Libration-Free Cargo Transfer of Floating Space Elevator

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Libration-Free Cargo Transfer of Floating Space Elevator

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Abstract

This paper studies the libration-free cargo transfer control of a partial space elevator where the main satellite may change its orbital state in the transfer period. The orbital motion of the main satellite in the climber transfer period is first studied. Then, a reduced-order libration-free dynamics of the partial space elevator is derived. Accordingly, a novel libration-free switching control strategy is proposed to stabilize the cargo transportation with two alternating controllers. The Controller I controls the cargo speed in the libration-free mode by a shrinking horizon model predictive control based on the reduced-order libration-free dynamic mode of the partial space elevator. This leads to high computational efficiency in control. Once the libration is induced by the cargo transfer, the control turns off the Controller I and activates the Controller II to suppress the libration to zero within one time step of the Controller I by a novel prescribed-time control law based on the fixed-time control scheme. The stability of the control is proved in the Lyapunov framework. The validity and effectiveness of the proposed control strategy are demonstrated by computation simulation. Simulation results reveal that the proposed control strategy is effective in keeping stable cargo transportation while ensuring the equilibrium state at the end of transportation.

Keywords: Partial space elevator, Libration-free transfer, Shrinking horizon model predictive control, Prescribed-time control

1 Introduction

Partial space elevator (PSE) is a promising technology for future cargo transportation to space stations and extra-large space structures due to its advantage in energy efficiency compared to the current space transportation by rockets [1-3]. The PSE is formed by one main satellite that is connected to a secondary satellite by a long tether as shown in Fig. 1 [4]. A climber moves along the tether to transfer cargoes between the main and secondary satellites. The cargo transportation will not only lead to the libration of PSE due to the Coriolis effect but also the variation of orbital states of the main satellite in terms of orbital radius and angular velocity. It is well-known that these will affect the stability of PSE in cargo transportation. Therefore, the in-depth knowledge of orbital and libration dynamics and the control strategy for stable cargo transfer are crucial for the successful operation of PSE.

The dynamic modeling and analysis of PSE have attracted many attentions from researchers. Lorenzini et al. [5] and Misra et al. [6] developed the two-piece dumbbell (TPDB) model for the PSE in a circular orbit. This model is widely used for the modeling of multibody tethered space systems, such as the sloshing fuel tethered satellites [7] and the motorized momentum exchange tether systems [8]. Based on the TPDB model, Woo and Misra [9] and Shi et al. [10] investigated the influences of the motion and parameters of climbers on the libration dynamic characteristics of PSE. Jung et al. [11] expanded the TPDB model by considering the orbital states of the main satellite and noted the orbital radius changes obviously in the cargo transfer period, even though the climber is moving at a constant speed. Li and Zhu [12, 13] proposed a high-fidelity model to include the influence of orbital states and elastic motion of tether by a nodal position finite element method. Their analysis shows that the classical TPDB model is accurate in describing the overall motions of the PSE and suitable for control

law development, provided the control law is robust. Besides the theoretical studies, Yamagiwa et al. [14] investigated the PSE from an engineering aspect for space demonstration.

In the control strategy development for stable cargo transportation of PSE, Kojima and Fujii [15] proposed a “Mission Function” control law to suppress the libration of PSE in the cargo transfer period by adjusting the climber speed only. Shi and Zhu [10] proposed a flexible tension control strategy based on sliding mode control to regulate the climber speed with feedback of libration dynamic characteristics for libration suppression. Wen et al. [16, 17] and Sun et al. [18, 19] proposed tension control laws to stabilize the deployment of two-body tethered satellites, which are applicable for the libration suppression of PSE. The above works reveal that the libration of PSE can be suppressed by adjusting the climber speed in the cargo transfer period. However, it is difficult for them to enforce multiple constraints simultaneously and ensure the full state stability of PSE at the end of the cargo transfer period, because the PSE is an underactuated system and is not in an equilibrium state in the transfer period due to the energy injection. Accordingly, open-loop [20, 21] or piece-wised optimal control laws [22] are applied to regulate the climber speed, due to their robust global planning capability. However, the computational load is heavy compared to other control schemes.

Although effective, it is worth pointing out that the aforementioned works assumed the center of mass of PSE is approximately at the main satellite in the cargo transfer period so that the cargo transportation does not change the orbital states of the main satellite. If the total mass of the secondary satellite and the cargo is comparable to the mass of the main satellite, the orbit of the main satellite is inevitably changed because the center of mass of the PSE is no longer at the main satellite. Furthermore, the motion of the climber will change the orbital states of the PSE and the main satellite, leading to the ascending or descending of the orbital radius of the main satellite. Therefore, the additional orbital states of the main satellite exacerbate the underactuated and non-equilibrium characteristics of the PSE. In the current paper, such a scenario is called the floating main satellite. Up to date, limited tether libration control strategies are

available in the literature for the PSE with a floating main satellite by simple adjustments. For instance, Zhong and Zhu [23] proposed a piecewise optimal control method to control the orbital parameters and the attitudes of an electrodynamic tethered system in the orbit transfer. Rao and Zhong [24] proposed a “three-stages” model predictive control scheme to simultaneously deal with libration suppression and orbital motion of a tether-tug system in deorbit process. However, a more generic control strategy is desired to achieve stable transfer of the PSE with a floating main satellite. Such a control strategy shall keep a stable state of PSE in the entire transfer period, while being computationally efficient. Shi and Zhu [4] tried a stable transfer control scheme for the PSE with two controllers acting alternately to control the thrust and tether deployment simultaneously or the tether deployment only at the secondary satellite. Although effective, the control law cannot guarantee the libration motion converging to zero in a short and fixed period. Fixed-time control theory can be used to solve this problem [25-27], but the control parameters must be adjusted following different prescribed periods.

To overcome the problem, this work models the motion of PSE by decomposing it into the global orbital motion with zero libration and the local libration motion by the Lagrangian equation. Based on the decomposed dynamics, a novel and computationally efficient switching control strategy is proposed to achieve a libration-free transfer. The cargo transfer is controlled by the primary Controller I, which is a computationally efficient shrinking horizon model predictive control (SHMPC). The controller assumes the PSE is always in the local vertical without any libration. The control action will switch to the secondary Controller II when the libration disturbance appears. The Controller II suppresses the libration to zero within one time step of the Controller I by a novel prescribed-time control (PTC) law based on the fixed-time control scheme [28-30]. The stability of the proposed control strategy is proved under the Lyapunov framework. Simulation results show that the proposed libration-free switching control strategy is highly effective in stabilizing the libration with high computational efficiency.

2 Dynamics of a PSE with Floating Main Satellite

2.1 Dynamics of a partial space elevator with floating main satellite

Consider a floating PSE shown in Fig. 1 [4] in an ideal central gravity field. The long-term environmental perturbations, such as solar radiation pressure, atmospheric drag, and gravity gradient are neglected in the current work. Moreover, the out-of-plane libration of the PSE is ignored too as their influence is high order. An Earth-centered inertial coordinate frame OXY is established to describe the libration motion of the PSE, see Fig. 1, where the main satellite (m_0), the secondary satellite (m_2), and the climber (m_1) are simplified as mass points, the mass and elasticity of the tether are ignored.

Fig. 1 Scheme of system description.

The spatial position of the main satellite is denoted by a vector $\mathbf{r}_0$ in the inertial frame. The true anomaly is denoted by ϑ . The tether (L_1) is the distance between m_0 and m_1 . Its libration is defined by the angle θ_1 that is measured from the vector $\mathbf{r}_0$ to L_1 . Similarly, the tether (L_2) is the distance between m_2 and m_1 , and its libration angle θ_2 is measured from the vector $\mathbf{r}_0$ to L_2 . The positions of the main satellite $\mathbf{r}_0$, climber $\mathbf{r}_1$, and secondary satellite $\mathbf{r}_2$ are defined as

$$\begin{aligned}
\mathbf{r}_0 &= r\mathbf{e}_r \\
\mathbf{r}_1 &= \mathbf{r}_0 - L_1 \cos \theta_1 \mathbf{e}_r - L_1 \sin \theta_1 \mathbf{e}_g \\
\mathbf{r}_2 &= \mathbf{r}_1 - L_2 \cos \theta_2 \mathbf{e}_r - L_2 \sin \theta_2 \mathbf{e}_g
\end{aligned} \tag{1}$$

where $\mathbf{e}_g$ is in the orbital motion direction of the main satellite with the angle ϑ , the unit vector $\mathbf{e}_r$ is in the main satellite's radius direction normal to $\mathbf{e}_g$.

Differentiating Eq. (1) yields the velocities of the main satellite, the climber, and the secondary satellite,

$$\mathbf{v}_0 = \frac{d\mathbf{r}_0}{dt} + \boldsymbol{\omega} \times \mathbf{r}_0, \quad \mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt} + \boldsymbol{\omega} \times \mathbf{r}_1, \quad \mathbf{v}_2 = \frac{d\mathbf{r}_2}{dt} + \boldsymbol{\omega} \times \mathbf{r}_2 \tag{2}$$

where $\boldsymbol{\omega} = [0 \quad 0 \quad \dot{\vartheta}]^T$.

Accordingly, the potential (U) and kinetic energy (K) of the PSE can be written as,

$$\begin{aligned}
U &= -\mu \left(\frac{m_0}{|\mathbf{r}_0|} + \frac{m_1}{|\mathbf{r}_1|} + \frac{m_2}{|\mathbf{r}_2|} \right) \\
K &= \frac{1}{2} m_0 \mathbf{v}_0^T \cdot \mathbf{v}_0 + \frac{1}{2} m_1 \mathbf{v}_1^T \cdot \mathbf{v}_1 + \frac{1}{2} m_2 \mathbf{v}_2^T \cdot \mathbf{v}_2
\end{aligned} \tag{3}$$

where μ is the gravitational constant of the Earth.

Introducing the Lagrangian function $L = K - U$ and the generalized coordinates $(q_1, q_2, q_3, q_4, q_5, q_6) = (\theta_1, \theta_2, L_1, L_2, r, \vartheta)$, the dynamics of PSE can be derived by the Lagrange equation,

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad (i = 1, 2, 3, 4, 5, 6) \tag{4}$$

Such that,

$$\begin{aligned}
Q_1 &= \frac{\mu \sin \theta_1}{r^2 L_1} + \frac{G_1^1 \sin \theta_1}{m_1 L_1} - \frac{G_1^2 \cos \theta_1}{m_1 L_1} + \frac{T_1 \sin \theta_1}{r m_0} - \frac{T_2 \sin(\theta_1 - \theta_2)}{L_1 m_1} \\
&\quad - \frac{2(\dot{\vartheta} + \dot{\theta}_1) L_1}{L_1} + \frac{2\dot{\vartheta}}{r} + \frac{F_1}{L_1 m_1}
\end{aligned} \tag{5}$$

$$\begin{aligned}
Q_2 &= \frac{G_1^2 \cos \theta_2 - G_1^1 \sin \theta_2}{L_2 m_1} + \frac{G_2^1 \sin \theta_2 - G_2^2 \cos \theta_2}{L_2 m_2} + \frac{T_1 \sin \theta_1}{r m_0} + \frac{2\dot{\vartheta}}{r} \\
&\quad + \frac{T_1 \sin(\theta_1 - \theta_2)}{L_2 m_1} - \frac{2(\dot{\vartheta} + \dot{\theta}_2) L_2}{L_2} - \frac{F_1 \cos(\theta_1 - \theta_2)}{L_2 m_1} + \frac{F_2}{L_2 m_2}
\end{aligned} \tag{6}$$

$$\begin{aligned} \ddot{L}_1 = \dot{L}_1 - \frac{\mu \cos \theta_1}{r} - \frac{G_1^1 \cos \theta_1 + G_1^2 \sin \theta_1}{m_1} \\ - \frac{T_1}{m_0} - \frac{T_1}{m_1} + \frac{\cos(\theta_1 - \theta_2)T_2}{m_1} + (2\dot{\theta}_1 + \dot{\theta}_1^2)L_1\dot{\theta}_1 \end{aligned} \quad (7)$$

$$\begin{aligned} \ddot{L}_2 = \dot{L}_2 + \frac{G_1^1 \cos \theta_2 + G_1^2 \sin \theta_2}{m_1} - \frac{G_2^1 \cos \theta_2 + G_2^2 \sin \theta_2}{m_1} \\ + (2\dot{\theta}_2 + \dot{\theta}_2^2)L_2\dot{\theta}_2 + \frac{T_1 \cos(\theta_1 - \theta_2)}{m_1} - \frac{T_1}{m_1} - \frac{T_2}{m_2} + \frac{F_1 \sin(\theta_1 - \theta_2)}{m_1} \end{aligned} \quad (8)$$

$$\ddot{r} = r\dot{\theta}^2 - \frac{\mu}{r^2} - \frac{T_1 \cos \theta_1}{m_0} \quad (9)$$

$$\dot{\theta} = -\frac{2\dot{\theta}}{r} - \frac{T_1 \sin \theta_1}{rm_0} \quad (10)$$

$$\begin{aligned} G_1^1 = -\frac{\mu m_1 (r - L_1 \cos \theta_1)}{|r_1|^3}, \quad G_1^2 = \frac{\mu m_1 L_1 \sin \theta_1}{|r_1|^3} \\ G_2^1 = -\frac{\mu m_2 (r - L_1 \cos \theta_1 - L_2 \cos \theta_2)}{|r_2|^3}, \quad G_2^2 = \frac{\mu m_1 (L_1 \sin \theta_1 + L_2 \sin \theta_2)}{|r_2|^3} \end{aligned} \quad (11)$$

where L_0 is the initial total length of two tethers, $\dot{L}_1$ and $\dot{L}_2$ are the relative velocities of the climber m_1 and the secondary satellite m_2 in tether direction, respectively. Equations (5) - (11) describe the dynamics of the PSE with a floating main satellite.

Assume the total tether length L_0 is constant yields

$$L_2 = L_0 - L_1, \dot{L}_2 = -\dot{L}_1, \ddot{L}_2 = -\ddot{L}_1 = u_L \quad (12)$$

The climber's acceleration can be regarded as a control input u_L . Such that, Eqs. (6) - (8) can be rewritten as

$$\begin{aligned} \ddot{\theta}_2 = \frac{G_1^2 \cos \theta_2 - G_1^1 \sin \theta_2}{(L_0 - L_1)m_1} + \frac{G_2^1 \sin \theta_2 - G_2^2 \cos \theta_2}{(L_0 - L_1)m_2} + \frac{T_1 \sin \theta_1}{rm_0} + \frac{2\dot{\theta}}{r} \\ + \frac{T_1 \sin(\theta_1 - \theta_2)}{(L_0 - L_1)m_1} + \frac{2(\dot{\theta}_1 + \dot{\theta}_2)L_1\dot{\theta}_1}{(L_0 - L_1)} - \frac{F_1 \cos(\theta_1 - \theta_2)}{(L_0 - L_1)m_1} + \frac{F_2}{(L_0 - L_1)m_2} \end{aligned} \quad (13)$$

$$\begin{aligned}
T_1 = & m_0 \{ r^2 G_1^1 [m_2 \sin \theta_2 \sin(\theta_1 - \theta_2) - m_1 \cos \theta_1] \\
& - r^2 G_1^2 [m_1 \sin \theta_1 + m_2 \cos \theta_2 \sin(\theta_1 - \theta_2)] \\
& + m_1 \{ -\mu(m_1 + m_2) \cos \theta_1 + r^2 \{ (m_1 + m_2) [L_1 (\dot{\theta}_1 + \dot{\theta}_2)^2 - u_L] \\
& - \cos(\theta_1 - \theta_2) \sin \theta_2 G_2^2 - \cos(\theta_1 - \theta_2) \cos \theta_2 G_2^1 \\
& + m_2 \cos \theta_1 \cos \theta_2 [(L_0 - L_1)(\dot{\theta}_1 + \dot{\theta}_2)^2 + u_L] \\
& + m_2 \sin \theta_1 \sin \theta_2 [(L_0 - L_1)(\dot{\theta}_1 + \dot{\theta}_2)^2 + u_L] \} \} \} / A
\end{aligned} \tag{14}$$

$$\begin{aligned}
T_2 = & \{ -2r^2 G_2^1 m_1 (m_1 + m_0) \cos \theta_2 - 2r^2 G_2^2 m_1 (m_1 + m_0) \sin \theta_2 \\
& + m_2 \{ 2r^2 G_1^1 [m_0 \sin \theta_1 \sin(\theta_1 - \theta_2) + m_1 \cos \theta_2] \\
& + \sin \theta_2 \{ 2r^2 G_1^2 (m_1 + m_0 \cos^2 \theta_1) \\
& + 2m_1 m_0 \{ r^2 [L_1 (\dot{\theta}_1 + \dot{\theta}_2)^2 - u_L] - \mu \cos \theta_1 \} \} \\
& - 2m_0 \cos \theta_1 \cos \theta_2 \{ r^2 G_1^2 \sin \theta_1 + m_1 \{ \mu \cos \theta_1 - r^2 [L_1 (\dot{\theta}_1 + \dot{\theta}_2)^2 - u_L] \} \} \\
& + 2r^2 m_1 (m_1 + m_0) [(L_0 - L_1)(\dot{\theta}_1 + \dot{\theta}_2)^2 + u_L] \} \} / (2r^2 A)
\end{aligned} \tag{15}$$

where $A = m_1(m_1 + m_2) + m_0 [m_2 \sin^2(\theta_1 - \theta_2) + m_1]$ and the generalized force,

$$\begin{aligned}
Q_1 &= [F_1 + F_2 \cos(\theta_1 - \theta_2)] L_1 \\
Q_2 &= F_2 L_2 \\
Q_3 &= F_2 \sin(\theta_1 - \theta_2) - T_1 \\
Q_4 &= -T_2 \\
Q_5 &= F_1 \sin \theta_1 + F_2 \sin \theta_2 \\
Q_6 &= F_1 L_1 + F_2 [L_1 \cos(\theta_1 - \theta_2) + L_2] - r(F_1 \cos \theta_1 + F_2 \cos \theta_2)
\end{aligned} \tag{16}$$

Here, F_1 and F_2 are the thrusts acting at the climber and the secondary satellite, respectively, T_1 and T_2 are the tensions in tethers L_1 and L_2 , respectively.

2.2 System in libration-free mode

The libration-free mode of a PSE is defined, the zero libration states are, such that

$$\theta_1 = \dot{\theta}_1 = \ddot{\theta}_1 = 0, \quad \theta_2 = \dot{\theta}_2 = \ddot{\theta}_2 = 0 \tag{17}$$

To keep the libration-free mode in the cargo transfer period, certain thrusts must be applied to cancel the disturbing effects caused by the moving climber. The state keeping thrust can be derived from Eqs. (5) and (6) by substituting the libration-free mode (17),

$$\begin{aligned} F_1 &= 2L_1\ddot{\phi} - 2\mu L_1 m_1 / r \\ F_2 &= -2\mu L_0 m_2 / r \end{aligned} \quad (18)$$

Substituting Eqs. (17) and (18) into Eqs. (5) - (11), the dynamics of the liberation-free system can be expressed as

$$\ddot{r} = r\ddot{\phi} - \frac{\mu}{r^2} - \frac{T_1^0}{m_0} \quad (19)$$

$$\ddot{\phi} = -\frac{2\mu\phi}{r} \quad (20)$$

$$\ddot{L}_1 = u_L \quad (21)$$

where

$$T_1^0 = \frac{m_0 m_1}{m_0 + m_1 + m_2} \left\{ \frac{\mu}{(r - L_1)^2} - \frac{\mu}{r^2} + \ddot{\phi} L_1 + \frac{m_2}{m_1} \left[\frac{\mu}{(r - L_0)^2} - \frac{\mu}{r^2} + \ddot{\phi} L_0 \right] - u \right\} \quad (22)$$

Equations (19) and (21) show that the radius of the main satellite will be influenced by the climber acceleration u_L and the tether length L_1 directly. However, the climber motion will not influence the orbital angular acceleration of the main satellite directly as shown by Eq. (20).

Thus, integration of Eq. (20) yields

$$\ddot{\phi} = \frac{\exp(c_0)}{r^2} \quad (23)$$

Here, c_0 is a constant and can be expressed as

$$c_0 = \ln(\ddot{\phi}_0) + 2 \ln(r_0) \quad (24)$$

where $\ddot{\phi}_0 = \ddot{\phi}(t_0)$ and $r_0 = r(t_0)$ are the initial $\ddot{\phi}$ and r , respectively.

Accordingly, the order of the libration-free dynamics is reduced from 3 to 2. It should be noted that the solution in Eq. (23) is true only in the liberation-free mode. Once the libration appears, the analytical solution of the main satellite's orbital angular velocity is no longer valid because the condition in Eq. (17) does not hold anymore. Therefore, once the libration is suppressed to zero under control and Eq. (23) becomes true again, the constant c_0 should be refreshed based on the new initial condition of $\ddot{\phi}$ and r .

3. Libration-free Transfer Control

3.1 Control scheme

The libration-free model shows a significant simplification of system dynamics in the cargo transfer period. To take this advantage, the control system is separated into two alternating parts: the libration-free subsystem and the libration subsystem, depending on the orbital and libration states of the PSE. Accordingly, a novel switching control strategy is proposed in Fig. 2 with three control inputs. It should note that $\theta = [\theta_1, \theta_2]^T$ in Fig. 2.

The Controller I is corresponding to the libration-free cargo transfer, where the climber movement is regulated through θ_1 using SHMPC. Once the libration of PSE appears, Controller I is deactivated and the Controller II is activated to keep the libration $\theta, \dot{\theta} = 0$ by eliminating possible external disturbances in a short using PTC but prescribed period, such that,

$$\Delta T = \begin{cases} t_{i+1} - t_i^s & \frac{t_{i+1} - t_i^s}{T_s} \geq \varepsilon \\ t_{i+1} - t_i^s + T_s & \frac{t_{i+1} - t_i^s}{T_s} < \varepsilon \end{cases} \quad (25)$$

where $0 < \varepsilon < 1$ is a constant, and $\varepsilon T_s \leq \Delta T < (\varepsilon + 1)T_s$.

It can be seen that the prescribed period depends on the PTC's starting time t_i^s and the predicted step of SHMPC T_s in the Controller I, where the subscript of t_i^s denotes the number of the time interval.

If no external disturbance acts on the climber or the secondary satellite, the PSE is kept libration-free by the keeping thrusts, see Eq. (18), and the Controller I is active to plan the optimal climber speed subjects to Eqs. (19) and (21) with angular velocity function of Eq. (23). Compared with the optimal control, the computational load is much lower than to plan the state trajectory for the full state dynamics in Eqs. (5) - (10). If the libration appears and exceeds a prescribed tolerance, the control switches to

the Controller II to suppress the libration in the prescribed period determined by Eq. (25). Simultaneously, the Controller I stops working and the thrust u_L follows the sequence planned in the previous time interval. By the end of the prescribed period, the libration will be eliminated. Then, the Controller I is activated, and the Controller II enters standby state.

Fig. 2 Diagram of SHMPC based switching control strategy

3.2 Shrinking Horizon Model Predictive Control for orbital transfer

The objective of Controller I is to ensure all the orbital states converge to an equilibrium state by the end of the cargo transfer period. Therefore, it is desired to use an SHMPC that the predictive horizon covers from the current state to the end of the transposition to regulate u_L robustly. Since the horizon window is fixed at each step proportionally, the horizon shrinks as the end of the mission approaches as shown in Fig. 3.

Fig. 3 Shrinking Horizon Model Predictive Control.

where n is the number of the control period. Accordingly, the general objective function can be written as:

$$\min J = \mathbf{x}^T(t_f) \mathbf{F} \mathbf{x}(t_f) + \int_{t_i}^{t_f} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}_L^T \mathbf{R} \mathbf{u}_L) dt \quad (26)$$

subjects to Eqs. (19), (21), (23) and

$$0 \leq T_1 \leq T_{1\max}, 0 \leq T_2 \leq T_{2\max}, \underline{a}_{1m} \leq u_L \leq \underline{a}_{1M} \quad (27)$$

where F , Q , and R are the positive semidefinite weight matrixes, $\mathbf{x} = [r, L_1]^T$, t_i is the current time, t_f is the predefined final time, $T_{1\max}$ and $T_{2\max}$ are the upper bounds of tension control inputs T_1 and T_2 , respectively, $\underline{a}_{1m}$ and $\underline{a}_{1M}$ are the lower and upper bounds of the climber's accelerations, respectively.

In the i -th interval, the prediction time horizon is $t_f - t_i = t_f - T_s (i - 1)$, and the respecting action period is $t_{i+1} - t_i = T_s$. The trajectory of the states and control input u_L are obtained by solving the nonlinear programming problem of Eq. (26), and the current states are used as the initial state.

3.3 Predefined-time control for libration suppression

To suppress the libration angles of θ_1 and θ_2 to the desired values in a prescribed period with limited control input, a new predefined time sliding mode control is proposed to deal with the unexpected libration with limited additional thrusts. The full libration dynamics in Eqs. (5) and (6) can be simplified under the small libration assumption such that,

$$\ddot{\boldsymbol{\theta}} = \mathbf{f} + \mathbf{g} \mathbf{u}$$

$$\mathbf{f} = \begin{bmatrix} -3g & 0 \\ 0 & -3g \end{bmatrix} \boldsymbol{\theta}, \quad \mathbf{g} = \begin{bmatrix} \frac{1}{L_1 m_1} & 0 \\ -1 & 1 \\ \frac{1}{(L_0 - L_1) m_1} & \frac{1}{(L_0 - L_1) m_2} \end{bmatrix} \quad (28)$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2]^T$, $\mathbf{u} = [u_1, u_2]^T$, u_1 and u_2 are the additional thrusts acting on the climber and the secondary satellite, respectively.

A sliding mode surface with predefined-time stability is designed as [29]

$$s = \theta + \text{sig}^{\frac{2}{2-k}} \left[\frac{k\Delta T}{\pi} \theta + \text{sig}^{\frac{2+k}{2}} (\theta) \right] \quad (29)$$

where $\text{sig}^n(\mathbf{x}) = [|x_1|^n \text{sign}(x_1), |x_2|^n \text{sign}(x_2), \dots, |x_i|^n \text{sign}(x_i)]^T$, ΔT is the predefined period, $k = \text{const} \in (0, 1)$.

Theorem 1. θ will converge to zero through the sliding mode surface s in the predefined period ΔT .

Proof. When $s_i = 0$, Eq. (29) can be written as

$$\theta_i = -\frac{\pi}{k\Delta T} \left[\text{sig}^{\frac{2-k}{2}} (\theta_i) + \text{sig}^{\frac{2+k}{2}} (\theta_i) \right], \quad (i = 1, 2) \quad (30)$$

Define $\eta = |\theta_i|^{k/2}$, then the first-order time derivation of η can be written as

$$\dot{\eta} = \frac{k}{2} \text{sig}^{\frac{k-2}{2}} (\theta) \theta \quad (31)$$

Substituting Eq. (30) into Eq. (31) gets

$$\dot{\eta} = -\frac{\pi\eta}{2\Delta T} (\eta^2 + 1) \quad (32)$$

Solving Eq. (32) leads to the settling time

$$T = \lim_{\eta \rightarrow \infty} \frac{2\Delta T}{\pi} \int_0^\eta \frac{1}{1+\eta^2} d\eta = \frac{2\Delta T}{\pi} \arctan(\eta) \Big|_0^\infty = \Delta T \quad (33)$$

Then, **Theorem 1** is proved.

Designing a predefined-time sliding mode control law as

$$u = \frac{-\pi}{k\Delta T} \mathbf{g}^{-1} \left[\Phi + \frac{k+2}{2} \text{diag} \left(\left| \theta \right|^{\frac{k}{2}} \theta \right) \right] - \mathbf{g}^{-1} \mathbf{f} \quad (34)$$

where

$$\Phi = \frac{2-k}{2} \left[\frac{\pi}{2k\Delta T} \left(2^{\frac{k}{2}} s^{1-k} + 2^{-\frac{k}{2}} s^{1+k} \right) \text{sign}(s) + \theta \right] \text{diag} \left(\left| \frac{k\Delta T}{\pi} \theta + \text{sig}^{\frac{2+k}{2}} (\theta) \right|^{\frac{k}{k-2}} \right) \quad (35)$$

Considering a candidate Lyapunov function as

$$V = \frac{1}{2} \mathbf{s}^T \cdot \mathbf{s} \quad (36)$$

Taking the derivative of V yields

$$\begin{aligned}
\dot{V} &= \mathbf{s}^T \cdot \dot{\mathbf{s}} \\
&= \mathbf{s}^T \cdot \left[\dot{\mathbf{s}} + \frac{k}{2-k} \text{diag} \left(\left[\frac{k\Delta T}{\pi} \dot{\mathbf{s}} + \text{sig}^{\frac{2+k}{2}}(\boldsymbol{\theta}) \right]^{\frac{k}{2-k}} \right) \right] \left[\frac{k\Delta T}{\pi} + \text{diag} \left(\left| \boldsymbol{\theta} \right|^{\frac{k}{2}} \right) \right] \dot{\mathbf{s}} \\
&= -\frac{\pi}{k\Delta T} \mathbf{s}^T \cdot \left[2^{\frac{k-2}{2}} \text{sig}^{1-k}(\mathbf{s}) + 2^{\frac{-k-2}{2}} \text{sig}^{1+k}(\mathbf{s}) \right] \\
&\leq -\frac{\pi}{k\Delta T} \left[\left(\frac{1}{2} \mathbf{s}^T \cdot \mathbf{s} \right)^{\frac{2-k}{2}} + \left(\frac{1}{2} \mathbf{s}^T \cdot \mathbf{s} \right)^{\frac{2+k}{2}} \right] \\
&\leq -\frac{\pi}{k\Delta T} \left(V^{\frac{2-k}{2}} + V^{\frac{2+k}{2}} \right) \\
&\leq 0
\end{aligned} \tag{37}$$

Therefore, the proposed control law is stable. The libration system approaches the predefined-time stability in the predefined period [31].

4. Simulation and Discussion

4.1 Dynamic characteristics

The dynamic characteristics of the PSE with a floating main satellite are analyzed with the following system parameters and initial states: $L_0 = 20 \text{ km}$, $m_0 = 50000 \text{ kg}$, $m_1 / m_0 = 0.01$, $m_1 / m_2 = 0.5$, $\dot{L}_1 = 0$, $\theta_1(0) = \theta_2(0) = 0$, $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$. All simulations were conducted by MATLAB with the RK-4 integrator. The time step is 0.001 s . The simulation results are shown in Figs. 4 - 10.

4.1.1 Effects of orbital variation of the main satellite

Let the climber move upward at constant speeds ranging from 0 to 20 m/s , such that, $L_1(0)/L_0 = 0.01$, $L_1(t_f)/L_0 = 0.99$ and $r(0) = 7100 \text{ km}$. The corresponding $\dot{\mathbf{s}}(0)$ can be obtained from Eq. (19). The comparisons of floating and fixed main satellites on the libration of PSE in the cargo transfer period are shown in Figs. 4 (a) & (c). The trends of θ_1 and θ_2 are similar to those in Ref. [10] where the main satellite is in a

fixed circular Kepler orbit. Figures 4 (b) and (d) show the differences of θ_1 and θ_2 between the floating and circular orbit cases. It shows that the difference θ_1 is no more than 6.7% with the peak appearing at the highest climber moving speed by the end of the transfer period. The difference of θ_2 is no more than 3%, which is smaller than that of θ_1 . The results of the climber moving downward are similar to the upward case and are not presented here to avoid redundancy.

Figure 5 shows the libration caused by the floating main satellite. When the main satellite radius varies near its initial state within the magnitude of 500 m , see Fig. 5 (a), the orbiting angular velocity varies in the same phase slightly, $\pm 1.5 \times 10^{-6} \text{ rad}$. The corresponding libration angles shown in Fig. 5 (c) and (d) vary by magnitudes of 10^{-4} rad . Although the position of the climber induces some influences on the libration of the climber and the secondary satellite, the overall libration angles lead by the floating main satellite is no more than 0.1% of that induced by the Coriolis effect which is slight. Here, it should be noted the floating magnitude $r - r_0$ of the main satellite in the numerical simulation is less than 0.1% which is also slight in the pure mathematic aspect. However, for a large space structure, such floating magnitude is concerned in the engineering aspect. The cases of the floating magnitudes of 100 m - 1000 m are studied. The results are similar to the case of 500 m . Thus, they are not presented to avoid redundancy.

In summary, the floating main satellite will induce additional libration motions, which is not mentioned in the literature. Although the magnitudes of the induced libration are small numerically, they may have adverse effects on the PSE structure from the engineering aspect, which should be studied in mission planning.

Fig. 4 Comparison of floating and fixed main satellites on libration. (a) Libration angle of the climber. (b) The gaps of θ_1 between fixed and floating cases. (c) Libration angle of the secondary satellite. (d) The gaps of θ_2 between fixed and floating cases.

Fig. 5 Effects of the floating main satellite on libration ($r - r_0 = 500$ m). (a) Variation of the main satellite radius. (b) Orbital angular velocity of the main satellite. (c) Libration angle of the climber. (d) Libration angle of the secondary satellite.

4.1.2 Libration vs. libration-free

The effects of libration on floating the main satellite are shown in Figs. 6 - 9. The climber moves upward at two different speed profiles with the initial and end conditions

$\dot{L}_1(0) = \dot{L}_1(t_f) = 0$. In the libration mode, the climber and the secondary satellite are free to libration. In libration-free mode, all libration is kept zero by applying the external forces shown in Eq. (18).

Let the climber move in a trapezoid speed profile as shown in Fig. 6. The acceleration and deceleration magnitudes are assumed the same as 0.01 m/s^2 and the accelerating and decelerating lengths are the same as $0.05L_0$, see Fig. 7(d). In such transfer mode, the magnitude of the libration angle of the main satellite peaks at 0.18 rad after the acceleration, if no libration control is applied. By the end of the transfer period after 0.5 orbit, the PSE continues libration with the magnitude of 0.06 rad . The variation of the main satellite radius is presented in Fig. 7 (b) and (c). Compared the orbital states in libration (solid line) and libration-free (dot line) modes, the magnitude and the changing trend of the radius r are obviously different. In the libration mode, the altitude of the main satellite reduces and fluctuates with a magnitude of 500 m due to the upward transportation of the climber in the libration mode. On the contrary, the main satellite radius increases and fluctuates with a magnitude of 1100 m in the libration-free mode. This is because, in the libration-free mode, external forces act at the climber and secondary satellite to suppress the libration, which is equivalent to an extra force acting on the center of mass of the PSE aligned with the PSE orbital motion, see Eq. (18). Thus, the radius of the center of mass will increase and, correspondingly, the radius of the main satellite increases. This is a characteristic of the libration-free mode. On the contrary, in the downward movement phase of the climber, the altitude of the main satellite will be reduced.

When the climber speed function is changed to hyperbola, such that

$$u_L = -\frac{1}{2} \left[L_1(0) - L_1(t_f) \right] \left(\frac{\pi}{t_f} t \right)^2 \cos \left(\frac{\pi}{t_f} t \right) \quad (38)$$

The variation of the main satellite orbit states is similar to those in the trapezoid speed profile as shown in Fig. 9. Compared with the previous case, the magnitudes of the libration angles and the radius r are smaller. This is because, the transfer period is doubled from 0.5 to 1 orbit in the hyperbola case, and the speed curve is smoother. The

comparison shows that by adjusting the climber speed profile, the orbital states of the main satellite can be controlled obviously.

In summary, (1) the main satellite's floating influents are slight on the libration, yet the libration of the climber and the secondary satellite will lead to concerned orbit changes of the main satellite. (2) The libration-free transfer leads to greater radius change of the main satellite than the climber transportation without libration suppression, as a cost of simplifying the model. (3) Different climber speed modes lead to different main satellite trajectories. Furthermore, (4) fixed speed function is difficult to ensure the full states converge to zero by the end of the transfer period. Thus, it is critical to regular the climber speed carefully.

Fig. 6 Climber speed - trapezoid profile.

Fig. 7 Climber moves in the trapezoid speed profile. (a) Libration angles. (b) Chang of main satellite's radius. (c) Orbital angular velocity of the main satellite. (d) Tether length of L_1 .

Fig. 8 Climber speed – hyperbola profile.

Fig. 9 Climber moves in the hyperbola speed profile. (a) Libration angles. (b) Chang of main satellite's radius. (c) Orbital angular velocity of the main satellite. (d) Tether length of L_1 .

4.2 Control case studies

The initial conditions of the system are given in Table 1.

Table 1 Initial states

Parameters	Values
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Initial orbital radius of the main satellite, $r(0)$ (m)	7.1×10^5
Initial anomaly angular velocity, $\dot{\mathcal{G}}(0)$ (rad / s)	0.00105561
Initial climber position, $L_1(0)$, (m)	200
Initial climber speed, $\dot{L}_1(0)$ (m/s)	0
Initial libration angle and angular velocity, $(\theta_1(0), \dot{\theta}_1(0))$	(0, 0)
Initial libration angle and angular velocity, $(\theta_2(0), \dot{\theta}_2(0))$	(0, 0)

The transfer period is set at 5952.2s, which equals to the initial orbital period. The control laws are implemented under the proposed libration-free switching control strategy as shown in Fig. 2. The constraint parameters of the SHMPC are given as:

$$T_{1\max} = T_{2\max} = 200 \text{ N}, \quad \ddot{L}_{1\min} = -\ddot{L}_{1\max} = -0.01 \text{ m/s}^2, \quad n = 20, \quad \text{and} \quad T_s = t/n = 297.61 \text{ s}.$$

The magnitude of the climber acceleration is not allowed too large to avoid overshoot. The control parameters of the prescribed time sliding mode control are defined as: $k = 0.04$ and the adapting ΔT follows Eqs. (25) with $\varepsilon = 0.7$. All simulations were carried out by MATLAB with the RK-4 integrator.

The simulation results are shown in Figs. 10 – 16. The solid line presents the cargo transfer in the ideal condition without libration and disturbance in the transfer period. By the end of the transfer period, the full states of the PSE converge to an equilibrium point successfully, see the solid lines in Figs. 10 and 12 – 14. This demonstrates the validity of SHMPC in achieving the stable cargo transfer of a PSE. In the ideal condition, only the Controller I works. The system dynamics is described by Eqs. (19) and (21), where the number of states is reduced from 6 to 2 and the computational efficiency has been improved greatly.

To validate the robust of the proposed control strategy and the prescribed time sliding mode control, a sudden disturbance is added on the climber at the time point of 2950 s. Figures 11 (a) and (d) show the long-term influence of the added disturbance.

To present the overall influence of the added disturbance, its influence on the secondary satellite is assumed eliminated by the state-keeping force of the secondary satellite. The added disturbance will increase the climber libration by the magnitude of 0.1 rad , if no additional control is added on the climber in addition to the state keeping force at the secondary satellite, see Fig. 11 (c). If Controller I is applied to control the climber speed based on the libration-free mode in presence of libration, the main satellite orbit states are obviously different from the ideal condition. Moreover, the libration will not converge to zero by the end of the transfer period.

When the disturbance occurs, the Controller II is activated while the Controller I begins to standby. The libration of the climber and the control inputs of the Controller II in the suppression phase from 2950s to 3273.71s are shown in Fig. 11(b), (c), and Fig. 16. In the beginning, there exist $(t_{i+1} - t_i^s)/T_s < \varepsilon$. Thus, the prescribed period is determined as $\Delta T = t_{i+1} - t_i^s + T_s = 323.71\text{s}$ per Eq. (25). Under the action of Controller II, the libration is suppressed to zero in the given period with limited additional control input successfully. This indicates the new prescribed time sliding mode control is effective to eliminate the sudden disturbance in the prescribed period.

The states after the suppression control are shown by the dashed lines in Figs. 10, 12 – 16. After the short suppression phase, the regulated climber speed shows obvious differences from the case without disturbance if one compares the solid and dashed lines in Figs. 12, 13, and 15. Such difference leads to different orbital state trajectories in comparison of the solid and dashed lines in Figs. 10 and 14. However, the overall trends of the orbital state curves are the same. By the end of the transfer period, the main satellite radius r and the orbital angular velocity $\mathcal{G}$ converge to 7100572.2 m and 0.00105435 rad/s , respectively. The difference of orbital states between the disturbed and ideal cases is negligible. This is because the libration is suppressed in a fast manner.

Finally, the computational efficiencies of the SHMPC in the libration mode (with full states) and the libration-free mode are compared. When the PSE operating in the libration-free mode for cargo transfer, the computational time in the first time interval,

see Fig. 3, is around 1.167 s. For the PSE not working in the libration-free transportation mode, the required computational time of the SHMPC subjects to the full-state in the same time interval is 51.68 s, which is about 40 times of the computational time in the libration-free transfer mode. This indicates the advantage of the libration-free transfer mode in reducing the computation burden.

Fig. 10 Main satellite radius.

Fig. 11 Prescribed-time control results. (a) Unsuppressed libration. (b) Suppressed libration with PTC. (c) Control input. (d) Main satellite radius with/without disturbing.

Fig. 12 Tether length of L_1

Fig. 13 Climber speed profile.

Fig. 14 Orbital angular velocity of the main satellite

Fig. 15 Control input of the SHMPC

Fig. 16 Total thrusts on the climber and the secondary satellite

5. Conclusions

This work first studies the dynamics of the PSE with a floating main satellite. It is found that (1) the libration of a PSE will lead to obvious orbital variations of the main satellite, while the floating main satellite influences the libration slightly; (2) the libration-free transfer leads to different radius change of the main satellite than the case without libration suppression; and (3) the orbital trajectory of the floating main satellite can be controlled by regulating the climber speed. Inspired by these discoveries, a switching control strategy for libration-free transfer is proposed. The cargo transfer is performed in the libration-free mode with a shrinking horizon model predictive control. Once the libration of the PSE is detected, the control is switched to a novel prescribed-time

control law to suppresses the libration within one time step of the controller I quickly and then the control switched back the shrinking horizon model predictive control. Numerical results demonstrate that the newly proposed libration-free switching control strategy is valid and computationally effective.

6. Declarations

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6.2 Conflicts of interest/Competing interests

The authors declare no conflict of interest in this article.

6.3 Availability of data and material

Data sharing does not apply to this article as no datasets were generated or analyzed during the current study.

6.4 Code availability (software application or custom code)

The custom codes in the current study are available from the corresponding author on reasonable request.

6.4 Authors' contributions

Dr. Gefei Shi conducted formal analysis and methodology development, coding, data processing, draft writing, acquired funding.

Dr. Zheng H. Zhu conducted methodology development and analysis, reviewing and editing of the paper, acquired funding.

6.5 Ethics approval

Not applicable.

6.6 Consent to participate

Not applicable.

6.7 Consent for publication

Not applicable.

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