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M. Awais

COMSATS University Islamabad, Attock Campus

Marium Bibi

COMSATS University Islamabad, Attock Campus

Aamir Ali (✉ aamir_ali@cuiatk.edu.pk)

COMSATS University Islamabad, Attock Campus

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Numerical Treatment for MHD Axisymmetric Rotating Bodewadt Rheology under Ohmic Heating and Viscous Dissipation Effects

M. Awais^a, Marium Bibi^a and Aamir Ali^{a,*}

^aDepartment of Mathematics, COMSATS University Islamabad, Attock Campus, Kamra Road, Attock, 43600, Pakistan.

*Email: aamir_ali@cuiatk.edu.pk, aamir_ali@ciit-attock.edu.pk

Abstract: The present research manuscript is intended to scrutinize the impacts of heat transfer in a Bodewadt flow above a penetrable disk numerically. The repercussions of internal heat generation and absorption, viscous dissipation, magnetic field, joule heating accompanying with variable properties of the fluid are also inspected as well. Within the recent past, multifarious scientists have prophesied that physically compatible solution of heat energy equation in a Bodewadt flow exist only when the disk is permeable. The final problem contains the variable θ_e which estimates the rate of dependence of fluid viscosity on the temperature. We transmuted the governing equations of heat transfer (accompanied by variable properties) and motion of fluid in to self-similar non-dimensional differential equations by utilizing the Von-Karman variables. The major result of this research study is that the velocity & temperature of the fluid surrounding the disk are substantially altered by taking numerous values of parameter of variable viscosity θ_e . Command of NDSolve in Mathematica software on the basis of Adams Bashforth method is employed to obtain numerical statistics from suggested mathematical model. Numerical results for velocity constituents (F, G, H) and temperature curves (θ) for distinct emerging parameters are illustrated graphically. Additionally, the values of co-efficient of skin friction and nusselt number are calculated for different emerging parameters and their behavior are exposed physically with the assistance of tables and bar charts.

Keywords: Bodewadt flow; Magnetic field; Heat transfer; Joule heating; Variable viscosity;

Viscous dissipation; Wall suction; Internal heat generation/absorption; Similarity solution;

1. Introduction

Three-dimensional flow problems produced by virtue of a rotating viscous fluid are experienced in a number of engineering-relevant, product-designing, industrial and scientific products and

procedures for instance centrifuges, viscometers, jet engines, turbines, mixing in chemical chambers, vacuum cleaners, disk drives of computers and turbo-machinery etc. A classic problem in the field of fluid mechanics is the problem of fluid flow that is generated by a disk which possesses large diameter and is revolving about its origin under the influences of constant angular velocity. Many years ago, Von-Karman [1] composed and resolved this problem. Anyone doesn't visualize an accurate circular flow close to the disk in this fluid problem since there is not applied pressure gradient that supplied the radial component of acceleration inwards and, as a consequence, fluid scrolled outside. Von-Karman [1] who was capable to transform the system of Navier-Stokes's equation into a set of ordinary differential equations with the assistance of appropriate similarity transformations. Cochran [2] noted some imprecisions in the solution of Von-Karman's problem and then developed an asymptotic solution of that problem. Ackroyd [3] elongated the problem of Von-Karman by accounting the impacts of injection/suction and illustrated series solution that comprises of the exponentially decomposing expressions. A similar problem is originated when the fluid revolves under the influences of constant angular velocity over a motionless plane is second fascinating problem. On the contrary, the fluid particles close to the disk moves in the direction of axis of rotation since its circumferential constituent of velocity is decelerated under the similar pressure gradient. Tabassum and Mustafa [4] modeled numerically the heat transfer and partial slip flow of reiner-rivlin non-newtonian fluid generated by a revolving disk. A distinctive illustration made by researchers in this regard is given in references [5-10]. Contrary to Von-Karman [1] problem's, Bodewadt [11] demonstrated that problem in 1940 by constructing boundary layer conditions. In Bodewadt's flow, fluid particles around the disk under the impact of radial pressure gradient directed to proceed inside towards the rotational axis where fluid is in the condition of stiff-body rotation. Bodewadt flow is also monitored in a tea cup when rotation in the fluid is urged by continuously stirring and then left the flow of fluid for a little while. Bodewadt flow is additionally perceived further in many procedures such as hurricanes, food-processing industries, tornadoes, turbo-machinery and chemical mixing chambers etc. Sahoo et al. [12] modeled numerically the heat transfer and steady revolving flow of a Reiner-Rivlin fluid using shooting and finite difference method. Sahoo et al. [13] obtained numerical solution of viscous fluid for steady Bodewadt flow using finite difference and Keller-box method. Rafiq et al. [14] examined numerical assessment of nanofluid for Bodewadt slip flow above a convectively heated permeable disk. Muhammad et al. [15] formulated entropy production in three-dimensional

Bodewadt flow of nanofluid over an infinite stationary disk. Some recent attempts related to Bodewadt flow can be seen in Refs. [16-27].

Heat transfer is the field of thermal engineering which deals with the use, generation, conversion and exchange of thermal energy between physical systems (i.e: heat energy). Transfer of heat is categorized within various mechanisms such as radiation, convection and conduction. Transfer of heat has broad range of applications in climate engineering, architecture, industries of chemical process, greenhouse effect and heat transfer in the body of human beings. There is an immense impact of temperature gradients on characteristics of fluid. Particularly, there is a direct relationship between temperature and viscosity in gases while an inverse relationship between temperature and viscosity is in liquids. Variable characteristics of fluid flows are often perceived in applications associated with aerodynamics & aero-acoustics. Takhar et al. [28] examined the impacts of fluid properties that depends upon temperature on boundary layer flow above a continuous moving surface. They considered two cases, namely, variable fluid viscosity and constant fluid properties. Salahuddin et al. [29] examined the rotating behavior of 2nd-grade fluid with mass & heat transfer effects between two parallel plates. Ahmed et al. [30] numerically modeled the unsteady MHD flow and heat transfer with variable viscosity of nanofluid in carbon nanotubes over a shrinking porous surface. Rafiq et al. [31] formulated the impacts of heat transfer in fluid flow produced by a revolving permeable disk with variable characteristics of fluid. Several attempts in this regime has been made by investigators includes [32-40].

The heat which is produced due to the flow of an electric current through a conductor is called joule heating, also known as ohmic heating. Joule heating have a wide range of applications in our daily life such as glowing of filament of an incandescent light bulb, electrical fuses, hotplate of electrical tabletop etc. Viscous dissipation is an irreversible procedure in which the work is done by a fluid on adjoining layers owing to the action of shear forces is converted into heat. Impacts of viscous dissipation plays a crucial role in natural convection in numerous devices that are subjected to large variations of gravitational force or which operate at high speed. Jawad et al. [41] discussed the analysis of transfer of heat and entropy generation in MHD flow of carbon nanotubes with the impacts of viscous dissipation and thermal radiation. Afridi et al. [42] described the chaos analysis in stagnation point flow with MHD, ohmic heating & fluid friction impact. Entropy analysis in MHD nanofluid due to a heated stretching surface with viscous dissipation & radiation

are analyzed by Sithole et al. [43]. Zubair et al. [44] investigated the entropy generation optimization in three-dimensional MHD flow between two parallel rotating plates of casson nanofluid with viscous dissipation & ohmic heating effects. The latest investigations in this regime include studies [45-52].

Magnetohydrodynamics describe the study of motion of electrically conducting fluid (e.g. liquid metals and plasmas) in the presence of a magnetic field. The key hypothesis behind magnetohydrodynamic is that magnetic fields can generate current in a moving conductive fluid, which sequentially produce a force on the fluid and also alter the magnetic field itself. MHD have broad variety of applications in the field of Astrophysics, Geophysics, Engineering and Magnetic drug targeting. Shah et al. [53] analyzed the radiative effect radiation in magnetohydrodynamic casson nanofluid flow along with entropy generation and chemical reactions above a nonlinearly stretching sheet. Awais et al. [54] explained the mathematical modeling of prandtl MHD melted fluid flow towards an inclined cylindrical surface. Khan et al. [55] explained the entropy optimization rate in magnetohydrodynamic flow of viscous liquids with chemical reactions due to curved stretchable surface. Alreshidi et al. [56] discussed the thermophoresis effects and brownian movement in an incompressible and time independent flow of MHD nanofluid due to permeable revolving disk. Bibi et al. [57] presented theoretical analysis of magnetohydrodynamic carreau fluid above a revolving disk with the aid of von-karman transformations. Some remarkable applications regarding magnetohydrodynamics are cited here [58-62].

Inspired by the recent work of Rafiq et al. [63] and Andersson and Rahman [17], we strive to formulate influences of heat transfer in a Bodewadt flow along with the impacts of joule heating, wall suction, internal heat generation and absorption, magnetic field and viscous dissipation when properties of fluids that are dependent on the temperature are considered. Our mathematical paradigm includes the influences of thermal conductivity and viscosity that are dependent on temperature of the fluid. Transformed governing mathematical model of equations will be examined numerically by implementing command of NDSolve in Mathematica software on the basis of Adams Bashforth method. Consequences of developing parameters will be described by plotting graphs, bar charts and displaying outcomes in the tables.

2. Mathematical formulation of problem:

Assume that a viscous fluid flow having variable characteristics of fluid above a penetrable disk

as depicted in Fig. 1 across origin. The disk is rotating around vertical axis with uniform angular velocity ω . The components of velocity (V_r, V_ϕ, V_z) are taken in the direction of cylindrical coordinates (r, ϕ, z) . The temperature of disk is supposed constant on T_w while the temperature of exterior flow is considered as T_∞ . A uniform magnetic field of strength B_0 is functional along the z -direction while the induced magnetic field is presumed to be negligible.

The equation of motion, Navier- Stoke's equation and thermal energy equation with internal heat generation and absorption, viscous dissipation & ohmic heating takes the form as mentioned below:

$\frac{\partial(rV_r)}{\partial r} + \frac{\partial(rV_z)}{\partial z} = 0,$	(1)
$\rho \left(V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\phi^2}{r} \right) + \sigma B_0^2 V_r = -\frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\phi\phi}}{r},$	(2)
$\rho \left(V_r \frac{\partial V_\phi}{\partial r} + V_z \frac{\partial V_\phi}{\partial z} + \frac{V_r V_\phi}{r} \right) + \sigma B_0^2 V_\phi = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\phi}) + \frac{\partial}{\partial z} \tau_{z\phi} + \frac{\tau_{r\phi} - \tau_{\phi r}}{r},$	(3)
$\rho \left(V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial}{\partial z} \tau_{zz}$	(4)

$\rho c_p \left(V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z} \right) = -\nabla \cdot [-k(T) \cdot \nabla T] + \sigma B_0^2 (V_r^2 + V_\phi^2) + Q^* (T - T_\infty)$ $+ \mu \left[2 \left\{ \left(\frac{\partial V_r}{\partial r} \right)^2 + \left(\frac{V_r}{r} \right)^2 + \left(\frac{\partial V_z}{\partial z} \right)^2 \right\} + \left(\frac{\partial V_\phi}{\partial z} \right)^2 + \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right)^2 + \left(r \frac{\partial}{\partial r} \left(\frac{V_\phi}{r} \right) \right)^2 \right],$	(5)
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Here (V_r , V_ϕ and V_z) in the above equations represents the constituents of fluid velocity in the radial, tangential and axial directions, (ρ) indicates the fluid density, (C_p) indicates the specific heat capacity, (B_0) represents the magnetic field intensity, $k(T)$ stands for thermal conductivity, (μ) indicates the dynamic viscosity, (T) denotes the temperature of fluid, (σ) represents the electrical conductivity, (T_∞) represents the free stream temperature and (Q^*) stands for coefficient of heat generation/absorption. The stress tensor components are given as:

$\tau_{rr} = \mu \left(2 \frac{\partial V_r}{\partial r} \right), \quad \tau_{\phi\phi} = \mu \left(\frac{2V_r}{r} \right), \quad \tau_{zz} = \mu \left(2 \frac{\partial V_z}{\partial z} \right),$ $\tau_{\phi z} = \tau_{z\phi} = \mu \left(\frac{\partial V_\phi}{\partial z} \right), \quad \tau_{rz} = \tau_{zr} = \mu \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right), \quad \tau_{r\phi} = \tau_{\phi r} = \mu \left(\frac{\partial V_\phi}{\partial r} - \frac{V_\phi}{r} \right),$	(6)
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We presume that viscosity (μ) manifests an inverse linear dependency of temperature. The dimensionless variables are mentioned below in order to acquire the similarity solution:

$\mu = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)},$	(7)
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Here (μ_∞) represents ambient viscosity of the fluid and γ denotes a constant. When we increase the temperature, the viscosity of the liquids decreases ($\gamma < 0$) whereas the viscosity of air and other gases increases ($\gamma > 0$). Equation (7) may also be rewritten in the form as given below:

$\frac{1}{\mu} = \alpha (T - T_e) \text{ where } T_e = T_\infty - \frac{1}{\gamma} \text{ and } \alpha = \frac{\gamma}{\mu_\infty}.$	
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We presumed thermal conductivity that is dependent on the temperature is of the shape

$k(T) = k_\infty \left(1 + \frac{\varepsilon(T - T_\infty)}{(T_w - T_\infty)} \right)$, in which K_∞ represents thermal conductivity at ambient. The suitable

boundary conditions for the considered flow model are defined as:

$\begin{aligned} \text{At } z = 0: V_r = 0, V_\phi = 0, V_z = -V_0, T = T_w, \\ \text{as } z \rightarrow \infty: V_r \rightarrow 0, V_\phi \rightarrow r\omega, T \rightarrow T_\infty. \end{aligned}$	(8)
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Note that the centrifugal force is balanced by the radial pressure gradient in the exterior flow.

Mathematically, we can write it as:

$\frac{\partial p}{\partial r} = \rho r \omega^2$	(9)
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2.1: Similarity Transformation:

Depending upon the scale of length $\sqrt{v_\infty/\omega}$, we write the similarity variable (too called dimensionless distance) as $\zeta = z \left(\frac{v_\infty}{\omega} \right)^{-\frac{1}{2}}$. As the scale of velocity is $\sqrt{v_\infty \omega}$ and scale of time is ω^{-1} , we introduce the dimensionless constituents of velocity (F, G, H) & profile of temperature θ without dimensions as follows:

$F(\zeta) = \frac{V_r}{r\omega}, G(\zeta) = \frac{V_\phi}{r\omega}, H(\zeta) = \frac{V_z}{\sqrt{v_\infty \omega}}, \theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}$	(10)
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By applying boundary layer approximations and then using formulas (10) in equations (1-5); we acquire:

$2F + H' = 0,$	(11)
$F'' - \frac{1}{\theta - \theta_e} F' \theta' + \frac{\theta - \theta_e}{\theta_e} M F + \frac{\theta - \theta_e}{\theta_e} (F^2 + H F' - G^2 + 1) = 0,$	(12)
$G'' - \frac{1}{\theta - \theta_e} G' \theta' + \frac{\theta - \theta_e}{\theta_e} M G + \frac{\theta - \theta_e}{\theta_e} (2FG + H G') = 0,$	(13)
$H \theta' - \left(\frac{1}{\text{Pr}} \right) [(1 + \varepsilon \theta) \theta'' + \varepsilon \theta'^2] - M E c (F^2 + G^2) - Q \theta + \left(\frac{\theta_e}{\theta - \theta_e} \right) E c [F'^2 + G'^2] = 0,$	(14)

and the boundary conditions are transformed into:

$\begin{aligned} H(\zeta) = -A, F(\zeta) = 0, G(\zeta) = 0, \theta(\zeta) = 1, \text{ as } \zeta \rightarrow 0 \\ F(\zeta) \rightarrow 0, G(\zeta) \rightarrow 1, \theta(\zeta) \rightarrow 0 \text{ as } \zeta \rightarrow \infty. \end{aligned}$	(15)
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In the above mentioned expressions (11-15), $A = \frac{V_0}{\sqrt{v_\infty \omega}}$ denotes the wall suction parameter,

$Pr = \frac{\mu_\infty c_p}{k_\infty}$ denotes the value of Prandtl number at the ambient, $Ec = \frac{r^2 \omega^2}{c_p (T_w - T_\infty)}$ is the Eckert

number, $M = \frac{\sigma B_0^2}{\rho \omega}$ denotes the magnetic field parameter, $\theta_e = \frac{T_e - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)}$ indicates the

dimensional constant and $Q = \frac{Q^*}{\rho c_p \omega}$ represents the parameter of heat generation/absorption.

2.2: Nusselt number and Skin friction component:

We define skin friction parameter as given below:

$C_f = \frac{\sqrt{\tau_r^2 + \tau_\phi^2}}{\rho(r\omega)^2}$	(16)
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Where τ_ϕ denotes the circumferential wall stress and τ_r represents radial wall stress that can be evaluated by utilizing the Newtonian formulas as:-

$\tau_r = \mu \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right)_{z=0} = r\omega \frac{\mu_\infty}{\gamma(T_w - T_\infty)\theta(0) + 1} \sqrt{\frac{\omega}{v_\infty}} F'(0)$	(17)
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$\tau_\phi = \mu \left(\frac{\partial V_\phi}{\partial z} + \frac{\partial V_z}{\partial r} \right)_{z=0} = r\omega \frac{\mu_\infty}{\gamma(T_w - T_\infty)\theta(0) + 1} \sqrt{\frac{\omega}{v_\infty}} G'(0)$	(18)
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By utilizing expressions (7 and 10) in expressions (16, 17 & 18), we obtain:

$C_f = Re_r^{-\frac{1}{2}} \frac{\theta_e}{\theta_e - \theta} \sqrt{F'(0)^2 + G'(0)^2}.$	(19)
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Here $Re_r = \frac{r\omega^2}{v_\infty}$ indicates the rotational Reynolds number. Additionally, we define the local

nusselt number as follows:

$Nu = \frac{Lq_w}{k(T_w - T_\infty)}$	(20)
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in which $L = \sqrt{\frac{v_\infty}{\omega}}$ denotes the length scale & $q_w = -k \left(\frac{\partial T}{\partial z} \right)_{z=0}$ represents the wall heat flux of the

current research problem. By using formulas (10) in expression (20), we get:

$Nu = -\theta'(0).$	(21)
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3. Numerical consequences and discussion:

In this portion, heat transfer influences in a Bodewadt flow with joule heating, viscous dissipation, magnetic field and internal heat generation and absorption that are evaluated by taking variable properties of fluid are discussed. Using Adams Bashforth method, numerical computations are obtained for velocity constituents & temperature profile for distinct values of the concerned parameters in the range $0.2 \leq Pr \leq 5.0$, $1.7 \leq A \leq 6.7$, $0.1 \leq M \leq 1.2$, $-100 \leq \theta_e \leq 100$, $-0.9 \leq Q \leq 3.6$, $0 \leq Ec \leq 1.2$, $0 \leq \varepsilon \leq 4.1$. The graphical illustration of velocity constituents (F, G, H) and temperature profile (θ) for a variety of suction strength parameter are illustrated in Figs. 2a-d. The impact of suction leads to betterment in the frigidty rate of wall at a vast scale that is beneficial in numerous technical processes. The direction of the axial flow is directed upward in the absenteeism of wall suction, no matter how much greater the quantity of roughness parameter may be. The direction of inward flow of radial velocity F as portrayed in Fig. 2a is diminished monotonically with an increment in the value of wall suction parameter A and this is escorted with a reduction of axial flow H as shown in Fig. 2c. However, the radial inward flow generated by the pressure gradient decelerates with an enhancement in the value of wall suction parameter A. The circumferential constituent of velocity G remains constant away from the disk whereas it is decreased significantly close to the disk due to the viscous drag as illustrated in Fig. 2b.

The axially independent radial pressure gradient compels the particles of the fluid close to the disk to move in the direction of rotational axis. For that reason, the direction of the axial flow in the Bodewadt flow is directed upwards as seen in Fig. 2c. The axially upward flow results into the non-physical solution of heat energy equation. Fig. 2c illustrates that axial component of velocity becomes constant when we applied an appropriate value of suction velocity. When we take $A=1$, the axial component of velocity H turns out to be negative revealing vertically downward locomotion of the fluid. All the three constituents of velocity manifest oscillations whose amplitude shrinks when axial distance increases. When $A \geq 1$, the direction of the axial flow is inverted such that the fluid instead moves towards the solid surface. Depth of heat penetration also decreases when suction velocity increases. The results of Fig. 2c depicts that when larger A is accounted than greater quantity of (cold) fluid is shifted in the direction of the disk. As a result, layer of thermal boundary is suppressed with an increment in the velocity of wall suction and boosts transfer of heat from the disk as portrayed in Fig. 2d. Lastly, the axially upward flow is a consequence of an unphysical solution to the problem of advection-diffusion. To overcome this physical incompatibility, we can reverse the direction of the axial flow through the application of sufficiently high suction velocity. Therefore, it can be stated that the effect of damping arising as a result of wall suction can overcome the absolute or convective type instabilities in bodewadt flow known from the published literature.

The variations of prandtl number (Pr) on dimensionless temperature profiles θ in two cases ($A = 5$ & $A = 8$) are depicted in Figs. 3a and 3b. It is suggested from the comparison of Figs. 3a and 3b that when velocity of wall suction increases than the deepness of heat penetration is much suppressed. It can be noted instantly that an enhancement in the values of prandtl number rises the temperature of the fluid in the vicinity of the disk. The graphs demonstrate that the thickness of layer of thermal boundary and fluid temperature decreases with an increment in the prandtl number on a fixed value of ζ . The reason behind this is that prandtl number (Pr) is termed as fraction of momentum diffusion to thermal diffusion. When thermal diffusivity reduces, automatically the value of prandtl number (Pr) increases and fluid temperature decreases. In other words, temperature and prandtl number have an inverse relationship. Prandtl number controls the relative thickening of layer of thermal boundary and momentum boundary layer. However, the quantity of $\theta'(0)$ increases considerably (in absolute sense) signaling a betterment in the rate of heat transition from the disk.

The influences of magnetic interaction parameter (M) on the velocity constituents (F, G, H) in (r, ϕ, z) directions and temperature curves (θ) can be visualized in Figs. 4a-d. The radial, circumferential and axial components of velocity are decreasing gradually with intensification of strength of magnetic field (M) as evinced in Figs. 4a, 4b & 4c. The physics behind this is that an increase in the magnetic parameter (M) has the impact of damping the velocity profiles of fluid. This is because the direction of flow that is normal to the transverse magnetic field will result in a

resistive force named as Lorentz force that is identical to the drag force which resist the fluid flow and as a consequence reduces the radial (F), axial (H) and tangential (G) velocities. Behavior of temperature curves (θ) for non- identical values of magnetic field parameter M is illustrated in Fig. 4d. Contrary to the profiles of velocity, temperature (θ) enhances when we increase the value of M. This is due to the fact that bigger drag co-efficient on the surface owing to sturdy magnetic field offers resistance to the particles of fluid. As a consequence produces heat owing to which temperature (θ) enhances.

Fig. 5a-d illustrates the change in velocity constituents (F, G, H) & temperature curve (θ) by changing the value of parameter ($\theta_e > 0$). In these figures, the value of prandtl number is kept persistent at $Pr = 1$ that is an adequate selection for air and numerous more gases. The value of

variable viscosity parameter (θ_e) elasticities the significance of the quantity $1/\gamma$ which is in direct proportion to the rate of dependence of viscosity on temperature of the fluid. When ($\theta_e \rightarrow \infty$), the viscosity of the fluid becomes equivalent to the ambient viscosity whereas the viscosity of fluid becomes a sturdy function of temperature as ($\theta_e \rightarrow 0$) when θ_e increases. Fig. 5c depicts that the axial velocity expedites in the downward direction as θ_e increases. As a consequence, this effect causes a reduction in the severity of the radial flow as illustrated in Fig. 5a. The value of entrainment velocity $H(\infty)$ also increases indicating a growth in the fluid volume that is sucked in the direction of the disk. The circumferential component of velocity G accelerated in the upward direction when the value of θ_e increases as shown in Fig. 5b. As a result, the depth of heat penetration is (slightly) diminished and rate of heat transmission is augmented when θ_e is increased as illustrated in Fig. 5d.

(c)	(d)
Fig. 5 (a-d): Components of velocity (F, G, H) & temperature profile (θ) versus variable viscosity parameter $\theta_e > 0$.	

In Figs. 6a-d, we calculate velocity constituents and temperature curves for various values of variable viscosity parameter θ_e when ($\theta_e < 0$). In such situation, the viscosity of the fluid has an inverse relation with temperature. Different to the outcomes of Fig. 5c, the axial component of velocity decreases when we increase the value of θ_e that in results to the thickening of layer of thermal boundary as illustrated in Fig. 6d. Fig. 6b reveals that the tangential velocity accelerates in the upward direction when the value of θ_e increases.

The influences of heat generation and absorption parameter (Q) and Eckert number (Ec) are demonstrated in Figs. 7a-b. In Fig. 7a, negative values of parameter (Q) are representing the heat absorption ($Q < 0$) and positive values are corresponding to heat generation ($Q > 0$). It is obvious from the Fig. 7a that an enhancement in the heat generation produces an increase in the thickness of layer of thermal boundary whereas an increase in the heat absorption yields a decrease in the thickness of layer of thermal boundary. An increment in the absorption of heat implies that the heat is withdrawn by some means from the system. That is the reason that why an enhancement in the parameter of heat absorption results a reduction in the thickness of layer of thermal boundary and an increment in the rate of heat transfer. Contrary to this, an increment in the positive values manifest strong heat generation that enhances the fluid temperature in accordance with the definition of heat generation. Fig. 7b portrays the influence of Eckert number on the temperature profile. An increase in the temperature is observed for greater values of viscous dissipation parameter (Ec). An Eckert number (Ec) is utilize to calculate the loss of energy during configuration of flow. It tells us the relationship between enthalpy difference and kinetic energy of liquid particles. For an augmentation in the values of Eckert number from 0 to 1.2, the mechanical energy of the fluid is transformed into thermal energy because of internal friction of molecules. Hence the temperature of the liquid increases.

Fig. 8a-d illustrates the change in velocity constituents and temperature curves for distinct values of parameter of variable thermal conductivity ($\mathcal{E}$). Fig. 8c shows that volumetric flow rate and axial velocity are decreasing functions of thermal conductivity parameter ($\mathcal{E}$). Such decrease in the axial flow intensity is indemnified by an increment in the inward radial velocity near the disk as illustrated in Fig. 8a. A cross over in the curves of velocity constituent G shown in Fig. 8b revealing that azimuthal velocity (G) increases away from the disk and decreases near the disk when parameter ($\mathcal{E}$) increases. Fig. 8d shows that temperature profiles (θ) becomes thicker when we change the values of parameter ($\mathcal{E}$). It is because of this reason that thermal conductivity is in direct proportion with the parameter ($\mathcal{E}$). Hence an increment in the values of parameter ($\mathcal{E}$) enhances the thermal condition of fluid that in result thickens the layer of thermal boundary.

Results of skin friction parameter and Nusselt number for distinct values of emerging parameters are presented in Tables 1 & 2. Further, bar charts displayed the tabular data of skin friction parameter and nusselt number for different emerging parameters in Figs. (9-17). Table 1 demonstrate the impact of A, ϵ, Q, Ec, M and θ_e against skin friction parameter. Table 1. shows that value of skin friction parameter decreases when we increase the value of A and θ_e whereas skin friction parameter increases when we increase the value of ϵ, Q, Ec and M . Table

2 highlighted the impact of A , ε , Q , Ec , M and θ_e against nusselt number. It is noticed that an increment in magnitude of A decays nusselt number while a direct trend is noticed for ε , Q , Ec , M and θ_e .

Fig. 9: Nu versus θ_e .

Fig. 10: Nu versus Ec .

Fig. 11: Nu versus Q .

Fig. 12: $Re_r^{1/2} C_f$ versus θ_e .

Fig. 13: $Re_r^{1/2} C_f$ versus M .

Fig. 14: $Re_r^{1/2} C_f$ versus Ec .

Fig. 15: $Re_r^{1/2} C_f$ versus A.

Fig. 16: Nu versus A.

Fig. 17: Nu versus M.

Table 1: Numerical values of Skin friction coefficient $Re_r^{1/2} C_f$ against several physical quantities.

A	Pr	ε	Q	Ec	M	θ_e	$Re_r^{1/2} C_f$ [0.5]	$Re_r^{1/2} C_f$ [0.6]	$Re_r^{1/2} C_f$ [0.7]
1.7	3.0	1.0	0.2	0.3	0.1	3.0	0.99618	0.84464	0.71454
2.2	3.0	1.0	0.2	0.3	0.1	3.0	0.92275	0.74344	0.59746
2.9	3.0	1.0	0.2	0.3	0.1	3.0	0.81986	0.61626	0.46271
6.0	1.0	0.2	0.4	0.5	0.1	2.5	0.48244	0.27581	0.15946
6.0	1.0	0.8	0.4	0.5	0.1	2.5	0.53268	0.30697	0.17785
6.0	1.0	1.6	0.4	0.5	0.1	2.5	0.59541	0.34909	0.20411
5.0	5.0	1.0	0.1	1.0	0.2	2.0	0.58866	0.37622	0.24477
5.0	5.0	1.0	1.3	1.0	0.2	2.0	0.62915	0.40536	0.26512
5.0	5.0	1.0	2.3	1.0	0.2	2.0	0.68613	0.44681	0.29429
5.0	2.0	1.0	0.4	0.1	0.2	5.0	0.46029	0.29011	0.18760
5.0	2.0	1.0	0.4	0.3	0.2	5.0	0.46838	0.29574	0.19141
5.0	2.0	1.0	0.4	0.6	0.2	5.0	0.48079	0.30442	0.19730

5.0	5.0	1.0	0.4	1.5	0.1	4.0	0.50815	0.31626	0.19840
5.0	5.0	1.0	0.4	1.5	0.2	4.0	0.51839	0.33009	0.21341
5.0	5.0	1.0	0.4	1.5	0.3	4.0	0.52582	0.34174	0.22689
3.0	1.0	1.0	0.4	0.5	0.1	2.0	1.16797	0.93249	0.73843
3.0	1.0	1.0	0.4	0.5	0.1	2.5	1.03534	0.81384	0.63608
3.0	1.0	1.0	0.4	0.5	0.1	3.5	0.90403	0.69860	0.53831

Table 2: Numerical values of Nusselt number Nu against several physical quantities.

A	Pr	ε	Q	Ec	M	θ_e	Nu [0.5]	Nu [0.6]	Nu [0.7]
1.7	3.0	1.0	0.2	0.3	0.1	3.0	0.74284	0.61841	0.50832
2.2	3.0	1.0	0.2	0.3	0.1	3.0	0.71498	0.49745	0.33924
2.9	3.0	1.0	0.2	0.3	0.1	3.0	0.46646	0.25852	0.14344
6.0	1.0	0.2	0.4	0.5	0.1	2.5	0.47184	0.27443	0.15924
6.0	1.0	0.8	0.4	0.5	0.1	2.5	0.70025	0.43585	0.26279
6.0	1.0	1.6	0.4	0.5	0.1	2.5	0.95422	0.68563	0.46178
5.0	5.0	1.0	0.1	1.0	0.2	2.0	0.14101	0.07617	0.05167
5.0	5.0	1.0	1.3	1.0	0.2	2.0	0.19661	0.11844	0.08708
5.0	5.0	1.0	2.3	1.0	0.2	2.0	0.28087	0.18274	0.14002
5.0	2.0	1.0	0.4	0.1	0.2	5.0	0.21264	0.08673	0.03639
5.0	2.0	1.0	0.4	0.3	0.2	5.0	0.26557	0.11808	0.05567
5.0	2.0	1.0	0.4	0.6	0.2	5.0	0.34767	0.16829	0.08680
5.0	5.0	1.0	0.4	1.5	0.1	4.0	0.17168	0.08626	0.05491
5.0	5.0	1.0	0.4	1.5	0.2	4.0	0.20713	0.12054	0.08804
5.0	5.0	1.0	0.4	1.5	0.3	4.0	0.23578	0.14982	0.11678
3.0	1.0	1.0	0.4	0.5	0.1	2.0	0.80021	0.73312	0.65965
3.0	1.0	1.0	0.4	0.5	0.1	2.5	0.80771	0.73534	0.65765
3.0	1.0	1.0	0.4	0.5	0.1	3.5	0.81378	0.73639	0.65488

4. Concluding remarks:

Impacts of heat transfer in a Bodewadt flow along with the consequences of viscous dissipation, heat generation/absorption, and magnetic field, joule heating and variable properties of the fluid are described numerically. The consequence of the current problem demonstrated that appropriate amount of wall suction is essential to restore well matched solutions of thermal energy equation physically. The key points of the current effort are appended as follows:

- Impact of wall suction has propensity to increase the heat transfer rate from the solid surface. Furthermore, charge of heat transfer seems to grow when viscosity approaches a constant value.
- Boundary layer is substantially thinned in Bodewadt's flow owing to the insertion of wall suction. All constituents of velocity & temperature are diminishing functions of variable viscosity parameter θ_e . Moreover, when higher θ_e is employed than the transformation in solution profiles with an increment in suction decreases.
- When parameter of variable thermal conductivity ε becomes greater, than deterioration in rate of heat transfer is observed.
- The magnetic field (M) shows the opposite impact on velocity profile which means the velocity of the fluid diminishes with an increment in the value of M. Reason is that the Lorentz forces are against the fluid flow. The temperature profiles improves with an increment in magnetic field.
- Opposite behavior of heat generation/absorption effects is observed over profile of temperature.

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