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A robust fractional model for improving industrial park water use efficiency under dual uncertainties

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Abstract:

As an existing form of high aggregation of industries, industrial parks, have problems with a large demand for water and sewage discharge. Therefore, developing an effective approach to improve the efficiency of industrial park water use is urgently needed. Originating from internal and external changes, complicated uncertainties may coexist in multiple system components and their correlations within industrial park water resource management. Previous studies ignored the uncertain probability related to water inflow, which is difficult to estimate for long-term decision-making problems. This research describes the exploitation of an inexact two-stage stochastic partial fractional programming (ITSPF) method for sustainable industrial park water supply, under dual uncertainties. ITSPF is developed via the synthesis of linear partial information, interval programming, two-stage stochastic programming, and fractional programming techniques. It could improve conventional industrial park water resource optimization by solving the uncertain probability of water inflow levels while optimizing the ratio issues. The results show that various water allocation plans within different inflow level probabilities can be generated by handling the trade-off between water consumption and system benefit. And the amount of reclaimed water

would be increased under the higher risk of water shortage. The comparisons of ITSPF results against the least-cost model and model with deterministic probability of water inflow levels demonstrated that ITSPF could not only result in higher resource-use efficiency, but also avoid missing possible solution sets and offer a pragmatic way for obtaining satisfactory alternatives by providing wider adjustable ranges.

Keywords: linear partial information, two-stage stochastic programming, fractional programming, industrial park water management, dual uncertainties

Highlights

- An inexact two-stage stochastic partial fractional (ITSPF) model was developed.
- It may handle dual uncertainties and ratio issues for industrial park water system.
- Various schemes under different probability of inflow levels were generated.
- Comparative cases were considered to validate the superiority of ITSPF.

1 Introduction

Global industrial water withdrawals are predicted to approach 2000 km³/yr by 2050, more than double the industrial water consumption in 2018 (920 km³/yr) (Burek et al., 2016; Wada et al., 2016; Boretti and Rosa, 2019; Bauer et al., 2020). The sustainability of industrial activities depends on the water availability, cost, and quality (WBCSD, 2006; Boysen et al., 2020). As an existing form of high aggregation of industries, industrial parks can obtain higher economic benefits, but have problems with large demands for water and sewage discharge. Among industrial park water-using networks, the case is usually treated as a water distribution issue through a superstructure-based model where water needs to be allocated, treated, and discharged optimally between the process units of various enterprises (Boix et al., 2015; Lawal et al., 2021). In industrial park water management systems, the decision

51 makers need to consider many complex processes because of the competition between
52 economic development and environmental protection. The periodical decisions in
53 industrial park water management issues are required with the passage of time,
54 resulting in a wide array of dynamic complexities. Furthermore, plenty of
55 uncertainties may coexist in multiple system components and their correlations within
56 an industrial park water management (Boix et al., 2012). These uncertainties may
57 stem from inner and outer interactions in water systems, such as variations in
58 technical or commercial parameters cause by technological advances, fluctuations in
59 water requirements cause by changing society and ecological systems, and shifts in
60 water-circulation patterns cause by climate change (Tan et al., 2007, 2010; Chew et al.,
61 2009; Tan et al., 2013; Khor et al., 2014; Bi et al., 2019). Thus, efficient optimization
62 tools for planning industrial park water resource systems under these complexities are
63 desired (Wang et al., 2016).

64 Over the past few decades, numerous studies have rapidly developed in the field of
65 industrial park water resource allocation optimization, which is generally optimized
66 through pinch technology and mathematical programming optimization (Yoo et al.,
67 2007; Boix et al., 2015; Kastner et al., 2015). The pinch analysis provides
68 easy-to-understand water allocated solutions due to its graphical concepts (Olesen and
69 Polley, 1996; El-Halwagi et al., 2003). Compared to pinch analysis, mathematical
70 programming approaches are well-adapted to solve large-scale and complex issues
71 (e.g., multi-period water exchange (Liao et al., 2007), multi-objective optimization
72 (Tiu and Cruz, 2017), multi-contaminant problems (Boix et al., 2011), etc.), which
73 have been well studied in the literature with the concept of superstructure (Boix et al.,
74 2015). Typically, practical industrial park water resource systems are complex
75 because of the conflicts among enterprises, natural resources along with social,

76 economic, and ecological factors. It is obvious that the optimal water allocation of an
77 industrial park through single-objective optimization is trivial (Ramos et al., 2016).
78 Multi-objective programming (MOP) approaches are useful to tackle these pressing
79 problems (Montastruc et al., 2013). In most applications, plenty of algorithms are
80 taken to explore the “most preferred” solution in industrial park water optimization to
81 circumvent the drawbacks of conventional linear programming approaches. (Mavrotas,
82 2009). One is converting multi-objective problems into single-objective problems
83 (Tiu and Cruz, 2017). The other is to obtain efficient solutions to the problem through
84 the Pareto front (Boix et al., 2011; 2012). The former algorithm requires subjective
85 and unrealistic assumptions to transform multiple conflicting objectives into a single
86 monetary measure. (Marler and Arora, 2010; Zhu et al., 2014; Ji et al., 2017). The
87 latter has a large computational effort and a lack of widely available software
88 (Mavrotas, 2009). Finally, MOP methods may encountered difficulties in considering
89 and assessing ratio issues, although it can balance conflicting objectives and
90 coordinate the interests of different stakeholders (Zhu and Huang, 2011).

91 Fractional programming (FP) can deal with conflict objective expressed as ratio
92 problem without considering subjective information, which can objectively avoid the
93 weaknesses of MOP methods (Stancu-Minasian, 1997). It is considered to be an
94 effective tool for ratio optimization, which can offer an unprejudiced method of
95 system efficiency and optimize marginal system benefits effectively (Mehra et al.,
96 2007; Zhu and Huang, 2013; Cui et al., 2015; Zhou et al., 2015; Zhou et al., 2016; Tan
97 and Zhang, 2018). In addition, the objective function in FP is the quotient of physical
98 and economic quantities which is proven to be a highly effective measure to meet the
99 needs of real-world problems in resource management, finance, transportation, and
100 production. A classic algorithm for FP was first developed by Charnes and Cooper

(1963) and the latest progress has been reported as well in Stancu-Minasian (2019). FP has been well studied in measuring the efficiency of water resource systems (Fasakhodi et al., 2010; Mani et al., 2016; Zhao et al., 2017). Nevertheless, this method has seldom been applied to address industrial park water management problems. Moreover, the uncertainties that are prevalent in industrial park water resource management cannot be veritably resolved in previous FP studies.

Industrial park water management problems contain many uncertain factors and inexact parameters which cannot be expressed accurately. Hence, many optimization methods have been proposed to address these uncertainties (Li et al., 2008; Aviso et al., 2010). Particularly, a large amount of researches drawing great attention to two-stage stochastic programming (TSP), which has been widely adopted for solving combined uncertainties and economic consequences of violating the committed targets in water resource management issues (Huang and Loucks, 2000; Xie et al., 2013; Zhou et al., 2015). It is considered an efficient approach for dealing with uncertainties expressed as probabilities, where an analysis of multi-stage decisions is desired to take recourse or for corrective actions (Khor et al., 2014; Cui et al., 2015; Zhou et al., 2015). But the existing TSP-based methods only address inflow probabilities, which are known as deterministic values. In industrial park water management problems, the probabilistic information of water inflow may be partially determined (Kofler, 2001). In practice, the water inflow is usually estimated by the number of random events within a certain period of time. Different water resource monitoring sectors have observation stations themselves to collect water inflow data. Therefore, the probability density functions of the future stream flow will be varied, which results in some uncertainties in the probability of the water inflow. Such compound uncertainties can be solved by introducing linear partial information (LPI),

in which the probability of water inflow levels fluctuates within a certain range and is expressed as an interval (Fan et al., 2012b; Chen and Wang, 2021). However, there is no further study addressing the dual uncertainties with FP, which can optimize the water use efficiency in industrial park water resources management.

Therefore, this research develop an inexact two-stage stochastic partial fractional (ITSPF) programming to tackle dual uncertainties and ratio issues involved in industrial park water resource management problems. Fractional programming is first incorporated with inexact two-stage stochastic partial programming to address multi-objective issues. The ITSPF method can balance two conflict aspects expressed as ratio issues and analyze various scenarios under corresponding probability distributions of water inflows. Thus, it can assist planners in identifying the optimal strategies of industrial park water resource management and gain in-depth insights into water allocation pattern, system failure risk, and system profit under different scenarios. A representative industrial park water resource management system is developed to further demonstrate the applicability of the developed method.

2 Complexities of industrial park water resources system

At present, the availability of water is critical for industrial park as the water demands for factories are increasing dramatically for their own development. For industrial park, surface water and underground water are considered as its potential sources. Underground water is regarded as an important water resource because of the stable quality and quantity. Overdraft of groundwater may cause adverse natural phenomena such as land subsidence, soil salinization, seawater intrusion, and catastrophic damages to ecological systems. Thus the exploitation of underground water is limited (Tan et al., 2013). The quality of surface water would lower than underground water due to its widely exposition to point and non-point pollution sources. Despite the

abundance of surface water, the severe contamination of it may be highly undesirable to water supply. Above all, the availability of water sources are limited cause by the polluted surface water and over exploitation of underground water. This will result in various kinds of negative impacts on the sustainable development of social-economic and ecological environment. Thus, the supplement of reclaimed water is desired which can facilitate the reduction of fresh water consumption. Reclaimed water is wastewater that has been properly treated by removing solids and impurities. The capacity of regeneration unit and the efficiency of sewage collection system are important factors for the availability of reclaimed water. Schematically, the water allocation pattern for an industrial park can be presented in Fig.1. For each water user plants, input water may be surface water, underground water and their own reclaimed water; the output water for each plant may be distributed to regeneration units or wastewater treatment plant.

Fig.1 Possible connections in an industrial park (WTP represents the wastewater treatment plant)

In the meantime, a great deal of conflicting goals, interactions and dynamics may

coexist within industrial park (Taskhiri et al., 2011). In industrial park water resources management problem, the amount of available water resources have random feature because of the temporal variation of water resources system. This will affect the optimal water resource management in industrial park. TSP can address uncertainties expressed as probabilistic distributions effectively (Huang and Loucks, 2000; Xie et al., 2013). But the existing TSP-based methods only address the inflow probabilities is known as deterministic values, which ignored the uncertain probability of water inflow level. By introducing LPI into TSP, the uncertain probability of water inflow levels would fluctuate within a certain range and express as interval, which is helpful to deal with the dual uncertainties related to random inflows in water resources management. According to the partial distribution probability mentions in equation (2.1), the solution of the inequality (1.7) is defined as extreme points. Therefore the results of extreme points p_1 , p_2 and p_3 would take the interval values of $[0.1, 0.2]$, $[0.5, 0.7]$ and $[0.2, 0.3]$, respectively (Fan et al., 2012b). Due to the sum of these extreme points is equal to 1, there are four combinations presented by A, B, C, D in Fig.2. Until now, the formulation of existed MOP methods for industrial park water management systems need to be converted into single objective problems or obtained the efficient solutions of the problem through Pareto front which is susceptible to subjective factors and large computational efforts. The essence of fractional programming is a ratio optimization problem, which can coordinate conflicting objectives and obtain the optimal scheme to improve system efficiency. Besides, multiple interval parameters, such as water availability, water demands, and operational costs, will further amplified the system complexities. Therefore, an inexact two-stage stochastic partial fractional industrial park water resource management (ITSPF-IPWRM) model based on the ITSPF method is desired to assist

192 decision makers in planning industrial park water resource management.

193

194 **Fig.2** Partial probability distribution of inflow (Fan et al., 2012b)

195 **3 Development of the ITSPF model**

196 **3.1 Methodology**

197 A particular type of stochastic partial information — LPI would be introduced to
 198 express the probability distribution as interval, which can solve the uncertainties of
 199 the probabilistic information to water inflow. Thus, an ITSPF method which combine
 200 interval programming, linear partial information, two-stage stochastic programming
 201 and fractional programming techniques will be proposed to address uncertain
 202 probability distribution described as interval parameters while measuring conflict
 203 objectives. The general ITSPF model can be calculated as follows :

$$204 \quad \max f^{\pm} = \frac{\sum_{j=1}^n c_j^{\pm} x_j^{\pm} + \sum_{h=1}^q \sum_{j=1}^n p_h e_j^{\pm} y_{jh}^{\pm} + \alpha^{\pm}}{\sum_{j=1}^n d_j^{\pm} x_j^{\pm} + \sum_{h=1}^q \sum_{j=1}^n p_h g_j^{\pm} y_{jh}^{\pm} + \beta^{\pm}} \quad (1.1)$$

205 subject to:

$$\sum_{j=1}^n a_{kj}^{\pm} x_j^{\pm} + \sum_{j=1}^n u_{kj}^{\pm} y_{jh}^{\pm} \leq b_k^{\pm}, \quad k = 1, 2, \dots, L, \quad p; \quad h=1, 2, \dots, L, \quad q \quad (1.2)$$

$$\sum_{j=1}^n a_{ij}^{\pm} x_j^{\pm} + \sum_{j=1}^n u_{ij}^{\pm} y_{jh}^{\pm} \geq w_{ih}^{\pm}, \quad i=1, 2, \dots, L, \quad I; \quad h=1, 2, \dots, L, \quad q \quad (1.3)$$

$$x_j^{\pm} \geq 0, \quad j = 1, 2, \dots, L, \quad t, \quad r+1, \dots, L, \quad r+l \quad (1.4)$$

$$y_{jh}^{\pm} \geq 0, \quad j = 1, 2, \dots, L, \quad t, \quad r+1, \dots, L, \quad r+l; \quad h=1, 2, \dots, L, \quad q \quad (1.5)$$

$$\sum_{h=1}^q p_h = 1 \quad (1.6)$$

$$HP \leq b \quad (1.7)$$

$$p_h \geq 0 \quad (1.8)$$

Where $x_j^{\pm}$ is the first-stage variable and $y_{jh}^{\pm}$ is the second-stage variable; the coefficients of first-stage variables are $a_{kj}^{\pm}$, $c_j^{\pm}$, $d_j^{\pm}$ and $a_{ij}^{\pm}$, and the coefficients of second-stage variables are $e_j^{\pm}$, $g_j^{\pm}$, $u_{kj}^{\pm}$ and $u_{ij}^{\pm}$, $\alpha^{\pm}$ and $\beta^{\pm}$ are constants, and $b_k^{\pm}$ are right-hand-side parameters, $w_{ih}^{\pm}$ is random right-hand-side parameters of constraint k under different probability levels (Lin and Huang, 2009). Equations (1.4) to (1.5) represent the non-negative decision variables.

The equations (1.6) to (1.7) represent the constraint of partial information. And the detailed partial probability distribution can be expressed as equation (2.1) (Kofler, 2001). Firstly, the probability p_j is fluctuate within δ and ε , where δ and ε are fixed numbers (i.e., $\delta \leq p_h \leq \varepsilon$). Moreover, the probability of p_j and p_i is equal to γ (i.e., $p_i + p_h = \gamma$). The information mention above can be shown as followings(Kofler, 2001):

$$\pi = \{P = (p_1, \dots, p_N) \in R^N \text{ s.t. } HP \leq b, \sum_{h=1}^N p_h = 1; p_h \geq 0, h = 1, \dots, N\} \quad (2.1)$$

where $H = (h_{ij})$ and $b = (b_i)$ are $s \times N$ and $s \times l$ fixed matrices, respectively. The partial distribution probability described as equation (2.1) can effectively address uncertain probability issues. When the $p_1^*, \dots, p_n^*$ satisfy n of the restrictions contained in $HP \leq b$ as independent equations and the restriction $\sum_{h=1}^N p_h = 1$ is eligible among these n equations, a solution $P^* = (p_1^*, \dots, p_n^*)$ of the inequality (1.7) is an extreme point (Kofler and Zweifel, 1991). Meanwhile, the expected value of a random variable X expressed as equation (2.1) can be defined within the expected interval $[E_{\min}(X), E_{\max}(X)]$, where $E_{\min}(x) = \min_{\{p^*\}} \sum_{h=1}^n p_j x_j$ and $E_{\max}(x) = \max_{\{p^*\}} \sum_{j=1}^n p_j x_j$. Therefore, ITSPF model would get its interval values when the probabilities inflow q_j reach the extreme points by constraints (1.6) to (1.7).

ITSPF can be addressed by a scenario based interactive algorithm where two deterministic sub-models are converted to get its lower and upper bounds. Firstly, extreme points of the linear partial information (LPI) will be obtained by constraints (1.6) to (1.8). Assuming $P^{l*} = (p_1^{l*}, \dots, p_n^{l*})$ ($l=1, \dots, r$) are the extreme points determined by equations (1.6) to (1.8). For one P^l , model (1) can convert to an inexact two-stage fractional (ITF) approach calculated as follows:

$$\max f^{l\pm} = \frac{\sum_{j=1}^n c_j^{\pm} x_j^{\pm} + \sum_{h=1}^q \sum_{j=1}^n p_h^{l*} e_j^{\pm} y_{jh}^{l\pm} + \alpha^{\pm}}{\sum_{j=1}^n d_j^{\pm} x_j^{\pm} + \sum_{h=1}^q \sum_{j=1}^n p_h^{l*} g_j^{\pm} y_{jh}^{l\pm} + \beta^{\pm}} \quad (3.1)$$

subject to:

$$\sum_{j=1}^n a_{kj}^{\pm} x_j^{\pm} + \sum_{j=1}^n u_{kj}^{\pm} y_{jh}^{l\pm} \leq b_k^{\pm}, \quad k = 1, 2, \dots, p; \quad h = 1, 2, \dots, q \quad (3.2)$$

$$246 \quad \sum_{j=1}^n a_{ij}^{\pm} x_j^{\pm} + \sum_{j=1}^n u_{ij}^{\pm} y_{jh}^{l\pm} \geq w_{ih}^{\pm}, \quad i=1, 2, L, I; h=1, 2, L, q \quad (3.3)$$

$$247 \quad x_j^{\pm} \geq 0, j=1, 2, L, t, r+1, L, r+l \quad (3.4)$$

$$248 \quad y_{jh}^{l\pm} \geq 0, j=1, 2, L, t, r+1, L, r+l; h=1, 2, L, q \quad (3.5)$$

249 Secondly, model (3) can be converted to two sub-models by interactive algorithm
 250 (Hladik, 2010). Since the decision variables are independent, the interactions between
 251 the bounds of $x_j^{\pm}$ (or $y_j^{\pm}$) and $f^{\pm}$ can be formulated by Zhu et al. (2014). To
 252 acquire solutions more precisely, sub-model corresponding to f^{-} is formulated first
 253 (Fan and Huang, 2012a):

$$254 \quad \max f^{l-} = \frac{\sum_{j=1}^r c_j^{-} x_j^{-} + \sum_{j=r+1}^n c_j^{-} x_j^{+} + \sum_{h=1}^q \sum_{j=1}^r p_h^{l*} e_j^{-} y_{jh}^{l-} + \sum_{h=1}^q \sum_{j=r+1}^n p_h^{l*} e_j^{-} y_{jh}^{l+} + \alpha^{-}}{\sum_{j=1}^r d_j^{+} x_j^{-} + \sum_{j=r+1}^n d_j^{+} x_j^{+} + \sum_{h=1}^q \sum_{j=1}^r p_h^{l*} g_j^{+} y_{jh}^{l-} + \sum_{h=1}^q \sum_{j=r+1}^n p_h^{l*} g_j^{+} y_{jh}^{l+} + \beta^{+}} \quad (4.1)$$

255 subject to:

$$256 \quad \sum_{j=1}^r |a_{kj}^{\pm}|^{+} \text{sign}(a_{kj}^{\pm}) x_j^{-} + \sum_{j=r+1}^n |a_{kj}^{\pm}|^{-} \text{sign}(a_{kj}^{\pm}) x_j^{+} + \sum_{j=1}^r |u_{kj}^{\pm}|^{+} \text{sign}(u_{kj}^{\pm}) y_{jh}^{l-} + \sum_{j=r+1}^n |u_{kj}^{\pm}|^{-} \text{sign}(u_{kj}^{\pm}) y_{jh}^{l+} \leq b_k^{-} \\ 257 \quad k=1, 2, L, p; h=1, 2, L, q \quad (4.2)$$

$$258 \quad \sum_{j=1}^r |a_{ij}^{\pm}|^{+} \text{sign}(a_{ij}^{\pm}) x_j^{-} + \sum_{j=r+1}^n |a_{ij}^{\pm}|^{-} \text{sign}(a_{ij}^{\pm}) x_j^{+} + \sum_{j=1}^r |u_{ij}^{\pm}|^{+} \text{sign}(u_{ij}^{\pm}) y_{jh}^{l-} + \sum_{j=r+1}^n |u_{ij}^{\pm}|^{-} \text{sign}(u_{ij}^{\pm}) y_{jh}^{l+} \geq w_{ih}^{+} \\ 259 \quad i=1, 2, L, I; h=1, 2, L, q \quad (4.3)$$

$$260 \quad x_j^{-} \geq 0, j=1, 2, L, t \quad (4.4)$$

$$261 \quad x_j^{+} \geq 0, j=r+1, L, r+l \quad (4.5)$$

$$262 \quad y_{jh}^{l-} \geq 0, j=1, 2, L, t; h=1, 2, L, q \quad (4.6)$$

$$263 \quad y_{jh}^{l+} \geq 0, j=r+1, L, r+l; h=1, 2, L, q \quad (4.7)$$

$$264 \quad \text{sign}(a_{ij}^{\pm}) = \begin{cases} 1 & \text{if } a_{ij}^{\pm} \geq 0 \\ -1 & \text{if } a_{ij}^{\pm} \leq 0 \end{cases} \quad (4.8)$$

265 $sign(a_{kj}^{\pm}), sign(u_{ij}^{\pm})$ and $sign(u_{kj}^{\pm})$ can be interpreted similarly. $|a_{ij}^{\pm}|^{\pm}$ is defined as:

$$266 \quad |a_{ij}^{\pm}|^{\pm} = \begin{cases} [|a_{ij}^{-}|, |a_{ij}^{+}|] & \text{if } a_{ij}^{\pm} \geq 0 \\ [|a_{ij}^{+}|, |a_{ij}^{-}|] & \text{if } a_{ij}^{\pm} \leq 0 \end{cases} \quad (4.9)$$

267 $|a_{kj}^{\pm}|^{\pm}, |u_{ij}^{\pm}|^{\pm}$ and $|u_{kj}^{\pm}|^{\pm}$ can be interpreted similarly.

268 The solutions of $x_{jopt}^{-} (j=1,2,...,r), x_{jopt}^{+} (j=r+1,...,n), y_{jhopt}^{l-} (j=1,2,...,r)$ and

269 $y_{jhopt}^{l+} (j=r+1,...,n)$ can be obtained from the sub-model mention above. Then the

270 interactive sub-model corresponding to f^{+} can be described as follows:

$$271 \quad \max f^{l+} = \frac{\sum_{j=1}^r c_j^{+} x_j^{+} + \sum_{j=r+1}^n c_j^{+} x_j^{-} + \sum_{h=1}^q \sum_{j=1}^r p_h^{l*} e_j^{+} y_{jh}^{l+} + \sum_{h=1}^q \sum_{j=r+1}^n p_h^{l*} e_j^{+} y_{jh}^{l-} + \alpha^{+}}{\sum_{j=1}^r d_j^{-} x_j^{+} + \sum_{j=r+1}^n d_j^{-} x_j^{-} + \sum_{h=1}^q \sum_{j=1}^r p_h^{l*} g_j^{-} y_{jh}^{l+} + \sum_{h=1}^q \sum_{j=r+1}^n p_h^{l*} g_j^{-} y_{jh}^{l-} + \beta^{-}} \quad (5.1)$$

272 subject to:

$$273 \quad \sum_{j=1}^r |a_{kj}^{\pm}|^{-} sign(a_{kj}^{\pm}) x_j^{+} + \sum_{j=r+1}^n |a_{kj}^{\pm}|^{+} sign(a_{kj}^{\pm}) x_j^{-} + \sum_{j=1}^r |u_{kj}^{\pm}|^{-} sign(u_{kj}^{\pm}) y_{jh}^{l+} + \sum_{j=r+1}^n |u_{kj}^{\pm}|^{+} sign(u_{kj}^{\pm}) y_{jh}^{l-} \leq b_k^{+}$$

$$274 \quad k=1, 2, L, p; h=1, 2, L, q \quad (5.2)$$

$$275 \quad \sum_{j=1}^r |a_{ij}^{\pm}|^{-} sign(a_{ij}^{\pm}) x_j^{+} + \sum_{j=r+1}^n |a_{ij}^{\pm}|^{+} sign(a_{ij}^{\pm}) x_j^{-} + \sum_{j=1}^r |u_{ij}^{\pm}|^{-} sign(u_{ij}^{\pm}) y_{jh}^{l+} + \sum_{j=r+1}^n |u_{ij}^{\pm}|^{+} sign(u_{ij}^{\pm}) y_{jh}^{l-} \geq w_{ih}^{-}$$

$$276 \quad i=1, 2, L, I; h=1, 2, L, q \quad (5.3)$$

$$277 \quad x_j^{+} \geq 0, j=1, 2, L, t \quad (5.4)$$

$$278 \quad x_j^{-} \geq 0, j=r+1, L, r+l \quad (5.5)$$

$$279 \quad y_{jh}^{l+} \geq 0, j=1, 2, L, t; h=1, 2, L, q \quad (5.6)$$

$$280 \quad y_{jh}^{l-} \geq 0, j=r+1, L, r+l; h=1, 2, L, q \quad (5.7)$$

$$281 \quad x_j^{+} \geq x_{jopt}^{-}, j=1, 2, L, r \quad (5.8)$$

$$282 \quad x_j^{-} \leq x_{jopt}^{+}, j=r+1, \dots, n \quad (5.9)$$

$$y_{jh}^{l+} \geq y_{jhopt}^{l-}, j = 1, 2, L, r \quad (5.10)$$

$$y_{jh}^{l-} \leq y_{jhopt}^{l+}, j = r+1, \dots, n \quad (5.11)$$

Thirdly, the above interactive sub-models can be solved by Charnes and Cooper (1963) directly (Appendix A). After that, repeat equations (4.1) to (5.1) for $l = 1, \dots, r$, and different interval solutions for model (1) under corresponding extreme points can be obtained. If all these requirements have been satisfied, a sets of optimal intervals can be obtained as $f^{\pm} = (f_{opt}^{l\pm}, f_{opt}^{2\pm}, \dots, f_{opt}^{r\pm})$. The final optimal solution for model (1) can be expressed as $f_{opt}^{\pm} = [\min_l(f_{opt}^{l-}), \max_l(f_{opt}^{l+})]$. The framework of ITSPF model is summarized in Fig.3.

292

293 **Fig.3** The framework of ITSPF model

294 3.2 Model formulation

295 In ITSPF-IPWRM, the goal is to maximize the economic efficiency of water

consumption. Moreover, uncertain probability are addressed in constraint (f). In detail, system net benefit of the ITSPF-IPWRM model can be described as follows:

(1) System net benefit

$$f_1^\pm = \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K NB_{jk}^\pm \times (T_{ijk}^\pm + IRW_{jk}^\pm) - \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H p_h^{l*} \times CT_{jk}^\pm \times D_{ijkh}^{l\pm} \quad (6.1)$$

where i is the index for water sources; j is the index for water user plants; k is index for planning periods; h is the index for probability level; $NB_{jk}^\pm$ is net benefit to user j per unit of water allocated in period k (yuan/m³); $T_{ijk}^\pm$ is fixed allocation target for water that is promised from source i to user j in period k (m³/day, the first-stage decision variable); $IRW_{jk}^\pm$ is the water supply targets during planning period k from recycled water to user j (m³/day); $CT_{jk}^\pm$ is reduction of net benefit to user j per unit of source i not delivered (yuan/m³, $CT_{jk}^\pm > NB_{jk}^\pm$); $D_{ijkh}^{l\pm}$ is amount by which water allocation target $T_{ijk}^\pm$ is not met when the seasonal flow is $q_{ih}^\pm$ with probability p_h^{l*} under extreme point P^{l*} (m³/day, the second-stage decision variable).

(2) Costs for water supply

$$f_2^\pm = \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H (T_{ijk}^\pm - D_{ijkh}^{l\pm} \times p_h^{l*}) \times WSC_{jk}^\pm \quad (6.2)$$

where $WSC_{jk}^\pm$ is unit cost of water supply from water source i during period k to user j (yuan/m³).

(3) Cost for reclaimed water supply

$$f_3^\pm = \sum_j \sum_k (IRW_{jk}^\pm \times CRIN_{jk}^\pm) \quad (6.3)$$

where $CRIN_{jk}^{\pm}$ is unit cost of water supply from reused water to user j during planning period k (yuan/m³).

(4) Cost for wastewater treatment

$$f_4^{\pm} = \sum_i^I \sum_j^J \sum_k^K \sum_h^H (T_{ijk}^{\pm} + IRW_{jk}^{\pm} - p_h^{l*} \times D_{ijkh}^{\pm}) \times CWWT_{jk}^{\pm} \times \beta_{jk}^{\pm} \quad (6.4)$$

where $CWWT_{jk}^{\pm}$ is unit cost of wastewater treatment for user j during period k (yuan/m³); $\beta_{jk}^{\pm}$ is the discharge coefficient of wastewater per unit consumption of water during period k for user j .

Therefore, the objective function of ITSPF-IPWRM model for industrial park water resource management could be calculated as follows:

$$\begin{aligned} \max f^{\pm} &= \frac{\text{System net benefit}}{\text{Total water consumption}} \\ &= \frac{f_1^{\pm} - f_2^{\pm} - f_3^{\pm} - f_4^{\pm}}{\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H (T_{ijk}^{\pm} + IRW_{jk}^{\pm} - D_{ijkh}^{l\pm})} \end{aligned} \quad (6.5)$$

Then equations (6.6) to (6.13) present the constraints of ITSPF-IPWRM model:

a) Water supply constraints

The total amount of water allocation to different plants should be lower than the supplying capacity of each water sources:

$$q_{ih}^{\pm} \geq \sum_{j=1}^J (T_{ijk}^{\pm} - D_{ijkh}^{l\pm}) \quad \forall j, k, h \quad (6.6)$$

where $q_{ih}^{\pm}$ is total water available for source i under water inflow level h (m³/day).

b) The amount of water consumption for different water user plants

The total water supply to the water user plants should be larger than the lowest water consumption:

$$\sum_{i=1}^I (T_{ijk}^{\pm} - D_{ijkh}^{l\pm}) + IRW_{jk}^{\pm} \geq INDW_{jk}^{\pm} \quad \forall j, k, h \quad (6.7)$$

where $INDW_{jk}^{\pm}$ is the amount of water demand for user j during period k (m^3/day).

c) Wastewater treatment capacity

$$\sum_i^I \sum_j^J (T_{ijk}^{\pm} - D_{ijkh}^{l\pm} + IRW_{jk}^{\pm}) \times \beta_{jk}^{\pm} \leq SWT_k^{\pm} \quad \forall j, k, h \quad (6.8)$$

where $SWT_k^{\pm}$ is the capacity for sewage treatment during period k (m^3/day).

d) Water supply capacity of waterworks

The water allocation should be lower than the maximum water supply ability of waterworks:

$$\sum_i^I \sum_j^J (T_{ijk}^{\pm} - D_{ijkh}^{l\pm}) \leq TWC_k^{\pm} \quad \forall j, k, h \quad (6.9)$$

where $TWC_k^{\pm}$ is the capacity for water treatment plant during period k (m^3/day).

e) Water reuses capacity

The wastewater after been treated can only be reused by themselves due to the limitation of reusing technology and water quality. For each water using plant, the recycled water should be lower than the capacity and the maximum treatment ability of of reused water:

$$IRW_{jk}^{\pm} \leq [\sum_{i=1}^I (T_{ijk}^{\pm} - D_{ijkh}^{l\pm}) + IRW_{jk}^{\pm}] \times \theta_{jk}^{\pm} \leq IRWT_{jk}^{\pm} \quad \forall j, k, h \quad (6.10)$$

where $IRWT_{jk}^{\pm}$ is the capacity for water recycling treatment to user j during period k (m^3/day); $\theta_{jk}^{\pm}$ is the recycling rate of water consumption during period k for user j .

f) Linear partial information constraint

$$\sum_{h=1}^H p_h^{l^*} = 1 \quad (6.11)$$

$$HP \leq b \quad (6.12)$$

where $P = (p_1, \dots, p_N) \in R^N$, $(h = 1 \dots N)$; $H = (h_{ij})$ and $b = (b_i)$ are $s \times N$ and $s \times l$ fixed matrices, respectively.

g) Technical constraint

$$T \max_{ijk}^{\pm} \geq T_{ijk}^{\pm} \geq D_{ijkh}^{l\pm} \geq 0 \quad \forall i, j, k, h \quad (6.13)$$

where $T \max_{ijk}^{\pm}$ is maximum allowable allocation amount from source i to user j in period k (m^3/day).

4 Application

4.1 Statement of problems

Electric power, electronic communication, machinery, food, beverage and pharmaceutical factories are considered the top six major water user plants in industrial park (The People's Government of Tianjin Binhai New Area, 2016). Assuming the planning horizon is fifteen-years and divide into three periods (five years each) to demonstrate the dynamic characteristics in the industrial park water system. Since the reclaimed water can satisfy the low quality of water demand processes, consider the wastewater of factories can be reused by themselves after simple treatment which can reduce the consumption of high quality water. Therefore, water will be supplied from the two sources (e.g., surface water and underground water) to each plant with different amount and operational cost. And the reclaimed water will not be considered in pharmaceutical factory because of the security of production and public acceptance barriers.

In this research, interval parameters are applied to solve imprecise uncertainties, which are related to many constraints such as water demands, the costs of water

supply, the capacities of wastewater treatment and reclaimed water supply, the availability of water resources and so on. The research data are sourced from comprehensive review of statistical yearbooks, governmental reports and many other documents. According to the current situation of industrial park water consumption, Table 1 lists the target water demand for six plants under different water sources. Table 2 presents the economic and operational parameters to each water user plant within industrial park. The industrial park development will obtain benefits, when it meets the need of water using plant. If the promised amount of water supply is not satisfy, it will cause a reduced net system benefit. Under the circumstances, water using plants will encounter higher water priced and production must be reduced to curtail its water demand (Huang and Loucks, 2000). In this research, surface water and groundwater are divided into low, medium and high water inflow levels due to its randomness and variability among different years. Table 3 lists the availability of surface water and underground water and their corresponding probabilities of stream inflow levels. And ITSPF-IPWRM will obtain the lower and upper bound of the objective function at extreme points as the probabilities of the stream inflow fluctuated within their intervals (Kofler, 2001; Fan et al., 2012b) (Table 4, i.e., S1, S2, S3 and S4).

Table 1 Water allocation targets for each plant from different water source (unit: m³/day)

Planning period	Water source	Plant					
		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
$k=1$	$i=1$	[2680,3580]	[1670,2540]	[1768,2458]	[1330,1870]	[1145,1921]	[1453,2363]
	$i=2$	[840,1730]	[865,1215]	[320,525]	[1150,2010]	[1370,2230]	[550,1200]
$k=2$	$i=1$	[3350,4260]	[2447,3137]	[2063,2793]	[2350,3040]	[2619,3449]	[2116,3096]
	$i=2$	[910,1820]	[730,1130]	[280,610]	[1359,2289]	[1860,2790]	[720,1570]
$k=3$	$i=1$	[4670,5700]	[3136,4046]	[2936,3816]	[3319,4089]	[3015,3995]	[3119,3989]
	$i=2$	[1070,2050]	[910,1510]	[540,940]	[1623,2273]	[2030,2680]	[940,1290]

¹($i=1$ is surface water, $i=2$ is underground water; $j=1$ is electric power factory; $j=2$ is electronic communication factory; $j=3$ is machinery factory; $j=4$ is food factory; $j=5$ is beverage factory; $j=6$ is medicine factory. And i, j represent the same in context.)

Table 2 Economic and operational parameters used in the optimization model

Planning period	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
Net benefit when water demand is satisfied (yuan/m ³)						
$k=1$	[335,390]	[285,340]	[275,330]	[265,320]	[250,305]	[225,280]
$k=2$	[380,435]	[310,365]	[300,355]	[280,335]	[270,325]	[255,310]
$k=3$	[435,490]	[350,405]	[330,385]	[310,365]	[300,355]	[285,340]
Penalty when water demand is not delivered (yuan/m ³)						
$k=1$	[430,485]	[395,450]	[375,430]	[370,425]	[345,400]	[340,395]
$k=2$	[475,530]	[405,460]	[390,445]	[385,440]	[360,415]	[355,410]
$k=3$	[530,585]	[445,500]	[420,475]	[400,455]	[385,440]	[370,425]
The amount of water demand (m ³ /d)						
$k=1$	[3384,5454]	[1036,1356]	[913,1033]	[3745,3845]	[3120,3210]	[1935,1985]
$k=2$	[4884,6084]	[2236,2306]	[1113,1903]	[5545,5745]	[4620,4820]	[2100,2150]
$k=3$	[5284,7484]	[3036,3336]	[2713,2803]	[6045,6245]	[5320,5520]	[3500,3550]

Table 3 Description of stream flow distribution (m³/d)

Inflow level			
	Low ($h=1$)	Medium ($h=2$)	High ($h=3$)
$i=1$	[10990,14090]	[11270,15770]	[23445,28645]
$i=2$	[4143.9,8543.9]	[4207.8,9107.8]	[11055.4,13355.4]

Table 4 Description of partial probability distribution

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Low ($h=1$)	0.1	0.1	0.2	0.2
Medium ($h=2$)	0.7	0.6	0.6	0.5
High ($h=3$)	0.2	0.3	0.2	0.3

²(Abbreviated scenario 1, scenario 2, scenario 3 and scenario 4 to S1, S2, S3 and S4, respectively.)

4.2 Results and analysis

Figs.4 to 5 depict the amount of utilization and shortage of each water source as the water change from low to high inflow level in each scenario. It can be seen that more water would be allocated into the system as the probability of high water inflow level

411 is getting higher under the same probability of low water inflow level. The reason is
412 that the system would have more water resources allocated to each enterprise as the
413 probability of high flow level increased. As for S2, for instance, $[2.81, 3.84] \times 10^3$
414 m^3/day , $[11.27, 14.95] \times 10^3 \text{ m}^3/\text{day}$ and $14.95 \times 10^3 \text{ m}^3/\text{day}$ of surface water would be
415 allocated in period 2 under low, medium and high water inflow levels, respectively
416 (Fig.4). Compared to S2, it can be seen that the surface water distribution for S1
417 would be $[43.20, 51.00]\%$, $[0, 4.80]\%$ and $[68.40, 70.00]\%$ lower than S2 under low,
418 medium and high water inflow levels, respectively. Moreover, the amount of water
419 shortage for S2 would be lower than S1 over periods under the same water inflow
420 level (Fig.5). For example, the amount of water shortage for S2 under low water
421 inflow level would be $[9.70, 17.50]\%$, $[10.60, 13.00]\%$ and $[0.70, 4.90]\%$ lower than
422 S1 in period 1, 2 and 3, respectively. Therefore, as higher probability of high water
423 inflow level, the less amount of water shortage would be. Under the condition of
424 sufficient water resource, the industrial park would be more inclined to use
425 high-quality water to meet the quality of production, and the consumption of
426 reclaimed water would be reduced. As illustrated in Fig.6, the utilization of reclaimed
427 water for S1 would be $[7.80, 8.89] \times 10^3 \text{ m}^3/\text{day}$, $[19.04, 20.03] \times 10^3 \text{ m}^3/\text{day}$ and $[24.18,$
428 $24.72] \times 10^3 \text{ m}^3/\text{day}$ over periods. And there would be $[6.39, 7.14] \times 10^3 \text{ m}^3/\text{day}$, $[17.48,$
429 $18.83] \times 10^3 \text{ m}^3/\text{day}$ and $[21.60, 22.87] \times 10^3 \text{ m}^3/\text{day}$ for S2 in period 1, 2 and 3,
430 respectively. Compared to S2, the risk of water shortage would be higher in S3 due to
431 the higher probability of low water inflow level and thus the water supply of S3 is
432 lower than S2. For example, the deficiency of surface water for S3 would be $[5.13,$
433 $9.22]\%$, $[25.00, 40.00]\%$ and $[77.33, 85.72]\%$ higher than S2 in period 1 under low,
434 medium and high water inflow levels, respectively. The industrial park would increase
435 the utilization of reclaimed water to meet their own production needs when the risk of

436 water shortage increased. In this case, the utilization of reclaimed water for S3 would
437 be $[8.03, 8.91] \times 10^3 \text{ m}^3/\text{day}$, $[19.96, 20.36] \times 10^3 \text{ m}^3/\text{day}$ and $[24.18, 24.87] \times 10^3 \text{ m}^3/\text{day}$,
438 which higher than S1 and S2. At last, the allocation of surface water for S4 would be
439 higher than S3 due to the probability of high water inflow level increased from 0.2 to
440 0.3. In general, the total underground water supply is lower than surface water. As for
441 S2 in period 2, for instance, the allocation of underground water would be $[25.50,$
442 $43.70]\%$, $[60.80, 62.70]\%$ and 60.80% lower than surface water under low, medium
443 and high water inflow levels, respectively. Therefore, surface water would be the
444 major supply source in industrial park. Following surface water, the trend of
445 underground water would be allocated in a similar way under the change of water
446 inflow level. For example, the allocation of underground water for S2 would be
447 $[26.68, 29.12]\%$, $[1.96, 2.38]\%$ and $[39.89, 67.81]\%$ higher than S1 in period 1 under
448 low, medium and high water inflow levels, respectively. And the amount of water
449 shortage would be $[29.62, 29.36]\%$, $[0, 68.18]\%$ and $[58.62, 63.24]\%$ lower than S1.
450 To sum up, the amount of water supply would increase with the probability of high
451 water inflow level when the probability of low inflow level is equal. In order to meet
452 the quality of production, the industrial park is more inclined to high quality water
453 and the water distribution for surface water and underground water would increase,
454 while the amount of reclaimed water would decrease. When the probability of low
455 water inflow level increased, the risk of water shortage would be higher and the
456 industrial park would increase the utilization of reclaimed water to satisfy its own
457 production needs. Surface water is the main water supply for the industrial park,
458 followed by underground water. Under the higher risk of water shortage, reclaimed
459 water could replace the high quality water to satisfy the process of low quality water
460 use and reduce low benefit caused by high water shortage. This could avoid the

dependence of high quality water, which is helpful for strengthening the stability and sustainability of industrial park water supply.

Fig.4 Optimal allocation schemes for different water sources in each scenario over periods ($k=1$ is period 1; $k=2$ is period 2; $k=3$ is period 3; $h=1$ is low inflow level; $h=2$ is medium inflow level; $h=3$ is high inflow level)

Fig.5 The amount of water shortage in each scenario over periods

Fig.6 The amount of reclaimed water in each scenario over periods

In order to clarify the amount of water allocation and shortage of each plant in different periods, the S1 under low water inflow level is taken as an example due to the water consumption for each plant in different scenario is similar. Fig. 7 presents the changes of water allocation and shortage in different periods for S1 under the low water inflow level. The water shortage of each plant would gradually increase over periods under the same water inflow level when surface water is used as the water supply source. For example, the amount of water shortage for electric power would be $[2.49, 3.30] \times 10^3 \text{ m}^3/\text{day}$ in period 1 under low water inflow level, which would be $[1.5, 9.8]\%$ and $[26.9, 29.4]\%$ lower than period 2 and 3. And the allocation of surface water would be $[0, 0.81] \times 10^3 \text{ m}^3/\text{day}$, $[0, 0.59] \times 10^3 \text{ m}^3/\text{day}$ and $[0, 1.26] \times 10^3 \text{ m}^3/\text{day}$ in period 1, 2 and 3, respectively. This is attribute to the amount of water available can not meet the increasing demand of electric power, resulting in a gradual increase in water shortage. Besides, the amount of water shortage for electric power would be the highest compared to other plants. For example, the deficiency of surface water for electronic communication, machinery and medicine factory would be $1.70 \times 10^3 \text{ m}^3/\text{day}$, $1.80 \times 10^3 \text{ m}^3/\text{day}$ and $[16.31, 66.31] \text{ m}^3/\text{day}$, respectively. And the amount of water shortage for food and beverage factory would be the lowest which the amount

of water allocation would reach the upper bound of its water demand and would be $1.33 \times 10^3 \text{ m}^3/\text{day}$ and $1.75 \times 10^3 \text{ m}^3/\text{day}$. The reason is that food and beverage factory would be the second and third largest water demand in the industrial park. And compared to electric power, the largest water-consuming factory in the park, the sewage discharge of food and beverage factory would be low. Therefore, the water demand of food and beverage factory would be preferentially satisfied. Reclaimed water would be used to replace high quality water (i.e., surface water and underground water) to meet the low water quality requirements in some processes of factory, when factories challenged by water shortage. Fig.8 illustrated the utilization of reclaimed water of each factory in S1. The amount of reclaimed water would gradually increased in period 1, 2 and 3. For example, the utilization of reclaimed water for electric power would be $[4.08, 4.65] \times 10^3 \text{ m}^3/\text{day}$, $4.88 \times 10^3 \text{ m}^3/\text{day}$ and $5.28 \times 10^3 \text{ m}^3/\text{day}$ over periods. For period 2 and 3, there would be $[5.10, 19.58]\%$ and $[13.71, 29.37]\%$ higher than period 1. Compared to other factories, electric power would have the largest utilization of reclaimed water. For example, the amount of reclaimed water for electric power, electronic communication, machinery, food and beverage factory would be $[4.08, 4.65] \times 10^3 \text{ m}^3/\text{day}$, $[1.04, 1.36] \times 10^3 \text{ m}^3/\text{day}$, $[0.91, 1.03] \times 10^3 \text{ m}^3/\text{day}$, $1.76 \times 10^3 \text{ m}^3/\text{day}$ and $[0, 0.09] \times 10^3 \text{ m}^3/\text{day}$ in S1 during period 1. In conclusion, the deficiency of water for each factory would gradually increase over periods under the same water inflow level due to the water availability can not satisfy the increasing water demand of factories. In order to satisfy their own water demand, they would increase the utilization of reclaimed water to replace the high quality water to satisfy the process of low quality water use .

Fig.7 The amount of water allocation and shortage for the six water user plants under low water inflow level in S1 during the planning horizon

Fig.8 The amount of reclaimed water for the six water user plants in S1 during the planning horizon

Fig.9 present the upper and lower bounds of the modeling objective function, for instance, net system benefits and recourse cost. In general, electric power factory has the highest net benefit over all planning periods and water inflows scenarios. For S1 during period 1, the total net benefit for electric power factory is $[4.10, 5.20] \times 10^6$ yuan, while electronic communication, machinery and medicine factory is $[1.30, 1.80] \times 10^6$, $[1.10, 1.30] \times 10^6$ and $[4.50, 5.60] \times 10^5$ yuan, respectively. Meanwhile, the net benefit would increase from period 1 to 3 for all scenario. For example, the net benefit

would be $[4.10, 5.20] \times 10^6$, $[4.70, 5.80] \times 10^6$ and $[7.10, 8.00] \times 10^6$ yuan for electric power in S2 from period 1 to 3, respectively. The more water allocates, the higher net benefit factory can get. Besides, the factories would have higher recourse cost under higher benefit. The recourse cost for electric power, electronic communication, machinery and medicine factory during period 1 in S1 would be $[4.30, 6.00] \times 10^5$, $[3.00, 3.40] \times 10^5$, $[2.40, 3.70] \times 10^5$ and $[0.18, 2.70] \times 10^4$ yuan, respectively. It can be seen that electric power factory should be ensured because it provide the highest benefit ($NB_{jk}^\pm$) when its water demand is met, whereas it is subject to the highest penalty if the promised water is not delivered. In comparison with that, medicine factory would correspond to lower benefit and penalties. Consequently, the allocation water to medicine factory should be diminished firstly in the case of inadequate water supplies.

Fig.9 Net system benefit and recourse cost for each factory during the three planning periods

Generally, the above results present different water allocation schemes under different scenarios, and different economic efficiency of water use would obtain (Fig.10). In detail, the objective function value would be [98.10, 126.80], [98.10, 124.10], [95.00, 122.20] and [95.70, 120.60] under S1, S2, S3 and S4, respectively. The objective function values gradually decrease as a result of higher probabilities for low inflow and lower probabilities of medium inflow. As the probabilities of low inflow levels for S1, S2, S3 and S4 would be 0.1, 0.1, 0.2 and 0.2, respectively; while the probabilities of medium inflow levels for S1, S2, S3 and S4 would be 0.7, 0.6, 0.6 and 0.5, respectively. Therefore, more water resources in S1 would be available. Above all, the final optimized value for the objective function would be $f_{opt}^{\pm} = [\min(f^{1-}, f^{2-}, f^{3-}, f^{4-}), \max(f^{1+}, f^{2+}, f^{3+}, f^{4+})] = [95.00, 126.80]$. It presents the range for the objective function over the planning horizon.

Fig.10 The economic efficiency of water use for ITSPF, ITPF, ITSPP and ITP model

4.3 Discussion

In this research, the proposed ITSPF-IPWRM method can obtain optimal efficiency between system benefit and water allocation in industrial park. The interval solutions present the dynamics of water allocation patterns which are sensitive to uncertain inputs. Multiple models are introduced for comparison to confirm the effectiveness

and reliability of ITSPF model. The proposed inexact two-stage fractional programming (ITFP) can tackle interval uncertainty except uncertain probability. Hence, an ITFP model is generated to address industrial park water resources management problem by introducing the mid-values to represent the interval probabilities. Moreover, the ITSPF model will convert to inexact two-stage stochastic partial programming (ITSPP) as the planners pay more attention to maximize system benefits. And the inexact two-stage programming (ITP) with deterministic probability distribution is also considered. Therefore, three optimization models that originated from the ITSPF model are established and listed in Table 5 and the optimal values of different optimization models are shown in Fig.11.

Table 5 Optimization models for comparing with ITSPF model

Model	Objective function	Constraints
ITFP	Maximize the economic efficiency of water use deterministic probability distribution	Same as the ITSPF
ITSPP	Maximize the system economic benefit with partial probability distribution	
ITP	Maximize the system economic benefit with deterministic probability distribution

Fig.11 Net benefit and water supply formulate by ITSPF, ITFP, ITSPP and ITP model

In terms of the system economic benefit as shown in Fig.11, the solution for ITSPF would be $[3.66, 5.41] \times 10^7$ yuan, while ITFP model would be $[3.81, 5.19] \times 10^7$ yuan.

The range of system economic benefit for ITSPP would be $[5.21, 7.18] \times 10^7$ yuan when consider the probability distribution partially. And $[5.25, 7.18] \times 10^7$ yuan

would be obtained formulating by ITP model. It can be seen that the values without considering partial probability distribution (i.e., ITFP and ITP model) are obviously located within ITSPF and ITSPP, respectively. As for water supply, there would be

$[3.85, 4.17] \times 10^5$ m³/day for ITSPF model in S1. And the amount of water supply would be increased to $[3.97, 4.36] \times 10^5$ m³/day as the probability of high stream

inflow ascend from 0.2 to 0.3. The value would go down to $[3.88, 4.06] \times 10^5$ m³/day

since the probability of low stream inflow would be higher. And the amount of water

allocation would be $[3.83, 4.18] \times 10^5$ m³/day in S4. Therefore, the final optimized

range for ITSPF model would be $[3.83, 4.36] \times 10^5$ m³/day. For ITPF model, the final amount of water allocation would be $[3.97, 4.25] \times 10^5$ m³/day, which is obviously located within ITSPF model. Moreover, the optimal water allocation would be $[5.71, 6.26] \times 10^5$ m³/day, $[5.71, 6.26] \times 10^5$ m³/day, $[5.75, 6.28] \times 10^5$ m³/day and $[5.75, 6.28] \times 10^5$ m³/day for ITSPF model in S1, S2, S3 and S4, respectively. It can be seen that the water allocation would be the same under the same probability of low stream inflow formulating by ITSPF mode. Therefore, the ITSPF results can provide wider adjustable ranges than ITSPF, thus ITSPF can offer more practical possibilities for obtaining satisfactory alternatives under various system conditions without missing possible solution set.

As depicted in Fig.10, the economic efficiency of water use for ITPF model would be $[96.10, 122.10]$. Although the ITPF method can generate feasible interval solution for practical problems with deterministic probability distribution, it would ignore many other feasible alternatives as the stochastic partial information is varied within their spectra and can not acquire the final fluctuating interval of the objective function when the parameters and decision variables fluctuate within their intervals. For ITSPF model, the objective function value would be $[91.22, 114.74]$, $[91.48, 114.74]$, $[91.17, 114.24]$ and $[91.27, 114.24]$ under S1, S2, S3 and S4, respectively. Therefore, the final optimized value for the objective function would be $[91.17, 114.74]$. And the optimal value for ITP model would be $[91.25, 114.24]$. Although the systems would have higher system economic benefit with mono-objective, it would consume a higher amount of water and present lower levels of system efficiency. For example, the system economic benefit for ITSPF and ITP is $[32.56, 42.13]\%$ and $[32.55, 43.26]\%$ higher than ITSPF, respectively. But their system efficiency would be $[4.03, 9.51]\%$ and $[3.95, 9.90]\%$ lower than ITSPF.

606 In comparison, the ITSPF model can address uncertain probability through generating
607 interval solutions under corresponding extreme points effectively. Uncertain cost
608 coefficients, resource availability, and constraint parameters can be totally addressed
609 in ITSPF. The ITSPF model can be transformed into an ITFP model and then obtained
610 decision intervals under each extreme point. On one hand, ITSPF advances
611 conventional FP by tackling probability distribution of water inflow expressed as
612 interval and also avoiding the loss of uncertain information during data input.
613 Meanwhile, ITSPF can obtain interval sets that offer plenty of decision alternatives in
614 response to uncertain inputs and wider adjustable ranges can be obtained which can
615 provide a great deal of real-world possibilities for obtaining satisfactory alternatives
616 under multiple system conditions. On the other hand, ITSPF is superior to existing
617 industrial water optimization methods which can simultaneously reflect the economic
618 efficiency of water use while tackling various uncertainties exist in system.

619 The ITSPF method firstly address the compound uncertainties expressed as the
620 integration of interval numbers, probability distributions and linear partial information
621 under the frame of fractional programming simultaneously. However, the ITSPF
622 method can not deal with fuzzy uncertainty, chance-constrained programming and
623 many other complicated uncertainties in practical environmental management
624 problems. Moreover, the ITSPF approach can not address other type of stochastic
625 information existing in general stochastic partial information. Hence, further studies
626 can concentrate on improving the proposed ITSPF approach to reflect more
627 uncertainties such as integrating with fuzzy flexible programming, chance constrained
628 programming, robust programming, etc. In addition, the proposed ITSPF approach
629 method is not limited to industrial park water resource planning, which is also useful
630 to address other types of resources management issues.

5 Conclusions

Inherent uncertainties of solving industrial park water resource management problems will cause great difficulties while considering conflict objectives of system. In this research, an inexact two-stage stochastic partial fractional programming has been established for industrial park water resource management to address dual uncertainties. The contribution of this research can be summarized as follows. Initially, dual uncertainties and ratio problems can be effectively reflected through the ITSPF model, which advance the existing optimization methods through integrating ITSPF model with FP. Additionally, the FP concept has been firstly applied to industrial park water resource management which advance in balancing conflict objectives by considering economic efficiency of water use and reflecting complicated interactions among various system factors. Eventually, it facilitates dynamic analysis for decisions of water allocation within a multi-source, multi-period and multiple uncertain water inflow levels context. The ITSPF model is transformed into two sub-models which reflect the dynamics and interactions between complicated uncertainties and decision processes under various representative scenarios. Therefore, ITSPF can permit in-depth insights of different stream inflow scenarios when the promised water-allocation targets are violated.

The ITSPF method has been applied to industrial park water resources management, which the uncertain probability of stream inflow is tackled by introducing linear partial information. The results indicated that ITSPF-IPWRM can help to identify desired water allocation schemes under compound complexities. Firstly, more high quality water would be allocated into the system and the amount of reclaimed water would be decreased as the probability of high stream inflow level is getting higher while the probability of low stream inflow level is equal. Since the probability of low

water inflow level is getting higher, the system would gain the higher risk of water shortage and the reclaimed water would be increased to meet the needs of water user plants. Secondly, reclaimed water would be chosen when the water demand of enterprises is getting higher and higher, which would be ascribed to its lower water allocation cost by the companies. Thirdly, the plant with highest benefit ($NB_{jk}^{\pm}$) should be ensured because it would gain the highest penalty if the promised water is not delivered. A comparative least-cost water allocation model and model without stochastic partial information have been considered. For one thing, the ITSPF generates feasible alternatives as the stochastic partial information, operational parameters and decision variables are varied within their spectra. For another, it can consume a lower amount of water and present higher levels of system efficiency. Particularly, more uncertainties such as fuzziness can be considered in the future studies. And ITSPF can be exploited in other resources management problems which is helpful for inexact analysis and relative comparisons of two magnitudes.

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Supplementary Material

Appendix A. Solving procedure of two-stage linear fractional model

The two-stage linear fractional model can be formulated as follows:

$$\max f = \frac{\sum_{j=1}^n c_j x_j + \sum_{h=1}^v \sum_{j=1}^n p_h e_j y_{jh} + \alpha}{\sum_{j=1}^n d_j x_j + \sum_{h=1}^v \sum_{j=1}^n p_h g_j y_{jh} + \beta}$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j + \sum_{j=1}^n u_{ij} y_{jh} \leq b_i, \quad i = 1, 2, L, m; \quad h = 1, 2, L, v$$

$$\sum_{j=1}^n a_{kj} x_j + \sum_{j=1}^n u_{kj} y_{jh} \leq w_{kh}, \quad k = 1, 2, L, K; \quad h = 1, 2, L, v$$

$$\sum_{h=1}^v p_h = 1$$

$$x_j \geq 0, \quad j = 1, 2, L, n$$

$$y_{jh} \geq 0, \quad j = 1, 2, L, n; \quad h = 1, 2, L, v$$

If the denominator is strictly positive, this model can be transformed to:

$$\max f = \sum_{j=1}^n c_j x_j^* + \sum_{h=1}^v \sum_{j=1}^n p_h e_j y_{jh}^* + \alpha r$$

Subject to:

$$\sum_{j=1}^n d_j x_j^* + \sum_{h=1}^v \sum_{j=1}^n p_h g_j y_{jh}^* + \beta r = 1$$

$$\sum_{j=1}^n a_{ij} x_j^* + \sum_{j=1}^n u_{ij} y_{jh}^* \leq b_i r, \quad i = 1, 2, L, m; \quad h = 1, 2, L, v$$

$$\sum_{j=1}^n a_{kj} x_j^* + \sum_{j=1}^n u_{kj} y_{jh}^* \leq w_{kh} r, \quad k = 1, 2, L, K; \quad h = 1, 2, L, v$$

$$x_j^* = x_j r$$

$$y_{jh}^* = y_{jh} r$$

$$\sum_{h=1}^v p_h = 1$$

$$x_j^* \geq 0, \quad j = 1, 2, L, n$$

$$y_{jh}^* \geq 0, \quad j = 1, 2, L, n; \quad h = 1, 2, L, v$$

The above mentioned model could be solved by branch and bound algorithm directly.

Statements & Declarations

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Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Author Contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Zhuoling Lan. The first draft of the

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