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Radiation shift from triple to quadruple frequency caused by the interaction of terahertz pulses with a nonlinear Kerr medium

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ABSTRACT

High-intensity optical radiation propagation in a transparent dielectric medium causes the phenomena of self-action of the pulse and the generation of radiation at triple frequencies due to the cubic nonlinearity of the medium. However, usually the quadratic nonlinear effects outshine the cubic ones in the anisotropic nonlinear crystals. In this work, we demonstrate that for certain experimental parameters the nonlinear effect of the third order can be stronger than one of the second order in MgO:LiNbO₃ crystal for terahertz frequency range. We experimentally and theoretically show that it can lead to the significant modification of the classical phenomenon of radiation generation at triple frequencies in the case when the pulse represents only one complete oscillation of the optical field. In the experiment it was shown that the phenomenon of generation of radiation at triple frequencies with respect to the frequency of the maximum spectral density in a nonlinear medium of the pulse disappears, and it is replaced by the generation of radiation at quadruple frequencies. The analysis demonstrates that this effect is based on the asymmetry of the initial spectrum of such extremely short pulses in terms of the number of oscillations.

Introduction

Research related to the application of terahertz (THz) radiation in various fields of science and industry has been actively carried out over the past 30 years. This is due to the fact that THz radiation is attractive for such areas as the detection of hidden explosives, diagnostics of food and pharmaceutical products, diagnostics and therapy of socially significant diseases¹, transmission and receiving of information, including wireless communication devices^{2,3}, as well as for various applications in space exploration⁴.

The very first source of electromagnetic THz radiation was created at the beginning of the last century by the Soviet scientist A.A. Glagoleva-Arkadyeva⁵. In recent years methods have been developed for efficient generation of high-intensity pulsed THz radiation, for example, the method of optical rectification⁶ or generation in plasma⁷⁻⁹. Under the excitation of high-intensity THz radiation, nonlinear effects of media begin to play a significant role¹⁰⁻¹². It is remarkable that most sources of pulsed terahertz radiation generate pulses containing only a few oscillations of the electrical field¹³. Therefore, nonlinear optics of few-cycle pulses is one of the most perspective and actively developing scientific directions in the field of THz photonics.

Recently, it has been theoretically predicted and experimentally confirmed that the coefficient of the nonlinear refractive index of materials in the terahertz frequency range can be several orders of magnitude higher than its value for the same materials in the visible and near-IR spectral ranges¹⁴⁻¹⁶. This nonlinearity has low-inertia mechanism, which means that high-speed THz photonics devices based on nonlinear effects are promising. The analysis of already studied nonlinear effects in the field of pulsed THz radiation can be found in review papers¹⁷⁻¹⁹. In addition to the investigation of the features of nonlinear phenomena in the THz frequency range, the study of new nonlinear materials for this range is actively developing, as well as optimizing existing ones in order to increase the efficiency of nonlinear processes for various applications. There are works that demonstrate modern advances in studies of the generation of the third harmonic of the THz frequency range in thin graphene layers^{20,21}, as well as in doped semiconductors using unipolar THz pulses²².

Few-cycle THz pulses are characterized by interesting features of nonlinear optics. For instance, a qualitative change in the nature of the familiar nonlinear effects. For example, it was shown in²³ that for pulses with such a small number of oscillations, the classical phenomenon of self-focusing may not be observed even when the critical self-focusing power is many times exceeds. Another classical phenomenon of generation of the second and third harmonics can be significantly modified in pulsed THz radiation as shown in^{24,25}.

In this study, the effect of the generation of the radiation at quadruple frequencies relative to the frequency of the power spectral density maximum due to interaction of a THz pulse containing only one full oscillation of the electrical field with a

cubic nonlinear medium, while the expected radiation at triple frequency is absent for the first time experimentally demonstrated. This effect is believed to be determined by the asymmetry of the spectrum of such a pulse and was demonstrated analytically previously in the study²⁵. In addition, based on theoretical calculations, the current work shows that in the THz frequency range, with a certain set of experimental parameters, it is possible to observe an excess of the cubic nonlinearity contribution over the quadratic one for the anisotropic nonlinear crystal, which is usually not observed in the visible frequency range. Due to this, within the framework of the work, it was possible to experimentally observe a pronounced cubic nonlinear effect in a anisotropic MgO:LiNbO₃ crystal. In the work, with the help of refined analytical calculations, the dependence of the position of the dip in the region of triple frequencies, as well as its amplitude relation to the peak in the region of quadruple frequencies, on the nonlinear properties of the material is demonstrated. Obtained analytical results are agreed with experimental data. On the basis of this phenomenon, a method for evaluating the nonlinear refractive index has been developed.

Results

Experimental results

The phenomenon of new frequencies generation during the interaction of a high-intensity THz pulse with a nonlinear medium is experimentally observe using a scheme for THz pulse generation with tilted wavefront in an MgO:LiNbO₃ crystal²⁶. An image of the THz radiation generation by the optical rectification method is shown in Fig.1(b).

Figure 1. (a) Generation of the THz pulse by optical rectification. (b) Photograph of the generation of a THz pulse by the optical rectification method in MgO:LiNbO₃ crystal. (c) Spectrum of the generated THz field. Black solid line is the experimental data for high pump intensity, red dotted line is the analytical results, blue dashes line shows the error for experimental data. (d) Spectrum of the generated THz field for lower pump intensity.

Thus, a THz pulse with an energy of up to 400 nJ and a duration of 1 ps is generated. The diameter of the radiation beam at the crystal output is 2 mm, and the radiation intensity reaches 10^8 W/cm².

As seen in Fig.1(c), no generation of radiation at triple frequencies ($3\nu_{max}$) is observed in the spectrum of pulsed THz radiation at the output of the medium during the interaction of the generated THz field with the crystal, which also has cubic nonlinearity. Moreover, in the THz radiation spectrum instead of the triple frequency, according to the frequency of

its spectral density maximum, a pronounced dip is observed. In this case, radiation of significant energy is generated at a quadruple frequency, which is not observed for similar experiments in the visible and near-IR frequency ranges. To explain this phenomenon, a theoretical study is carried out.

Figure 1(d) shows the spectrum of a THz pulse generated at a lower pump intensity. As can be seen, such a low intensity is not enough to observe the nonlinear phenomenon of generation of a new frequency. This confirms the fact that the phenomenon of generation of a quadruple frequency instead of the expected triple one which is observed in Fig. 1(c), has purely nonlinear origin.

Analytical results

On the basis of the proposed theoretical model (see Methods), analytical studies of the nonlinear interaction of THz radiation with a medium with quadratic and cubic nonlinearity were carried out, which corresponded to the experiment. The temporal dependence of the THz pulse field at the input of the optical medium (2, 4) is presented on the Fig. 2(a). Figure 2(b) demonstrates spectrum modulus of such THz pulse as well as the normalized modulus of changes in this spectrum due to quadratic and cubic nonlinearity of the medium (11, 12) in the THz frequency range. Figure 2(c) demonstrates the spectrum modulus of the THz pulse after its propagation in nonlinear medium (9). The maximum of the spectrum modulus corresponds to the frequency of ν_{max} equals to 1 THz ($\tau = 10^{-12}$ s). Parameters in the equations was chosen for the lithium niobate crystal used in the experiment: $I = 10^8 \text{ W/cm}^2$, $N_0 = 5.15$, $n_{el} = 2.26$, $\omega_0 = 5.6 \cdot 10^{12} \text{ s}^{-1}$, $a_1 = 5.15 \cdot 10^{-8} \text{ cm}$, $m = 1.5 \cdot 10^{-22} \text{ g}$, $q = 4.8 \cdot 10^{-10} \text{ Fr}$, $\alpha_T = 1.48 \cdot 10^{-5} \text{ K}^{-1}$, $S = 4.64 \text{ g/cm}^3$, $N = 2.96 \cdot 10^{22} \text{ cm}^{-3}$, $k_B = 1.38 \cdot 10^{-16} \text{ erg} \cdot \text{K}^{-1}$. All parameters, the dependencies of which are shown in the figures, are normalized to their maximum values.

Figure 2. (a) Electric field of a THz single-cycle pulse. (b) The THz pulse spectrum modulus at the input of the medium (red curve), modulus of second-order nonlinear contribution (green dashed curve), modulus of third-order nonlinear contribution (blue dotted curve). (c) Modulus of the THz pulse spectrum at the input of the medium (red curve) and the spectrum of the THz pulse at the output of the medium with (blue dotted curve) and without (green dashed curve) taking into account the second-order nonlinearity. The position of the maximum frequency ν_{max} of the THz radiation is indicated.

It can be seen from Fig. 2(b) that for a single-cycle pulse, the quadratic nonlinearity leads to the generation of tripled frequencies with respect to the maximum of the single-cycle pulse spectrum at the input of the nonlinear medium instead of doubled ones, and the cubic nonlinearity leads to the generation of quadruple frequencies instead of tripled ones. The first effect was described in²⁴, and the second one in²⁵. As can be seen from Fig. 2(c), at the output of the nonlinear medium in the radiation spectrum in the region of tripled frequency with respect to the maximum spectral density of the initial pulse a pronounced dip is observed (second dotted vertical line), which corresponds qualitatively to the experimental results (see Fig. 1(c)). Radiation in the medium with quadratic and cubic nonlinearity consideration as well as with only cubic nonlinearity

consideration is generated at higher frequencies with a maximum in the spectrum shifted to the region of quadruple frequencies of the maximum spectral density of a single-cycle pulse at the input of the nonlinear medium (third dotted vertical line). The analytical expression for the spectrum at the output (9) makes it possible to estimate the position of the extrema of the function by taking the derivative $\partial G/\partial \nu$ of (9). For example, for the parameters of the medium as in Fig. 2(c), the position of the minimum and maximum in the region of triple and quadruple frequencies, respectively, is 3.34 and 4.24, and the ratio of their amplitudes is 2.14. With a decrease in the nonlinearity parameters μ_2 and μ_3 , the ratio between the aforementioned maximum and minimum changes in direct proportion.

It should be noted that Fig. 2(c) demonstrates a negligible contribution of the quadratic nonlinearity with respect to the cubic one, which is not typical for anisotropic crystals in the optical range. To explain this phenomenon, the nonlinear coefficients μ_2 and μ_3 were calculated from Eq. (3) depending on the radiation intensity (Fig. 3).

Figure 3. Dependence of quadratic (μ_2) and cubic (μ_3) nonlinear coefficients on the THz pulse intensity.

As can be seen from the Fig. 3, there is indeed a range of intensities at which the contribution of the quadratic nonlinearity is higher than that of the cubic one as expected. However, in our experiment the intensity was 10^8 W/cm^2 , and for this intensity the nonlinear coefficients can be calculated as $\chi_2 = 2.7 \cdot 10^{-6} \text{ cm}^{1/2} \cdot \text{s} \cdot \text{g}^{-1/2}$, so that $\mu_2 = 2.11 \cdot 10^{-4}$, while $\mu_3 = 6.21 \cdot 10^{-3}$. It can be seen that cubic nonlinearity contribution is more than 30 times larger than quadratic one. This makes it possible to observe a cubic nonlinear effect in a anisotropic crystal. This can be explained by the fact that in the THz range, the value of nonlinear refractive index n_2 significantly exceeds the value for the visible and IR frequency ranges¹⁶. For lithium niobate, this excess is 4 orders of magnitude¹⁵.

In our opinion, such a paradoxical effect of the disappearance of the third harmonic generation in a medium with cubic nonlinearity with the transfer of radiation generation to the region of quadruple frequencies with respect to the maximum spectral density of the optical pulse may be associated with the strongly pronounced asymmetry of the spectrum of a single-cycle pulse.

Figure 2(c) shows the position of the average frequency $\langle \nu \rangle = \int_{-\infty}^{+\infty} \nu |G(\nu)| d\nu = 1.46 \nu_{\max}$ of the spectrum of a single-cycle pulse and the triple one. The figure demonstrates that the average frequency of the spectrum of a single-cycle pulse is slightly shifted relative to the maximum of the spectral density to the "blue" region, and the position of the maximum in the radiation spectrum generated in a medium with cubic nonlinearity at high frequencies becomes equal to 3.3 from the average frequency.

Nevertheless, with an increase in the number of periods in the pulse, the effect of the disappearance of radiation at tripled frequencies and the appearance of radiation at quadrupled ones disappears. Figure 4 represents solutions (9) considering only cubic nonlinearity for a three-cycle pulse. It can be seen that even with such a small increase in the number of total field oscillations in the initial pulse, its spectrum becomes noticeably more symmetric and the effect of radiation generation at triple frequencies ($3\nu_{\max}$) practically "returns".

As a result of the above theoretical discussion of the theory, let us compare the results of the experiment and the corresponding theoretical calculations. With the available characteristics of the crystal (which correspond to a crystal thickness of 0.5 cm) and the parameters of THz radiation $I = 10^8 \text{ W/cm}^2$, $\tau = 10^{-12} \text{ s}$, the experimental and theoretical curves are in close agreement, which is shown in Fig. 1(c), and occurs for $n_2 = 8 \cdot 10^{-11} \text{ cm}^2/\text{W}$. This is the measured value of the

Figure 4. (a) Shape of the pulse with several oscillations electric field. (b) Modulus of the pulse spectrum at the input of the medium (red curve) and the spectrum of the pulse at the output of the medium (blue dotted curve). The position of the maximum frequency ν_{max} of the THz radiation is indicated.

nonlinear refractive index of the MgO:LiNbO₃ crystal in this work. It is very close to the measurement result of the authors of²⁷ $n_2 = 4 \cdot 10^{-11} \text{ cm}^2/\text{W}$. Note that in¹⁵ the theoretical estimate of this coefficient was $5 \cdot 10^{-11} \text{ cm}^2/\text{W}$, which is in full agreement with the result of our measurement.

Conclusion

In the present work, we have shown for the first time experimentally that for the single-cycle THz pulse the phenomenon of radiation generation at triple frequencies with respect to the frequency of the spectral density maximum in a nonlinear medium disappears, and it is replaced by the generation of radiation at quadruple frequencies. Previously, this phenomenon was described analytically in²⁵. In current work, we continued this study and derived analytical formulas that allow us to predict the position of the dip in the region of triple frequencies and the peak in the region of the quadruple frequencies, and also revealed the direct dependence of their ratio on the nonlinear properties of the medium. Furthermore, we revealed, that this phenomenon is related to the strong asymmetry of the spectrum of a single-cycle pulse. In addition, it was found that in the THz frequency range, with a certain set of experimental parameters, it is possible to observe an excess of the cubic nonlinearity contribution over the quadratic one in a anisotropic lithium niobate crystal, which is usually not observed in the visible frequency range. Due to this fact it was possible to observe a pronounced cubic nonlinear effect in MgO:LiNbO₃ crystal. We used the studied nonlinear effect to measure the nonlinear refractive index of the MgO:LiNbO₃ crystal in the THz spectral range to be $n_2 = 8 \cdot 10^{-11} \text{ cm}^2/\text{W}$. This result obtained are in complete agreement with other works.

Methods

Experimental setup

Experimental setup scheme is shown in Fig.5. Radiation from femtosecond laser system based on a regenerative amplifier (pulse duration is 30 fs, central wavelength is 790 nm, pulse energy is 2 mJ, repetition rate is 1 kHz) is divided into two beams with a beam splitter with 98:2 ratio. Pump radiation goes through optical attenuator into TERA-AX THz generator. Pump radiation after diffraction grating focused on MgO:LiNbO₃ crystal by spherical mirror. Generated THz radiation collimated by parabolic mirror and goes out of the system. After that THz radiation focused on 1 mm thick ZnTe crystal by another parabolic mirror for detection. Probe beam after beamsplitter goes to delay line and intersects with THz pulse at ZnTe crystal and measured by electro-optical setup.

Generation of THz radiation by the optical rectification method occurs in the entire volume of interaction of the tilted wavefront with the crystal. Thus, THz radiation is generated in the entire volume of crystal as well during the pump radiation propagation through the medium with both quadratic nonlinearity, due to which the THz radiation generation process occurs, and cubic nonlinearity, due to which radiation is expected to be generated at triple frequencies with respect to the THz radiation. A schematic diagram of THz radiation generation is shown in Fig.1(a). As a result, a diverging THz radiation beam with a Gaussian profile is formed, directed perpendicular to the crystal cut. Then, THz radiation is collimated using a parabolic mirror with a focal length of 25 mm. The THz field amplitude is measured using a standard electro-optical detection scheme in a 1 mm thick ZnTe crystal.

Figure 5. Schematic diagram of the experimental setup. Standard electro optical scheme for detecting the THz field using a 1 mm thick ZnTe crystal. The THz radiation source is a lithium niobate crystal, pumped by pulsed laser radiation. BS - beamsplitter, OA - optical attenuator, PBS - polarizing beamsplitter, $\lambda/2$ - half-wavelength plate, TERA-AX - terahertz generator, M - mirrors, DG - diffraction grating, SM - spherical mirror, PM - parabolic mirror, DL - delay line, D - diaphragm, ZnTe - 1 mm thick zinc telluride crystal, $\lambda/4$ - quarter-wavelength plate, WP - wollaston prism, BD - balanced detector.

Analytical model

The dynamics of the THz radiation electric field in a transparent dielectric medium with inertialess quadratic and cubic nonlinearity as well as absorption and amplification consideration can be analyzed on the basis of the equation²⁸:

$$\frac{\partial E}{\partial z} + \frac{N_0}{c} \frac{\partial E}{\partial t} - a \frac{\partial^3 E}{\partial t^3} - (\Gamma - \gamma)E + g_1 E \frac{\partial E}{\partial t} + g_2 E^2 \frac{\partial E}{\partial t} = 0, \quad (1)$$

where E is electric field, z is the propagation direction, t is the time, c is the speed of light in vacuum, N_0 and a are the empirical constants characterizing the dependence of the linear refractive index of the medium on the frequency ν of the form $n(\nu) = N_0 + 4\pi^2 c a \nu^2$, Γ is the amplification coefficient, γ is the absorption coefficient, $g_1 = \chi_2/c$ and $g_2 = 2n_2/c$, are the parameters characterizing the quadratic and cubic nonlinearity of the medium response respectively, χ_2 is quadratic susceptibility and n_2 is the nonlinear refractive index coefficient of the medium (CGS units).

The field at the input of a nonlinear medium (at $z = 0$) is considered in the form of a single-cycle pulse (see Fig.2(a)) of the form:

$$E(t, 0) = E_0 \frac{t}{\tau} e^{-\left(\frac{t}{\tau}\right)^2} \quad (2)$$

where E_0 is the amplitude of the pulse electric field, τ is its duration. Figure 1(a) shows that the THz pulse observed in the experiment in the far diffraction zone has one and a half cycle. However, writing the boundary condition in the form of a single-cycle pulse (2), we consider that during the propagation in the crystal, as well as at its output the THz pulse is a single-cycle one and only in the far zone, due to diffraction, it acquires a half-wave²⁹. This effect, which is practically not observed for pulses with a large number of oscillations, is clearly pronounced for a single-cycle pulse, which becomes one and a half cycle in the far diffraction zone³⁰.

For further analysis, it is useful to rewrite Eq. (1) in normalized variables $\tilde{E} = E/E_0$, $\tilde{t} = t/\tau$, $\tilde{z} = z/L_{pulse}$, where $L_{pulse} = c\tau/N_0$ is the longitudinal size of the THz pulse in the medium:

$$\frac{\partial \tilde{E}}{\partial \tilde{z}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} - \mu_0 \frac{\partial^3 \tilde{E}}{\partial \tilde{t}^3} - \mu_1 \tilde{E} + \mu_2 \tilde{E} \frac{\partial \tilde{E}}{\partial \tilde{t}} + \mu_3 \tilde{E}^2 \frac{\partial \tilde{E}}{\partial \tilde{t}} = 0 \quad (3)$$

Here, $\mu_0 = 1/2 \cdot \Delta n_{disp}/N_0$. $\Delta n_{disp} = 4\pi^2 ac v_{max}^2$ characterizes the change in the refractive index due to dispersion, where $v_{max} = \sqrt{2}/\tau$ is the spectral density maximum frequency at the input of the nonlinear medium, $\mu_1 = \frac{c\tau}{N_0}(\Gamma - \gamma)$ characterizes the amplification and absorption of the medium; $\mu_2 = \frac{\chi_2 E_0}{N_0}$ represents the quadratic nonlinearity contribution, where $\chi_2 = \frac{m\omega_0^2 a_l \alpha_T}{32\pi^2 q N k_B} (n_{0,v}^2 - 1)^2$, $\mu_3 = 4\Delta n_{nl}/N_0$, $\Delta n_{nl} = 1/2 n_2 E_0^2 = n_2' I$ characterizes the change in the refractive index of the medium due to the cubic nonlinearity, n_2' is the coefficient of the nonlinear refractive index of the medium (in SI units), I is intensity of THz pulse, $n_{0,v}^2 = \sqrt{1 + N_0^2 - n_{el}^2}$ is the vibrational contribution to the low-frequency refractive index, n_{el} is the refractive index in the range with nonresonant electronic contribution (800 nm), ω_0 is the fundamental vibration frequency, a_l is the lattice constant, m is the reduced mass of the vibrational mode, q is the effective charge of the chemical bond, α_T is the thermal expansion coefficient, N is the number density of vibrational units and k_B is the Boltzmann constant.

The normalized boundary condition (2) takes the form

$$\tilde{E}(\tilde{t}, 0) = \tilde{t} e^{-\tilde{t}^2} \quad (4)$$

The contribution of dispersion, as is known, leads to a change in the temporal structure of the pulse and does not affect its spectrum. In addition, it can be seen from the experimental data that no dispersive spreading of the pulse is observed; therefore, the dispersive contribution can be neglected.

The dependence of the absorption spectrum in the studied spectral range has no characteristic features³¹. The result of taking into account the absorption coefficient leads to a monotonic decrease in the power spectrum of the output THz radiation. In this regard, absorption can be neglected in calculations.

For ease of taking into account the amplification of the THz pulse, in the calculations we consider the case when the energy of the pulse interacting with the medium corresponds to the energy of the pulse at the output from the medium. Therefore, in the propagation dynamics equation, the amplification term can also be omitted.

Thus, for an analytical calculation of the dynamics of a THz pulse during propagation, one can focus on the last two terms of Eq. (3). Assuming the values μ_2 and μ_3 are small, the solution to Eq. (3) can be found in the form of a series:

$$\tilde{E}(\tilde{t}, \tilde{z}) = \tilde{E}^{(0)}(\tilde{t}, \tilde{z}) + \mu_2 \tilde{E}^{(1, nl2)}(\tilde{t}, \tilde{z}) + \mu_3 \tilde{E}^{(1, nl3)}(\tilde{t}, \tilde{z}) \quad (5)$$

It is easy to show that with boundary conditions (2), the terms in such a solution take the form

$$\tilde{E}^{(0)}(\tilde{t}, \tilde{z}) = e^{-(\tilde{t}-\tilde{z})^2} (\tilde{t}-\tilde{z}) \quad (6)$$

$$\tilde{E}^{(1, nl2)}(\tilde{t}, \tilde{z}) = \tilde{z}(\tilde{t}-\tilde{z})(2(\tilde{t}-\tilde{z})^2 - 1)e^{-2(\tilde{t}-\tilde{z})^2} \quad (7)$$

$$\tilde{E}^{(1, nl3)}(\tilde{t}, \tilde{z}) = \tilde{z}(\tilde{t}-\tilde{z})^2(2(\tilde{t}-\tilde{z})^2 - 1)e^{-3(\tilde{t}-\tilde{z})^2} \quad (8)$$

Here, $\tilde{E}^{(0)}$ is the solution of equation (3) for the "zero" approximation, i.e. without taking into account the nonlinearity of the medium, $\tilde{E}^{(1, nl2)}$ and $\tilde{E}^{(1, nl3)}$ are the change in the shape of the pulse, associated with quadratic and cubic nonlinearity respectively.

Accordingly, the field (5) spectrum $G(\tilde{v}, \tilde{z}) = 1/\sqrt{2\pi} \int_{-\infty}^{+\infty} \tilde{E}(\tilde{t}, \tilde{z}) \cdot e^{i\tilde{v}\tilde{t}} d\tilde{t}$ where $\tilde{v} = v\tau$ can be represented as

$$\tilde{G}(\tilde{v}, \tilde{z}) = \tilde{G}^{(0)}(\tilde{v}, \tilde{z}) + \mu_2 \tilde{G}^{(1, nl2)}(\tilde{v}, \tilde{z}) + \mu_3 \tilde{G}^{(1, nl3)}(\tilde{v}, \tilde{z}) \quad (9)$$

$$\tilde{G}^{(0)}(\tilde{v}, \tilde{z}) = \frac{ie^{i\tilde{z}\tilde{v}-\tilde{v}^2/4}\tilde{v}}{2\sqrt{2}} \quad (10)$$

$$\tilde{G}^{(1, nl2)}(\tilde{v}, \tilde{z}) = -\frac{i\tilde{v}\tilde{z}(\tilde{v}^2 - 4)e^{-\tilde{v}^2/8+i\tilde{v}\tilde{z}}}{64} \quad (11)$$

$$\tilde{G}^{(1,n/3)}(\tilde{v}, \tilde{z}) = \frac{\tilde{z}\tilde{v}^2(\tilde{v}^2 - 18)e^{-\tilde{v}^2/12 + i\tilde{v}\tilde{z}}}{648\sqrt{6}} \quad (12)$$

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Author contributions statement

A.T. conceived the experiment. A.I. and A.T. conducted the experiment. A.I. and M.M. analysed the experimental results. I.A. and S.K. developed the theoretical model. I.A. and M.G. obtained analytical results and describe them. All authors reviewed the manuscript.

Additional information

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Data availability: All data generated or analyzed during this study are included in this published article and its supplementary information files.