \|\dot{\varsigma}(\theta)\|^2 \} \\ &\leq \varrho \sup_{-\chi \leq \theta \leq 0} \mathbb{E} \{ \|\varsigma(\theta)\|^2 + \|\dot{\varsigma}(\theta)\|^2 \}, \end{aligned} \quad (25)$$

where $\varrho = \max \{ \varrho_1 + \varrho_2 + \varrho_3, \varrho_4 + \varrho_5 + \varrho_6 \}$,

$$\begin{aligned} \varrho_1 &= \max_{i \in \mathcal{N}} \lambda_{\max}(P_i), \\ \varrho_2 &= \sum_{v=1}^m \alpha_{1,v} \max_{i \in \mathcal{N}} \lambda_{\max}(Q_{v,i}), \\ \varrho_3 &= \sum_{v=1}^m \alpha_{2,v} \lambda_{\max}(Q_v), \\ \varrho_4 &= \sum_{v=1}^m \alpha_{2,v} \max_{i \in \mathcal{N}} \lambda_{\max}(\mathcal{R}_{v,i}), \\ \varrho_5 &= \sum_{v=1}^m \alpha_{3,v} \lambda_{\max}(\mathcal{R}_v), \\ \varrho_6 &= \alpha_4 \lambda_{\max}(U), \\ \chi &= \max \{ d_v, h_M \}, v = 1, 2, \dots, m. \end{aligned}$$

Combining (24) with (25), we can get that

$$\mathbb{E} \{ \|\varsigma(t)\|^2 \} \leq \sum_{v=1}^m e^{2\alpha h_M} \max \left\{ \frac{\theta_1}{\lambda_{\min}(P_i)}, \frac{\theta_2 \varepsilon}{\lambda_{\min}(Q_{v,i}) e^{2\alpha d_v}} \right\} \varrho e^{-2\alpha t} \mathbb{E} \{ \|\varsigma_0\|_c^2 \}, \quad v = 1, 2, \dots, m. \quad (26)$$

Hence, by Definition 1, the master-slave systems (1) and (2) are exponentially synchronous in the mean square. $\square$

Remark 2. In the proof of Theorem 1, we introduced mode-dependent single/double integral terms and considered the mode-dependent sampled-state information to construct functionals. For example, since dif-

ferent U_i is selected for distinct system mode, $\int_{t_k}^t e^{2\alpha s} \dot{\varsigma}^T(s) (\sum_{j=1}^N r_{ij} U_j) \dot{\varsigma}(s) ds$ is consequentially involved in $\mathcal{LV}_4(\varsigma(t), i, t)$. To achieve the stability result, we need to cope with it. As seen clearly from (18), $\int_{t_k}^t e^{2\alpha s} \dot{\varsigma}^T(s) (\sum_{j=1}^N r_{ij} U_j) \dot{\varsigma}(s) ds$ is efficiently eliminated by $-\int_{-h_M}^0 \int_{t+\theta}^t e^{2\alpha s} \dot{\varsigma}^T(s) U \dot{\varsigma}(s) ds d\theta$. Similarly, it can be seen from (16) and (17), the introduction of $\sum_{v=1}^m \int_{-d_v}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \dot{\varsigma}^T(s) Q_v \dot{\varsigma}(s) ds d\theta$ and $\sum_{v=1}^m \int_{-d_v}^0 \int_{\theta}^0 \int_{t+\beta}^t e^{2\alpha(s-\beta)} \dot{\varsigma}^T(s) \mathcal{R}_v \dot{\varsigma}(s) ds d\theta$ is to ensure distinct $Q_{v,i}$ and $\mathcal{R}_{v,i}$ can be applied to different system mode, that's the reason why the mode-independent double/triple integral terms are constructed in the Lyapunov functionals.

Remark 3. In Theorem 1, one of the innovation of Lyapunov functionals $\mathcal{V}(\varsigma(t), i, t)$ in (15) lies in that some mode-dependent two-sided functions i.e., $\mathcal{V}_5, \mathcal{V}_6, \mathcal{V}_7$ which not only fully utilize the interval message from $\varsigma(t_k)$ to $\varsigma(t)$ but also consider the information from $\varsigma(t)$ to $\varsigma(t_{k+1})$. The looped-functional approach has been used to analyze system stability in [42]. However, Ref. [42] did not consider the influence of Markovian parameters on the construction of functionals. To bridge this gap, this paper constructs mode-dependent looped-functional.

To make comparison, on the basis of Theorem 1, a simplified delayed system is proposed in which Markovian parameters is not considered.

Consider the following master system:

$$\mathcal{M} : \begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}x(t-d) + \mathcal{W}\eta(\mathcal{D}x(t)), \\ q(t) = \mathcal{C}x(t), \end{cases} \quad (27)$$

and slave system

$$\mathcal{N} : \begin{cases} \dot{y}(t) = \mathcal{A}y(t) + \mathcal{B}y(t-d) + \mathcal{W}\eta(\mathcal{D}y(t)) + u(t), \\ p(t) = \mathcal{C}y(t), \end{cases} \quad (28)$$

where $u(t) = \mathcal{K}(q(t_k) - p(t_k))$, $t_k \leq t < t_{k+1}$, d is the constant time-delay. Hence, the error system (3) reduces to

$$\dot{\varsigma}(t) = \mathcal{A}\varsigma(t) + \mathcal{B}\varsigma(t-d) + \mathcal{W}\xi(\mathcal{D}\varsigma(t), y(t)) - \mathcal{K}\mathcal{C}\varsigma(t_k). \quad (29)$$

For simplicity, we use nomenclature for vectors and matrices in the following.

$$\begin{aligned} \omega_3 &= \frac{1}{d} \int_{t-d}^t \varsigma(s) ds, & \omega_4 &= \frac{2}{d^2} \int_{t-d}^t \int_{t-d}^s \varsigma(u) du ds, & \alpha_5 &= \frac{e^{2\alpha d} - 1}{2\alpha}, \\ \bar{\varsigma}^T(t) &= [\varsigma^T(t), \varsigma^T(t-d), \omega_3^T(t), \omega_4^T(t), \varsigma^T(t_k), \varsigma^T(t_{k+1}), \dot{\varsigma}^T(t), \xi^T(\mathcal{D}\varsigma(t), y(t))], \\ \bar{e}_{\bar{t}} &= [0_{n \times (\bar{t}-1)n}, I_{n \times n}, 0_{n \times (8-\bar{t})n}], \quad \bar{t} = 1, 2, \dots, 8. \end{aligned}$$

Corollary 1. Given scalars $\alpha > 0$, δ , and γ , the error system (29) is exponentially stable if there exist positive definite matrices $P, Q, \mathcal{R}, U$, matrices $Z, X, Y, S, M_1, M_2, M_3, G, L$, and diagonal matrices

$\mathcal{V}_1 > 0, \mathcal{V}_2 > 0$ such that the following conditions hold for $h_k \in \{h_m, h_M\}$:

$$\check{\Xi}_1(h_k) = \begin{bmatrix} \bar{\Phi}_1 + \bar{\Phi}_2 + h_k \bar{\Phi}_3 & \sqrt{d} \bar{\Xi} \\ * & \bar{\mathcal{R}} \end{bmatrix} < 0, \quad (30)$$

$$\check{\Xi}_2(h_k) = \begin{bmatrix} \bar{\Phi}_1 + \bar{\Phi}_4 + h_k \bar{\Phi}_5 & \sqrt{d} \bar{\Xi} \\ * & \bar{\mathcal{R}} \end{bmatrix} < 0, \quad (31)$$

where

$$\begin{aligned} \bar{\pi}_1 &= [\bar{e}_5^T - \bar{e}_1^T, \bar{e}_1^T - \bar{e}_6^T]^T, \quad \bar{\pi}_2 = [\bar{e}_1^T - \bar{e}_5^T, \bar{e}_1^T - \bar{e}_6^T]^T, \quad \bar{\pi}_3 = [\bar{e}_5^T, \bar{e}_6^T]^T, \\ \bar{\pi}_4 &= [\bar{e}_7^T, \bar{e}_7^T]^T, \quad \bar{\pi}_5 = [\bar{e}_7^T, 0]^T, \quad \bar{\pi}_6 = [0, \bar{e}_7^T]^T, \quad \bar{\pi}_7 = \bar{e}_1 - \bar{e}_2, \\ \bar{\pi}_8 &= \bar{e}_1 + \bar{e}_2 - 2\bar{e}_3, \quad \bar{\pi}_9 = \bar{e}_1 - \bar{e}_2 - 6\bar{e}_3 + 6\bar{e}_4, \quad \bar{\pi}_{10} = [\bar{e}_1^T - \bar{e}_5^T, 0]^T, \\ \bar{\pi}_{11} &= [0, \bar{e}_1^T - \bar{e}_6^T]^T, \quad \bar{\pi}_{12} = [\bar{e}_1^T, \bar{e}_2^T, \bar{e}_3^T, \bar{e}_4^T]^T, \\ \bar{\pi}_{13} &= [\bar{e}_5^T, \bar{e}_6^T]^T, \quad \bar{\pi}_{14} = \bar{e}_1 - \bar{e}_5, \quad \bar{\pi}_{15} = \bar{e}_1 - \bar{e}_6, \\ \bar{\Phi}_1 &= \bar{e}_1^T (2\alpha P + e^{2\alpha d} Q) \bar{e}_1 - \bar{e}_2^T Q \bar{e}_2 + \alpha_5 \bar{e}_7^T \bar{\mathcal{R}} \bar{e}_7 + \bar{e}_7^T U \bar{e}_7 + \bar{\pi}_{13}^T 2\alpha h_M^2 Z \bar{\pi}_{13} \\ &\quad + \text{sym}\{\bar{e}_1^T P \bar{e}_7 + \bar{\pi}_1^T (Y \bar{\pi}_2 + S \bar{\pi}_3) + \bar{\pi}_{14}^T 2\alpha X \bar{\pi}_{15} + \bar{e}_7^T X \bar{\pi}_{15} + \bar{\pi}_{14}^T X \bar{e}_7 \\ &\quad + (\delta \bar{e}_1^T G + \bar{e}_7^T G + \gamma \bar{e}_5^T G)(-\bar{e}_7 + \mathcal{A} \bar{e}_1 + \mathcal{B} \bar{e}_2 + \mathcal{W} \bar{e}_8 - \mathcal{K} \mathcal{C} \bar{e}_5) \\ &\quad + \bar{\pi}_{12}^T (M_1 \bar{\pi}_7 + M_2 \bar{\pi}_8 + M_3 \bar{\pi}_9)\}, \\ \bar{\Phi}_2 &= \text{sym}\{\bar{e}_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_1 \bar{e}_8\} - 2\bar{e}_8^T \mathcal{V}_1 \bar{e}_8, \\ \bar{\Phi}_3 &= \text{sym}\{(\bar{\pi}_5^T + 2\alpha \bar{\pi}_{10}^T)(Y \bar{\pi}_2 + S \bar{\pi}_3) + \bar{\pi}_{10}^T Y \bar{\pi}_4\} + \bar{\pi}_{13}^T Z \bar{\pi}_{13}, \\ \bar{\Phi}_4 &= \text{sym}\{\bar{e}_1^T \mathcal{D}^T \mathcal{S} \mathcal{V}_2 \bar{e}_8\} - 2\bar{e}_8^T \mathcal{V}_2 \bar{e}_8, \\ \bar{\Phi}_5 &= \text{sym}\{(\bar{\pi}_6^T + 2\alpha \bar{\pi}_{11}^T)(Y \bar{\pi}_2 + S \bar{\pi}_3) + \bar{\pi}_{11}^T Y \bar{\pi}_4\} - \bar{\pi}_{13}^T Z \bar{\pi}_{13}, \\ \bar{\Xi} &= [\bar{\pi}_{12}^T M_1 \quad \bar{\pi}_{12}^T M_2 \quad \bar{\pi}_{12}^T M_3], \\ \bar{\mathcal{R}} &= \text{diag}\{-\mathcal{R} \quad -3\mathcal{R} \quad -5\mathcal{R}\}. \end{aligned}$$

Proof. Introduce simplified two-sided looped-functional,

$$V(t) = \sum_{\epsilon=1}^7 V_\epsilon(t), \quad (32)$$

with

$$\begin{aligned} V_1(t) &= e^{2\alpha t} \zeta^T(t) P \zeta(t), \\ V_2(t) &= \int_{t-d}^t e^{2\alpha(s+d)} \zeta^T(s) Q \zeta(s) ds, \\ V_3(t) &= \int_{-d}^0 \int_{t+\theta}^t e^{2\alpha(s-\theta)} \zeta^T(s) \mathcal{R} \zeta(s) ds d\theta, \\ V_4(t) &= \int_{t_k}^t e^{2\alpha s} \zeta^T(s) U \zeta(s) ds, \end{aligned}$$

$$\begin{aligned}
V_5(t) &= 2e^{2\alpha t} \mu_1^T(t) X \mu_2(t), \\
V_6(t) &= (t_{k+1} - t)(t - t_k) e^{2\alpha t} \zeta_3^T Z \zeta_3, \\
V_7(t) &= 2e^{2\alpha t} \zeta_1^T(t) [Y \zeta_2(t) + S \zeta_3].
\end{aligned}$$

According to the derivative of $V(t)$ along the trajectory of the error system (29), we can obtain

$$\begin{aligned}
\dot{V}_1(t) &= 2e^{2\alpha t} \bar{\zeta}^T(t) (\bar{e}_1^T \alpha P \bar{e}_1 + \bar{e}_1^T P \bar{e}_7) \bar{\zeta}(t), \\
\dot{V}_2(t) &= e^{2\alpha t} \bar{\zeta}^T(t) (e^{2\alpha d} \bar{e}_1^T Q \bar{e}_1 - \bar{e}_2^T Q \bar{e}_2) \bar{\zeta}(t), \\
\dot{V}_3(t) &= e^{2\alpha t} \bar{\zeta}^T(t) \{ \text{sym}[\bar{\pi}_{12}^T (M_1 \bar{\pi}_7 + M_2 \bar{\pi}_8 + M_3 \bar{\pi}_9)] + \alpha_5 \bar{e}_7^T \mathcal{R} \bar{e}_7 \\
&\quad + d \sum_{\kappa=1}^3 \frac{1}{2\kappa - 1} \bar{\pi}_{12}^T M_\kappa^T \mathcal{R}^{-1} M_\kappa \bar{\pi}_{12} \} \bar{\zeta}(t), \\
\dot{V}_4(t) &= e^{2\alpha t} \bar{\zeta}^T(t) \bar{e}_7^T U \bar{e}_7 \bar{\zeta}(t), \\
\dot{V}_5(t) &= 2e^{2\alpha t} \bar{\zeta}^T(t) (\bar{\pi}_{14}^T 2\alpha X \bar{\pi}_{15} + \bar{e}_7^T X \bar{\pi}_{15} + \bar{\pi}_{14}^T X \bar{e}_7) \bar{\zeta}(t), \\
\dot{V}_6(t) &= e^{2\alpha t} \bar{\zeta}^T(t) \bar{\pi}_{13}^T \{ 2\alpha h_M^2 Z + [(t_{k+1} - t) - (t - t_k)] Z \} \bar{\pi}_{13} \bar{\zeta}(t), \\
\dot{V}_7(t) &= 2e^{2\alpha t} \bar{\zeta}^T(t) \{ \bar{\pi}_1^T (Y \bar{\pi}_2 + S \bar{\pi}_3) + (t_{k+1} - t) [(\bar{\pi}_5^T + 2\alpha \bar{\pi}_{10}^T) (Y \bar{\pi}_2 + S \bar{\pi}_3) \\
&\quad + \bar{\pi}_{10}^T Y \bar{\pi}_4] + (t - t_k) [(\bar{\pi}_6^T + 2\alpha \bar{\pi}_{11}^T) (Y \bar{\pi}_2 + S \bar{\pi}_3) + \bar{\pi}_{11}^T Y \bar{\pi}_4] \} \bar{\zeta}(t).
\end{aligned}$$

In addition, for any free weighting matrix G , and scalars γ and δ , we always have:

$$0 = 2e^{2\alpha t} [\delta \varsigma^T(t) G + \varsigma^T(t) G + \gamma \varsigma^T(t_k) G] [-\dot{\varsigma}(t) + \mathcal{A} \varsigma(t) + \mathcal{B} \varsigma(t - d) + \mathcal{W} \xi(\mathcal{D} \varsigma(t), y(t)) - \mathcal{K} \mathcal{C} \varsigma(t_k)],$$

and similarly the following inequality is satisfied:

$$\begin{aligned}
0 &\leq 2e^{2\alpha t} \frac{t_{k+1} - t}{h_k} [\varsigma(t)^T \mathcal{D}^T \mathcal{S} \mathcal{V}_1 \xi(\mathcal{D} \varsigma(t), y(t)) - \xi(\mathcal{D} \varsigma(t), y(t))^T \mathcal{V}_1 \xi(\mathcal{D} \varsigma(t), y(t))] \\
&\quad + 2e^{2\alpha t} \frac{t - t_k}{h_k} [\varsigma(t)^T \mathcal{D}^T \mathcal{S} \mathcal{V}_2 \xi(\mathcal{D} \varsigma(t), y(t)) - \xi(\mathcal{D} \varsigma(t), y(t))^T \mathcal{V}_2 \xi(\mathcal{D} \varsigma(t), y(t))].
\end{aligned}$$

Furthermore, referring to the proof of Theorem 1, the ASDC gain matrix in (29) is

$$\mathcal{K} = G^{-1} L. \quad (33)$$

□

4 Numerical Examples

In this section, two examples are given to demonstrate the reduced conservatism of the synchronization criterion and the effectiveness of the ASDC design approach.

Example 1. Consider a two mode ($i \in \mathcal{N} = 1, 2$) Chua's circuit with multiple time delays as the master

system:

$$\begin{cases} \dot{x}_1(t) = a_i[x_2(t) - m_i x_1(t) + g(x_1(t))] - c_{1,i}x_1(t - d_1) - c_{2,i}x_1(t - d_2), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) - c_{1,i}x_1(t - d_1) - c_{2,i}x_2(t - d_2), \\ \dot{x}_3(t) = -b_i x_2(t) + c_{1,i}[2x_1(t - d_1) - x_3(t - d_1)] + c_{2,i}[2x_1(t - d_2) - x_3(t - d_2)]. \end{cases}$$

with the nonlinear characteristics

$$g(x_1(t)) = \frac{1}{2}(m_i - m_0)(|x_1(t) + 1| - |x_1(t) - 1|).$$

Choose parameters $m_0 = -\frac{1}{7}$, $m_1 = \frac{2}{7}$, $m_2 = \frac{3}{7}$, $a_1 = 9$, $a_2 = 8$, $b_1 = 14.28$, $b_2 = 14.52$, $c_{1,1} = 0.1$, $c_{1,2} = 0.09$, $c_{2,1} = 0.09$, $c_{2,2} = 0.11$, and the constant time-delay $d_1 = 1$, $d_2 = 0.9$. It can be found that the corresponding system parameters are as follows:

$$\begin{aligned} \mathcal{A}_i &= \begin{bmatrix} -a_i m_i & a_i & 0 \\ 1 & -1 & 1 \\ 0 & -b_i & 0 \end{bmatrix}, \quad \mathcal{W}_i = \begin{bmatrix} a_i(m_i - m_0) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \mathcal{B}_{1,i} &= \begin{bmatrix} -c_{1,i} & 0 & 0 \\ -c_{1,i} & 0 & 0 \\ 2c_{1,i} & 0 & -c_{1,i} \end{bmatrix}, \quad \mathcal{B}_{2,i} = \begin{bmatrix} -c_{2,i} & 0 & 0 \\ -c_{2,i} & 0 & 0 \\ 2c_{2,i} & 0 & -c_{2,i} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -0.5 & 0.5 \\ 1 & -1 \end{bmatrix}. \end{aligned}$$

with $\eta_1(x_1(t)) = \frac{1}{2}(|x_1(t) + 1| - |x_1(t) - 1|)$ pertaining to sector $[0, 1]$, and $\eta_2(x_2(t)) = \eta_3(x_3(t)) = 0$. Choosing $\mathcal{C}_1 = \mathcal{C}_2 = [1 \ 0 \ 0]$, $\delta = 0.5$, $\gamma = 2$, $h_m = 10^{-5}$. Table 1 presents the maximum values of h_M for different α by using Theorem 1. It can be found that the choice of α has an impact on the value of h_M . Specifically, a larger α implies that the master-slave systems has faster synchronization speed, which corresponds to a smaller value of h_M .

Table 1: Maximum values of h_M for different α

α	0.1	0.2	0.3	0.4	0.5
h_M	0.2340	0.2050	0.1596	0.1347	0.1300

Choosing $\alpha = 0.3$ and $h_M = 0.1596$, by solving the LMIs (9)–(13), the gain matrix $\mathcal{K}_1$, $\mathcal{K}_2$ can be computed as:

$$\begin{aligned} \mathcal{K}_1 &= [4.9445 \ 1.2194 \ -4.3946]^T, \\ \mathcal{K}_2 &= [4.3834 \ 1.2246 \ -4.5036]^T. \end{aligned}$$

Assuming $x(t) = [0.2 \ 0.3 \ 0.2]^T$ and $y(t) = [-0.3 \ -0.1 \ 0.4]^T$, $t \in [-1, 0]$ as the initial conditions of the master-slave systems. The Markovian jump mode and the corresponding state trajectories of the error system (3) without ASDC are displayed in Figure. 1(a). We can conclude that the error system (3) itself is

unstable. On the basis of obtained ASDC by Theorem 1, Figure. 1(b) displays the Markovian jump mode and corresponding state trajectories of the error system (3). Furthermore, Figure. 1(c) displays the control input under the action of ASDC and the aperiodic sampling interval, from which one can get that the closed-loop system is stable. Figure. 1(d) shows the displayed chaotic behavior. Therefore, the ASDC can successfully achieve the master-slave systems exponential synchronization.

Furthermore, consider the Chua's circuit with a single time-delay and without Markovian jump parameters, that is, let $\mathcal{A} = \mathcal{A}_1$, $\mathcal{B} = \mathcal{B}_{1,1}$, $\mathcal{W} = \mathcal{W}_1$, $\mathcal{C} = \mathcal{C}_1$, $d = 1$, and keep $\eta_\sigma(x_\sigma(t))$, δ , γ and h_m unchanged. Based on Corollary 1, the specific numerical comparison is shown in Table 2. From comparison with [19] and [20], it can be seen that our method can enlarge the upper bound of the sampling period h_M . Therefore, the method developed in this paper significantly reduces the conservativeness.

Take it one step further, choosing $d = 0$, $\alpha = 0$, Table 3 shows the comparison of our result with some previous results when considering the Lur'e system without time-delay. It is clear that the results obtained in this paper are better than the existing results.

Table 2: Maximum values of h_M for different α

α	0.1	0.2	0.3	0.4	0.5
[19]	0.3247	0.2941	0.2658	0.2396	0.2154
[20]	0.4381	0.3919	0.3519	0.3164	0.2847
This paper	0.4902	0.4880	0.4823	0.4743	0.4648

Table 3: Maximum values of h_M for $\alpha=0$ for different methods

Methods	[16]	[17]	[18]	[19]	This Paper
h_M	0.17	0.21	0.3914	0.3981	0.4540

Example 2. Use the master-slave systems (1) and (2) with the following parameters:

$$\mathcal{A}_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -1.1 & 0 & 0 \\ 0 & -1.1 & 0 \\ 0 & 0 & -1.1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -0.5 & 0.5 \\ 1 & -1 \end{bmatrix},$$

$$\mathcal{W}_1 = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.24 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}, \quad \mathcal{W}_2 = \begin{bmatrix} 1.24 & -1.5 & 0 \\ 1.2 & 0.9 & 1 \\ 0 & 2.1 & 1.6 \end{bmatrix}, \quad \mathcal{C}_1 = \mathcal{C}_2 = \mathcal{D} = I.$$

Based on above parameters, the system is reduced to a three neurons neural network with two mode. Moreover, the activation functions is $\eta_\sigma(x_\sigma(t)) = \frac{1}{2}(|x_\sigma(t) + 1| - |x_\sigma(t) - 1|)$, $\sigma = 1, 2, 3$, and thus $s_1 = s_2 = s_3 = 1$. Denote $\delta = 0.5$, $\gamma = 2$, $h_m = 10^{-5}$. Using Theorem 1, the relationship between the maximum values of h_M and α is displayed in Table 4. Choosing $\alpha = 0.2$ and $h_M = 0.3288$, by solving the LMIs (9)–(13), the

Figure 1: Open-loop state trajectory of the error system, closed-loop state trajectory of the error system, sampled-data control input and aperiodic sampled-data interval, chaotic behaviors of master system and slave system

gain matrix $\mathcal{K}_1$, $\mathcal{K}_2$ can be computed as:

$$\mathcal{K}_1 = \begin{bmatrix} 2.4299 & -1.0771 & 0.5392 \\ 0.4121 & 1.4925 & 0.4596 \\ -1.0102 & 1.5581 & 1.1123 \end{bmatrix}, \mathcal{K}_2 = \begin{bmatrix} 1.1822 & -0.2435 & -0.2448 \\ 1.1023 & 0.8558 & 0.9797 \\ 0.4682 & 0.4595 & 2.1018 \end{bmatrix}.$$

Assuming the initial conditions of the master-slave systems are $x(0) = [0.4 \ 0.3 \ 0.8]^T$ and $y(0) = [0.2 \ 0.4 \ 0.9]^T$, respectively. The Markovian jump mode and the corresponding state trajectories of the error system (3) are displayed in Figure. 2(a) when there is no ASDC. We can get that the error system (3) itself is unstable. On the basis of the obtained ASDC, by Theorem 1, Figure. 2(b) displays the Markovian jump mode and the corresponding state trajectories of the error system (3). Furthermore, Figure. 2(c) displays the control input under the action of ASDC and aperiodic sampling interval, from which one can get that the closed-loop system is stable. Figure. 2(d) shows the displayed chaotic behavior. Therefore, the ASDC

Figure 2: Open-loop state trajectory of the error system, closed-loop state trajectory of the error system, sampled-data control input and aperiodic sampled-data interval, chaotic behaviors of master system and slave system

can successfully stabilize the unstable error system. The master-slave systems exponential synchronization is achieved.

Furthermore, consider the neurons neural network without Markovian jump parameters, that is, let $\mathcal{A} = \mathcal{A}_1$, $\mathcal{W} = \mathcal{W}_1$, $\mathcal{C} = \mathcal{C}_1$, and keep $\mathcal{D}$, $\eta_\sigma(x_\sigma(t))$, δ , γ and h_m unchanged. Baed on Corollary 1, Table 5 shows the comparison of our result with some previous results, our method can enlarge the upper bound of the sampling period h_M . Therefore, the approach obtained in this article is effective and can ensure a lower conservativeness.

Table 4: Maximum values of h_M for different α

α	0.1	0.2	0.3	0.4	0.5
h_M	0.3395	0.3288	0.3171	0.3049	0.2919

Table 5: Maximum values of h_M for different methods

Methods	[19]	[21]	[22]	[27]	This Paper
h_M	0.3250	0.3818	0.9121	0.9213	0.9272

5 Conclusion

The presented paper has proposed the mean square exponential synchronization criterion and the ASDC design method for the MJCLSs with multiple time delays. A novel construction of Lyapunov functionals is given by introducing mode-dependent single/double integral terms and mode-dependent sampled-state information. The estimation of the expectation of the solution plays a vital role in obtaining the exponential stability criterion in the mean square sense. On the basis of the stability criterion, a mode-dependent ASDC has been presented. Finally, a Chua’s circuit and a neural network have been provided as numerical simulation examples to demonstrate the effectiveness of the raised methods.

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Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Conflict of Interest

The authors declare that they have no conflict of interest.

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