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Monitoring Non-Parametric Profiles using Adaptive EWMA Control Chart

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ABSTRACT

To monitor the quality of a process in statistical process control (SPC), considering a functional relationship between a dependent variable and one or more independent variables which is denoted as profile monitoring is becoming an increasingly common approach. Most of the studies in the SPC literature have dealt with cases in which the functional relationship has the same form in the in-control (IC) and out-of-control (OC) situation or considered parametric approaches. Non-parametric profiles, which have different relationship in OC conditions is also very common. This paper designs a novel control chart to monitor not only the regression parameters but also the variation of the profile in phase II applications, using an adaptive approach. Adaptive control charts adjust the final statistic with regard to information of previous samples and the proposed method considers the relative distance of the chart statistic to the control limits as a tendency index and provides some outcomes about the process condition. The results of Monte Carlo simulations show the superiority of the proposed monitoring scheme in comparison with the common non-parametric control chart.

Keywords: Adaptive methods, Control chart, Profile monitoring, Non-parametric profiles, Statistical Process Control (SPC).

1. Introduction

1.1 Background

Employing statistical process control (SPC) scheme for monitoring industrial processes has been widely developed throughout the past five decades. The main aim of this approach is to improve the final quality of the products by reducing unfavorable conditions. By controlling the process of manufacturing a product, SPC detects internal system problems and inconsistencies and provides some solutions to improve the production process with the aim of some major tools involving Pareto chart, control chart, cause-and-effect diagram, scatter diagram, check sheet, stratification and histogram [1, 2].

In monitoring quality characteristics in industrial processes, control charts have been considered as an acceptable and appropriate tool since the 1920s, in both phase I and II processes. The aim of phase I is to identify a suitable estimation of process parameters and on the other hand, one tries to conduct on-line monitoring of the process in phase II. So, the major object of the second phase is related to the process's changes from IC (in-control) to OC (out-of-control). To evaluate a control chart in phase II, the well-known criteria i.e., ARL (average run length) is usually employed. It measures the average number of generated statistics to reach an OC or IC signal (hereafter denoted with ARL_I (ARL_O), respectively) [3]. More details about phase II analysis of SPC could be found in Zou, Zhang⁴ and Mahmoud, Saad⁵.

To develop a control chart in SPC, two major approaches could be utilized. Using of the univariate or multivariate distribution of the quality characteristics is the first one while, as a secondary and state-of-the-art method, the relationship between two or more quality characteristics as a regression model between the dependent and independent variables could be representative of the process condition. Profile monitoring is usually referred to Monitoring of such a relationship in which investigation the validation of it over time is the main aim [6].

It could be observed a wide range of applications for the profile monitoring in industrial process; for example, wood composites [7], health-care monitoring [8], remote laser welding [9], system calibration [10], semiconductor products [11], energy management [12], pharmaceutical sector [13], chemical gas sensors [14], 3D printing [15], compressor discharge temperature [16] and so forth. Different profile types have been considered as the IC model of the mentioned applications and it could be demonstrated that linear [17-21], non-linear [22, 23], logistic [24-26], Poisson [27, 28], circular and cylindrical [29], mixture profiles [30], multivariate [31] and even a complex relationship [32] are reputable in the SPC field. Because the idea of this paper has been extended for phase II monitoring of non-parametric profiles; hence, a short literature

review of these types of control charts only has been provided here. More details could be found in Maleki, Amiri³³.

1.2 Literature review of non-parametric profiles

Questing the literature revealed that most of the existing profiles monitoring control charts are designed in the same way as the quality characteristics' schemes have been designed; i.e., based on some fundamental assumptions or parametric schemes, the aim of which is whether these conclusions are established or not. As a common assumption in profile monitoring [10, 11, 17, 19, 34], parametric schemes constrain the OC form of the profiles to follow the IC formulation while it is not always a reasonable assumption because the OC model may not be easily determined, especially in complicated process, such as non-linear models and it is often invalid in practical applications. Beside it may increase the values of ARL_I and non-compatible products.

To overcome these problems, several non-parametric control charts have been extended for monitoring quality characteristics. As a general approach in most of the developed approaches, a distribution-independent (*distribution-free*) statistic such as Mann-Whitney statistic is defined and the performance has been evaluated in a robust condition against distribution by considering a few assumptions to it [35]. Six different categories of non-parametric statistics of quality characteristics have been introduced in Hou and Yu³⁶ and interested readers are referred to them. Due to main aim of this paper which is to monitor the profiles that have different IC and OC shape, the other literature of non-parametric method in monitoring quality characteristics are neglected and this type of profiles which is called non-parametric profiles hereafter is thus chosen for the focus of the literature review.

In spite of the fact that there are several non-parametric control charts for monitoring quality characteristics, few studies can be found for non-parametric profile monitoring, especially in phase II applications. Following paragraphs provide a comprehensive review of the related literature in which the most focuses lie in the researches with different IC and OC models. Hereafter, similar to the main focus of this study, the profiles that have different IC and OC shapes are denoted as non-parametric profiles.

As the fundamental research, Williams, Woodall³⁷ extended five metrics in monitoring non-linear profiles in phase I, based on measuring the deviations from the IC situation. Although these methods created a parametric control chart, they can be also considered as non-parametric as allowing the user to not assign a prespecified distribution of parameters. Considering this work, Zou, Tsung³⁸ developed a non-parametric multivariate EWMA (exponentially weighted moving average) statistic called NEWMA (non-parametric EWMA) for phase II profile monitoring. In this method, the coefficients and standard deviation parameters are firstly scaled and thereafter are taken into account as the previous samples to construct an MEWMA

statistic. To reach a distribution free statistic in error variance, a GLR (general likelihood ratio) test has been proposed by them. Because there were not any competitive methods, they designed two simple and naïve statistics based on the Zou, Tsung¹¹'s work in simple linear profiles. On the ground of simulations results of Zou, Tsung³⁸'s paper, it can be counted as the most fundamental control chart in phase II of non-parametric profile monitoring.

Extension of this work to other non-parametric conditions entailing random explanatory variables and profiles within auto-correlation is found in Qiu and Zou³⁹ and Qiu, Zou⁴⁰. Dealing with auto-correlation non-parametric profiles, three similar frameworks have been proposed in Hung, Tsai⁴¹, Chuang, Hung⁴² and Li, Pan⁴³. The main contribution of these papers included employing machine learning techniques, especially SVR (support vector regression), to IC model to fit a new IC regression curve. The proposed frameworks are very comprehensive and easy to implement; however, no performance comparisons were reported.

Designing EWMA3 statistics proposed by Kim, Mahmoud¹⁷ and considered as one of the most common approaches in simple linear profiles for non-parametric method has been carried out in Zhang, He⁴⁴. By consideration of changing the linear IC model to quadratic form, they compared the EWMA3 approaches, denoted with KMW by them, with some hypothesis tests including LRT (likelihood ratio test), F and T^2 . The same approach has been utilized in Shang, Wang²⁷ by considering the binomial and Poisson IC models. Similar to Zhang, He⁴⁴, Zi, Zou⁴⁵ developed the non-parametric control chart for monitoring linear profiles. In this approach, Wilcoxon rank estimator was employed instead of common least square approach and the simulation results approved the favorable properties of this approach.

Because non-parametric methods usually deal with complicated profiles, wavelet transformation has been employed to simplify the complexity of monitoring and enhancing the identification ability of OC sources [46, 47]. Also, to ease the computations of wavelets in non-linear profiles, PCA (principal component analysis) techniques has been suggested in Paynabar, Jin⁴⁸. Although this approach has not been established for the non-parametric profiles due to consideration of Gaussian distribution and some other relations, it can apply in the non-parametric wavelet-based methods. To increase the complexity of non-parametric profiles, Yang, Zou⁴⁹ considered profiles with time-varying sample sizes and random explanatory variables. To monitor this type of profiles, they designed a control chart with variable control limits in which the OC signal is triggered with the conditional probability that there is no alarm before the current time point. Jones, Abdel-Salam⁶ provided some useful guidelines for the implementation of the non-parametric control charts in profile monitoring.

1.3 Literature review of adaptive control charts in profile monitoring

In the previous literature of profile monitoring, there are several memory-type control charts especially for linear profiles. The idea of ranked set sampling was proposed by Riaz, Mahmood⁵⁰ and Huwang, Lin⁵¹ in which the samples are taken as a memory to be ranked by some criteria. Employing of this rank, a set of sorted samples generates very sensitive statistic to small shifts. Employing of Variable Sampling Interval (VSI) idea could be observed in profile monitoring [24, 52-55]. The main idea of VSI is that the samples' interval is not constant and could be identified regarding to the OC occurrence probability.

As a general term, these approaches could be referred as the adaptive control charts in which the sample size, statistic magnitude, interval between the sampling and other parameters are adjusted regarding to the estimation of the occurred shifts. Although several applications of adaptive control charts have been extended in monitoring quality characteristics [56-59], it was observed that little attention has been paid to this idea in profile monitoring. Also in profile monitoring, the existing works were extended for linear profiles [24, 50, 52, 55, 60, 61].

To the best of authors knowledge, except Mohammadzadeh, Yeganeh²⁴ and Jeong, Kim⁶¹, which has been developed based on the logistic and non-linear profiles, other adaptive methods in monitored linear profiles while there are not any approaches for complicated models such as non-linear and non-parametric. The main reason for this may be due to the weak performance and incompatibility with nonparametric limitations of adaptive approaches. Also, the complexity of parameter tuning could stir up some troubles in complicated profiles.

1.4 The aims and innovations of the study

In the present study, a novel adaptive approach is developed for non-parametric profile monitoring in which there are not any research based on the adaptive control charts in phase II SPC applications. The proposed approach, whose purpose is to improve the detection ability of NEWMA chart statistics, proposed by Zou, Tsung³⁸, in term of ARL criterion, is developed based on a generally different scheme as compared to older adaptive mentioned methods. It considers the relative distance of NEWMA chart statistics to the OC situations as the major criteria in OC detection and then, converts the obtained relative distance to an EWMA statistics. To reach the best performance in phase II, i.e., minimum ARL_1 in a constant ARL_0 , a heuristic designing procedure of the parameters are also illustrated in this paper. The performance evaluation is conducted through extensive non-parametric simulations in addition to a real example. In addition to the NEWMA approach as the base statistic, some other non-parametric and adaptive based competitors are defined to show the superiority of it of the proposed method. Although the proposed method has been

extended based on the NEMWA statistic, it is able to combine with other base statistics in phase II monitoring. The main contributions of this study could be summarized as follows:

- Proposing an adaptive control chart in profile monitoring.
- Reaching the optimum control chart parameters with a heuristic approach.
- Combination of the proposed adaptive approach with NEWMA statistic for monitoring non-parametric profiles.
- Computing the detection ability of the proposed method in term of ARL in comparison with some non-parametric and adaptive conventional methods.

The remainder of this paper is organized such as follows: the concept of NEWMA control chart is illustrated in Section 2. The proposed method of this study is clarified in Section 3. Section 4 provides the simulations for performance evaluation while some sensitivity analyses are reported in the next section. The real example is implemented in Section 6, and finally, the conclusions and recommendations for the future researches are presented in Section 7.

2. NEWMA Statistic for non-parametric profiles

Because we focus on the monitoring of profiles that could be well summarized by non-parametric regression model in phase II, the notations and formulations are extended based on the NEWMA scheme [38] as the most fundamental control chart in this field. Suppose that for the j^{th} ($j \geq 1$) random sample gathered over time, we have a set of observations entailing a n_j -variate response vector ($\mathbf{y}_j$) and $p \times n_j$ matrix of explanatory ($\mathbf{X}_j$) variables with sample size n_j and for simplicity, it is supposed that the sample sizes are the same in all the profiles, so n_j is replaced with n . Also the assumption of the same values in explanatory variable is very common in the related literature (i.e., using $\mathbf{X}$ instead of $\mathbf{X}_j$) so we have $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n)$. By these assumptions, the general IC model can be given by:

$$\begin{aligned} y_{ij} &= g_0(x_i) + \varepsilon_{ij}, \\ \varepsilon_{ij} &: N(0, \sigma_0^2), \\ i &= 1, 2, \dots, n, \\ j &= 1, 2, \dots, \end{aligned} \tag{1}$$

where g_0 is the IC regression function and σ_0^2 is the IC parameter obtained from phase I or previous information about the process, which are assumed to be known in phase II. Considering $\mathbf{G}_0 = (g_0(\mathbf{x}_1), g_0(\mathbf{x}_2), \dots, g_0(\mathbf{x}_n))$, to monitor the above model at the j^{th} sampling time with the vector of response variables $\mathbf{Y}_j = (y_{1j}, y_{2j}, \dots, y_{nj})$, Zou, Tsung³⁸ defined the following statistic:

$$\begin{aligned} \mathbf{Z}_j &= \frac{\mathbf{Y}_j - \mathbf{G}_0}{\sigma_0}, \\ \hat{\sigma}_j^2 &= \frac{1}{n} (\mathbf{Z}_j - \mathbf{WZ}_j)' (\mathbf{Z}_j - \mathbf{WZ}_j). \end{aligned} \quad (2)$$

In Equation (2), $\mathbf{W}$ is a symmetrical $n \times n$ smoothing matrix utilized in local linear estimator of response variables and obtained based on the explanatory variables, so it is fixed and constant in each new generated profile. To compute smoothing matrix, a symmetric probability density function and a bandwidth, denoted by h_E are defined and each element of $\mathbf{W}$ are calculated based on them. For brevity, the formulas are not reported here but a MATLAB function for computation of $\mathbf{W}$ is provided by the authors and available upon request. For reaching optimum value of bandwidth, a relationship based on the explanatory variables was introduced in Equation (10) of [Zou, Tsung](#)³⁸. In that Equation, a constant parameter, denoted by c , is defined and the simulations are conducted based on different values of c .

For developing the MEWMA chart statistic, [Zou, Tsung](#)³⁸ first transformed the $\hat{\sigma}_j^2$ to a standard normal variable:

$$\begin{aligned} \mathcal{G}_j &= \varphi^{-1} \{ \psi(n\hat{\sigma}_j^2; \mathbf{I}_{n \times n} - \mathbf{V}) \}, \\ \mathbf{V} &= \mathbf{W}' + \mathbf{W} - \mathbf{W}'\mathbf{W}. \end{aligned} \quad (3)$$

In this transformation, ϕ^{-1} is the inverse of the standard normal CDF (cumulative distribution function) and φ can be approximated with the chi-square distribution in the way that:

$$\begin{aligned} \frac{n\hat{\sigma}_j^2 - c_3}{c_1} &: \chi_{c_2}^2, \\ c_1 &= \sqrt{\frac{\text{tr}(\mathbf{A}^2)}{\text{tr}(\mathbf{A}^3)}}, \\ c_2 &= \text{tr}(\mathbf{A}^3), \\ c_3 &= \text{tr}(\mathbf{A}) - \sqrt{\text{tr}(\mathbf{A}^2) \cdot \text{tr}(\mathbf{A}^3)}, \\ \mathbf{A} &= \mathbf{I} - \mathbf{V}. \end{aligned} \quad (4)$$

Then, $\mathbf{U}_j$, denoted as $(\mathbf{Z}_j, \mathcal{G}_j)'$, which is an $(n+1)$ -variate vector and the $(n+1)$ -dimensional symmetric covariance matrix is defined as $\Sigma = \begin{pmatrix} \mathbf{V} & 0 \\ 0 & 1 \end{pmatrix}$. The proposed EWMA charting statistic then could be calculated as:

$$\begin{aligned} \mathbf{E}_j &= \lambda \mathbf{U}_j + (1 - \lambda) \mathbf{E}_{j-1}, \\ \mathbf{E}_0 &= (0, 0, \dots, 0)_{1 \times (n+1)}, \\ 0 &< \lambda \leq 1. \end{aligned} \quad (5)$$

And finally, we have a positive chart statistic:

$$Q_j = E'_j \sum E_j, \quad (6)$$

where λ is the EWMA constant (here is equal to 0.2). Considering Equation (6), an OC signal is triggered if Q_j is located beyond the IC region ($Q_j > L \frac{\lambda}{2-\lambda}$), where L is assigned based on desired ARL_0 .

3. The proposed method

The major idea of the current study is to propose an adaptive approach that is established based on the tendency (relative distance) of obtained statistics to the OC situations. As mentioned in Aly, Saleh⁵⁷ and Haq and Khoo⁵⁹, an adaptive control chart adjusts the chart statistic based on the shift size of the process in the current time. Following this idea, the proposed tendency or relative distance is computed according to the ratio of the statistic's distance from upper control limit (UCL) and lower control limit (LCL). So, the tendency of the j^{th} sample (T_j) is defined as:

$$T_j = \frac{UCL - Q_j}{Q_j - LCL}. \quad (7)$$

In NEWMA approach, LCL is zero, so the above formula transforms to:

$$T_j = \frac{UCL - Q_j}{Q_j}. \quad (8)$$

Basically, the lower the charting statistics are, the greater tendency would be. For better understanding, Figure 1 depicts the tendency values for three different NEWMA statistics when LCL and UCL are 0 and 2, respectively. For example, if the 1th NEWMA statistic is 0.5, T_1 is calculated as 3.

Insert Figure 1 about here

Naturally, such an OC situation in an NEWMA control chart will cause more samples with lower tendency values; in other words, the average of tendencies should be decreased in case an OC situation has been observed. So, the average of tendencies (T_j^*) is firstly computed by the following Equation and then, it is mapped to a specific rate for applying in an adaptive approach.

$$T_j^* = \frac{T_1 + T_2 + \dots + T_j}{j}. \quad (9)$$

The aim of this paper is to use the proposed tendency index in Equation (8) as an auxiliary adaptive approach for adjustment of the final NEWMA statistic. To have a quicker detection signal in OC condition in the proposed adaptive approach, the NEWMA statistic (Q_j) is increased (decreased) when the tendency

index (T_j) is decreased (increased), respectively. The updating of the NEWMA statistic in the j^{th} sample is done by using the tendency rate (defined by Tr_j) as follows:

$$Q_j^* = Q_j \times Tr_j. \quad (10)$$

To have a more sensitive control chart against OC shifts, it is expected that Tr_j to be lower than 1 when T_j^* has a large value (IC situation); note though, it can be greater than 1 when T_j^* has a small value (OC situation). Hence, a mapping function or relationship between Tr_j and T_j^* will be required to use the proposed adaptive approach. This approach has been carried out in previous works by definition of a score function [56, 57], improved estimators [59], warning limits [52, 58] and so forth.

To compute Tr_j , a completely linear function is not employed in this paper and a hinge function (called mapping function in this paper), as shown in Figure 2, is utilized. A hinge function hinders very small rates causing very late signalling especially in large shifts so it is not allowed to have smaller tendency rates than lower limit (L_L) even the average of tendencies is greater than a predefined value b . In addition to a and L_L , two parameters entailing B and upper limit U_L have to be defined for obtaining the mapping function formulation. The dash line means that it is very unlikely to be a sample in that region.

Insert Figure 2 about here

Considering the known values for the a , b , L_L and U_L (designing of these parameters are discussed later), an OC signal will be triggered when Q_j^* is greater than UCL. This approach adapts the NEWMA statistic magnitude based on the previous samples in a way that the chart statistic becomes so large to easily reach an OC signal. It is shown that the proposed approach is able to gain tangibly lower ARL_I values rather than NEWMA in the simulation studies.

For better understanding, the procedure of ARL_I simulations is depicted in Figure 3. In this procedure, the OC profiles are generated to reach a signal in $MaxIt$ times and the average of run lengths (signalling time) is reported as ARL_I . Similar to ARL_I , the standard deviation of run length ($SdRL_I$) is computed for each predefined shift in the process.

Insert Figure 3 about here

In this approach, the designing procedure involves selection a proper value for of a , b , L_L and U_L . Due to complex close form formulation, the control limits are adjusted based on the simulation in this paper; it is a common approach in previous profile monitoring studies [52, 55, 62].

The procedure of designing comprises three main directions. First, the value of UCL is only related to the ARL_0 or equivalently, it is similar with the NEWMA approach. Considering the ration of sample in IC data generation for the coefficients i.e., a and b is the second one, and keeping the slope of mapping function at a predefined value is the last.

To this aim, 10000 IC profiles are generated and T_j ($T_1, T_2, \dots, T_{9999}$ and T_{10000}) are computed. Then, the 45% and 95% quantile of these data are considered as proper values for a and b . These experimental values have been obtained after several investigations in different IC profiles. To adjust L_L and U_L , three following aims should be satisfied:

- $L_L < 1 < U_L$,
- $-3 < \text{slope of mapping function} < -1$,
- The final ARL_0 is supposed to be very close to its desired value.

The reason behind the first condition is that the proposed method tries to have greater statistics in OC situations. The second condition suggests a reasonable mapping function between Tr_j and T_j^* . It has been obtained by several simulation studies in different IC profiles. Finally, the last one is the natural aim of each control chart in phase II application.

By three above conditions, we should be trying to reach the best values of L_L and U_L . In the above formula, L_L is expected to be applied in IC condition while the chart statistic will become greater with U_L in OC situation. In other words, the chart statistic is multiplied by a value which is less (greater) than one in large (small) tendencies (or IC (OC) situation), respectively. The second condition has been suggested by several investigations in different IC profiles and the last one is the main criteria for phase II applications. The following steps summarize the designing procedure.

- **Step 1:** Adjust UCL to reach desired ARL_0 . It is the same as the NEWMA approach.
- **Step 2:** Generate 10000 IC profiles and store $T_1, T_2, \dots, T_{9999}$ and T_{10000} .
- **Step 3:** Calculate the 45% and 95% quantile of these data as a and b .
- **Step 4:** Calculate L_L and U_L with regard to the three above conditions.
- **Step 5:** Obtain the mapping function formula.

4. Simulation results

To compare our proposed method, three different simulation setups in which the IC model respectively has the polynomial, exponential and linear form are adjusted here based on the non-parametric profiles simulated in Zou, Tsung³⁸ and Zhang, He⁴⁴. The IC model of these scenarios are defined in Table 1 in the j^{th} generated profile ($i = 1, 2, \dots, n$).

Insert Table 1 about here

Note that in all the models $\sigma_0 = 1$. The competitive methods for the first and second scenarios are NEWMA, NM (naïve MEWMA) and PM (parametric MEWMA). All of these methods have been defined in Zou, Tsung³⁸. It is noteworthy to mention that the NM and PM have not been extended for non-

parametric profiles so one would not be surprised with their weak performance. Also, our proposed method has been denoted with ANEWMA hereafter.

Zou, Tsung³⁸ evaluated NEWMA approach under three different setups of bandwidth constant (denoted as c) equal to 1, 1.5 and 2. This parameter pertains to the W in Equation 2 and for brevity, we only consider the last situation ($c = 2$) in the first and second scenarios. To start the designing procedure (discussed in previous section), the values of UCL are considered as the values reported in Tables 2 and 4 of Zou, Tsung³⁸, i.e. it is computed with $L \frac{\lambda}{2 - \lambda}$. In scenario 3, the competitive methods are KMW (it is the same as EWMA3 approach in Kim, Mahmoud¹⁷) and NEWMA with $c = 1$ (it was not simulated in Zhang, He⁴⁴ and the results have been obtained based on our simulations). To reach ARL_0 equal to 200 in NEWMA approach, L is set to 19.58 and 20.75 for $n = 13$ and 25 (the initial values of UCL in our procedure). It is worthwhile to mention that Zhang, He⁴⁴ did not propose a clear manner in determination of control limits of KMW. But we speculate that they adjusted each separate chart to reach the same ARL_0 . Note that simulations were not conducted for the case of $n = 5$, this is due to the inability of NEWMA method to signal in some shifts when dealing with small sample sizes (for more details, see Remark 1 in Zou, Tsung³⁸).

We tried to define similar models as those in Zou, Tsung³⁸ and Zhang, He⁴⁴; however, due to some inconsistency in the results of second scenario in Zou, Tsung³⁸, there are some differences in this case. The OC models in scenario 1 are defined as:

- (I): $y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_{ij}$.
- (II): $y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \varepsilon_{ij}$.
- (III): $y_{ij} = 1 + 2x_i + 3x_i^2 + \beta_1 \sin(2\pi\beta_2 x_i) + \varepsilon_{ij}$.

In this scenario, six different OC shifts have been employed based on the Table 2.

Insert Table 2 about here

In the second scenario, the OC models are:

- (I): $y_{ij} = 1 - \beta_1 \exp(-x_i^{\beta_2}) + \varepsilon_{ij}$.
- (II): $y_{ij} = 1 - \exp(-x_i) + \beta_1 \cos(\pi\beta_2(x_i - 0.5)) + \varepsilon_{ij}$.
- (III): $y_{ij} = 1 + \beta_1 - \exp(x_i) + \beta_2 x_i^2 + \varepsilon_{ij}$.

In this scenario, six different OC shifts have been employed based on the Table 3.

Insert Table 3 about here

In the third scenario, Zhang, He⁴⁴ supposed that the IC model changed to quadratic form by adding the $\gamma(x_i^{*2} - \eta)$ term to the IC model in the way that $\gamma = \delta\sigma$. Note that the EWMA constant is assumed equal to 0.2 in all scenarios.

Considering ARL_0 equal to 200, the values of UCL in each scenario based on the Zou, Tsung³⁸'s method (NEMWA approach) are reported in Table 4. Note that the results of first and second scenario are the same as Tables 2 and 4 in Zou, Tsung³⁸. Also, the design parameters for each scenario were gathered in Table 4.

Insert Table 4 about here

In the following subsections, our proposed method is compared with other competitors based on the ARL_I criteria in each of the predefined scenarios.

4.1 The simulated ARL_I values for scenario 1

The results of ARL_I with respect to the above OC models are gathered in Table 5. These results reveal that the proposed method (ANEWMA) is effective for all of the shifts. For large shifts in the process parameter, its performance is roughly better than competing methods. As can be seen in the results of model (II) and (III), the more complicated the OC model is, the better the outperformance.

Insert Table 5 about here

4.2 The simulated ARL_I values for scenario 2

Three different types of OC models with six different shifts are defined in this subsection based on Table 3 (it is the same as Table 3 in Zou, Tsung³⁸). We may be asked about the changing of the third OC model from Zou, Tsung³⁸'s proposed model ($y_{ij} = \frac{1}{1 + \beta_1 x_i^{\beta_2}} + \varepsilon_{ij}$). This is due to our inability to produce the same results with Zou, Tsung³⁸'s paper. We even re-simulated the results of the NM method (PM was not able to detect these types of shifts) to make sure there was nothing wrong with our simulations, but unfortunately those results were also not obtained. It may be possible that the third model was written incorrectly in that paper.

The results of ARL_I with respect to the above OC models are gathered in Table 6. These results reveal that the proposed method is effective for all of the shifts and the same conclusions with scenario 1 can be drawn and only in few cases ANEWMA is not the best approach.

Insert Table 6 about here

4.3 The simulated ARL_I values for scenario 3

The results of ARL_I for these types of shifts for $n = 13$ and 25 are reported in Tables 7 and 8, respectively. In these tables, the results of EWMA3 approach are shown with KMW notation.

Insert Table 7 about here

Insert Table 8 about here

The superiority of the NEWMA to KMW is predictable due to the linear structure of the KMW control chart. Also, NEWMA has better performance than the other competitors reported in [Zhang, He](#)⁴⁴ when $n = 13$. However, when $n = 25$, the ANEWMA outperformed the other competitors in small shifts only, while the NEWMA is the best in moderate and large shifts. Since the ANEWMA naturally improves the detection ability of NEWMA, it gives logically the best performance in all the shifts.

4.4 The simulated $SdRL_I$ values for scenario 3

To evaluate control charts in phase II, $SdRL_I$ could be used in addition to ARL_I [63-65] and such a proper approach is expected to have lower $SdRL_I$ values. Figure 4 illustrates the $SdRL_I$ values for ANEWMA, NEWMA and KMW schemes. We can see that the proposed approach was able to reduce $SdRL_I$ for ANEWMA chart in comparison with NEWMA chart. The same results have been generated by other scenarios but they have been omitted for the sake of brevity.

Insert Figure 4 about here

5. Sensitivity analysis

In this section, two different simulations are provided to show the superiority of the proposed method. In the first simulation, because there are not any adaptive control charts for non-parametric profiles, two conventional adaptive approaches are prepared for monitoring non-parametric profiles. In the first approach, following the [Zou, Tsung](#)¹¹ and [Mohammadzadeh, Yeganeh](#)²⁴, VSI scheme is implemented and the proposed adaptive method in [Haq](#)⁶⁰ is the second one. These methods hereafter called VSINEWMA and HQNEWMA incorporate with NEWMA statistic to have a fair comparison and For brevity, the details of designing have not provided here.

To show the effectiveness of the proposed adaptive method, the NM statistic [38] are combined with the proposed adaptive approach (denoted with ANM (Adaptive NM)) in the second simulation, the purpose of which is to compare the detection ability of ANM and NM. To this aim, tendency rate based on Equations 9 and 10 is multiplied on NM statistic instead of NEWMA statistic. The designing procedure is the same as the ANEWMA and for the brevity the details are neglected. Also, the results of the first scenario are only reported due to page limit.

5.1 Comparison of ANEWMA with other adaptive approaches

Figure 5 reports the ARL_1 values of the first scenario for model (I) and (II) when n is 20 and 40. Note that the ANEWMA results have been gathered from Table 5. The superiority of the proposed method over two adaptive competitors is obvious from the following simulations. Except the shift type (v) in model (I) when $n = 20$, ANEWMA has the minimum ARL_1 value in all of the simulations. Also, VSINEWMA has nearly been better than HQNEWMA in these simulations.

Insert Figure 5 about here

5.2 The effect of proposed adaptive approach in combination with NM statistic

Figure 6 reports the ARL_1 values of the first scenario for model (I) and (II) when n is 20 and 40. Note that the ANEWMA and NM results have been gathered from Table 5. In all of the shifts, ANM had the better performance over NM control chart which is indicator of the capability of the proposed adaptive approach. Also, ANEWMA outperformed ANM in all the shifts which is due to the better detection ability of NEWMA against NM. The superiority of the proposed method over two adaptive competitors is obvious from the following simulations. Except the shift type (v) in model (I) when $n = 20$, ANEWMA has the minimum ARL_1 value in all of the simulations. Also, VSINEWMA has nearly been better than HQNEWMA in these simulations.

Insert Figure 6 about here

The above two simulations revealed two main findings. First, the proposed method not only outperformed non-parametric and parametric control charts but also had the superiority over conventional adaptive methods. It may be possible to reach better performance in the Haq⁶⁰'s method by using the optimisation of the parameters regarding to the non-parametric profiles; however, it is not in the scope of this paper and would be valuable for the future researches.

6. Illustrative example

A real case is provided in this section to show the practical application of the proposed scheme. It was selected from the semiconductor device fabrication in which there is deep reactive ion-etching process. The aim of this process is to check an etched wafer by an electron microscopy scanner in scientific labs. Employing of this wafer is needed to run complicated mechanical- chemical reactions for a complex automotive system [11, 34, 38, 66]. In these reactions, one key factor is to determine the profile of a trench in the system because it can change the amount outcome of the downstream operations and also the final quality of the product.

To establish a suitable monitoring approach for such a trench profile, Zou, Tsung¹¹ suggested a new method based on the profile monitoring control charts. By using the Scanning Electronic Microscope (SEM) data rather direct measurements, they were able to reach a proper monitoring scheme.

They suggested an IC polynomial model (Equation (11)) for this problem with the following transformation $z_i = x_i$ and $z_i^2 = x_i^2 - \sum_{i=1}^{11} \frac{x_i^2}{11}$.

$$\begin{aligned} y_{ij} &= 1.55 + 0z_i + 0.62z_i^2 + \varepsilon_{ij}, \\ i &= 1, 2, \dots, 11; \quad j = 1, 2, \dots, \\ x_i &= -2.5 : 0.5 : 2.5, \\ \varepsilon_{ij} &: N(0, 0.16), \end{aligned} \tag{11}$$

where the dependent and independent variables are the shape of the profile. Due to complexity of getting real data, the new samples were generated by simulations in Zou, Tsung¹¹. To have an ARL_0 equal to 370, UCL_{NEWMA} was set at 2.01 ($L = 18.00$) when $c = 1.5$ [38]. So, the design parameters (as described in Section 3) are $a = 2.3$, $b = 4.6$, $L_L = 0.46$, and $U_L = 2.85$. The mapping function's intercept and slope were obtained as 5.24 and -1.04 and 1.41 by the proposed designing scheme.

By changing the standard deviation to 0.48 from 0.4 (it is equal to a shift magnitude 1.2σ), the OC profiles were generated. Table 9 gathers the details of the 15 OC generated profiles. As can be seen, NEWMA was able to trigger an OC signal in 15th sample while ANEMWA only needed to 7 samples for this aim.

Insert Table 9 about here

For better understanding, the computations of Tr_j (in the second and last sample) are illustrated here.

In the second sample, $T_2 = \frac{2.01 - 0.366}{0.366} = 4.49$, $T^*_2 = 4.8$, $Tr_2 = \max(0.46, 5.24 - 1.04 \times 4.8) = 0.46$ and $Q^*_2 = 0.37 \times 0.46 = 0.17$. In a similar way, we have, $T_7 = \frac{2.01 - 1.69}{1.69} = 0.19$, $T^*_7 = 2.88$, $Tr_7 = \max(0.46, 5.24 - 1.04 \times 2.88) = 2.24$ and $Q^*_7 = 1.69 \times 2.24 = 3.79$. The charting statistic in ANEWMA and NEWMA for the above data are depicted in Figure 7 with green and black lines, respectively. The values of Tr_j are shown in red.

Insert Figure 7 about here

Conclusions

In this paper, an effective adaptive approach for improvement of the non-parametric profile monitoring in Phase II was proposed. That is, the adaptive approach was combined with the NEWMA control chart

proposed by Zou, Tsung³⁸, to enhance the OC detection in Phase II monitoring. To compute the control limits, a simulation-based designing procedure was proposed based on the Monte Carlo theory. The results showed that the proposed method not only improved the performance of NEWMA approach in phase II, but also outperformed some of the existing control charts, such as NM, EWMA3 and other adaptive methods, for most shifts in terms of ARL criterion. In these simulations, three different scenarios (IC models) entailing polynomial, exponential and linear have been investigated to show the robustness of the proposed scheme. In the case study, an example from the semiconductor device fabrication was implemented in which remarkable ability of the proposed method in detection of OC shifts was demonstrated. For future researches, we suggest the development of a control chart based on the combination of this approach and other adaptive control schemes.

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I certify that no party having a direct interest in the results of the research supporting this article has or will confer a benefit on me or on any organization with which I am associated and, I certify that all financial and material supports for this research and work are clearly identified in the title page of the manuscript.

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Table 1. The IC model of different scenarios in the j^{th} generated profile

Scenarios	1	2	3
IC model	$y_{ij} = 1 + 2x_i + 3x_i^2 + \varepsilon_{ij}$	$y_{ij} = 1 - e^{-x_i} + \varepsilon_{ij}$	$y_{ij} = 3 + 2x_i^* + \varepsilon_{ij}$ $x_i^* = x_i - \bar{x}$
x_i	$\frac{i - 0.5}{n}$	$\frac{i - 0.5}{n}$	$2 + \frac{(8 - 2)(i - 1)}{n - 1}$
n	20, 40	20, 40	13, 25

Table 2. The OC shifts in scenario 1

Shift type	Model (I)				Model (II)				Model (III)		
	β_0	β_1	β_2	σ	β_0	β_1	β_2	σ	β_1	β_2	σ
(i)	1.0	2.0	3.1	1.0	0.8	4.4	-3.0	1.0	0.1	1.0	1.0
(ii)	1.0	2.1	3.1	1.0	0.8	4.4	-3.0	1.0	0.2	1.0	1.0
(iii)	1.1	2.1	3.1	1.0	0.8	4.4	-3.2	1.0	0.2	0.8	1.0
(iv)	1.0	2.0	3.0	1.0	1.0	4.4	-3.2	1.0	0.2	1.3	1.0
(v)	1.0	2.0	3.0	0.7	0.8	4.4	-3.0	1.1	0.3	1.5	1.0
(vi)	1.1	2.1	3.1	1.1	0.8	4.5	-3.0	1.1	0.3	1.5	1.1

Table 3. The OC shifts in scenario 2

Shift type	Model (I)			Model (II)			Model (III)		
	β_1	β_2	σ	β_1	β_2	σ	β_1	β_2	σ
(i)	1.00	1.30	1.00	0.20	3.00	1.00	0.05	0.01	1.00
(ii)	1.00	1.50	1.00	0.30	3.00	1.00	0.05	0.05	1.00
(iii)	1.10	1.00	1.00	0.20	2.00	1.00	0.05	0.00	1.05
(iv)	1.30	1.00	1.00	0.30	2.00	1.00	0.00	0.30	1.00
(v)	1.20	1.00	1.10	0.20	4.00	1.10	0.00	0.10	1.10
(vi)	1.00	1.20	0.70	0.20	4.00	1.30	0.50	0.00	1.10

Table 4. The control chart parameters in each scenario ($ARL_0 = 200$)

Scenario	1		2		3	
n	20	40	20	40	13	25
c	2	2	2	2	1	1
UCL	1.74	1.83	1.74	1.83	2.18	2.31
a	2	1.99	2	1.99	1.71	1.55
b	4	3.98	4	3.98	3.18	2.87
L_L	0.65	0.54	0.65	0.54	0.45	0.47
U_L	4.95	3.7	4.95	3.7	2	1.84
Intercept	9.25	6.86	9.25	6.86	3.78	3.44
Slope	-2.15	-1.59	-2.15	-1.59	-1.04	-1.04

Table 5. ARL_1 values for different shifts in scenario 1

Model	$n = 20$						$n = 40$						
	NEWMA			NM	PM	ANEWMA	NEWMA			NM	PM	ANEWMA	
	$c = 1$	$c = 1.5$	$c = 2$				$c = 1$	$c = 1.5$	$c = 2$				
I	(i)	150.10	144.70	140.00	171.30	137.10	81.54	120.60	115.30	110.10	162.30	103.10	55.43
	(ii)	66.60	60.80	56.70	98.20	54.20	29.53	37.90	34.00	31.80	79.40	29.20	16.43
	(iii)	20.70	18.30	17.50	33.10	16.50	9.54	11.20	10.30	9.80	22.40	9.10	6.56
	(iv)	30.50	27.40	27.00	43.70	26.50	11.64	17.50	16.20	15.50	30.50	14.80	7.93
	(v)	8.20	6.60	5.90	*	5.30	4.64	4.20	3.70	3.50	*	3.20	2.69
	(vi)	12.5	11.4	11	17.3	10.8	6.92	7.4	7.00	6.7	11.8	6.3	4.75
II	(i)	104.60	104.20	113.40	132.20	199.70	48.64	66.50	64.00	67.50	114.20	200.10	25.63
	(ii)	89.40	87.50	92.50	119.20	154.10	38.75	55.20	50.30	52.10	99.30	120.20	24.32
	(iii)	78.80	75.70	79.20	109.30	121.00	29.15	45.00	42.40	42.80	89.30	84.90	16.36
	(iv)	24.20	22.10	21.10	38.90	22.80	11.54	12.70	11.60	11.20	26.20	12.20	7.53
	(v)	24.50	23.70	22.60	35.80	25.70	10.52	13.90	13.00	12.80	24.40	14.00	7.28
	(vi)	22.4	20.60	20.30	32.30	22.50	8.51	12.60	12.00	11.60	21.70	12.60	5.52
III	(i)	106.80	103.10	102.30	137.20	119.60	48.53	69.90	66.70	66.20	122.00	81.30	35.27
	(ii)	37.80	35.60	34.80	60.10	47.10	15.32	20.00	18.40	18.00	43.10	43.10	9.03

(iii)	35.80	32.20	31.30	57.70	31.50	13.64	19.00	16.90	16.50	40.60	40.60	8.98
(iv)	37.30	36.20	38.20	58.10	51.80	15.01	19.50	18.30	18.60	41.70	41.70	10.41
(v)	18.00	18.30	20.40	29.50	27.40	9.93	9.70	9.50	10.00	18.30	18.30	6.43
(vi)	11.2	11.1	11.4	15.5	13.4	5.89	6.8	6.6	6.6	10.6	10.6	4.9

* denotes that the related approach has not been able to detect that type of shift.

Table 6. ARL_1 values for different shifts in scenario 2

model		$n = 20$					$n = 40$				
		NEWMA			NM	ANEWMA	NEWMA			NM	ANEWMA
		$c = 1$	$c = 1.5$	$c = 2$			$c = 1$	$c = 1.5$	$c = 2$		
I	(i)	144.50	139.90	134.20	168.30	100.42	110.30	103.10	100.60	155.00	54.31
	(ii)	99.10	93.00	86.20	131.20	25.64	63.80	58.70	55.10	113.30	24.85
	(iii)	113.70	109.10	102.20	143.00	52.61	79.50	72.70	67.40	129.10	35.79
	(iv)	19.90	17.80	16.60	31.80	9.92	10.60	9.80	9.40	21.00	6.93
	(v)	17.90	16.60	15.70	25.40	7.68	10.30	9.70	9.20	17.10	6.92
	(vi)	8.2	6.5	5.8	*	4.46	4.2	3.70	3.5	*	3.03
II	(i)	39.40	40.90	45.90	59.30	21.41	20.30	20.10	21.40	43.30	12.41
	(ii)	17.90	18.10	20.60	27.10	9.83	9.70	9.40	9.90	18.40	7.52
	(iii)	37.90	35.20	34.40	60.20	18.62	20.00	18.40	17.80	43.40	10.19
	(iv)	17.30	16.00	15.50	27.00	9.01	9.40	8.80	8.70	18.30	6.38
	(v)	17.40	17.80	19.00	24.10	8.42	9.80	9.90	10.30	16.10	7.93
	(vi)	5.4	5.00	4.90	8.50	2.85	3.50	3.30	3.30	6.00	1.91
III	(i)	136.43	128.94	120.98	156.64	89.42	108.31	96.15	92.94	146.81	50.33
	(ii)	102.42	105.19	108.32	148.42	49.67	75.42	68.42	66.31	125.40	27.53
	(iii)	64.09	61.78	63.81	71.51	23.62	39.42	39.60	36.51	55.22	19.32
	(iv)	42.41	38.11	35.63	66.27	16.41	21.42	19.91	19.51	50.19	11.51
	(v)	27.49	26.42	25.19	41.62	11.51	15.89	14.79	13.91	29.42	9.64
	(vi)	4.91	3.81	3.79	5.5	3.85	2.94	2.74	2.65	4.20	2.67

Table 7. ARL_1 values for different shifts in scenario 3 when $n = 13$

η	δ								Method
	0.025	0.050	0.075	0.10	0.15	0.20	0.30	0.50	
0.50	87.94	25.09	11.79	7.24	4.18	2.99	2.07	1.20	NEWMA
	99.88	33.87	15.33	9.21	4.93	3.38	2.19	1.31	KMW
	42.22	17.53	8.66	6.67	3.91	2.31	1.37	1.00	ANEWMA
1.00	95.02	28.08	12.93	8.14	4.63	3.28	2.20	1.40	NEWMA
	115.12	46.76	21.62	12.58	6.30	4.16	2.52	1.61	KMW
	62.73	17.50	10.47	6.87	4.74	2.69	1.54	1.00	ANEWMA
1.50	104.03	36.18	15.57	9.32	5.08	3.66	2.36	1.60	NEWMA
	135.00	64.18	31.71	18.38	8.47	5.24	2.94	1.80	KMW
	59.53	19.70	12.06	8.59	4.79	2.94	1.71	1.10	ANEWMA
2.00	107.70	36.63	17.63	10.51	5.44	3.85	2.52	1.75	NEWMA

	157.66	91.66	50.91	29.04	12.40	6.91	3.39	1.88	KMW
	72.61	23.69	11.78	8.82	5.11	3.38	1.85	1.24	ANEWMA
	119.04	42.27	20.19	11.17	6.08	4.10	2.66	1.86	NEWMA
2.50	173.51	124.74	78.02	46.93	18.48	9.00	3.77	1.91	KMW
	77.61	24.98	13.49	9.24	5.88	3.80	1.93	1.43	ANEWMA
	120.81	47.94	21.16	12.68	6.33	4.30	2.78	1.89	NEWMA
3.00	185.24	150.37	108.27	69.96	25.26	10.58	3.87	1.42	KMW
	82.22	25.85	13.78	9.84	6.34	4.19	2.26	1.45	ANEWMA
	116.57	49.59	22.12	12.53	6.62	4.43	2.81	1.91	NEWMA
3.5	187.90	163.05	123.14	78.84	27.55	10.89	3.93	1.91	KMW
	85.98	29.03	14.66	10.69	6.16	4.05	2.18	1.46	ANEWMA
	115.99	47.63	21.02	11.82	6.30	4.36	2.74	1.89	NEWMA
4	186.84	153.71	108.74	70.44	24.92	10.56	3.91	1.91	KMW
	74.27	28.96	14.49	9.39	6.45	4.03	2.21	1.46	ANEWMA
	118.92	46.69	19.82	11.84	6.16	4.19	2.64	1.84	NEWMA
4.5	172.67	122.59	78.28	46.87	18.70	8.91	3.74	1.91	KMW
	75.61	26.36	12.49	9.29	6.23	3.35	2.03	1.32	ANEWMA
	113.03	40.75	17.88	10.25	5.53	3.90	2.52	1.77	NEWMA
5	155.49	91.49	50.08	28.94	12.27	6.92	3.42	1.88	KMW
	65.43	21.97	12.68	8.92	5.50	3.44	1.79	1.31	ANEWMA

Table 8. ARL_1 values for different shifts in scenario 3 when $n = 25$

η	δ								Method
	0.025	0.05	0.08	0.10	0.15	0.20	0.30	0.35	
0.50	59.11	16.09	8.09	5.27	3.22	2.37	1.86	1.00	NEWMA
	73.75	21.14	10.06	6.29	3.64	2.60	1.83	1.02	KMW
	36.37	13.24	8.04	5.18	2.76	2.62	2.04	1.24	ANEWMA
1.00	71.02	19.16	8.99	5.77	3.53	2.61	1.94	1.02	NEWMA
	94.56	31.58	14.49	8.47	4.64	3.20	2.10	1.16	KMW
	38.34	14.64	8.91	5.93	4.81	3.52	2.63	1.52	ANEWMA
1.50	78.50	22.10	10.08	6.63	3.88	2.83	2.01	1.09	NEWMA
	119.29	48.61	22.36	13.08	6.44	4.15	2.46	1.41	KMW
	48.57	15.63	9.14	7.13	3.24	3.77	2.31	1.46	ANEWMA
2.00	88.80	24.91	11.58	7.15	4.22	3.05	2.08	1.23	NEWMA
	145.96	78.33	39.65	22.50	9.63	5.60	2.86	1.56	KMW
	57.30	16.85	9.88	7.11	3.41	2.06	1.77	1.28	ANEWMA
2.50	92.78	28.77	12.98	7.78	4.54	3.27	2.16	1.37	NEWMA
	169.95	117.23	72.62	41.92	15.32	7.25	3.11	1.57	KMW
	58.64	18.10	10.48	7.85	3.86	2.28	1.15	1.11	ANEWMA
3.00	101.36	27.81	13.66	8.21	4.68	3.34	2.20	1.46	NEWMA
	185.12	153.22	105.28	64.41	20.00	8.18	3.18	1.58	KMW
	69.71	19.37	11.18	8.94	4.52	2.49	1.29	1.23	ANEWMA
3.5	103.04	30.46	13.48	8.28	4.69	3.32	2.21	1.44	NEWMA

η	δ								Method
	0.025	0.05	0.08	0.10	0.15	0.20	0.30	0.35	
	186.83	150.01	107.13	64.76	20.13	8.10	3.16	1.57	KMW
	63.45	19.58	10.57	8.38	4.29	2.72	1.37	1.21	ANEWMA
	89.06	28.81	12.70	7.85	4.53	3.21	2.16	1.38	NEWMA
	170.40	118.56	72.39	42.00	15.60	7.29	3.10	1.58	KMW
4	56.52	18.82	10.44	8.49	4.54	3.35	2.19	1.44	ANEWMA
	85.83	26.00	11.50	7.15	4.19	3.01	2.08	1.22	NEWMA
	147.63	77.19	39.26	22.03	9.67	5.57	2.87	1.56	KMW
	60.10	16.76	9.62	7.22	4.12	2.84	2.25	1.24	ANEWMA
4.5	80.03	22.03	10.38	6.44	3.84	2.84	2.02	1.10	NEWMA
	117.41	48.97	22.55	12.96	6.38	4.14	2.45	1.43	KMW
	47.53	15.93	8.72	6.63	3.87	2.95	2.15	1.21	ANEWMA

Table 9. The details of 15 OC generated profiles in illustrative example

j	y_{ij}												Q_j	T_j	Tr_j	$Q^*_{\cdot j}$
1	3.88	2.88	1.50	-0.30	-0.10	-0.14	1.03	1.06	1.37	3.11	3.37	0.33	5.11	0.46	0.15	
2	3.71	3.16	2.12	0.97	0.39	-0.28	0.51	0.22	1.67	2.62	4.42	0.37	4.49	0.46	0.17	
3	3.67	2.79	1.78	0.19	0.23	0.77	0.21	0.47	1.61	2.35	4.09	0.42	3.76	0.61	0.26	
4	3.81	2.47	1.62	1.27	0.37	0.79	-0.82	0.40	1.51	2.08	3.26	0.46	3.33	0.90	0.42	
5	4.17	2.77	1.53	0.81	-0.26	-0.24	0.10	0.29	1.55	3.62	3.64	0.57	2.52	1.25	0.71	
6	4.19	1.98	2.04	0.15	0.26	-0.30	0.40	0.63	1.37	3.90	3.57	1.13	0.79	1.78	2.00	
7	3.85	3.77	0.84	0.89	-0.36	0.49	0.31	0.93	1.26	2.60	4.58	1.69	0.19	2.24	3.79	
8	2.78	1.70	1.59	0.31	0.01	-0.72	-0.28	0.43	1.05	2.06	3.67	0.64	-	-	-	
9	3.42	3.12	0.92	1.49	-0.02	-0.70	-0.14	1.07	1.90	2.56	4.01	1.33	-	-	-	
10	4.18	1.78	1.12	-0.26	-0.06	0.46	0.50	1.72	1.48	1.44	4.69	1.62	-	-	-	
11	4.49	2.20	1.50	0.99	0.34	0.33	0.10	0.60	1.44	2.10	4.56	0.92	-	-	-	
12	3.88	2.81	0.98	0.10	0.11	-0.12	0.73	0.91	1.65	1.79	3.41	0.65	-	-	-	
13	3.97	2.31	2.01	1.26	-0.12	0.42	0.82	0.77	2.17	2.99	3.98	0.95	-	-	-	
14	4.30	2.57	1.20	0.79	0.17	-0.18	1.01	0.73	2.71	2.34	4.15	1.83	-	-	-	
15	4.63	3.79	1.54	0.24	0.54	-0.63	0.02	0.75	2.11	3.17	3.86	2.73	-	-	-	

Figures

Figure 1

The tendency value for three different NEWMA statistics when LCL and UCL are 0 and 2

Figure 2

The proposed linear mapping function for obtaining Tr_j

Figure 3

The framework for ARL1 and SdRL1 computations in the proposed method

Figure 4

The results of SdRL1 for the scenario 3

Figure 5

The results of ARL1 for ANEWMA, VSINEWMA and HQNEWMA methods (the figures on top and bottom are for $n = 20$ and 40 respectively)

Figure 6

The results of ARL1 for ANEWMA, NM and ANM methods (the figures on top and bottom are for $n = 20$ and 40 respectively)

Figure 7

The chart statistic of ANEWMA (green) and NEWMA (black) control charts for the simulated data