Optimization of redundant degree of freedom in robot milling considering chatter stability

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Manuscript Title

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Abstract:

Aiming at the redundant degree of freedom problem of six axis industrial robot performing five axis milling task, a new optimization method of robot milling redundant degrees of freedom considering chatter stability was proposed. Taking the limit cutting depth of robot milling regenerative chatter as the optimization objective, and combined with the constraints of robot motion performance (avoiding joint limit, singular pose and collision), a redundant degree of freedom optimization model considering chatter stability was established. The milling experimental results showed that the new optimization method of robot milling redundant degrees of freedom considering chatter stability proposed in this paper can effectively avoid robot milling regenerative chatter and improve machining quality and efficiency.

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Highlights:

1. A new optimization method of redundant degrees of freedom in robotic milling considering chatter stability is proposed to determine the optimal redundant degrees of freedom when a six axis industrial robot performs a five axis milling task.
2. Experiments verify the feasibility of this method to avoid robotic milling regenerative chatter and improve machining quality and efficiency, It provides a theoretical basis for the setting of redundant degrees of freedom in robotic machining CAM system.
3. The optimal redundancy degree of freedom significantly improves the chatter stability of robot high-speed milling.

Optimization of redundant degree of freedom in robot milling considering chatter stability

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Abstract:

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1 Introduction

With the advantages of large working space, high flexibility and low cost, industrial robots have been gradually applied in the field of milling, such as the processing of automobile inspection tools, ship hulls and large aircraft parts [1]. However, the development of CAM systems for robot machining is far behind that for machine tool machining.

In recent years, some mainstream commercial off-line programming software such as RobotStudio and KUKA-Sim have successively launched robot machining modules, which can convert the G code generated by five-axis milling modules in CAD/CAM software into robot program, realizing automatic programming of complex parts machining[2]. Series industrial robots used for milling usually have 6 degrees of freedom, while the tool point pose information in G code only contains position X , Y , Z and orientation A and B . Therefore, the six-axis industrial robot performing five-axis milling tasks will lead to one redundant degree of freedom, namely the degree of freedom rotating around the axis of the tool[3]. At present, the commercial off-line programming software needs to be manually set by users to complete the transformation of robot program. However, there is a lack of theoretical basis for setting and optimizing the redundant degrees of freedom.

The optimization of redundant degrees of freedom in robot machining has become an important research content in recent years. Scholars have put forward different optimization objectives, such as joint limit, flexibility, Cartesian stiffness and so on. Zhu et al.[4]proposed the joint limit performance index and took it as the optimization objective to make each joint axis of the robot away from the joint limit angle through the optimal

selection of redundant degrees of freedom. Zargarbashi et al.[5] took Jacobian condition number as the optimization objective of flexibility, aiming to make better use of redundant degrees of freedom to avoid robot singularity. In order to improve the robot milling accuracy, Chen et al.[6] took the normal stiffness as the optimization objective and the joint limit as the constraint condition to optimize the redundant degrees of freedom and feed direction. In order to improve the production efficiency of robot milling, Xiong et al.[7] took the feed direction stiffness as the optimization objective and the joint limit and singularity as the constraints to optimize the redundant degrees of freedom of the five axis milling industrial robot. Lu et al.[8] proposed a joint motion planning algorithm based on redundant degree of freedom optimization to realize smooth and collision free machining. Xiao [9] established a multi-objective optimization model of joint limit, flexibility and collision to determine the redundant degrees of freedom of a six-axis industrial robot in five-axis milling applications.

Aiming at the optimization of redundant degrees of freedom in robot machining, the optimization objectives proposed in the existing literature mainly focus on improving the kinematics of the robot, avoiding collision and improving the Cartesian stiffness of the robot end effector, ignoring the chatter stability of robot machining. However, some researchers[10-12] found through experimental research that regenerative chatter is the main reason for the instability of robot high-speed milling process. Chatter will not only affect the machining quality, reduce the tool life, and even damage the robot. Celikag et al.[13] also confirmed the view that regenerative chatter plays a dominant role in robot milling from two aspects of theoretical analysis and experimental research. Mejri et al.[14] found through experimental modal analysis that the change of robot pose will cause the dynamic change of tool tip, and then affect the stability of machining process. On this basis, Mousavi et al.[15-18] studied the influence of redundant degrees of freedom on milling stability based on the robot dynamics model. The results show that the transition from unstable zone to stable zone can be realized by controlling redundant degrees of freedom without modifying cutting parameters. Therefore, it is very necessary to consider the chatter stability when setting the redundant degrees of freedom in the robot machining CAM system. However, the optimization of redundant degrees of freedom with the ultimate cutting depth of robot milling regenerative chatter as the optimization objective has not been published.

Taking the limit cutting depth of regenerative chatter in robot milling as the optimization objective and the motion performance of the robot as the constraint condition, this paper establishes an optimization model of redundant degrees of freedom in robot milling considering chatter stability, which provides a theoretical basis for the setting of redundant degrees of freedom in robot machining cam system. The feasibility and effectiveness of the model are verified by milling experiments.

2 Problem description of redundant degrees of freedom in robot milling

A serial robot has redundant degrees of freedom when the operational space dimension (n) is larger than the degree t of the task performed(t). The redundant degrees of freedom(r_n) is determined as follows

$$r_n = n - t \quad (1)$$

Fig.1 Comparison of pose optimization methods

Fig.1(a) shows the process of G code conversion robot program in the robot machining module of commercial off-line programming software.

First, generate G code: use CAD/CAM software for geometric modeling, tool path planning and generate G code.

Then, reverse the Euler angle: the cutter location $CL(x, y, z, i, j, k)$ data described in the workpiece coordinate system can be obtained from the G code, where (x, y, z) is the coordinate of the tool tip point and (i, j, k) is the unit vector of the tool axis direction. The pose of the robot tool coordinate system T relative to the workpiece coordinate system W is represented by a six-dimensional vector $(x, y, z, \alpha, \beta, \gamma)$, where (α, β, γ) is a Z-Y-Z Euler angle. The homogeneous transformation matrix of the tool coordinate system relative to the workpiece coordinate system is

$${}^wT_r = \text{trans}(x, y, z) \text{rot}(z, \alpha) \text{rot}(y, \beta) \text{rot}(z, \gamma) \quad (2)$$

Where, **trans** and **rot** are translation matrix and rotation matrix respectively.

Therefore, when the robot executes the robot program converted by G code for milling, the tool position data and matrix must meet the following requirements: Therefore, when executing the robot program converted by G code for milling, the cutter location data and matrix wT_r have to satisfy :

$$\begin{bmatrix} i & z \\ j & y \\ k & x \\ 0 & 1 \end{bmatrix} = {}^wT_r \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3)$$

It can be deduced from equation (3) that

$$\begin{bmatrix} i & j & k \end{bmatrix}^T = \begin{bmatrix} -\cos \alpha \sin \beta & -\sin \alpha \sin \beta & \cos \beta \end{bmatrix}^T \quad (4)$$

From equation (4), it can be seen that for a given $CL(x, y, z, i, j, k)$, only the former five components of the

six-dimensional vector $(x, y, z, \alpha, \beta, \gamma)$ can be determined, while the Euler angle γ is arbitrary. Therefore, when a six-axis industrial robot performs a five-axis milling task, a redundant degree of freedom will be generated, which is quantified by the Euler angle γ .

Thirdly, remove redundancy: the Euler angle γ needs to be set when solving the robot inverse kinematics, that is, the process of removing redundancy. This value should be given by the optimization process.

Finally, conversion code: the transformed robot program is processed and simulated, and the executable optimized robot program is output.

3 Redundant degree of freedom optimization model considering chatter stability

As shown in Fig.1, the traditional method takes joint limit, flexibility and Cartesian stiffness as the optimization objectives to determine the redundant degrees of freedom γ . However, the chatter stability of robotic machining has not been considered. Therefore, a new redundant degree of freedom optimization method considering chatter stability is proposed in this paper.

The optimal redundant degree of freedom γ is determined by maximizing the limit cutting depth $a_{\text{plim}}(\gamma)$ of the regenerative chatter in the robotic milling to avoid the occurrence of regenerative chatter. In addition, it is necessary to avoid the joint angles of the robot from approaching the limit position, being in singular pose and collision. The redundant degree of freedom optimization model considering chatter stability was established

$$\begin{aligned} & \max a_{\text{plim}}(\gamma) \\ & \text{s.t.} \begin{cases} K(\mathbf{J}) > \eta \\ |T \cdot L| > r_a + r_b \\ \theta_{\min} \leq \theta_j \leq \theta_{\max}, \theta_j = f^{-1}(\gamma), j = 1, 2, \dots, 6 \end{cases} \end{aligned} \quad (5)$$

Where $a_{\text{plim}}(\gamma)$ is the regeneration chatter limit cutting depth; $K(\mathbf{J})$ is the reciprocal of the conditional number of the normalized Jacobian matrix of the robot; η is a user-defined flexibility value. T is the distance between two bounding boxes; L is the detection axis; r_a and r_b are the projections of radius of bounding boxes A and B on the detection axis; θ_i is the joint angle of the robot; $f^{-1}()$ represents inverse kinematics; and $\theta_{\min}$ and $\theta_{\max}$ are joint limits.

4 Solution of redundant degree of freedom optimization model

4.1 The limit cutting depth of the regenerative chatter in the robotic milling

4.1.1 Dynamic model of robot milling system

As shown in Fig.2, the dynamic model of the robot milling system can be simplified into a 2-DOF vibration system in the x -axis and y -axis directions. For a given configuration p_0 in Cartesian space, the dynamic model of the tool tip can be expressed by equation (6):

Fig. 1 Robotic milling dynamic model

$$\begin{cases} M_x(p_0)\ddot{\delta}_x(t) + C_x(p_0)\dot{\delta}_x(t) + K_x(p_0)\delta_x(t) = F_x(t) \\ M_y(p_0)\ddot{\delta}_y(t) + C_y(p_0)\dot{\delta}_y(t) + K_y(p_0)\delta_y(t) = F_y(t) \end{cases} \quad (6)$$

where $M_x, M_y, K_x, K_y, C_x, C_y$ are the modal mass, modal stiffness and modal damping of the system along the two directions(x, y), respectively. $\delta(t)$ is vibration displacement of tool point; $F(t)$ is the cutting force.

4.1.2 Regenerative chatter dynamic model

According to the milling theory proposed in reference[19], the dynamic cutting force in the milling process is

$$\mathbf{F}(t) = \frac{1}{2} K_c \mathbf{D}(t) a_p [\delta(t) - \delta(t - \tau)] \quad (7)$$

Where, K_c is the cutting force coefficient; τ is the delay time between two cutter teeth; a_p is the cutting depth; δ is the vibration displacement of the tool tip. $\mathbf{D}(t)$ is time-varying directional dynamic milling force coefficient. $\mathbf{D}(t)$ can be extended into a Fourier series, which is approximately expressed as its average component $\mathbf{D}_0$.

$$\mathbf{D}_0 = \frac{N}{2\pi} \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix} \quad (8)$$

Where N is the number of tool teeth.

Time-varying directional dynamic milling force coefficient is

$$\begin{aligned} \alpha_{yx} &= \frac{1}{2} [-\sin 2\phi - \phi + r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{yx} &= \frac{1}{2} [-\sin 2\phi - \phi + r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{xy} &= \frac{1}{2} [-\sin 2\phi + 2\phi + r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{xx} &= \frac{1}{2} [-\cos 2\phi - 2r\phi - r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \end{aligned} \quad (9)$$

Where ϕ is the tool's rotational angle; r is ratio of radial and tangential cutting force coefficients; ϕ_{st} is the entry angle; ϕ_{ex} is the exit angle.

Substitute the harmonic solution $\delta(t): \delta(t) = \Delta e^{i\omega t}$ into equation (7)

$$[-M\omega^2 + C\omega i + K]\Delta e^{i\omega t} = \frac{1}{2} K_c \mathbf{D}_0 a_p (1 - e^{-i\omega\tau}) \Delta e^{i\omega t} \quad (10)$$

4.1.3 The limit cutting depth of the regenerative chatter

The frequency response function of the tool tip is

$$\mathbf{H}(\omega) = [-\mathbf{M}\omega^2 + i\mathbf{C}\omega + \mathbf{K}]^{-1} \quad (11)$$

By substituting the frequency response function into equation (10)

$$\left[\mathbf{I} - \frac{1}{2} K_c \mathbf{D}_0 \mathbf{a}_p (1 - e^{-i\omega\tau}) \right] \mathbf{H}(\omega) \Delta = 0 \quad (12)$$

If the determinant below is zero, then equation (12) has a nontrivial solution.

$$\det \left[\mathbf{I} - \frac{1}{2} K_c \mathbf{D}_0 \mathbf{a}_p (1 - e^{-i\omega\tau}) \right] \mathbf{H}(\omega) = 0 \quad (13)$$

Since the frequency response function is complex, the characteristic equation (13) has real and imaginary parts, and the eigenvalue at chatter frequency is given by

$$\Lambda = \Lambda_R + \Lambda_I i = -\frac{N}{4\pi} \mathbf{a}_p K_c (1 - e^{-i\omega_c\tau}) \quad (14)$$

It can be obtained by solving equation (14)

$$e^{-i\omega_c\tau} = \cos(\omega_c\tau) - i \sin(\omega_c\tau) \quad (15)$$

$$\mathbf{a}_p = \frac{2\pi}{NK_c} \left[\frac{\Lambda_R (1 - \cos(\omega_c\tau) + \Lambda_I \sin(\omega_c\tau))}{(1 - \cos(\omega_c\tau))} + i \frac{\Lambda_I (1 - \cos(\omega_c\tau) - \Lambda_R \sin(\omega_c\tau))}{(1 - \cos(\omega_c\tau))} \right] \quad (16)$$

Where the imaginary part of $\mathbf{a}_p$ must be equal to zero because $\mathbf{a}_p$ is a real number.

The real part of $\mathbf{a}_p$ gives the limit cutting depth (a_{plim}) in chatter frequency (ω_c), which separates the stable and instable zone. If κ is defined as

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\omega_c\tau)}{1 - \cos(\omega_c\tau)} \quad (17)$$

Therefore, limit cutting depth of the regenerative chatter (a_{plim}) can be written as

$$a_{\text{plim}} = -\frac{2\pi\Lambda_R}{NK_t} (1 + \kappa^2) \quad (18)$$

4.1.4 The influence of redundant degree of freedom γ on the limit cutting depth $a_{\text{plim}}(\gamma)$

As shown in Fig.3, it is a block diagram of the regenerative chatter in the robotic milling. It can be seen from the figure that the change of the robot's pose will cause the dynamic changes of the tool tip point, which will affect the stability of the processing. Therefore, for each robot pose with the same cutting parameters, the limit cutting depth $a_{\text{plim}}(\gamma)$ is determined according to equation (18), $a_{\text{plim}}(\gamma)$ is a function of the redundant degree of freedom γ .

4.2 The robotic flexibility index

The condition number of the normalized Jacobian matrix of the robot is used to measure the closeness of the robot to the singularity[20], which is expressed as

$$C(\mathbf{J}) = \text{cond}(\mathbf{J}) = \|\mathbf{J}(\theta)\| \|\mathbf{J}(\theta)^{-1}\| \quad (19)$$

Where J is Jacobian matrix, $C(J) \in [1, +\infty)$.

The robotic flexibility index is expressed as

$$K(J) = 1 / C(J) \quad (19)$$

The closer the $K(J)$ value is to 1, the better the flexibility. The $K(J)$ value is close to 0, indicating that the joint angle of the robot at this time is close to the singularity position.

Fig.3 Regenerative chatter block diagram in robotic machining

4.3 Robot collision free condition

Fig. 4 OBB bounding box detection pump

The OBB bounding box intersection detection is used to determine whether there is a collision between the detected robot link and the detected object[21]. To test whether the bounding boxes intersect, it is necessary to project the center and radius of the bounding boxes onto the detection axis L . If the distance between the centers of the bounding boxes on the axis is greater than the sum of the radius, the bounding boxes are separated. T is the distance between two bounding boxes, L is the detection axis which is one of the 15 separation axes, a_i and b_i are half the side lengths of bounding boxes A and B , respectively, and A^i and B^i as the side direction vectors of bounding boxes A and B , r_a and r_b are the projections of radius of bounding boxes A and B on the detection axis, as shown in Figure 4.

The detection axis and the projections of the radius of the bounding box on the detection axis are respectively

$$L = \begin{cases} A_i & i = 1, 2, 3 \\ B_j & j = 1, 2, 3 \\ A_i \times B_j & i = 1, 2, 3, j = 1, 2, 3 \end{cases} \quad (21)$$

$$r_a = \sum_{i=1}^3 |a_i A_i \cdot L|, \quad r_b = \sum_{j=1}^3 |b_j B_j \cdot L| \quad (22)$$

The projection distance between the center points of the boxes is $|T \cdot L|$, so the condition that the bounding boxes do not intersect is if and only if

$$|T \cdot L| > r_a + r_b \quad (23)$$

4.4 Algorithm for solving redundant degree of freedom optimization model

After obtaining the parameters of the redundant degree of freedom optimization model, the discrete optimization search algorithm is used to solve the optimization model to determine the optimal redundant degree of freedom γ . The algorithm flow chart is shown in Fig.5.

Firstly, the redundant degree of freedom interval of the robot is selected as 5° , and the limit cutting depth $a_{\text{plim}}(\gamma)$ corresponding to γ is determined according to section 4.1.4. Then, all Euler angles γ are traversed. Finally the maximum limit cutting depth $a_{\text{plimax}}(\gamma)$ and its corresponding optimal redundant degree of freedom are output.

Fig.5 Flow chart of the algorithm for pose optimization

5 Milling experimental validation

5.1 Construction of experimental platform

As shown in Fig. 6, ABB IRB1200 six degree of freedom industrial robot is used as the experimental platform and a vibration signal acquisition system (model 3560-B, Brüel & Kjær, Denmark), a pulse analysis software (Brüel & Kjær, Denmark), a modal hammer (Model: 086C01, PCB Inc., USA), and an acceleration sensor (Model:

356A24, PCB Inc., USA) were used for data collection from the milling system.

Fig.6 the experimental platform for robotic milling

5.2 Construction of simulation platform RobotYY

Robotyy is an off-line programming simulation software for six axis industrial robot milling developed by the author on python, QT and OpenGL platforms. It mainly has modules such as off-line programming post-processing, machining simulation and collision detection. The software interface is shown in Fig.7.

Fig.7 Software interface

The post-processor interface is shown in Fig.8. G code (NC format) is converted into robot program through post-processing algorithms such as motion instruction conversion, coordinate calculation and posture calculation. In addition, the post-processing module can also predict FRF of other estimated pose according to the imported

FRF (TXT format) of measured pose.

Fig.8 Post-processor interface

As shown in Fig.9, the machining simulation module can perform real-time milling processing simulation based on the robot program generated by the post-processing module, which can more intuitively check whether the robot program is correct.

Fig.9 Motion simulation of robot

The collision detection module can avoid the situation that a collision is not found in the machining simulation due to the viewing angle limitation, while a collision occurs in the actual machining. As shown in Fig. 10, when there is no collision, the bounding box is green. when the collision occurs, that is, when the bounding boxes intersect, the bounding box will change from green to red.

Fig.10 Collision detection simulation

5.3 Experimental verification of redundant degrees of freedom optimization

5.3.1 Prediction of frequency response function

In recent years, two methods have been used to predict the frequency response function of the tool tip of the robot: (1) Data modeling based on experiment[22-24]; (2) Theoretical modeling based on finite element method[17]. Due to the complexity of the robot structure, material properties, transmission system layout, etc., it is difficult to establish an accurate and reliable theoretical model to describe the dynamic characteristics of the robot tool tip. In practical applications, the modal hammering experiment is usually used to obtain the frequency response function of the robot tool tip, and the experimental results are more reliable. Note that there are an infinite number of robot poses in the robot workspace. It is very time-consuming to obtain the frequency response function of the tool tip point in each robot pose through experiments. Therefore, this paper uses the frequency response function of the measured poses to accurately and efficiently predict the frequency response function of other estimated poses, based on the inverse distance weighting method. This method is simple and efficient, and has good interpolation effect for uniformly distributed sample points[24].

Firstly, establish the set of frequency response functions of measured poses. The influence of rotating axis 1 on the FRF of the tool tip is negligible, so the angle of axis 1 in all measured tool position is 0° . In the base coordinate system, the tool positions are equally spaced at 100 mm intervals on the XZ plane, as shown in Fig.11. The same tool position corresponds to 5 redundant postures, as shown in Fig.12, a total of 100 test poses.

Fig.11 Arrangement of robot tool positions in impact tests

Fig.12 Five postures corresponding to the same tool position

Then, determine the number of samples m . Since the influence of the measured pose FRF on the estimated pose decreases with the increase of distance d_i , the measured pose is sorted according to distance d_i , and the frequency response function of the nearest m measured poses is selected to predict the frequency response function of the estimated pose. In this paper, M is determined as 10.

$$d_i = \sqrt{(q_{j1} - q_{i1})^2 + (q_{j2} - q_{i2})^2 + L + (q_{j6} - q_{i6})^2} \quad (24)$$

Where d_i is the Euclidean distance between the estimated pose(q_j) and the measured pose(q_i) in the joint space.

After that, the weight λ_i of each test point is calculated. The weight λ_i is a function of the reciprocal of the distance, and satisfies the following relationship:

$$\sum_{i=1}^m \lambda_i = 1 \quad (25)$$

$$\lambda_i = \frac{\frac{1}{d_i}}{\sum_{i=1}^m \frac{1}{d_i}} \quad (26)$$

Finally, predict the frequency response function. The frequency response function $H_j(\omega)$ of any estimated pose in the robot workspace can be predicted based on the frequency response function $H_i(\omega)$ of the m measured poses, which is determined by

$$H_j(\omega) = \sum_{i=1}^m \lambda_i H_i(\omega) \quad (27)$$

Fig.13 Frequency response function of robot tool tip

In this paper, only the first dominant mode of the tool tip point of the robot is considered, and the contribution of other additional modes is ignored. As shown in Fig.13, it is the frequency response function curve of the tool position (400mm, 0mm, 100mm) corresponding to the pose 1. The peak picking method is used to identify the modal parameters.

Fig. 14 comparison of frequency response functions between prediction and experiment

Table 1 shows the comparison between the predicted parameter and identified parameter of any three poses. It is concluded from the table 1 that the relative error of most of the data is less than 20%, which proves that the inverse distance weighting method can meet the prediction accuracy of the frequency response function. In addition, it can be seen from Fig. 14 that the predicted FRF matches well with the experimental FRF.

Table 1 Comparisons between predicted and identified parameters

		X direction			Y direction		
		predicted	experimental	error (%)	predicted	experimental	error (%)
Pose 1	natural frequency f (Hz)	190	199	-4.74	189	191	-1.06
	damping ratio ξ (%)	10.54	8.54	18.98	11.78	9.42	20.03
	modal stiffness K (N/m $\times 10^6$)	1.43	1.6	-11.89	0.34	0.39	-14.71
Pose 2	natural frequency f (Hz)	193	190	1.55	190	189	0.53
	damping ratio ξ (%)	10.1	11.05	-9.41	13.05	15.08	-15.56
	modal stiffness K (N/m $\times 10^6$)	0.95	1.12	-17.89	0.22	0.19	13.64
Pose 3	natural frequency f (Hz)	194	193	0.52	191	190	0.52
	damping ratio ξ (%)	11.6	9.59	17.33	12.73	15.79	-24.04
	modal stiffness K (N/m $\times 10^6$)	1.16	1.48	-27.59	0.29	0.26	10.34

5.3.2 Stability circle diagram

In order to study the influence of the redundant degree of freedom on the stability of the robotic milling, the

position of the tool tip point remains unchanged, and the redundant degree of freedom γ is changed, that is, it rotates around the z-axis of the tool coordinate system. At each change of 5° , the frequency response function of tool point was predicted, and a total of 38 groups of frequency response curves were predicted. The peak picking method was used to identify modal parameters, drawing the stability lobe diagram, as shown in Fig.15.

Fig.15 Stable lobe diagram

In Fig.15, the limit cutting depth of the same spindle speed (12000 RPM as an example) and its corresponding redundant degrees of freedom were taken to draw the circle diagram $a_{plim}(\gamma)$, as shown in Fig.16. The radius of the circle represents the limit cutting depth, and the radian of the circle represents the redundant degree of freedom. The discrete optimization search algorithm in Section 4.4 is used to solve the redundant degree of freedom optimization model equation (5) to determine the optimal redundant degree of freedom $\gamma_{opt}=115^\circ$, and the maximum limit cutting depth $a_{plimax}=1.345mm$.

Fig.16 Stability circle diagram

5.3.3 Experimental verification

In order to verify the influence of redundant degrees of freedom on the milling stability of the robot, the 6061-T6 aluminum alloy machining experiment is carried out with ABB IRB1200 robot under the optimal redundant degrees of freedom of 115° and redundant degrees of freedom of 0° , as shown in Fig.17. The Cutting parameters in experimental machining are shown in Table 2.

(a) $\gamma=115^\circ$

(b) $\gamma=0^\circ$

Fig.17 Experiment machining of robot milling

Table 2 Cutting parameters in experimental machining

redundancy	Spindle speed	Feed rate	Depth of cut	predicted	experimental
115°	12000rpm	1mm / s	1mm	stable	stable
0°	12000rpm	1mm / s	1mm	instable	instable

Fig. 18(a) shows the acceleration signal spectrum at the optimal redundant degree of freedom of 115°, and the frequencies 201 Hz and 402 Hz are the rotating frequency and cutter tooth passing frequency respectively. In Fig. 18 (b), in addition to the rotating frequency and cutter tooth passing frequency, there is also a peak value of 341 Hz, which is not the cutter tooth passing frequency or its harmonic, but the regenerative chatter frequency. The experimental results are consistent with the predicted results, as shown in Table 2.

Fig. 18 Spectrum diagram of acceleration signal

Another indicator to measure the stability of robot milling is the surface quality of the machining parts. When chatter occurs, obvious chatter marks will be left on the surface of parts. As shown in Fig.19, the surface quality of the optimal redundant degree of freedom of 115° ($R_a=2.5\mu\text{m}$) is significantly better than that of the redundant degree of freedom of 0° ($R_a=9.2\mu\text{m}$), which also shows that the stability of the optimal redundant degree of freedom 115° is significantly better than that of the redundant degree of freedom 0°.

Fig.19 Surface comparison of parts machined

6. Conclusions

(1) A new optimization method of redundant degrees of freedom in robotic milling considering chatter stability is proposed to determine the optimal redundant degrees of freedom when a six axis industrial robot performs a five axis milling task. Experiments verify the feasibility of this method to avoid robotic milling regenerative chatter and improve machining quality and efficiency, It provides a theoretical basis for the setting of redundant degrees of freedom in robotic machining CAM system.

(2) The maximum limit cutting depth of 1.345mm corresponding to the optimal redundancy degree of freedom (115°) is 1.323 times higher than the minimum limit cutting depth of 0.579mm corresponding to the redundancy degree of freedom (0°), which significantly improves the chatter stability of robot high-speed milling.

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