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Research Article

Keywords: Interval-valued q-rung orthopair fuzzy sets, Yager operator, decision-making method

Posted Date: March 3rd, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1364390/v1>

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An integrated group decision-making method for hypertension risk management under interval-valued q-rung orthopair fuzzy

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Abstract: This paper develops a new group decision-making (GDM) method for multi-attribute group decision-making (MAGDM) problems with unknown weights, which integrates Yager operator, CRITIC and WASPAS decision methods under interval-valued q-rung orthopair fuzzy sets (IVq-ROFS). The main contributions of this paper are concluded as below: (1) We first extend the distance measurement to interval-valued q-rung fuzzy numbers (IVq-ROFNs), and proves its properties. And we develop a new score function which is used to measure IVq-ROFNs, and the new score function overcomes the deficiencies of the existing score function. (2) We extend Yager weighted average operator (IVq-ROFYWA) and Yager weighted geometric average operator (IVq-ROFYWG) under IVq-ROFS, and we discuss the properties of IVq-ROFYWA and IVq-ROFYWG operators. (3) We propose a new method to derive expert weights of different alternatives, which can avoid the deficiency of overall decision-making perspective. And we extend deriving attribute weights method based on CRITIC under IVq-ROFS. (4) A new integrated MAGDM method is proposed. The MAGDM method integrates Yager operator, score function, distance measurement, expert weights deriving method, CRITIC method, and WASPAS method. (5) Finally, the group decision-making algorithm is used for hypertension risk management. The results are consistent with experts' opinions. And comparing and analyzing results show that the proposed MAGDM method is effective and feasible.

Keywords: Interval-valued q-rung orthopair fuzzy sets; Yager operator; decision-making method

1. Introduction

Decision-making (DM) is widespread in human production and life [1-2], and the DM problem which includes multiple alternatives with various attributes is a multi-attributes decision-making (MADM). And the MADM problem, involving multiple people, is called the multi-attribute group decision-making (MAGDM) problem [3-4]. Since Zadeh had proposed fuzzy sets (FS) [5], fuzzy theories and methods have been widely used in MADM and MAGDM [1-4]. And Atanassov proposed the intuitionistic fuzzy set (IFS) to express approval and disapproval by membership degree ($u \geq 0$) and non-membership degree ($v \geq 0$) respectively [6], which provided more freedom for group decision-making applications. For example, Kumar and Chen [7] studied Einstein's MADM method based on intuitionistic fuzzy numbers (IFNs), and Yager et al. [8] focused on the MADM of generalized IFS. Yager proposed the Pythagorean fuzzy set (PFS) [9], and further proposed q-Rung orthopair fuzzy sets (q-ROFS) in 2017

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[10]. The q-ROFS not only integrates IFS and PFS, but also extends the application field and provide decision-makers with more free representation [8-9]. Although the IFS can express the expert's approval, disapproval, and hesitation, the uncertainty is often represented by an interval valued. For this reason, the researchers proposed interval-valued q-Rung orthopair fuzzy sets (IVq-ROFS) which have been used to solve the MAGDM problems [11][12]. For example, Wang et al. [12] proposed a new MAGDM method based on IVq-ROFS for selecting green suppliers, Jin et al. [13] made use of IVq-ROFS assessing the risk of improved tool change manipulators, Limei Liu et al. [14] proposed a large-scale green supplier selection method under IVq-ROFS. These group decision-making methods only work when the expert weights and attribute weights are given. This paper develops an integrated MAGDM method with unknown weights under IVq-ROFS.

In MAGDM, operators are often used to aggregate the information of experts, and some classic operators are developed, such as OWA, OWG, Frank, Einsten, Hamy, Heronian, Bonferroni, Yager and etc [15-16]. Researchers have been proposed some aggregation in different fuzzy sets, for examples, Wei et al. [17] developed HPFIWB and HPFIGWBM operators under hesitant Pythagorean fuzzy sets (HFS), which are applied to the MAGDM problems. Liu et al. [18] studied q-ROFWGMSM and q-ROFWGGMSM operator under q-ROFS. Wang et al. [12] proposed q-RIVOFWA, q-RIVOWYG, q-RIVOFHM, q-RIVOFHM and q-RIVOFHM operators based on the Hamy operator, and he also put forward the q-RIVOFWHM operator, q-RIVOFDHM operator and q-RIVOFWDHM operator under q-ROFS. Jin et al. [13] proposed IVq-ROFWMSM operator and IVq-ROFWG operator under IVq-ROFS, Ju et al. proposed [19] IVq-ROFWA and IVq-ROFOWA operators. Among the operators, the Yager operator [15] can be employed to aggregate data and effectively control/reduce uncertainty in expert's evaluation, so as to yield dense information. Therefore, some Yager operators have been extended and applied to decision-making. Liu etc[14] proposed q-RPFYWA operator, q-RPFYOWA operator, q-RPFYWG operator, q-RPFYWG operator, and q-RPFYWG operator under q-ROFS. Muhammad Akram and Peng et al. [21] studied the MADM model based on the Pythagorean fuzzy Yager aggregation operator. However, so far, no scholars have proposed Yager operator under IVq-ROFS.

Although operators can aggregate information, it is failed to work when attributes have special relationships. Therefore, researchers have proposed a large number of decision methods, such as TOPSIS, MOORA, COPRAS, MACBETH, and MABAC, etc [22]. Among them, the TOPSIS [4] determines the priority according to the information of the decision matrix from distance of the positive ideal (PIS) and the negative ideal (NIS). Brauers and Zavadskas proposed MOORA [23]. Zavadskas and Kaklauskas developed COPRAS method to solve the problem [24]. Bana proposed MACBETH method [25]. Pamucar proposed MABAC [26]. The WASPAS [27] combines the weighted sum model (WSM) and the weighted product model (WPM). The advantages of integrating the two are: the WSM method can reflect the difference more quickly, and the WPM method is not susceptible to the influence of extreme numbers, so the data collected by the WASPAS method is more accurate. Katarzyna Rudnik et al. [28] worked out the OFN-WASPAS method. R. Davoudabadi et al. [29] developed a new method of aggregation and ranking based on WASPAS and TOPSIS methods under IFS. However, each alternative can only

be evaluated comprehensively when the weight is known. If the attribute weight is unknown, researchers use the CRITIC to calculate the objective weight [30], Peng and Huang proposed the interval value fuzzy soft decision intelligent medical management evaluation method based on CoCoSo and CRITIC [31]. However, no scholar has studied the application of CRITIC method and WASPAS method under IVq-ROFS.

Distance, entropy, and similarity of measurement methods are used to compare and analyze fuzzy numbers. Researchers have conducted many results. Peng and Liu [32] summarized the q-rung orthopair measurement method. According to reference point theory, researchers have taken a further step to develop positive ideal distance (PIS), negative ideal distance (NIS), and compromise distance (CS) [4]. As a result, the NIS examines the result of people and total opposition, while the CS uses parameters to establish the relationship between the PIS and NIS. The selected parameters need to be judged according to risk preference, and more decisions are required. Yue [33] proposed a method based on the ideal distance to obtain the similarity for the expert weights, who also [34] proposed a mapping method to derive expert weight. However, this method does not consider the difference of experts in evaluating different alternatives. In order to meet the decision requirement for deviations in different alternatives, this article has developed a new method to acquire expert weight of different alternative based on the similarity, making up for the shortcomings of the existing methods. At the same time, in order to compare IVIFNs, the researchers introduced score functions, for examples, Li et al. [35] put forward an information-based score function for calculating score of interval valued intuitionistic fuzzy numbers (IVIFNs), Chen [36] improved the score function with a new IVIFS, Bai [37], and Gong and Ma [38] proposed score functions for calculating score of IVIFNs. However, the existing score functions have deficiency that IVIFNs cannot be compared. In order to overcome deficiencies, we will propose a new scoring function for IVq-ROFNs.

Sometimes, the expert weight and attribute weight are unknown, but there are not the GDM method to capture expert weight and attribute weight under IVq-ROFS. Therefore, this paper proposes an integrated GDM which combine Yager operator, CRITIC, WASPAS method. The contributions of this paper are as follows:

- (1) A new distance measurement is proposed. Inspired by Ref [32], the distance measurement is proposed under IVq-ROFNs. And the PIS, NIS, and CS also are extended, which are used to derived expert weight.
- (2) A new score function is proposed. Inspired by Ref [36], a new interval-valued fuzzy number score function is proposed, which makes up for the limitation of Ref [36], [37], [38], and works very well.
- (3) Inspired by Ref [20], this paper proposes Yager operator, IVq-ROFYWA and IVq-ROFYWG under IVq-ROFS, and illustrate their properties.
- (4) An integrating GDM method is developed that combines IVq-ROFYWA, IVq-ROFYWG operators and CRITIC, and WASPAS methods. The proposed GDM method can derive expert weight of different alternatives, which has a good application when the experts have bias in evaluating different alternatives. Then the IVq-ROFYWA operator and expert weights are used to aggregate the expert judgement matrices. And modified WASPAS method is used to rank alternatives based on CRITIC method which is used to derive attribute weight under IVq-ROFS. And the proposed GDM only requires expert judgment matrices based on the problem, therefore, it greatly

improves the efficiency of decision-making when the expert and the attribute weights are unknown.

(5) The proposed GDM method is used to the early warning management of hypertension diseases. According to the patient's test results and risk factors, the GDM method is used to calculate the risk level according to expert judgment matrices. The result is consistent with expert's prediction, which proves the effectiveness of the method proposed in this paper. The experimental results confirm that the proposed GDM is highly adaptable, comparing and analysis results further proves that the MAGDM method is feasible and effective.

This article is organized as follows. Section 2 introduces basic knowledge. Section 3 describes score function and distance measurement, Section 4 presents the IVq-ROFYWA and IVq-ROFYWG. Section 5 gives the integrated GDM method. Section 6 expounds the application of the GDM method in hypertension management. Section 7 is the conclusion.

2. Preliminary

2.1 IVq-ROFS

Definition 1 [11]. Let X be the universe of discourse, the $IVq - ROFS$ A on X is defined as shown in formula (1).

$$A = \{ \langle x, u_A(x), v_A(x) \rangle \mid x \in X \} \quad (1)$$

In formula (1), the membership function is a mapping of interval values that satisfies: $u_A(x) = [u_A^-(x), u_A^+(x)] \subseteq [0,1]$, and the non-membership function is a mapping of interval values that satisfies: $v_A(x) = [v_A^-(x), v_A^+(x)] \subseteq [0,1]$, and at the same time satisfies: $0 \leq (u_A^+(x))^q + (v_A^+(x))^q \leq 1, (q \geq 1)$. The hesitation is shown in formula (2).

$$\pi_A(x) = [\pi_A^-(x), \pi_A^+(x)] = [\sqrt[q]{1 - (u_A^+(x))^q - (v_A^+(x))^q}, \sqrt[q]{1 - (u_A^-(x))^q - (v_A^-(x))^q}] \quad (2)$$

Definition 2 [11]. Let $a = ([u^-, u^+], [v^-, v^+])$, $a_1 = ([u_{a_1}^-, u_{a_1}^+], [v_{a_1}^-, v_{a_1}^+])$ and $a_2 = ([u_{a_2}^-, u_{a_2}^+], [v_{a_2}^-, v_{a_2}^+])$ be three $IVq - ROFNs$, where $q \geq 1$. Then the IVq -ROFNs operations of addition, multiplication, scalar multiplication, power show as formula (3), (4), (5) and (6).

$$a_1 \oplus a_2 = \left(\left[\sqrt[q]{(u_{a_1}^-)^q + (u_{a_2}^-)^q - (u_{a_1}^-)^q (u_{a_2}^-)^q}, \sqrt[q]{(u_{a_1}^+)^q + (u_{a_2}^+)^q - (u_{a_1}^+)^q (u_{a_2}^+)^q} \right], \left[v_{a_1}^- v_{a_2}^-, v_{a_1}^+ v_{a_2}^+ \right] \right) \quad (3)$$

$$a_1 \otimes a_2 = \left(\left[u_{a_1}^- u_{a_2}^-, u_{a_1}^+ u_{a_2}^+ \right], \left[\sqrt[q]{(v_{a_1}^-)^q + (v_{a_2}^-)^q - (v_{a_1}^-)^q (v_{a_2}^-)^q}, \sqrt[q]{(v_{a_1}^+)^q + (v_{a_2}^+)^q - (v_{a_1}^+)^q (v_{a_2}^+)^q} \right] \right) \quad (4)$$

$$\lambda a = ([\sqrt[q]{1 - (1 - (u^-)^q)^\lambda}, \sqrt[q]{1 - (1 - (u^+)^q)^\lambda}], [(v^-)^\lambda, (v^+)^\lambda]) \quad (5)$$

$$a^\lambda = ([(u^-)^\lambda, (u^+)^\lambda], [\sqrt[q]{1 - (1 - (v^-)^q)^\lambda}, \sqrt[q]{1 - (1 - (v^+)^q)^\lambda}]) \quad (6)$$

2.2 Yager operator

The Yager operator can control the uncertainty of some experts' evaluation data, and obtain dense information. Under the environment of dense information, it is used for the aggregation of experts' information.

Definition 3[15]. For any two real numbers $a, b \in [0,1]$, Yager sum $\oplus_Y$ and Yager product $\otimes_Y$ are shown in formula (7) and (8).

$$a \oplus_Y b = \min \left[1, (a^p + b^p)^{\frac{1}{p}} \right], p \in [1, +\infty] \quad (7)$$

$$a \otimes_Y b = 1 - \min \left\{ 1, [(1-a)^p + (1-b)^p]^{\frac{1}{p}} \right\}, p \in [1, +\infty] \quad (8)$$

Yager confirmed that when $p = 1$, $(\oplus_Y, \otimes_Y)$ will be transformed into $(\oplus, \otimes)$, that is, t-norms operation and t-conorms operation are performed on a and b . When $p \rightarrow +\infty$, $(\oplus_Y, \otimes_Y)$ will be transformed into $(\vee, \wedge)$, that is, take the larger and smaller operations of a and b .

3. Interval distance and score function

3.1 Distance measurement

Distance is used to compare two IVq-ROFNs. Inspired by Peng [32], a new interval-valued q-rung orthopair distance is proposed that shows as Definition 4.

Definition 4. For any two IVq-ROFNs $a_1 = ([u_{a_1}^-, u_{a_1}^+], [v_{a_1}^-, v_{a_1}^+])$ and $a_2 = ([u_{a_2}^-, u_{a_2}^+], [v_{a_2}^-, v_{a_2}^+])$ where $q \geq 1$, the distance is shown in formula (9).

$$d(a_1, a_2) = \frac{(|(u_{a_1}^-)^q - (u_{a_2}^-)^q - ((v_{a_1}^-)^q - (v_{a_2}^-)^q)| + |(u_{a_1}^+)^q - (u_{a_2}^+)^q - ((v_{a_1}^+)^q - (v_{a_2}^+)^q)|)}{4} \quad (9)$$

Theorem 1. Formula (13) satisfies the following properties:

- (1) $0 \leq d(a_1, a_2) \leq 1$;
- (2) $d(a_1, a_2) = d(a_2, a_1)$;
- (3) $d(a_1, a_2) = 0$ only if $a_1 = a_2$;
- (4) If $a_1 \subseteq a_2 \subseteq a_3$, then $d(a_1, a_2) \leq d(a_1, a_3)$ and $d(a_2, a_3) \leq d(a_1, a_3)$;

Since the property (2) and (3) are easy to be proved, this paper only gives the proof of property (1) and (4).

Proof. (1) $d(a_1, a_2) = \frac{1}{4} \left[\begin{aligned} & |(u_{a_1}^-)^q - (u_{a_2}^-)^q - [(v_{a_1}^-)^q - (v_{a_2}^-)^q]| + \\ & |(u_{a_1}^+)^q - (u_{a_2}^+)^q - [(v_{a_1}^+)^q - (v_{a_2}^+)^q]| \end{aligned} \right]$

$$\begin{aligned}
&= \frac{1}{4} \left[\left| (u_{a_1}^-)^q - (v_{a_1}^-)^q + (v_{a_2}^-)^q - (u_{a_2}^-)^q \right| + \right. \\
&\quad \left. \left| (u_{a_1}^+)^q - (v_{a_1}^+)^q + (v_{a_2}^+)^q - (u_{a_2}^+)^q \right| \right] \\
&\leq \frac{1}{4} \left[\left| \max[(u_{a_1}^-)^q - (v_{a_1}^-)^q] + \max[(v_{a_2}^-)^q - (u_{a_2}^-)^q] \right| + \right. \\
&\quad \left. \left| \max[(u_{a_1}^+)^q - (v_{a_1}^+)^q] + \max[(v_{a_2}^+)^q - (u_{a_2}^+)^q] \right| \right] \leq \frac{1}{4} \times (2 + 2) = 1
\end{aligned}$$

When the two IVq-ROFNs are exactly the same, the distance is obviously 0, and $0 \leq d(a_1, a_2) \leq 1$, the property (1) is proved.

$$\begin{aligned}
(4) \quad d(a_1, a_2) &= \frac{1}{4} \left\{ \left| (u_{a_1}^-)^q - (u_{a_2}^-)^q - [(v_{a_1}^-)^q - (v_{a_2}^-)^q] \right| + \right. \\
&\quad \left. \left| (u_{a_1}^+)^q - (u_{a_2}^+)^q - [(v_{a_1}^+)^q - (v_{a_2}^+)^q] \right| \right\} \\
&= \frac{1}{4} \left\{ \left((v_{a_1}^-)^q - (v_{a_2}^-)^q - [(u_{a_1}^-)^q - (u_{a_2}^-)^q] \right) + \right. \\
&\quad \left. \left((v_{a_1}^+)^q - (v_{a_2}^+)^q - [(u_{a_1}^+)^q - (u_{a_2}^+)^q] \right) \right\} \\
d(a_1, a_3) &= \frac{1}{4} \left\{ \left| (u_{a_1}^-)^q - (u_{a_3}^-)^q - [(v_{a_1}^-)^q - (v_{a_3}^-)^q] \right| + \right. \\
&\quad \left. \left| (u_{a_1}^+)^q - (u_{a_3}^+)^q - [(v_{a_1}^+)^q - (v_{a_3}^+)^q] \right| \right\} \\
&= \frac{1}{4} \left\{ \left((v_{a_1}^-)^q - (v_{a_3}^-)^q - [(u_{a_1}^-)^q - (u_{a_3}^-)^q] \right) + \right. \\
&\quad \left. \left((v_{a_1}^+)^q - (v_{a_3}^+)^q - [(u_{a_1}^+)^q - (u_{a_3}^+)^q] \right) \right\} \\
d(a_1, a_3) - d(a_1, a_2) &= \frac{1}{4} \left[\left((u_{a_3}^-)^q - (u_{a_2}^-)^q + (v_{a_2}^-)^q - (v_{a_3}^-)^q \right) + \right. \\
&\quad \left. \left((u_{a_3}^+)^q - (u_{a_2}^+)^q + (v_{a_2}^+)^q - (v_{a_3}^+)^q \right) \right] \geq 0
\end{aligned}$$

Similarly, it can be obtained $d(a_2, a_3) \leq d(a_1, a_3)$, so the property (4) is proved.

In decision-making, the ideal points are often used to compare the various alternatives. Positive ideal distance (PIS), Negative ideal distance (NIS), and Compromise distance (CS) are defined under IVq-ROFS.

Definition 5. Let $a = ([u^-, u^+], [v^-, v^+])$ be a IVq-ROFNs, the NIS can be calculated by formula (10).

$$NIS(a) = \frac{1}{4} (|(u^-)^q - ((v^-)^q - 1)| + |(u^+)^q - ((v^+)^q - 1)|) \quad (10)$$

Definition 6. Let $a = ([u^-, u^+], [v^-, v^+])$ be a IVq-ROFNs, and PIS can be calculated by formula (11).

$$PIS(a) = \frac{1}{4} (|(u^-)^q - (1 - (v^-)^q)| + |(u^+)^q - (1 - (v^+)^q)|) \quad (11)$$

Definition 7. Let $a = ([u^-, u^+], [v^-, v^+])$ be a IVq-ROFNs, and CS can be calculated by formula (12).

$$CS(a_1) = \vartheta \times PIS + (1 - \vartheta)NIS \quad (12)$$

In formula (12), $0 \leq \vartheta \leq 1$. When $\vartheta = 0$, CS is equal to NIS; when $\vartheta = 1$, CS is equal to PIS. $\vartheta > 0.5$ indicates that experts prefer low risk, $\vartheta < 0.5$ indicates that experts prefer high risk, and $\vartheta = 0.5$ indicates experts prefer compromise.

Example 1. Let $a_1 = ([0.85, 0.95], [0.1, 0.2])$ be IVq-ROFNs, and $q=3$, we can get $NIS(a_1) = 0.87$; $PIS(a_1) = 0.13$. When $\vartheta = 0.7$, $CS(a_1) = 0.35$, the result is close to $PIS(a_1)$, which indicates expert is prefer to high risk. When $\vartheta = 0.2$, $CS(a_1) = 0.72$, the result is close to $NIS(a_1)$ that indicates expert is prefer low risk. When $\vartheta = 0.5$, $CS(a_1) = 0.5$, the result is between $PIS(a_1)$ and $NIS(a_1)$, indicating that the expert is prefer to compromise.

3.2 Score function

The score function is often used to calculate the score of the alternatives, which is used to compare and rank the alternatives under MADM or MAGDM problem. The proposed score function has been well used to rank alternatives under IVIFS [36] [37][38], but they have some deficiencies.

Definition 8 [36]. Let $a = ([u^-, u^+], [v^-, v^+])$ be a IVIFNs, the score function proposed by Chen is shown in formula (13).

$$S_C(a) = \frac{1}{2}(u^- - v^- + u^+ - v^+) + 1 \quad (13)$$

Example 2. Let $a_1 = ([0.15, 0.2], [0.15, 0.2])$ and $a_2 = ([0.3, 0.4], [0.3, 0.4])$ are two IVIFNs, $S_C(a_1) = S_C(a_2) = 1$, the S_C cannot compare these two IVq-IFNs.

Definition 9 [37]. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFNs, the definition of the score function proposed by Bai is shown in formula (14).

$$S_B(a) = \frac{1}{2}[u^- + u^+ + u^-(1 - u^- - v^-) + u^+(1 - u^+ - v^+)] \quad (14)$$

Example 3. Let $a_3 = ([0.5, 0.5], [0.3, 0.3])$ and $a_4 = ([0.4, 0.4], [0.1, 0.1])$ be two IVq-IFNs, the $S_B(a_3) = S_B(a_4) = 0.6$, the S_B cannot compare them.

Definition 10 [38]. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVIFNs, the score function of $S_{GM}(a)$ proposed by Gong and Ma as shown in formula (15).

$$S_{GM}(a) = \frac{1}{2}(v^+ + v^- - u^+ - u^-) + \frac{u^+ + u^- + 2[(u^+)(u^-) - (v^+)(v^-)]}{v^+ + v^- + u^+ + u^-} \quad (15)$$

Example 4. For two IVIFNs $a_5 = ([0.4, 0.6], [0.4, 0.6])$ and $a_6 = ([0.2, 0.5], [0.2, 0.5])$, the $S_{GM}(a_5) = S_{GM}(a_6) = 0.5$, it can be seen that the S_{GM} cannot compare the two IVq-IFNs.

These score functions have been used to compare IVIFNs. In order to overcome the deficiencies, we propose a new score function for IVq-ROFNs inspired by Ref [39].

Definition 11. Let $a = ([u^-, u^+], [v^-, v^+])$ be an IVq-ROFNs, the score function of the IVq-ROFNs is shown in formula (20), ($q > 1, 0 < \alpha < 1$) .

$$S(a) = \frac{1}{2}[\alpha(\sqrt[q]{u^-} + \sqrt[q]{1 - v^-}) + (1 - \alpha)(\sqrt[q]{u^+} + \sqrt[q]{1 - v^+})] \quad (16)$$

Let $a_1 = ([u_{a_1}^-, u_{a_1}^+], [v_{a_1}^-, v_{a_1}^+])$ and $a_2 = ([u_{a_2}^-, u_{a_2}^+], [v_{a_2}^-, v_{a_2}^+])$ be two IVq-ROFNs, the proposed score function satisfies following properties.

- (1) If $a_1 > a_2$, then $S(a_1) > S(a_2)$, if $a_1 < a_2$, then $S(a_1) < S(a_2)$, if $a_1 = a_2$, then $S(a_1) = S(a_2)$;
- (2) $S(a_1) \in [0,1]$;
- (3) $S(a_2) = 0$ if $a_2 = ([0,0], [1,1])$;
- (4) $S(a_1) = 1$ if $a_1 = ([1,1], [0,0])$;
- (5) $S(a)$ aims at α and β that have the same monotonicity.

The properties (2), (3), and (4) of the score function are easy to be proved. The property of (1) and (5) are proved by the following.

Proof. (1) $S(a_1) = \frac{1}{2} [\alpha(\sqrt[q]{u_{a_1}^-} + \sqrt[q]{1 - v_{a_1}^-}) + (1 - \alpha)(\sqrt[q]{u_{a_1}^+} + \sqrt[q]{1 - v_{a_1}^+})]$

$$S(a_2) = \frac{1}{2} [\alpha(\sqrt[q]{u_{a_2}^-} + \sqrt[q]{1 - v_{a_2}^-}) + (1 - \alpha)(\sqrt[q]{u_{a_2}^+} + \sqrt[q]{1 - v_{a_2}^+})]$$

$$S(a_1) - S(a_2) = \frac{1}{2} \left[\frac{\alpha(\sqrt[q]{u_{a_1}^-} - \sqrt[q]{u_{a_2}^-}) + \alpha(\sqrt[q]{1 - v_{a_1}^-} - \sqrt[q]{1 - v_{a_2}^-}) + (1 - \alpha)(\sqrt[q]{u_{a_1}^+} - \sqrt[q]{u_{a_2}^+}) + (1 - \alpha)(\sqrt[q]{1 - v_{a_1}^+} - \sqrt[q]{1 - v_{a_2}^+})}{(1 - \alpha)(\sqrt[q]{u_{a_1}^+} - \sqrt[q]{u_{a_2}^+}) + (1 - \alpha)(\sqrt[q]{1 - v_{a_1}^+} - \sqrt[q]{1 - v_{a_2}^+})} \right]$$

When $a_1 > a_2$, there is $u_{a_1}^- > u_{a_2}^-$, $u_{a_1}^+ > u_{a_2}^+$, $v_{a_1}^- < v_{a_2}^-$, $v_{a_1}^+ < v_{a_2}^+$, then $S(a_1) - S(a_2) > 0$, that is $S(a_1) > S(a_2)$;

When $a_1 < a_2$, there is $u_{a_1}^- < u_{a_2}^-$, $u_{a_1}^+ < u_{a_2}^+$, $v_{a_1}^- > v_{a_2}^-$, $v_{a_1}^+ > v_{a_2}^+$, then $S(a_1) - S(a_2) < 0$, that is $S(a_1) < S(a_2)$;

When $a_1 = a_2$, there is $u_{a_1}^- = u_{a_2}^-$, $u_{a_1}^+ = u_{a_2}^+$, $v_{a_1}^- = v_{a_2}^-$, $v_{a_1}^+ = v_{a_2}^+$, then $S(a_1) - S(a_2) = 0$, that is $S(a_1) = S(a_2)$;

The property (1) is proved.

(5) The partial derivative of α .

$$S'_\alpha(a) = \frac{1}{2} (\sqrt[q]{u^-} + \sqrt[q]{1 - v^-})$$

It can be found that $S'_\alpha(a) > 0$, $S(a)$ monotonically increase about the parameter α . Therefore, $S(a)$ has monotonicity for the parameters α , the property (5) is proved.

Example 5. When $q = 3$, $\alpha = 0.5$, using formula (16), we can get $S(a_1) = 0.74793$, $S(a_2) = 0.78439$, $S(a_3) = 0.84080$, $S(a_4) = 0.85115$, $S(a_5) = 0.79012$, $S(a_6) = 0.77513$, so they can be distinguished and overcome the deficiency of S_C , S_B , and S_{GM} . In order to further verify the deficiencies, there are 12 IVq-ROFNs which are compared by the score functions, and the results are shown in Table 1.

Table 1 Comparison results of each scoring function

IVIFNs	S_C	S_{GM}	S_B	S
$a_7 = ([0.2, 0.6], [0.2, 0.4])$ $a_8 = ([0.3, 0.5], [0.1, 0.5])$	$h_7 = h_8$	$h_7 < h_8$	$h_7 < h_8$	$h_7 < h_8$
$a_9 = ([0.3, 0.5], [0.2, 0.4])$ $a_{10} = ([0.35, 0.45], [0.25, 0.35])$	$h_9 = h_{10}$	$h_9 = h_{10}$	$h_9 < h_{10}$	$h_9 < h_{10}$
$a_{11} = ([0.3, 0.6], [0.2, 0.3])$	$h_{11} = h_{12}$	$h_{11} < h_6$	$h_{11} < h_{12}$	$h_{11} < h_{126}$

$a_{12}=[(0.4, 0.5],[0.1, 0.4])$				
$a_{13}=[(0.1, 0.4],[0.2, 0.5])$ $a_{14}=[(0.2, 0.3],[0.3, 0.4])$	$h_{13}=h_{14}$	$h_{13}=h_{14}$	$h_{13}<h_{14}$	$h_{13}<h_{14}$
$a_{15}=[(0.0, 0.0],[0.1, 0.1])$ $a_{16}=[(0.0, 0.0],[0.9, 0.9])$	$h_{15}>h_{16}$	$h_{15}=h_{16}$	$h_{15}=h_{16}$	$h_{15}>h_{16}$
$a_{17}=[(0.6, 0.6],[0.4, 0.4])$ $a_{18}=[(0.4, 0.4],[0.1, 0.1])$	$h_{17}<h_{18}$	$h_{17}<h_{18}$	$h_{17}=h_{18}$	$h_{17}<h_{18}$

It can be seen from Table 1 the proposed score function can make up for the deficiencies of S_C , S_B , and S_{GM} . The comparison of IVq-ROFNs is shown in Definition 12.

Definition 12. Let $a_1 = ([u_{a_1}^-, u_{a_1}^+], [v_{a_1}^-, v_{a_1}^+])$ and $a_2 = ([u_{a_2}^-, u_{a_2}^+], [v_{a_2}^-, v_{a_2}^+])$ be two IVq-ROFNs, and the comparison of them is as follows:

- (1) If $S(a_1) > S(a_2)$, then $a_1 > a_2$,
- (2) If $S(a_1) < S(a_2)$, then $a_1 < a_2$,

If $S(a_1) = S(a_2)$, then we use accuracy function, which is defined in Ref[11]: $H(a) = \frac{1}{2}((u_a^-)^q + (u_a^+)^q + (v_a^-)^q + (v_a^+)^q)$, ($q \geq 1$), to calculate the accuracy value. If $H(a_1) > H(a_2)$, then $a_1 > a_2$, If $H(a_1) < H(a_2)$, then $a_1 < a_2$, If $H(a_1) = H(a_2)$, then $a_1 = a_2$.

4. IVq-ROFYWA operator and IVq-ROFYWG operator

On the basis of the Yager operator in Section 2.2, Section 4.1 extends Yager operations and studies their properties, Section 4.2 explains the IVq-ROFYWA operator and its related properties, Section 4.3 introduces the IVq-ROFYWG operator and its properties.

4.1 IVq-ROFY operator

According to formulas (7) and (8), the basic operation of Yager, inspired by Liu [14], this section defines Yager operation of addition, multiplication, scalar multiplication, and power about IVq-ROFNs, and their properties.

Definition 13. Let $a = ([u^-, u^+], [v^-, v^+])$, $a_1 = ([u_1^-, u_1^+], [v_1^-, v_1^+])$, $a_2 = ([u_2^-, u_2^+], [v_2^-, v_2^+])$ be three IVq-ROFNs, and $\delta > 0$, Yager operations are shown in formulas (17), (18), (19), (20).

$$a_1 \oplus_Y a_2 = \left(\left[\sqrt[q]{\min\left\{1, [(u_1^-)^{qp} + (u_2^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [(u_1^+)^{qp} + (u_2^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \left[\sqrt[q]{1 - \min\left\{1, [(1 - (v_1^-)^q)^p + (1 - (v_2^-)^q)^p]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, [(1 - (v_1^+)^q)^p + (1 - (v_2^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \right) \quad (17)$$

$$a_1 \otimes_Y a_2 = \left(\left[\sqrt[q]{1 - \min\left\{1, [(1 - (u_1^-)^q)^p + (1 - (u_2^-)^q)^p]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, [(1 - (u_1^+)^q)^p + (1 - (u_2^+)^q)^p]^{\frac{1}{p}}\right\}} \right], \left[\sqrt[q]{\min\left\{1, [(v_1^-)^{qp} + (v_2^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [(v_1^+)^{qp} + (v_2^+)^{qp}]^{\frac{1}{p}}\right\}} \right] \right) \quad (18)$$

$$\delta a = \left(\begin{array}{c} \left[\sqrt[q]{\min\{1, [\delta(u^-)^{qp}]^{\frac{1}{p}}\}}, \sqrt[q]{\min\{1, [\delta(u^+)^{qp}]^{\frac{1}{p}}\}} \right], \\ \left[\sqrt[q]{1 - \min\{1, [\delta(1 - (v^-)^q)^p]^{\frac{1}{p}}\}}, \sqrt[q]{1 - \min\{1, [\delta(1 - (v^+)^q)^p]^{\frac{1}{p}}\}} \right] \end{array} \right) \quad (19)$$

$$a^\delta = \left(\begin{array}{c} \left[\sqrt[q]{1 - \min\{1, [\delta(1 - (u^-)^q)^p]^{\frac{1}{p}}\}}, \sqrt[q]{1 - \min\{1, [\delta(1 - (u^+)^q)^p]^{\frac{1}{p}}\}} \right], \\ \left[\sqrt[q]{\min\{1, [\delta(v^-)^{qp}]^{\frac{1}{p}}\}}, \sqrt[q]{\min\{1, [\delta(v^+)^{qp}]^{\frac{1}{p}}\}} \right] \end{array} \right) \quad (20)$$

It is easy to verify that the Yager addition, multiplication, scalar multiplication and power satisfy the basic definition of Yager operator under IVq-ROFS.

Theorem 2. Let $a = ([u^-, u^+], [v^-, v^+])$, $a_1 = ([u_1^-, u_1^+], [v_1^-, v_1^+])$, $a_2 = ([u_2^-, u_2^+], [v_2^-, v_2^+])$ be three IVq-ROFNs, and $\delta, \delta_1, \delta_2 > 0$, satisfies the following six properties, as shown in formulas (21)-(26).

$$(1) a_1 \oplus_Y a_2 = a_2 \oplus_Y a_1 \quad (21)$$

$$(2) a_1 \otimes_Y a_2 = a_2 \otimes_Y a_1 \quad (22)$$

$$(3) \delta(a_1 \oplus_Y a_2) = \delta a_1 \oplus_Y \delta a_2 \quad (23)$$

$$(4) (a_1 \otimes_Y a_2)^\delta = (a_2)^\delta \otimes_Y (a_1)^\delta, (\delta > 0) \quad (24)$$

$$(5) (\delta_1 + \delta_2)a = \delta_1 a \oplus_Y \delta_2 a \quad (25)$$

$$(6) a^{\delta_1} \otimes_Y a^{\delta_2} = a^{(\delta_1 + \delta_2)} (\delta_1, \delta_2 > 0) \quad (26)$$

The property (3) is proved by the following, and other properties are easy to be deduced. This paper does not give a detailed derivation.

Proof. (3) According to formulas (17) and (19), the following formulas can be obtained:

$$\begin{aligned} \delta(a_1 \oplus_Y a_2) &= \delta \left(\begin{array}{c} \left[\sqrt[q]{\min\{1, [(u_1^-)^{qp} + (u_2^-)^{qp}]^{\frac{1}{p}}\}}, \sqrt[q]{\min\{1, [(u_1^+)^{qp} + (u_2^+)^{qp}]^{\frac{1}{p}}\}} \right], \\ \left[\sqrt[q]{1 - \min\{1, [(1 - (v_1^-)^q)^p + (1 - (v_2^-)^q)^p]^{\frac{1}{p}}\}}, \right. \\ \left. \sqrt[q]{1 - \min\{1, [(1 - (v_1^+)^q)^p + (1 - (v_2^+)^q)^p]^{\frac{1}{p}}\}} \right] \end{array} \right) \\ &= \left(\begin{array}{c} \left[\sqrt[q]{\min\{1, [\delta(u_1^-)^{qp} + \delta(u_2^-)^{qp}]^{\frac{1}{p}}\}}, \sqrt[q]{\min\{1, [\delta(u_1^+)^{qp} + \delta(u_2^+)^{qp}]^{\frac{1}{p}}\}} \right], \\ \left[\sqrt[q]{1 - \min\{1, [\delta(1 - (v_1^-)^q)^p + \delta(1 - (v_2^-)^q)^p]^{\frac{1}{p}}\}}, \right. \\ \left. \sqrt[q]{1 - \min\{1, [\delta(1 - (v_1^+)^q)^p + \delta(1 - (v_2^+)^q)^p]^{\frac{1}{p}}\}} \right] \end{array} \right) \\ \delta a_1 \oplus_Y \delta a_2 &= \left(\begin{array}{c} \left[\sqrt[q]{\min\{1, [\delta(u_1^-)^{qp}]^{\frac{1}{p}}\}}, \sqrt[q]{\min\{1, [\delta(u_1^+)^{qp}]^{\frac{1}{p}}\}} \right], \\ \left[\sqrt[q]{1 - \min\{1, [\delta(1 - (v_1^-)^q)^p]^{\frac{1}{p}}\}}, \sqrt[q]{1 - \min\{1, [\delta(1 - (v_1^+)^q)^p]^{\frac{1}{p}}\}} \right] \end{array} \right) \oplus_Y \end{aligned}$$

$$\begin{aligned}
& \left(\begin{array}{c} \left[\sqrt[q]{\min\left\{1, [\delta(u_2^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [\delta(u_2^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \\ \left[\sqrt[q]{1 - \min\left\{1, [\delta(1 - (v_2^-)^q)^p]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, [\delta(1 - (v_2^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \end{array} \right) \\
&= \left(\begin{array}{c} \left[\sqrt[q]{\min\left\{1, [\delta(u_1^-)^{qp} + \delta(u_2^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [\delta(u_1^+)^{qp} + \delta(u_2^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \\ \left[\sqrt[q]{1 - \min\left\{1, [\delta(1 - (v_1^-)^q)^p + \delta(1 - (v_2^-)^q)^p]^{\frac{1}{p}}\right\}}, \right. \\ \left. \sqrt[q]{1 - \min\left\{1, [\delta(1 - (v_1^+)^q)^p + \delta(1 - (v_2^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \end{array} \right) \\
&= \delta(a_1 \oplus_Y a_2) \\
&\delta(a_1 \oplus_Y a_2) = \delta a_2 \oplus_Y \delta a_1 \text{ (3) is proved.}
\end{aligned}$$

4.2 IVq-ROFYWA operator

According to the algorithm of addition, multiplication, scalar multiplication, and power of the IVq-ROFNs, the IVq-ROFYWA is defined.

Definition 14. Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ be a set of IVq-ROFNs, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is a weight vector and satisfy $\sum_{i=1}^n \omega_i = 1, \omega_i \in [0,1], (i = 1,2, \dots, n)$. IVq-ROFYWA is defined as formula (27).

$$\text{IVq-ROFYWA}(a_1, a_2, \dots, a_n) = \bigoplus_{i=1}^n \omega_i a_i \quad (27)$$

Theorem 3. Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ ($i = 1,2, \dots, n$) be a set of IVq-ROFNs, and the aggregating value of IVq-ROFYWA is also a IVq-ROFNs, as shown formula (28).

$$\begin{aligned}
& \text{IVq-ROFYWA}(a_1, a_2, \dots, a_n) \\
&= \left(\begin{array}{c} \left[\sqrt[q]{\min\left\{1, [\sum_{i=1}^n \omega_i (u_i^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [\sum_{i=1}^n \omega_i (u_i^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \\ \left[\sqrt[q]{1 - \min\left\{1, [\sum_{i=1}^n \omega_i (1 - (v_i^-)^q)^p]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, [\sum_{i=1}^n \omega_i (1 - (v_i^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \end{array} \right) \quad (28)
\end{aligned}$$

Proof. When $n=2$,

$$\omega_i a_i = \left(\begin{array}{c} \left[\sqrt[q]{\min\left\{1, [\omega_i (u_i^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [\omega_i (u_i^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \\ \left[\sqrt[q]{1 - \min\left\{1, [\omega_i (1 - (v_i^-)^q)^p]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, [\omega_i (1 - (v_i^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \end{array} \right)$$

$$\text{IVq-ROFYWA}(a_1, a_2) = \omega_1 a_1 \oplus \omega_2 a_2$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{\min\left\{1, [\omega_1 (u_1^-)^{qp} + \omega_2 (u_2^-)^{qp}]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, [\omega_1 (u_1^+)^{qp} + \omega_2 (u_2^+)^{qp}]^{\frac{1}{p}}\right\}} \right], \\ \left[\sqrt[q]{1 - \min\left\{1, [\omega_1 (1 - (v_1^-)^q)^p + \omega_2 (1 - (v_2^-)^q)^p]^{\frac{1}{p}}\right\}}, \right. \\ \left. \sqrt[q]{1 - \min\left\{1, [\omega_1 (1 - (v_1^+)^q)^p + \omega_2 (1 - (v_2^+)^q)^p]^{\frac{1}{p}}\right\}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{\min \left\{ 1, [\sum_{i=1}^2 \omega_i (u_i^-)^{qp}]^{\frac{1}{p}} \right\}}, \sqrt[q]{\min \left\{ 1, [\sum_{i=1}^2 \omega_i (u_i^+)^{qp}]^{\frac{1}{p}} \right\}} \right], \\ \left[\sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^2 \omega_i (1 - (v_i^-)^q)^p]^{\frac{1}{p}} \right\}}, \sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^2 \omega_i (1 - (v_i^+)^q)^p]^{\frac{1}{p}} \right\}} \right] \end{array} \right) \quad (29)$$

From formula (29), $IVq - ROFYWA(a_1, a_2) = \omega_1 a_1 \oplus_Y \omega_2 a_2$. When $n=2$, the formula (28) is satisfied.

When $n=k$, the formula (28) is satisfied,

$$IVq - ROFYWA(a_1, a_2, \dots, a_k) = \oplus_{i=1}^k \omega_i a_i$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{\min \left\{ 1, \left[\sum_{i=1}^k \omega_i (u_i^-)^{qp} \right]^{\frac{1}{p}} \right\}}, \sqrt[q]{\min \left\{ 1, \left[\sum_{i=1}^k \omega_i (u_i^+)^{qp} \right]^{\frac{1}{p}} \right\}} \right], \\ \left[\sqrt[q]{1 - \min \left\{ 1, \left[\sum_{i=1}^k \omega_i (1 - (v_i^-)^q)^p \right]^{\frac{1}{p}} \right\}}, \sqrt[q]{1 - \min \left\{ 1, \left[\sum_{i=1}^k \omega_i (1 - (v_i^+)^q)^p \right]^{\frac{1}{p}} \right\}} \right] \end{array} \right)$$

Then when $n=k+1$, there is

$$IVq - ROFYWA(a_1, a_2, \dots, a_{k+1}) = (\oplus_{i=1}^k \omega_i a_i) \oplus \omega_{k+1} a_{k+1}$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{\min \left\{ 1, [\sum_{i=1}^k \omega_i (u_i^-)^{qp}]^{\frac{1}{p}} \right\}}, \sqrt[q]{\min \left\{ 1, [\sum_{i=1}^k \omega_i (u_i^+)^{qp}]^{\frac{1}{p}} \right\}} \right], \\ \left[\sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^k \omega_i (1 - (v_i^-)^q)^p]^{\frac{1}{p}} \right\}}, \sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^k \omega_i (1 - (v_i^+)^q)^p]^{\frac{1}{p}} \right\}} \right] \end{array} \right) \oplus_Y$$

$$\left(\begin{array}{c} \left[\sqrt[q]{\min \left\{ 1, [\omega_{k+1} (u_{k+1}^-)^{qp}]^{\frac{1}{p}} \right\}}, \sqrt[q]{\min \left\{ 1, [\omega_{k+1} (u_{k+1}^+)^{qp}]^{\frac{1}{p}} \right\}} \right], \\ \left[\sqrt[q]{1 - \min \left\{ 1, [\omega_{k+1} (1 - (v_{k+1}^-)^q)^p]^{\frac{1}{p}} \right\}}, \sqrt[q]{1 - \min \left\{ 1, [\omega_{k+1} (1 - (v_{k+1}^+)^q)^p]^{\frac{1}{p}} \right\}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{\min \left\{ 1, [\sum_{i=1}^{k+1} \omega_i (u_i^-)^{qp}]^{\frac{1}{p}} \right\}}, \sqrt[q]{\min \left\{ 1, [\sum_{i=1}^{k+1} \omega_i (u_i^+)^{qp}]^{\frac{1}{p}} \right\}} \right], \\ \left[\sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^{k+1} \omega_i (1 - (v_i^-)^q)^p]^{\frac{1}{p}} \right\}}, \sqrt[q]{1 - \min \left\{ 1, [\sum_{i=1}^{k+1} \omega_i (1 - (v_i^+)^q)^p]^{\frac{1}{p}} \right\}} \right] \end{array} \right) \quad (30)$$

It can be seen that formula (30) and formula (28) is also satisfied. Therefore, Theorem 3 is proved.

The IVq-ROFYWA operator also satisfies Idempotency, Boundedness, Monotonicity, and Commutativity.

Theorem 4 (Idempotency). Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$, $a = ([u^-, u^+], [v^-, v^+])$ ($i = 1, 2, \dots, n$) is a set of IVq-ROFNs, and $a_i = a$, then formula (31) holds.

$$IVq - ROFYWA(a_1, a_2, a_3, \dots, a_n) = a \quad (31)$$

Theorem 5 (Boundedness). Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ ($i = 1, 2, \dots, n$) be a set of IVq-ROFNs, then formula

(32) holds.

$$a_{min} \leq ROFYWA(a_1, a_2, a_3, \dots, a_n) \leq a_{max} \quad (32)$$

In formula (32), $a_{max} = \max(a_i)$, $a_{min} = \min(a_i)$.

Theorem 6 (Monotonicity). Suppose $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$, $a_i^* = ([u_i^{*-}, u_i^{*+}], [v_i^{*-}, v_i^{*+}])$ ($i = 1, 2, \dots, n$) be two sets of IVq-ROFNs, and satisfy $a_i \leq a_i^*$, then formula (33) holds.

$$IVq - RVOYWA(a_1, a_2, a_3, \dots, a_n) \leq IVq - RVOYWA(a_1^*, a_2^*, a_3^*, \dots, a_n^*) \quad (33)$$

Theorem 7 (Commutativity). Suppose $(\alpha'_1, \alpha'_2, \dots, \alpha'_n)$ is a set of IVq-ROFNs that indicates any permutation of $(\alpha_1, \alpha_2, \dots, \alpha_n)$, then

$$IVq - RVOYWA(\alpha_1, \alpha_2, \dots, \alpha_n) = IVq - RVOYWA(\alpha'_1, \alpha'_2, \dots, \alpha'_n) \quad (34)$$

It is easy to prove Idempotency, Boundedness, Monotonicity, and Commutativity by formulas (17), (18), (19), (20).

4.3 IVq-ROFYGM operator

According to the addition, multiplication, number multiplication, and power operations of the IVq-ROFNs of the Yager operator defined in Section 4.1, this paper proposes interval-valued q-rung orthopair Yager weighted geometric mean operator and their properties.

Definition 15. Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ ($i = 1, 2, \dots, n$) be a set of IVq-ROFNs, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be a weight vector, $\sum_{i=1}^n \omega_i = 1$, $\omega_i \in [0, 1]$, ($i = 1, 2, \dots, n$). The IVq-ROFYWG is defined as formula (35).

$$IVq\text{-}ROFYWG(a_1, a_2, \dots, a_n) = \bigotimes_{i=1}^n a_i^{\omega_i} \quad (35)$$

Theorem 8: Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ ($i = 1, 2, \dots, n$) be a set of IVq-ROFNs, and the aggregation value of IVq-ROFYWA is also a IVq-ROFNs. The aggregation result is shown in formula (36).

$$IVq - ROFYWG(a_1, a_2, \dots, a_n) = \left(\left[\sqrt[q]{1 - \min\left\{1, \left[\sum_{i=1}^n \omega_i (1 - (u_i^-)^q)^p\right]^{\frac{1}{p}}\right\}}, \sqrt[q]{1 - \min\left\{1, \left[\sum_{i=1}^n \omega_i (1 - (u_i^+)^q)^p\right]^{\frac{1}{p}}\right\}} \right], \left[\sqrt[q]{\min\left\{1, \left[\sum_{i=1}^n \omega_i (v_i^-)^{qp}\right]^{\frac{1}{p}}\right\}}, \sqrt[q]{\min\left\{1, \left[\sum_{i=1}^n \omega_i (v_i^+)^{qp}\right]^{\frac{1}{p}}\right\}} \right] \right) \quad (36)$$

The proof process is similar to Theorem 4. This paper does not give a detailed proof.

The IVq-ROFYWG operator also satisfies (1) Idempotency; (2) Boundedness; (3) Commutativity; (4) Monotonicity.

Theorem 9 (Idempotency). Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$, $a = ([u^-, u^+], [v^-, v^+])$ ($i = 1, 2, \dots, n$) be a set of IVq-ROFNs, satisfying $a_i = a$, then formula (37) holds.

$$IVq - ROFYWG(a_1, a_2, a_3, \dots, a_n) = a \quad (37)$$

Theorem 10 (Boundedness). Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$ ($i = 1, 2, \dots, n$) be a set of IVq-ROFNs, then formula (38) holds.

$$a_{min} \leq IVq - ROFYWG(a_1, a_2, a_3, \dots, a_n) \leq a_{max} \quad (38)$$

Where $a_{max} = \max(a_i), a_{min} = \min(a_i)$

Theorem 11 (Monotonicity). Let $a_i = ([u_i^-, u_i^+], [v_i^-, v_i^+])$, $a_i^* = ([u_i^{*-}, u_i^{*+}], [v_i^{*-}, v_i^{*+}]) (i = 1, 2, \dots, n)$ be two sets of IVq-ROFNs, and satisfy $a_i \leq a_i^*$, then formula (39) holds.

$$IVq - ROFYWG(a_1, a_2, a_3, \dots, a_n) \leq IVq - ROFYWG(a_1^*, a_2^*, a_3^*, \dots, a_n^*) \quad (39)$$

Theorem 12 (Commutativity). Let $(\alpha'_1, \alpha'_2, \dots, \alpha'_n)$ be a set of IVq-ROFNs and any permutation of $(\alpha_1, \alpha_2, \dots, \alpha_n)$, then formula (40) holds.

$$IVq - RVOYWG(\alpha_1, \alpha_2, \dots, \alpha_n) = IVq - RVOYWG(\alpha'_1, \alpha'_2, \dots, \alpha'_n) \quad (40)$$

It is easy to prove Idempotency, Boundedness, Monotonicity, and Commutativity of the IVq-ROFYWG operator by using formulas (17), (18), (19), (20), there are ignored.

5. GDM method based on modified WASPAS

The operating environment of the decision-making method is described in 5.1, the expert weight method is introduced in Section 5.2, the attribute weight solution method based on the CRITIC method is described in Section 5.3, the WASPAS method is described in Section 5.4, and the GDM method is given in Section 5.5.

5.1 Decision environment description

For k experts, m alternatives with n attributes, and GDM problems with unknown experts' weights and unknown attribute weights can be described in mathematical form. The specific description is given below.

Suppose that k experts are invited to give decision data of m alternatives with n attributes, the task is to select the optimal solution among the alternatives based on the data of the experts. The attribute weights and expert weights are unknown. But the sum of them is equal to 1. The set of k experts is $E = \{e_1, e_2, \dots, e_k\}$, and the expert weights satisfy $\sum_{t=1}^k \lambda_t = 1, \lambda_t \geq 0 (t = 1, 2, \dots, k)$. The set of m alternatives is $Y = \{y_1, y_2, \dots, y_m\}$, each alternative contains n identical attributes, the attribute set is represented as $C = \{C_1, C_2, \dots, C_n\}$, and the attribute weight of the weight satisfies $\sum_{j=1}^n \omega_j = 1, \omega_j \geq 0$. For different experts, different alternatives and attribute values conduct the decision matrix. The decision matrix of each expert is $A^{(t)} = (a_{ij}^{(t)})_{m \times n}$, ($i = 1, 2, \dots, m; j = 1, 2, \dots, n; t = 1, 2, \dots, k$), where $a_{ij}^{(t)}$ represents the decision value of the t-th decision maker for the attribute j in the alternative i, and $a_{ij}^{(k)} = ([u_{a_{ij}}^-, u_{a_{ij}}^+], [v_{a_{ij}}^-, v_{a_{ij}}^+])^{(t)}$. The decision matrix of the t-th expert is shown in (41).

$$A^{(t)} = \begin{bmatrix} ([u_{a_{11}}^-, u_{a_{11}}^+], [v_{a_{11}}^-, v_{a_{11}}^+])^{(t)} & \dots & ([u_{a_{1n}}^-, u_{a_{1n}}^+], [v_{a_{1n}}^-, v_{a_{1n}}^+])^{(t)} \\ \vdots & \ddots & \vdots \\ ([u_{a_{m1}}^-, u_{a_{m1}}^+], [v_{a_{m1}}^-, v_{a_{m1}}^+])^{(t)} & \dots & ([u_{a_{mn}}^-, u_{a_{mn}}^+], [v_{a_{mn}}^-, v_{a_{mn}}^+])^{(t)} \end{bmatrix} \quad (41)$$

In the decision-making process, if the attributes include benefit type Ω_1 and cost type Ω_2 , then $A^{(t)}$ needs to be standardized to obtain the decision matrix $R^{(t)}$ as formula (42).

$$r_{ij}^{(t)} = \begin{cases} \frac{a_{ij}^{(t)} - \min(a_{ij}^{(t)})}{\max(a_{ij}^{(t)}) - \min(a_{ij}^{(t)})}, & a_{ij}^{(t)} \in \Omega_1 \\ \frac{\max(a_{ij}^{(t)}) - a_{ij}^{(t)}}{\max(a_{ij}^{(t)}) - \min(a_{ij}^{(t)})}, & a_{ij}^{(t)} \in \Omega_2 \end{cases} \quad (42)$$

After calculating the weights of experts, the IVq-ROFYWA operator is utilized to aggregate $R^{(t)}$ to obtain the decision matrix R. Then, using the CRITIC method to get the attribute weights of R, and the WASPAS method is employed to calculate alternative scores that represents the best solution.

5.2 Deriving expert weights of different alternatives

Ref [33] describes a deriving expert weights method based on similarity, the similarity is calculated by NIS dividing sum of PIS and NIS. After dividing the similarity of each expert by the similarity, the weight of each can be obtained. This method has been used to get the expert weight in MAGDM problem. However, when the decision matrices $R^{(1)}$ and $R^{(2)}$ given by two experts are shown in Table 2 and Table 3, it is easy to see that the alternative y_1 given by the expert e_1 and the alternative y_1 given by expert e_2 has obvious differences, and the alternative y_2 given by expert e_1 is also significantly different from the alternative y_2 given by expert e_2 , but the e_1 and e_2 weights calculated by Ref [33] both are 0.5, so the weights of the two experts are exactly the same. From the matrices given by the experts, it is obvious that e_1 and e_2 have obvious differences, which cannot fully reflect the objective judgment of the expert. At the same time, in real life, experts are susceptible to their own and external environmental influences when judging different options, such as the psychological state of experts, the manifestations of different options, the surrounding environment, and the relationship between group preferences. Therefore, experts give different options that obviously show different weights.

Table 2 Decision matrix $R^{(1)}$

	C_1	C_2	C_3
y_1	([0.02, 0.14],[0.12, 0.93])	([0.54, 0.87],[0.02, 0.52])	([0.59, 0.82],[0.76, 0.79])
y_2	([0.13, 0.78],[0.11, 0.61])	([0.63, 0.78],[0.01, 0.36])	([0.08, 0.20],[0.38, 0.41])
y_3	([0.70, 0.90],[0.10, 0.20])	([0.09, 0.85],[0.65, 0.99])	([0.51, 0.87],[0.36, 0.59])
y_4	([0.01, 0.05],[0.04, 0.94])	([0.75, 0.90],[0.53, 0.76])	([0.19, 0.31],[0.01, 0.53])

Table 3 Decision matrix $R^{(2)}$

	C_1	C_2	C_3
y_1	([0.13, 0.78],[0.11, 0.61])	([0.63, 0.78],[0.01, 0.36])	([0.08, 0.20],[0.38, 0.41])
y_2	([0.70, 0.90],[0.10, 0.20])	([0.09, 0.85],[0.65, 0.99])	([0.51, 0.87],[0.36, 0.59])
y_3	([0.01, 0.05],[0.04, 0.94])	([0.75, 0.90],[0.53, 0.76])	([0.19, 0.31],[0.01, 0.53])
y_4	([0.02, 0.14],[0.12, 0.93])	([0.54, 0.87],[0.02, 0.52])	([0.59, 0.82],[0.76, 0.79])

Inspired by the Ref [33], this paper proposes a new deriving experts weights method of different alternatives based on similarity. First the positive and negative ideal reference points are found between experts, then the positive and negative ideal distances are calculated according to similarity. Next, the expert weights are calculated. For k-th experts, the weights of m alternatives are solved respectively to obtain an expert weights matrix. The steps are as follows:

- (1) Calculate positive and negative ideal distances. First the positive and negative ideal point are calculated by

formula (43) and (44), then positive and negative ideal distance are calculated by formulas (45) and (46)

$$d_j^{+*} = \left(\left[\max_{1 \leq i \leq m} \left(u_{r_{ij}^{(t)}}^- \right), \max_{1 \leq i \leq m} \left(u_{r_{ij}^{(t)}}^+ \right) \right], \left[\min_{1 \leq i \leq m} \left(v_{r_{ij}^{(t)}}^- \right), \min_{1 \leq i \leq m} \left(v_{r_{ij}^{(t)}}^+ \right) \right] \right) \quad (43)$$

$$d_j^{-*} = \left(\left[\min_{1 \leq i \leq m} \left(u_{r_{ij}^{(t)}}^- \right), \min_{1 \leq i \leq m} \left(u_{r_{ij}^{(t)}}^+ \right) \right], \left[\max_{1 \leq i \leq m} \left(v_{r_{ij}^{(t)}}^- \right), \max_{1 \leq i \leq m} \left(v_{r_{ij}^{(t)}}^+ \right) \right] \right) \quad (44)$$

$$d_{ij}^{+(t)} = d(r_{ij}^{(t)}, d_j^{+*}) \quad (45)$$

$$d_{ij}^{-(t)} = d(r_{ij}^{(t)}, d_j^{-*}) \quad (46)$$

(2) Calculate the similarity $sim_{ij}^{(t)}$ of experts, as shown in formula (47), and combine them to form a similarity matrix $SIM^{(t)} = [sim_{ij}^{(t)}]$.

$$sim_{ij}^{(t)} = \frac{d_{ij}^{-(t)}}{d_{ij}^{-(t)} + d_{ij}^{+(t)}} \quad (47)$$

(3) Calculate the sum of the similarity of each alternative, as shown in formula (48).

$$sim_i^{(t)} = \sum_{j=1}^n sim_{ij}^{(t)} \quad (48)$$

(4) Calculate the expert weight value $\lambda_i^{(t)}$ ($t = 1, 2, \dots, k; i = 1, 2, \dots, m$) of each alternative, and use formula (49) to normalize $d_i^{(t)}$ to determine the weight value of each alternative of each expert.

$$\lambda_i^{(t)} = \frac{sim_i^{(t)}}{\sum_{t=1}^k sim_i^{(t)}} \quad (49)$$

(5) Solve the expert weight matrix λ of different alternatives. Combining the weight value $\lambda_i^{(t)}$ ($t = 1, 2, \dots, k; i = 1, 2, \dots, m$) of each alternative calculated in step (4), the expert weight of different alternatives can be obtained, as shown in formula (50).

$$\lambda = \begin{bmatrix} \lambda_1^{(1)} & \dots & \lambda_1^{(k)} \\ \vdots & \ddots & \vdots \\ \lambda_m^{(1)} & \dots & \lambda_m^{(k)} \end{bmatrix} \quad (50)$$

For the decision matrices $R^{(1)}$ and $R^{(2)}$ given by the expert e_1 and e_2 , the proposed method is used to calculate the expert weight matrix of different alternatives as shown in Table 4.

Table 4 Calculation results of the weights of experts

	$e_1(t = 1)$	$e_2(t = 2)$
$\lambda_1^{(t)}$	0.4916	0.5084
$\lambda_2^{(t)}$	0.4171	0.5829
$\lambda_3^{(t)}$	0.5466	0.4534
$\lambda_4^{(t)}$	0.5452	0.4548

It can be seen from Table 4 that the weights of experts of different alternatives are inconsistent, which can reflect the true judgment of the experts. The literature [33] gives a weight for the expert to judge the whole problem, and it cannot reflect the change of the expert due to the external environment. This paper proposes that the method of weighting for decision-makers of alternatives is more adaptable when the weights are unknown and the decision-makers have obvious differences in the judgment of different alternatives.

5.3 Deriving attribute weights based on CRITIC method

In order to aggregating the different alternatives of R, the attribute weight should be known. Inspired by [40], the modified CRITIC method is used to solve attribute weight under IVq-ROFS. The CRITIC method includes five steps: (1) Calculate the attributes weighted average of R, (2) Calculate the standard deviation of the attribute, (3) Calculate the correlation coefficient between each attribute; (4) Calculate the index, (5) Get the weight of each attribute. The steps of deriving attribute weight are following.

(1) Calculate the attributes weighted average of R. The weighted average of each attribute is calculated by formula (51), where the weight is: $\omega_i = \frac{1}{m} (i = 1, 2, \dots, m)$.

$$\bar{r}_j = IVq - ROFYWA(r_{1j}, r_{2j}, \dots, r_{mj}) \quad (51)$$

(2) Calculate the standard deviation of the attribute σ_j . The standard deviation of each attribute is calculated by formula (52).

$$\sigma_j = \sqrt{\frac{1}{m} \sum_{i=1}^m d(r_{ij}, \bar{r}_j)^2} \quad (52)$$

(3) Solve the correlation coefficient between the two attributes. the correlation coefficient ρ_{jk} between every two attributes can be determined by formula (53), and $\bar{r}_j$ and $\bar{r}_k$ represent the average value of the jth and kth attributes, respectively.

$$\rho_{jk} = \frac{\sum_{i=1}^m d(r_{ij}, \bar{r}_j) d(r_{ik}, \bar{r}_k)}{\sqrt{\sum_{i=1}^m d(r_{ij}, \bar{r}_j)^2 \sum_{i=1}^m d(r_{ik}, \bar{r}_k)^2}} \quad (53)$$

(4) Calculate the index value N_j of the attribute. The index N_j ($j = 1, 2, \dots, n$) of each attribute can be calculated by formula (54).

$$N_j = \sigma_j \sum_{k=1}^n (1 - \rho_{jk}) \quad (54)$$

(5) Calculate the attribute weight. The attribute weight is calculated by formula (55).

$$w_j = \frac{N_j}{\sum_{i=1}^n N_j} \quad (55)$$

5.4 GDM based on WASPAS method

According to this article's explanation of decision maker weights, Yager set settlement sub, based on CRTIC attribute weight, WASPAS method, a new group decision-making method, as shown in Figure 1, the method steps are as follows:

- (1) Determine the value of q . According to the decision matrix $R^{(t)}$ given by each expert, the q value is determined by definition of IVq-ROFS.
- (2) The decision matrix $A^{(t)}$ is standardized, and the matrix $R^{(t)}$ is obtained.
- (3) Calculate expert weight matrix λ of different alternatives.

First, the positive and negative ideal distances are calculated by formula (45) and (46), then the $sim_{ij}^{(t)}$ of experts is calculated by formula (47), and the expert weight value $\lambda_i^{(t)}$ ($t = 1, 2, \dots, k; i = 1, 2, \dots, m$) of each alternative are calculated by formula (49). The expert weight matrix λ of different alternatives are calculated by formula (50).

- (4) Aggregate the expert decision-making matrices. Aggregating the $R^{(1)}, R^{(2)}, \dots, R^{(k)}$ to $R = (r_{ij})_{m \times n}$ by formula (56). The expert weight matrix λ are obtained by step (3).

$$r_{ij} = IVq - ROFYWA(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(t)}) \quad (56)$$

- (5) Calculate the attribute weight of R .

The modified CRITIC method is used to solve the attribute weight which have described in section 5.3. And the attribute weight is calculated by formula (55).

For the aggregating matrix R , the various alternatives can be further aggregated. Inspired by[27], the WASPAS method is used to aggregate alternatives under IVq-ROFNs. The steps of modified WASPAS method are as follows:

- (6) Calculate the weighted average $Q_i^{(1)}$ of the alternative. Use formula (57) to calculate the weighted average of each alternative, where ω_j ($j=1, 2, \dots, n$) is the attribute weight.

$$Q_i^{(1)} = IVq - ROFYWA(r_{i1}, r_{i2}, \dots, r_{in}), (i = 1, 2, \dots, m) \quad (57)$$

- (7) Calculate the weighted geometric mean $Q_i^{(2)}$ of the alternative. The formula (58) is sued to calculated weighted geometric average, where ω_i ($i=1, 2, \dots, n$) is the attribute weight.

$$Q_i^{(2)} = IVq - ROFYWG(r_{i1}, r_{i2}, \dots, r_{in}); (i = 1, 2, \dots, m) \quad (58)$$

- (8) Calculate the compromise r_i . The compromise value r_i is calculated by formula (59) where $0 \leq \lambda \leq 1$.

$$r_i = \lambda Q_i^{(1)} + (1 - \lambda) Q_i^{(2)}; (i = 1, 2, \dots, m) \quad (59)$$

- (9) Calculate the score Sr_i of the alternative r_i as shown in formula (60).

$$Sr_i = S(r_i) \quad (60)$$

- (10) Sort Sr_i and get the optimal alternatives. The best solution is the one with the largest score.

Figure 1 The processing flow chart of the group decision method

6. Case study

The case of hypertension detection and prediction management system verifies the proposed GDM, and section 6.2 analyzes parameters sensitivity. Section 6.3 and Section 6.4 respectively compare and analyze the operators and decision-making methods.

6.1 Case

With the development of information technology, the prevention and management of hypertension adopts information for community management. Although the existing management system meets the medical management, the efficiency needs to be improved, and the prediction results of existing systems still need to be analyzed and judged by experts. In order to develop community hypertension management system and provide personalized monitoring and prediction of hypertension risk more accurately, the residents' susceptibility to hypertension is divided into five levels: normal, low risk, medium risk, high risk and extremely high risk. And the green, blue, yellow, orange and red is used to warning community doctor or resident. Each color and the corresponding risk level are shown in Table 5, they are regarded as five alternatives $(y_1, y_2, y_3, y_4, y_5)$. For each alternative, the warning level is related to many indices of hypertensive patients, the current blood pressure measurement value, family

genetic history, change trend of multiple blood pressure measurements, age and relevant risk factors is regarded as five mainly indices for managing hypertension resident. These five indices are represented by C1, C2, C3, C4, and C5. All indices are efficiency type.

Table 5 The meaning of the alternative early warning

alternatives	Risk level	Warning color
y ₁	Very high risk of illness in recent years	Red
y ₂	High risk of illness	Orange
y ₃	At risk of illness	Yellow
y ₄	Not susceptible	Blue
y ₅	Normal	Green

During the system design process, 5 hypertension experts (e_1, e_2, e_3, e_4, e_5) are hired to analyze the physical examination data. In the early stage of system implementation, experts judge the residents' hypertension risks based on their own experience. Because the hypertension risk prediction value is not a determined value, but an interval value, the judgment value given by experts is expressed by IVq-ROFNs. In this case, there is resident who is 50 years old, and examination results: systolic blood pressure is 135mmHg, diastolic blood pressure is 87mmHg, and blood pressure has been rising in the past 5 years, has family genetic history, no related diseases, normal living habits. Five experts judgement values of hypertension risk for the resident are shown in Table 6 to Table 10:

Table 6 Decision matrix given by Expert 1 $A^{(1)}$

	C1	C2	C3	C4	C5
y ₁	([0.85,0.95],[0.1,0.2])	([0.8,0.9],[0.1,0.2])	([0.85,0.95], [0.1,0.2])	([0.7,0.8],[0.2,0.3])	([0.4,0.5],[0.5,0.6])
y ₂	([0.9,0.95],[0.1,0.2])	([0.75,0.8],[0.2,0.3])	([0.85,0.9], [0.1,0.15])	([0.65,0.7],[0.3,0.35])	([0.5,0.6],[0.4,0.5])
y ₃	([0.8,0.9],[0.1,0.2])	([0.6,0.7],[0.3,0.4])	([0.8,0.95], [0.1,0.2])	([0.6,0.7],[0.3,0.4])	([0.3,0.4],[0.6,0.7])
y ₄	([0.5,0.6],[0.4,0.5])	([0.6,0.75],[0.2,0.3])	([0.7,0.8], [0.2,0.3])	([0.5,0.6],[0.4,0.5])	([0.3,0.4],[0.6,0.7])
y ₅	([0.4,0.5],[0.5,0.6])	([0.5,0.6],[0.4,0.5])	([0.6,0.7], [0.3,0.4])	([0.5,0.6],[0.4,0.5])	([0.1,0.2],[0.8,0.9])

Table 7 Decision matrix given by Expert 2 $A^{(2)}$

	C1	C2	C3	C4	C5
y ₁	([0.7,0.8],[0.2,0.3])	([0.8,0.9],[0.1,0.2])	([0.8,0.9], [0.1,0.2])	([0.65,0.75],[0.3,0.35])	([0.5,0.6],[0.4,0.5])
y ₂	([0.9,0.95],[0.1,0.2])	([0.75,0.85],[0.2,0.3])	([0.85,0.9], [0.1,0.15])	([0.6,0.7],[0.3,0.4])	([0.4,0.5],[0.5,0.6])
y ₃	([0.8,0.9],[0.1,0.2])	([0.65,0.7],[0.2,0.3])	([0.75,0.8], [0.2,0.3])	([0.7,0.8],[0.2,0.3])	([0.35,0.5],[0.5,0.65])
y ₄	([0.7,0.8],[0.2,0.3])	([0.6,0.7],[0.3,0.4])	([0.7,0.8], [0.2,0.3])	([0.2,0.3],[0.7,0.8])	([0.2,0.3],[0.7,0.8])
y ₅	([0.75,0.85],[0.2,0.3])	([0.55,0.65],[0.35,0.4])	([0.6,0.7], [0.3,0.4])	([0.1,0.2],[0.8,0.9])	([0.3,0.4],[0.6,0.7])

Table 8 Decision matrix given by Expert 3 $A^{(3)}$

	C1	C2	C3	C4	C5
y ₁	([0.85,0.9],[0.1,0.2])	([0.7,0.8],[0.2,0.3])	([0.85,0.9], [0.1,0.2])	([0.65,0.75],[0.25,0.35])	([0.4,0.5],[0.5,0.6])
y ₂	([0.9,0.95],[0.1,0.2])	([0.8,0.9],[0.1,0.2])	([0.85,0.95], [0.1,0.15])	([0.65,0.8],[0.2,0.3])	([0.5,0.6],[0.4,0.5])
y ₃	([0.85,0.9],[0.1,0.2])	([0.6,0.7],[0.3,0.4])	([0.75,0.85], [0.2,0.3])	([0.6,0.7],[0.3,0.4])	([0.4,0.5],[0.5,0.6])
y ₄	([0.85,0.9],[0.1,0.2])	([0.4,0.5],[0.5,0.6])	([0.2,0.3], [0.7,0.8])	([0.4,0.5],[0.5,0.6])	([0.3,0.4],[0.6,0.7])
y ₅	([0.7,0.85],[0.2,0.3])	([0.2,0.3],[0.8,0.9])	([0.1,0.2], [0.8,0.9])	([0.3,0.4],[0.6,0.7])	([0.3,0.4],[0.6,0.7])

Table 9 Decision matrix given by Expert 4 $A^{(4)}$

	C1	C2	C3	C4	C5
y ₁	([0.85,0.9],[0.1,0.2])	([0.7,0.8],[0.2,0.3])	([0.85,0.95], [0.1,0.15])	([0.6,0.75],[0.3,0.35])	([0.45,0.5],[0.5,0.55])
y ₂	([0.9,0.95],[0.1,0.2])	([0.75,0.8],[0.2,0.3])	([0.85,0.95], [0.1,0.15])	([0.7,0.75],[0.2,0.3])	([0.4,0.5],[0.5,0.6])
y ₃	([0.75,0.8],[0.2,0.3])	([0.6,0.75],[0.3,0.4])	([0.8,0.9], [0.1,0.2])	([0.6,0.7],[0.3,0.4])	([0.35,0.45],[0.5,0.65])
y ₄	([0.85,0.9],[0.1,0.2])	([0.4,0.5],[0.5,0.6])	([0.5,0.6], [0.4,0.5])	([0.6,0.7],[0.3,0.4])	([0.4,0.5],[0.5,0.6])
y ₅	([0.7,0.85],[0.2,0.3])	([0.3,0.4],[0.6,0.7])	([0.4,0.5], [0.5,0.6])	([0.45,0.5],[0.5,0.6])	([0.35,0.4],[0.5,0.6])

Table 10 Decision matrix given by expert 5 $A^{(5)}$

	C1	C2	C3	C4	C5
y ₁	⟨[0.8,0.9],[0.1,0.2]⟩	⟨[0.7,0.8],[0.2,0.3]⟩	⟨[0.8,0.9],[0.1,0.2]⟩	⟨[0.6,0.75],[0.3,0.35]⟩	⟨[0.4,0.5],[0.5,0.6]⟩
y ₂	⟨[0.9,0.95],[0.1,0.2]⟩	⟨[0.75,0.85],[0.2,0.3]⟩	⟨[0.8,0.95],[0.1,0.2]⟩	⟨[0.7,0.8],[0.2,0.3]⟩	⟨[0.5,0.6],[0.4,0.5]⟩
y ₃	⟨[0.7,0.85],[0.1,0.2]⟩	⟨[0.65,0.75],[0.3,0.4]⟩	⟨[0.8,0.9],[0.1,0.2]⟩	⟨[0.6,0.7],[0.2,0.3]⟩	⟨[0.4,0.5],[0.5,0.6]⟩
y ₄	⟨[0.8,0.9],[0.1,0.2]⟩	⟨[0.5,0.6],[0.4,0.5]⟩	⟨[0.6,0.7],[0.3,0.4]⟩	⟨[0.6,0.65],[0.3,0.4]⟩	⟨[0.35,0.45],[0.5,0.6]⟩
y ₅	⟨[0.75,0.85],[0.2,0.3]⟩	⟨[0.4,0.5],[0.5,0.6]⟩	⟨[0.5,0.6],[0.4,0.5]⟩	⟨[0.3,0.4],[0.6,0.7]⟩	⟨[0.35,0.4],[0.5,0.6]⟩

According to the known conditions, Since all indices are efficiency type. The entire decision-making process includes the following 10 steps:

(1) Determine the a q value. The q is 3 which make all values of Table 6 to Table 10 satisfy the definition of IVq-ROFS.

(2) Calculate the standardized decision matrix $R^{(t)}$. Since they are all efficiency indices, $R^{(t)} = A^{(t)}$, as shown in Tables 6 to 10.

(3) Calculate expert weight matrix λ of different alternatives.

First, the positive and negative ideal distances are calculated by formula (45) and (46), then the $sim_{ij}^{(t)}$ of experts is calculated by formula(47), and the expert weight value $\lambda_i^{(t)}$ ($t = 1, 2, \dots, k; i = 1, 2, \dots, m$) of each alternative are calculated by formula(49). The expert weight matrix λ of different alternatives are calculated by formula(50), and the result is shown as following.

$$\lambda = \begin{bmatrix} 0.2244 & 0.2124 & 0.1662 & 0.2058 & 0.1914 \\ 0.2436 & 0.2051 & 0.1751 & 0.2028 & 0.1734 \\ 0.2276 & 0.2594 & 0.1466 & 0.2030 & 0.1634 \\ 0.1878 & 0.2506 & 0.2122 & 0.1951 & 0.1543 \\ 0.1314 & 0.1971 & 0.2580 & 0.2242 & 0.1894 \end{bmatrix}$$

(4) Expert decision-making moments are assembled. Use formula (63) to gather the decision matrices of five experts to obtain matrix R, as shown in Table 11.

Table 11 Assembly matrix R

	C1	C2	C3	C4	C5
y ₁	⟨[0.82,0.90],[0.13,0.23]⟩	⟨[0.75,0.85],[0.17,0.26]⟩	⟨[0.83,0.92],[0.1,0.19]⟩	⟨[0.65,0.76],[0.27,0.34]⟩	⟨[0.44,0.53],[0.48,0.57]⟩
y ₂	⟨[0.90,0.95],[0.10,0.20]⟩	⟨[0.76,0.84],[0.19,0.29]⟩	⟨[0.84,0.93],[0.1,0.16]⟩	⟨[0.66,0.75],[0.25,0.34]⟩	⟨[0.47,0.57],[0.45,0.54]⟩
y ₃	⟨[0.79,0.88],[0.13,0.23]⟩	⟨[0.62,0.72],[0.28,0.38]⟩	⟨[0.78,0.89],[0.16,0.25]⟩	⟨[0.63,0.73],[0.27,0.36]⟩	⟨[0.36,0.48],[0.53,0.65]⟩
y ₄	⟨[0.78,0.85],[0.24,0.32]⟩	⟨[0.54,0.65],[0.41,0.50]⟩	⟨[0.63,0.72],[0.44,0.51]⟩	⟨[0.52,0.60],[0.50,0.58]⟩	⟨[0.33,0.43],[0.60,0.69]⟩
y ₅	⟨[0.67,0.80],[0.32,0.41]⟩	⟨[0.45,0.54],[0.58,0.66]⟩	⟨[0.52,0.61],[0.54,0.61]⟩	⟨[0.40,0.47],[0.60,0.69]⟩	⟨[0.32,0.39],[0.59,0.69]⟩

(5) Calculate the attribute weight of the matrix R. Use the CRITIC method described in section 5.3 to calculate the attribute weights of the aggregation matrix R. Using formula (58) to calculate the attribute weight, the results are as follows:

$$\omega_1 = 0.04737, \quad \omega_2 = 0.28571, \quad \omega_3 = 0.26277, \quad \omega_4 = 0.30496, \quad \omega_5 = 0.09919$$

(6) Calculate the weighted average $Q_i^{(1)}$ of the alternatives. The weighted average $Q_i^{(1)}$ of the alternative is calculated by formula (57), and the results are as follows:

$$Q_{y_1}^{(1)} = \langle [0.566, 0.687], [0.199, 0.306] \rangle > Q_{y_2}^{(1)} = \langle [0.569, 0.687], [0.199, 0.306] \rangle$$

$$Q_{y_3}^{(1)} = \langle [0.551, 0.670], [0.203, 0.315] \rangle > Q_{y_4}^{(1)} = \langle [0.530, 0.642], [0.236, 0.354] \rangle$$

$$Q_{y_5}^{(1)} = \langle [0.516, 0.623], [0.267, 0.391] \rangle >$$

(7) Calculate the weighted average $Q_i^{(2)}$ of the alternative. The weighted average $Q_i^{(2)}$ of the alternative is calculated by formula (58), and the results are as follows:

$$Q_{y_1}^{(2)} = < [0.558, 0.680], [0.205, 0.312] > \quad Q_{y_2}^{(2)} = < [0.561, 0.679], [0.205, 0.312] >$$

$$Q_{y_3}^{(2)} = < [0.542, 0.661], [0.209, 0.320] > \quad Q_{y_4}^{(2)} = < [0.519, 0.631], [0.241, 0.358] >$$

$$Q_{y_5}^{(2)} = < [0.504, 0.609], [0.272, 0.396] >$$

(8) When $\lambda = 0.5$, the compromise value of $Q_i^{(1)}$ and $Q_i^{(2)}$ are calculated by formula (59), the result is as follows:

$$r_1 = < [0.5620, 0.6840], [0.2020, 0.3090] > \quad r_2 = < [0.5650, 0.6830], [0.2020, 0.3090] >$$

$$r_3 = < [0.5470, 0.6660], [0.2060, 0.3175] > \quad r_4 = < [0.5250, 0.6370], [0.2385, 0.3560] >$$

$$r_5 = < [0.5100, 0.6160], [0.2695, 0.3935] >$$

(9) Score of each alternative is calculated. The score of each alternative is calculated by formula (60), where the values of α and β are 0.5, the results are shown following.

$$S(r_1) = 0.87949 \quad S(r_2) = 0.87975 \quad S(r_3) = 0.87439 \quad S(r_4) = 0.86097 \quad S(r_5) = 0.84923$$

(10) Ranking and selection of alternatives.

According to the score result of the alternative, the ranking result is: $S(r_2) > S(r_1) > S(r_3) > S(r_4) > S(r_5)$, so the ranking of alternatives is: $y_2 > y_1 > y_3 > y_4 > y_5$, the y_2 is the optimal alternative, Therefore, the the future hypertension risk of the resident of is an orange warning, the resident has a high hypetension risk and needs to consult a doctor on time, in order to prevent hypertension in time. This result is consistent with the judgment of five hypertension experts, and so the proposed MAGDM is feasible and effective in this paper.

6.2 Parameter sensitivity analysis

In order to further verify the feasibility of the proposed MAGDM method in this paper, this subsection mainly studies the influence of the parameters for ranking of alternatives.

(1) When $p=2$, the q takes 2, 3, 4, and 5, $\alpha \geq 0.1$, and the ranking results of the alternative are as shown in Table 13. It can be seen from the Table 13 that the ranking result of the alternative does not change.

Table 13 Ranking of alternatives under different q values

q	Score sort	Alternative ranking	Result
2	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
3	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
4	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
5	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning

(2) When $p=2$, $q=2,3,3,5$, the $\alpha = 0.5$, and the ranking results of the alternative are as shown in Figure 2. From t Figure 2, it can be seen that when q is not the same, the ranking result of alternatives does not change.

Figure 2 Alternative ranking results under different q values

(3) When $q=3$, the p takes 1, 2, 3, 4, and 5 respectively, and the $\alpha \geq 0.1$, The score ranking and the ranking results of the alternatives are shown in Table 14. From the table, it can be seen that the ranking results of the alternatives do not change with the increase of p .

Table 14 The ranking of alternatives under different p values

p	Score sort	Alternative ranking	Result
1	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
2	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
3	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
4	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
5	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning

(4) When $q=3$, $p=1,2,3,4,5,6$, the $\alpha = 0.5$, the ranking results of the alternative are shown in Figure 3. From the Figure 3, it can be seen that when p is changed from 1 to 4, the ranking of the alternative does not change .

Figure 3 Alternative ranking results under different p values

(5) When $p=2,3,4,5$, and $q=2, 3, 4, 5$, $\alpha = 0.5$, the score results of the alternative are shown in Figure 4. From Figure 4, it can be seen that when p and q changes, the ranking of the alternatives is not change.

Figure 4 Sorting results of alternatives under different p and q

(6) When the $p=2, 3, 4, 5$, and the $q=2, 3, 4, 5$, and the $\alpha \geq 0.1$, the ranking result of alternative is shown in Table 15. It can be seen that when p and q changes, the ranking of the alternative remains unchanged.

Table 15 The ranking of alternatives under different p and q

Alternative ranking	$p=2$	$p=3$	$p=4$	$p=5$
$q=2$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$
$q=3$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$
$q=4$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$
$q=5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	$y_2 > y_1 > y_3 > y_4 > y_5$

6.3 Comparative analysis of operators

In order to further verify the effectiveness and feasibility of the proposed MAGDM in this paper, the IVq-RIVOFWHM[12], IVq-ROFWMSM[13], IVq-RIVOFFWA[12], IVq-RIVOFFWA[41], and IVq-RIVOFFGA[41] operator are compared with IVq-ROFYWA operator. And for operators, $q=3$. Especially, for q-RIVOFWHM, $x = 2$, for the IVq-ROFWMSM operator, $k = 2$, for IVq_RIVOFFWA and IVq_RIVOFFGA operators, $\theta = 2$. And comparing results of 5 operators are shown in Table 16.

Table 16 Comparison results under different operators

Operator	Score	ranking	result
IVq-RIVOFWHM ^[12]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
IVq-ROFWMSM ^[13]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
IVq-RIVOFWA ^[12]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
IVq-RIVOFFWA ^[41]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
IVq-RIVOFFGA ^[41]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
IVq-ROFYWA	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning

From Table 16, the ranking results of proposed IVq-ROFYWA operator is in the same as IVq-RIVOFWHM, IVq-ROFWMSM, IVq-RIVOFWA, IVq-RIVOFFWA, and IVq-RIVOFFGA operator. And the warning lever is orange, which is consistent with given by 5 hypertension experts, so the results verify that the proposed operator is effective and feasible.

6.4 Comparative analysis of decision-making methods

In order to further verify the effectiveness and feasibility of the proposed WASPAS in this article, we compare WASPAS with MOORA, COPRAS, MACBETH, MABAC, ARAS mehtod[13] [23][24][25][26]. The comparing results are shown in Table 17.

Table 17 Comparison results under different decision-making methods

Method	Score sort	ranking	Result
MOORA ^[23]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
COPRAS ^[24]	$Sr_5 > Sr_2 > Sr_1 > Sr_4 > Sr_3$	$y_2 > y_1 > y_3 > y_4 > y_5$	Green warning
MACBETH ^[25]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
MABAC ^[26]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
ARAS ^[13]	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning
WASPAS	$Sr_2 > Sr_1 > Sr_3 > Sr_4 > Sr_5$	$y_2 > y_1 > y_3 > y_4 > y_5$	Orange warning

According to the results in Table 17, it can be found that proposed WASPAS is different from the COPRAS which indicates that the warning level is green warning. The main reason is that the COPRAS method is suitable for solving the MADM problem in which benefit-type attributes and cost-type attributes exist at the same time, and the presented case only has benefit attributes so that the ranking results will be different. However, the ranking result of WASPAS is same as MOORA, MACBETH, MABAC, ARAS method that shows WASPAS is effective and feasible.

In order to verify the characteristics of the WASPAS method, this paper will use WSM and WPM to replace the WASPAS method in the proposed group decision making calculations. When $\lambda = 0.5$, compare and analyze their ranking results are shown in Figure 5.

Figure 5 Attribute weights under different models

From the results in Figure 5, it can be found that the results of WSM model and WPM model are consistent with WASPAS in this case. When $\lambda = 0.5$, the results of WASPAS always fluctuates between the results of WSM and the results of WPM, and the order of the alternative remains unchanged, when $\lambda > 0.5$, the results of WASPAS are biased toward WSM, and the WASPAS is more sensitive, when $\lambda < 0.5$, the result of WASPAS is biased toward WPM, the WASPAS is relatively stable. It further verifies the more accuracy of WASPAS than WSM and WPM.

7. Conclusion

The proposed MAGDM method integrates IVq-ROFYWA, IVq-ROFYWG operators, CRITIC, WASPAS method, which provides a new decision-making tool for people, and experts have more freedom when making fuzzy judgments of uncertainties. The integrated group decision-making method not only considers the uncertainties of different experts, but also provides much more freedom for expert under IVq-ROFS, and the proposed MAGDM also takes into account the impact of variability and relevance on indices. The main contribution are following: (1) A new distance measurement is proposed under IVq-ROFS. And the PIS, NIS, and CS also are extended. (2) A new score function is proposed under IVq-ROFS, which makes up for the limitation of Ref [36], [37], [38], and works very well. (3) the IVq-ROFYWA and IVq-ROFYWG operator are proposed under IVq-ROFS, and their properties are studied. (4) An integrated MAGDM method is proposed. the proposed MAGDM method combines IVq-ROFYWA, IVq-ROFYWG operator and CRITIC, and WASPAS method. Expert weight and attribute weight are unknown in our proposed MAGDM method, the expert weight of different alternatives can be derived by similarity, which has a good application when the experts have bias in evaluating different alternatives. And the attribute weight is derived by modified CRITIC method under IVq-ROFS. And the modified WASPAS method is used to rank alternatives under IVq-ROFS. So the proposed MAGDM only requires experts judgment matrices based on the

problem under IVq-ROFS, therefore, it greatly improves the efficiency of decision-making when the expert and the attribute weight are unknown. (5) The proposed GDM method is used to the early warning management of hypertension. The implementation results are is consistent with expert's prediction, which proves that the proposed GDM is effective. And Comparing and analysising results further proves that the MAGDM method is feasible and effective.

Although the proposed GDM method can solve high density data decision making problem, but it is only suitable for few experts decision-making environments. At the same time, the actual application requires a large amount of data verification and analysis. Therefore, the future studies will further integrate big data [44] and artificial intelligence to deal with complex intelligent decision-making problems with unknown weights [45].

Funding This work was supported in part by the Department of Shenzhen Local Science and Technology Development under Grant 2021Szvup052; Department of Education of Jiangxi Province under Grant GJJ202105; the National Natural Science Foundation of China (Nos.61862042, 61762062); the Science and Technology Innovation Platform Project of Jiangxi Province (No. 20181BCD40005); the Primary Research & Development Plan of Jiangxi Province (Nos. 20192BBE50075, 20192BEL50041, 20181ACE50033); the Jiangxi Province Natural Science Foundation of China (Nos. 20192BAB207019); the Graduate Innovation Fund Project of Jiangxi Province(Nos.YC2019-S100, YC2019-S048, YC2020-S028, YC2020-S092, YC2020-S083); the Practice Innovation Training Program of Jiangxi Province for College Students (Nos. 20190403041, 20190402125, 2020CX160). The innovation of this paper has been pre-print as arXiv in “A Integrating Interval-valued generalized orthogonal fuzzy group decision-making method and its application in hypertension management” by the authors of this paper.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed Consent Written informed consent was obtained from all the participants prior to the enrollment (or for the publication) of this study (or case report).

References

- [1] Bhattacharyya S, Valeriani D, Cinel C, Citi L, Poli R. Anytime collaborative brain-computer interfaces for enhancing perceptual group decision-making[J]. Sci Rep. 2021 Aug 20;11(1):17008.
- [2] Liguó Fei, Yuqiang Feng. Intuitionistic fuzzy decision-making in the framework of Dempster–Shafer structures[J]. International Journal of Intelligent Systems, 2021,36(10):5419-5448
- [3] Depeng Kong, Tianqing Chang, Jiali Pan, Na Hao, Kehu Xu, Lei Zhang, GuoZhen Yang. A decision variable-based combinatorial optimization approach for interval-valued intuitionistic fuzzy MAGDM[J]. Information Sciences,2019, 484:197-218
- [4] Akram, M., Adeel, A. TOPSIS Approach for MAGDM Based on Interval-Valued Hesitant Fuzzy N-Soft Environment[J]. Int. J. Fuzzy Syst. 2019, 21:993–1009.

- [5] Zadeh L.A. Fuzzy sets[J]. *Information & Control*, 1965, 8(3):338-353.
- [6] Atanassov K.T. Intuitionistic fuzzy sets[J]. *Fuzzy Sets and Systems*, 1986, 20 (1):87-96.
- [7] Kamal Kumar, Shyi-Ming Chen. Multiattribute decision making based on the improved intuitionistic fuzzy Einstein weighted averaging operator of intuitionistic fuzzy values[J]. *Information Sciences*. 2021, 571:206-224.
- [8] Feng Feng, Hamido Fujita, Muhammad Irfan Ali, Ronald R. Yager, Xiaoyan Liu. Another View on Generalized Intuitionistic Fuzzy Soft Sets and Related Multiattribute Decision Making Methods[J]. *IEEE Transactions on Fuzzy Systems*, 2019, 27(3):474-488
- [9] Yager R.R. Pythagorean membership grades in multicriteria decision making[J]. *IEEE Transactions on Fuzzy Systems*, 2014, 22(4):958-965.
- [10] Yager, R.R., Naif Alajlan. Approximate reasoning with generalized orthopair fuzzy sets[J]. *Information Fusion*, 2017, 38:65-73.
- [11] Joshi B. P. Singh A., et al., Interval valued q-rung orthopair fuzzy sets and their properties[J]. *J. Intell. Fuzzy Syst.*, 2018, 35(5):5225–5230.
- [12] Wang J, Gao H, Wei G, Wei Y. Methods for Multiple-Attribute Group Decision Making with q-Rung Interval-Valued Orthopair Fuzzy Information and Their Applications to the Selection of Green Suppliers[J]. *Symmetry*. 2019, 11(1):56.
- [13] Chuanxi Jin, Yan Ran, Genbao Zhang. Interval-valued q-rung orthopair fuzzy FMEA application to improve risk evaluation process of tool changing manipulator. *Applied Soft Computing Journal*, 2021, 104:107192
- [14] Liu L, Cao W, Shi B, Tang M. Large-Scale Green Supplier Selection Approach under a Q-Rung Interval-Valued Orthopair Fuzzy Environment[J]. *Processes*, 2019, 7(9):573.
- [15] Yager, R.R. Aggregation operators and fuzzy systems modeling[J]. *Fuzzy Sets Syst.* 1994, 67:129–145.
- [16] Xindong Peng, Zhigang Luo, A review of q-rung orthopair fuzzy information: bibliometrics and future directions[J], *Artificial Intelligence Review*,
- [17] Yang, W., Wang, C., Liu, Y. et al. Hesitant Pythagorean fuzzy interaction aggregation operators and their application in multiple attribute decision-making[J]. *Complex Intell. Syst.* 2019, 5:199–216.
- [18] Peide Liu, Yumei Wang. Multiple attribute decision making based on q-rung orthopair fuzzy generalized Maclaurin symmetric mean operators[J]. *Information Sciences*, 2020, 518:181-210
- [19] Ju Y, Luo C, Ma J, Gao H, Gonzalez EDS, Wang A. Some interval-valued q-rung orthopair weighted averaging operators and their applications to multiple-attribute decision making. *Int J Intell Syst.* 2019; 34(10):2584-2606
- [20] Liu, G.M. Specific Types of q-Rung Picture Fuzzy Yager Aggregation Operators for Decision-Making[J]. *International Journal of Computational Intelligence Systems*, 2020, 13(1):1072–1091
- [21] Akram, M., Peng, X. & Sattar, A. Multi-criteria Decision-Making Model Using Complex Pythagorean Fuzzy Yager Aggregation Operators[J]. *Arab J Sci Eng*, 2021, 46:1691–1717.
- [22] Alsalem, M.A., Alamoodi, A.H., Albahri, O.S. et al. Multi-criteria decision-making for coronavirus disease 2019 applications: a theoretical analysis review. *Artif Intell Rev*, 2022. <https://doi.org/10.1007/s10462-021-10124-x>
- [23] Brauers, W.K.; Zavadskas, E.K. The MOORA method and its application to privatization in a transition economy[J]. *Control Cybern.* 2006, 35:445–469.
- [24] Zavadskas EK, Kaklauskas A. The new method of multicriteria complex proportional assessment of projects[J]. *Technological and Economic Development of Economy*, 1994, 1(3):131–139
- [25] Tavares, R.S., Angulo-Meza, L., Rangel, L.A.D. et al. Interdisciplinary graduate programs: application of the

MACBETH multicriteria method for assessing their performances. *Ann Oper Res*, 2021. <https://doi.org/10.1007/s10479-021-04108-x>.

- [26] Pamucar D, Cirovic G. The selection of transport and handling resources in logistics centers using Multi-Attributive Border Approximation area Comparison (MABAC)[J]. *Expert Syst Appl* 2015;42(6):3016-3028
- [27] Zavadskas, E. K., Turskis, Z., Antucheviciene, J., & Zakarevicius, A. Optimization of weighted aggregated sum product assessment[J]. *Elektronika ir elektrotechnika*, 2012,122(6):3-6.
- [28] Katarzyna Rudnik, Grzegorz Bocewicz, Aneta Kucińska-Landwójtowicz, et al. Ordered fuzzy WASPAS method for selection of improvement projects[J]. *Expert Systems with Applications*, 2021,169:114471
- [29] Davoudabadi, R., Mousavi, S.M. Mohagheghi, V. A new last aggregation method of multi-attributes group decision making based on concepts of TODIM, WASPAS and TOPSIS under interval-valued intuitionistic fuzzy uncertainty[J]. *Knowl Inf Syst*, 2020,62:1371–1391.
- [30] Peng, X., Zhang, X. Luo, Z. Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation[J]. *Artif Intell Rev*, 2020,53:3813–3847.
- [31] Peng, X., Krishankumar, R., Ravichandran, K. S. A novel interval-valued fuzzy soft decision-making method based on CoCoSo and CRITIC for intelligent healthcare management evaluation.[J] *Soft Computing*, 2021, 25(6): 4213–4241.
- [32] Peng X, Liu L. Information measures for q-rung orthopair fuzzy sets[J]. *Int J Intell Syst*. 2019;34:1795-1834.
- [33] Z. Yue , Deriving decision maker's weights based on distance measure for interval-valued intuitionistic fuzzy group decision making[J]. *Expert Syst. Appl*. 2011,38:11665–11670.
- [34] Z. Yue, Approach to group decision making based on determining the weights of experts by using projection method[J]. *Appl. Math. Modell*. 2012,36 2900–2910.
- [35] Wei, AP, Li, DF., Lin, PP. et al. An information-based score function of interval-valued intuitionistic fuzzy sets and its application in multiattribute decision making[J]. *Soft Comput*, 2021,25:1913–1923.
- [36] S.H. Chen, Autocratic multiattribute group decision making for hotel location selection based on interval-valued intuitionistic fuzzy sets[J]. *Information Sciences*, 2018, 427:77-87.
- [37] Zhi-yong Bai. An Interval-Valued Intuitionistic Fuzzy TOPSIS Method Based on an Improved Score Function[J]. *The Scientific World Journal*[J]. 2013:1-6. <https://doi.org/10.1155/2013/879089>
- [38] Gong R, Ma LY, A new score function and accuracy function of interval-valued intuitionistic fuzzy number and its application[J]. *Syst Eng Theor Pract*, 2019,39(2):463–475
- [39] S-M. Chen, K-Y. Tsai, Multiattribute decision making based on new score function of interval-valued intuitionistic fuzzy values and normalized score matrices, *Information Sciences* 2021,575:714-731
- [40] Diakoulaki, D., Mavrotas, G., Papayannakis, L. Determining objective weights in multiple criteria problems: The critic method[J]. *Computers & Operations Research*,1995, 22(7):763–770.
- [41] Yager RR. On some new classes of implication operators and their role in approximate reasoning. *Information Sciences*, 2004, 167(1):193-216.
- [42] Thomas U, Claudio B, Fadi C, et al. International Society of Hypertension global hypertension practice guide lines[J]. *J Hypertens*, 2020, 75:1334-1357
- [43] Williams B, Mancia G, Spiering W, et al. 2018 ESC/ ESH Guidelines for the management of arterial hypertension: The Task Force for the management of arterial hypertension of the European Society of Cardiology (ESC) and the European Society of Hypertension (ESH)[J]. *J Hypertens*, 2018, 36(10):1953-2041.
- [44] Ming Tang, Huchang Liao, From conventional group decision making to large-scale group decision making:

What are the challenges and how to meet them in big data era? A state-of-the-art survey[J], Omega, 2021,102141

[45] Li-Xin Wang, Fast Training Algorithms for Deep Convolutional Fuzzy Systems With Application to Stock Index Prediction[J], IEEE Transactions on fuzzy systems, 2020, 28(7):1301-1314.

[46] Benitng Wan, Shufen Zhou, A integrating critic-waspas group decision making method under interval-valued q-rung orthogonal fuzzy enviroment, arXiv, 2022.1.