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A control method with considering error coupling in milling multi feature thin-walled parts

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Abstract: The error coupling analysis is important for machining accuracy of thin-walled parts. However, most of the error coupling analysis methods only based on multiple error sources with a single feature, without considering the coupling relationship of these errors between features. This paper no longer focuses on analyzing the machining error of a single feature, but discusses the error coupling phenomenon between each other and interrelated features. First, interactive factors during machining multiple features are analyzed. Subsequently, according to the main interactive factors, the models about geometric interaction and machining interaction of multi feature processing are established. Afterwards, based on the comprehensive error model obtained through the transformation of the coordinate systems, an efficient error control method of multi feature thin-walled parts is proposed. Finally, the approach has been validated through an illustrative case study using a sample part with complex machining features.

Keywords: Multi feature thin-walled parts, Error coupling, Error compensation, Posture adjustment

1. Introduction

Thin walled parts are widely used in aerospace industry considering their advantages in their light weight, good stiffness and high strength[1]. However, the machining of thin-walled parts is the most difficult problem in metal cutting. For the complex structures, such parts often have many machining features. These features are

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related to machining process, dimension and tolerance. They restrict and benchmark each other. As the process going on, the errors among the features will interact and coupling. In this situation, interactive manufacturing features play important role in the analysis of thin-walled parts processing quality. It makes it more difficult to predict and control the machining error of thin-walled parts.

To solve the above-mentioned problems during machining multi-feature thin walled parts, many researchers have made much detailed study about this complex process problem. Pralay Pal [2] analyzed the optimization algorithm needed for the interactive function of various datum / tolerances of multi -feature parts, and provided a more accurate machining strategy for NC machining process by using spatial reasoning method. According to the geometric information and boundary conditions of multiple features, Mohammad Givchi[3] simplified the features to arrange the processing sequence considering the interaction between feature volume and adjacent features. Ma [4] established the calculation model of instantaneous cutting point and system stiffness in multi-feature machining, a method of machining deformation homogenization was proposed to control the machining error of thin-walled parts. Tobias dipper [5] constructed a feature information model related to each other in multi feature machining. Combined with the process information, a three-dimensional information model is formed and applied to the NC machining process. Deja[6] presented a feature based reasoning approach for generating machining sequences in terms of part setups and the assignment of machine alternatives. The approach assumes a heavy reliance on a data input model incorporating functional requirements for parts and in particular GD&T references. Wan[7] proposed a machining-feature requirement-driven workpiece-holding scheme to constrain the degrees of freedom (DOFs) of the workpiece according to the various machining features.

These models and strategies can obtain the optimized cutting method and sequence relative to the multi-feature thin-walled parts. Process optimization method try to eliminate or greatly decrease the interaction between multi feature based on the angle of datum / tolerance and stiffness. However, it is ignored error coupling caused by the force, heat and stress in multi-operation and multi -feature manufacturing processes.

About the influence factors of error coupling of thin-walled parts, the researchers at home and abroad mainly focuses on the error coupling of datum transformation[8, 9], thermal coupling[10, 11], stress coupling[12-14], etc. Liu et al.[15] studied a variety of error fields generated in the multi position clamping of thin-walled parts, and built a 3D error coupling model using the state space theory. Huang et al.[16, 17] followed the concept of datum flow chain (DFC), regarded each station of multi-feature machining as a series of discrete events. The transmission, coupling and superposition of errors in the manufacturing process were analyzed. Control theory was also introduced to optimize the system, so as to improve process parameters and reduce product errors. In addition, Ge[18] proposed an integrated error compensation method based on on-machine measurement (OMM) inspection for thin web parts machining. Du[19] supplied a comprehensive error compensation method that includes three major error sources, which are geometric error, thermal error, and force-induced error.

The abovementioned coupling theories have limitations and particularity. Error

models are developed to solve the coupling effect of different error sources on specific machining features. Although some researches involve multi process processing, they only focus on the problem that the same single feature flows in different processes. Besides, they ignore that in the current feature cutting process, the cutting force, heat and stress will affect the related features, and make the quality change again, which will affect the final machining accuracy of thin-walled parts. These are the important shortcomings of thin-walled part deformation error models, and lead to the inaccuracy of predictive models. In this way, some new models and theories about machining multiple features need to be studied to improve the error prediction model of thin-walled parts. The interactive influence of multiple features can be divided into two classifications: geometric interactions and machining interactions[20]. This paper mainly analyzes the error coupling and control method of multiple features in the machining process.

2. Interactive factors of multi feature processing

The interactive factors in multi feature machining can be divided into two classes as shown in Fig.1. The first is geometric error Δ_g resulting from locating and clamping, which is the error generated before machining. Locating datum plane is the reference of parts processing. If there is error in datum plane, it will affect the further processing of parts. Clamping deformation can further change the locating position and shape of the thin-walled part. The second class is machining error from machining, which is the error encountered during processing. If permanent distortions of part and fixture are ignored, the error Δ_m from machining is composed of (1) the error caused by force / heat generated in the cutting process of associated features; and (2) the error related to elastic deformation due to the smaller and smaller stiffness with the removal of materials.

The error variation, the tiny deviation of dimension, shape, and position for a machining feature can be expressed by some torsor parameters. Such torsor parameters are then called small displacement torsors (SDT). The SDT can be used directly in its generic form to represent potential variations along and about all three Cartesian axes. If the SDT is used to express the difference between the actual and ideal quality of the key quality feature, a machining error vector, denoted as Δ_{tol} , can be written as:

$$\Delta_{tol} = S_{actual} - S_{ideal} = \begin{bmatrix} \Delta x & \Delta \alpha \\ \Delta y & \Delta \beta \\ \Delta z & \Delta \gamma \end{bmatrix} \quad (1)$$

Here, S_{actual} represents the actual machined feature; S_{ideal} is the nominal machined feature. Δx , Δy and Δz are the translational vectors of the feature in the

directions x, y, and z, $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$ are the rotating vectors around x, y, and z.

According to the error sources, the vector of the resultant surface error can also be obtained as:

$$\Delta_{tol} = \Delta_g + \Delta_m \quad (2)$$

Fig.1 Interaction of multi-feature machining

(a) Machining features and datum number of complex thin-walled parts

(b) Double clamping of complex thin-walled parts

Fig.2 Features and positioning mode of the complex thin-walled part

3. Geometric interaction of multi-feature machining

In the process of machining thin-walled parts, the tolerance / datum between features will affect each other. However, the tolerance / datum plane is the reference for machining parts. If there are errors in the datum plane, the further machining of parts will be affected. As shown in Fig.2, the complex thin-walled part is clamped twice with different datum planes to mill different features. Its geometric features such as planes, holes and cylindrical surfaces are labeled in Fig.2 (a). Figure 2 (b) shows the clamping and positioning method when machining different features. Different machining features can share the same datum, and the machined features can also be the machining datum of other quality features. Therefore, different machining features of the same part

can influence each other through datum. The features of complex thin-walled parts interact through positioning and datum. The relationship between the 10 features of the sample part is illustrated in Fig.3.

Fig.3 Relationship between datum and machining features

According to Fig.3, the relationship between datum and machining features is defined as $H = \{H_i\}$, $i = 1, 2, \dots, N$. Where N is the number of clamping. H_i is the relationship between datum and machining feature in the i th installation.

$$H_i = \{D_{i1}, D_{i2}, D_{i3}, A_{ij}, \xi_{ijk}\} \quad (3)$$

In equation (3), $j = 1, 2, 3$ is the number of machining features, and $k = 1, 2, 3$ is the locating surface. D_{i1}, D_{i2}, D_{i3} is the first positioning / datum, the second positioning / datum plane and the third positioning / datum plane respectively. A_{ij} denotes machining the j th feature in the i th clamping. ξ_{ijk} is the attribute of machining feature and positioning / datum plane. When $\xi_{ijk} = 0$, the machining feature is parallel to the positioning / datum plane. When $\xi_{ijk} = 90$, the locating surface is perpendicular to the machining feature; When $\xi_{ijk} = \alpha$, there is an angle relationship between machining features and location / datum; When $\xi_{ijk} = 1$, there is only dimension relationship between machining features and location / datum.

There are two kinds of errors between machining feature and positioning / datum. That is, the error Δ_{pv} caused by the incorrect position of the positioning part (unideal fixture position and manufacturing error of fixture), and the variation value Δ_{wv} belongs to the size of the part. The error between the machining feature and the positioning / reference plane can be expressed as

$$\Delta_g = \Delta_{pv} + \Delta_{wv} \quad (4)$$

When the position of the positioning element is deviated, the position and direction of the part will change, and the machining error will also be affected by these changes. Compared with the ideal positioning, this change shows the change of datum. After the positioning datum changes, any point on the machining feature of the part will also change. Considering the uneven datum plane and clamping deformation, the fitting surface is obtained by fitting the changed discrete points to express the nonideal surface features. The three locating surfaces are expressed by fitting plane equations. The locating surface is expressed by plane equation.

$$A_d x + B_d y + C_d z + D_d = 0 \quad (5)$$

The datum plane represented by the plane equation is

$$A_j x + B_j y + C_j z + D_j = 0 \quad (6)$$

Where A_d, B_d, C_d, D_d and A_j, B_j, C_j, D_j are the constants of the plane equations.

(1) When the machining feature is parallel to the positioning / datum plane, that is, when $\xi_{ijk} = 0$, the equal error between the positioning surface and the machining feature is shown in Fig.4.

Fig.4 Error caused by positioning / datum plane parallel to machining feature

In the unified coordinate system, it is assumed that the normal vector n_j of the datum plane needs to rotate α around the x -axis, β around the y -axis, and γ around the z -axis to coincide with the normal vector n_d of the positioning plane. Namely

$$\cos \alpha = \frac{B_j B_d + C_j C_d}{\sqrt{B_j^2 + C_j^2} \sqrt{B_d^2 + C_d^2}} \quad (7)$$

$$\cos \beta = \frac{A_j A_d + C_j C_d}{\sqrt{A_j^2 + C_j^2} \sqrt{A_d^2 + C_d^2}} \quad (8)$$

$$\cos \gamma = \frac{A_j A_d + B_j B_d}{\sqrt{A_j^2 + B_j^2} \sqrt{A_d^2 + B_d^2}} \quad (9)$$

The variation of positioning / datum plane will lead to the position change of machining features. Suppose that the ideal coordinate of any point on the machining

feature is $M(x,y,z)$, and it actually becomes $M'(x',y',z')$ after positioning. If point M' simply rotates angle α anticlockwise around the x' axis, then M' and x' coincide, then

$$\begin{cases} y = y' \cos \alpha - z' \sin \alpha \\ z = y' \sin \alpha + z' \cos \alpha \end{cases} \quad (10)$$

The matrix form is as follows

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (11)$$

That is

$$[M] = [R_x][M'] \quad (12)$$

Where

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad (13)$$

Similarly, the rotation matrices of simple rotation around the y' -axis (rotation angle is β) and simple rotation around the z' -axis (rotation angle is γ) are as follows

$$[R_y] = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}, [R_z] = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (14)$$

Suppose that the translation of $M(x,y,z)$ and $M'(x',y',z')$ along the three axes is

$(\Delta x, \Delta y, \Delta z)$, the translation matrix can be written as

$$T_M = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (15)$$

Then the relation between $M(x,y,z)$ and $M'(x',y',z')$ is

$$[M] = [R_x][R_y][R_z][M'] + T_M \quad (16)$$

It can be expressed as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (17)$$

The general error produces a small rotation angle, so the above equation can be simplified as follows

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (18)$$

Then the error of any point of the part caused by the positioning / datum plane can be computed as

$$\Delta_g = \begin{bmatrix} \Delta x_g \\ \Delta y_g \\ \Delta z_g \end{bmatrix} = \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (19)$$

(2) When the machining feature is perpendicular to the positioning / datum plane, that is, $\xi_{ijk} = 90^\circ$, the vertical error between the positioning plane and the machining feature is shown in Fig.5.

Fig.5 Error caused by positioning / datum perpendicular to machining feature

Fig.6 Rotation of machined features due to datum / locating plane errors

As shown in Fig.6, when the machining feature is parallel or vertical to the positioning / datum plane, according to the geometric characteristics of the part and positioning element, the rotation angle ΔR of the machining parallel feature and vertical feature caused by the positioning / datum plane error is the same. The analysis

process is consistent with the above parallel machining features. Therefore, when machining vertical features such as holes and cylinders, the relationship between the ideal position of any point on the feature and the actual position is as follows

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (20)$$

The error is the same as Eq. (21).

$$\Delta_g = \begin{bmatrix} \Delta x_g \\ \Delta y_g \\ \Delta z_g \end{bmatrix} = \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (21)$$

4 Interaction between features caused by machining

Milling process is a very complex process. The problem involved is not simple structural analysis or thermal analysis, but the coupling effect of various physical fields. Therefore, in this kind of analysis, the function and interaction of each physical quantity need to be considered. Among them, the deformation caused by clamping is an important factor affecting the machining accuracy in the multi feature machining of thin-walled parts. It can make the adjacent machining feature produce elastic deformation by clamping force, which leads to the machining error of the adjacent feature in the cutting process. At the same time, the cutting force and heat in cutting metal is a common physical phenomenon. It may influence the features connected with it through force, heat and stress. The calculation of machining deformation under these coupling effects involves many related theories, including elastic-plastic mechanics theory, heat transfer theory, advanced dynamics theory, vibration mechanics theory and so on. This section mainly analyzes the influence of clamping force and cutting force on multi feature machining of thin-walled parts.

4.1 Influence caused by clamping force

For a part, under a certain positioning scheme, as shown in Fig.8, the clamping deformation of feature surface can be expressed as the displacement of feature surface in x, y and z directions of spatial coordinate system, and the displacement is related to the clamping force in the three directions. Suppose $n_i = [n_{ix}, n_{iy}, n_{iz}]^T$ is the inner normal unit vector of the contact between the part surface and the ith positioning element. $\tau_i = [\tau_{ix}, \tau_{iy}, \tau_{iz}]^T$ and $\sigma_i = [\sigma_{ix}, \sigma_{iy}, \sigma_{iz}]^T$ are two orthogonal unit vectors of the tangent of the part surface at the contact point as shown in Fig .7. Then, the three directions of force generated by the positioning element on the part at the contact point are as follows

$$F_i = [F_{ni}, F_{\tau i}, F_{\sigma i}]^T = [f_{ni} \cdot n_i, f_{\tau i} \cdot \tau_i, f_{\sigma i} \cdot \sigma_i]^T \quad (22)$$

In which, $f_i = [f_{ni}, f_{\tau i}, f_{\sigma i}]^T$ is the contact force at the contact point in the normal and two orthogonal tangential directions. The tangential forces are the friction forces at the contact point. When the part is in contact with the positioning element, the contact deformation will occur at the contact point due to the clamping force, which will make the part deviate from the original position. It is assumed that K_i is the contact stiffness of the i th positioning clamping element and the part. According to the *Hertz* contact theory, the relationship between the contact force f_i and the displacement Δd_i of the locating point is as follows

$$[f_i] = [K_i] \cdot [\Delta d_i] \quad (23)$$

$$K_i = \frac{\text{Force at point } i}{\text{Normal deformation of point } i} \quad (24)$$

Fig.7 Fixture distribution of workpiece fixture system

Fig.8 Workpiece-fixture contact deformation point cloud

Due to the contact deformation between the positioning point and the part, the part deviates from the ideal installation position, which leads to the clamping error. In this case, the offset of the part is defined as Δq , and the orientation matrix of the positioning element is represented by Jacobian matrix J . If the offset of the positioning contact point is Δd , then there is

$$\Delta q = J^{-1} \cdot \Delta d \quad (25)$$

Where

$$J = \begin{bmatrix} \frac{\partial d_1}{\partial x} & \frac{\partial d_1}{\partial y} & \frac{\partial d_1}{\partial z} & \frac{\partial d_1}{\partial \alpha} & \frac{\partial d_1}{\partial \beta} & \frac{\partial d_1}{\partial \gamma} \\ \frac{\partial d_2}{\partial x} & O & & & & \frac{\partial d_2}{\partial \gamma} \\ M & & & & & M \\ \frac{\partial d_m}{\partial x} & L & & & & \frac{\partial d_n}{\partial \gamma} \end{bmatrix} \quad (26)$$

Fig.9 Clamping deformation of the thin-walled part

On the other hand, the simulation software is used to analyze the clamping deformation of thin-walled parts, and the results are shown in Fig.9. The normal component of generating points on machined surface adjacent to the clamping surface is the machining error. Suppose that $q'_i(x'_i, y'_i, z'_i)$ is the actual coordinate value of the surface at a certain time, and the corresponding ideal coordinate value is $q_i(x_i, y_i, z_i)$. It can be seen from Fig.10 that the machining error of the surface caused by the clamping force is:

$$\Delta q_i = (q'_i - q_i) \cdot n_i \quad (27)$$

In which, i is the number of nodes.

Fig.10 Schematic diagram of position variation of surface points

4.2 Influence caused by cutting force

For complex thin-walled parts, they are more sensitive to the force, heat and stress in the machining process, so the interaction of their features is more prominent. The influence mode is shown in Fig.11. Under the interaction of tool and part, machined feature ① is affected by post machined feature ②. The characteristic plane is extruded in normal and tangential directions, resulting in normal warping deformation and tangential torsion deformation. The deformation is shown in Fig.11. The influence of tangential torsion deformation on the flatness of planar features is relatively small,

which can be ignored generally, while the influence of normal deformation is relatively large.

According to the static equilibrium equation, the deformation of in-process thin-walled part is:

$$\begin{bmatrix} \Delta x_{im} \\ \Delta y_{im} \\ \Delta z_{im} \end{bmatrix} = \begin{bmatrix} K_{ix}^{-1} & 0 & 0 \\ 0 & K_{iy}^{-1} & 0 \\ 0 & 0 & K_{iz}^{-1} \end{bmatrix} \begin{bmatrix} F_{ix} \\ F_{iy} \\ F_{iz} \end{bmatrix} \quad (28)$$

Where Δx_{im} , Δy_{im} , and Δz_{im} is the deformation of point i in three directions under the action of cutting force. F_{ix} , F_{iy} and F_{iz} is the cutting force. K_{ix} , K_{iy} and K_{iz} are the stiffness of the thin-walled part in three directions positions, respectively.

Fig.11 Multi feature machining of the thin-walled part

5. Attitude adjustment accuracy control based on error model

5.1 The comprehensive error model based on SDT

According to the analysis and modeling of the two error sources in Section 3 and Section 4, the comprehensive error model can be expressed based on SDT in the key feature coordinate system. The key feature is the important machining quality that must be guaranteed for the final machining of parts. However, the geometric error is modeled in the workpiece coordinate system while the force-induced error is calculated in the key feature coordinate system. To realize the comprehensive error compensation, the error models need to be unified into a coordinate system. In order to facilitate the adjustment of part pose, the workpiece coordinate system is selected here. Then the comprehensive error can be obtained by rotation and translation between coordinate systems as:

$$\Delta_i = \begin{bmatrix} 1 & \Delta\psi & \Delta\theta & d_{ix} \\ \Delta\psi & 1 & \Delta\phi & d_{iy} \\ -\Delta\theta & \Delta\phi & 1 & d_{iz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{if} + \Delta x_{im} \\ \Delta y_{if} + \Delta y_{im} \\ \Delta z_{if} + \Delta z_{im} \\ 1 \end{bmatrix} + \begin{bmatrix} \Delta x_{ig} \\ \Delta y_{ig} \\ \Delta z_{ig} \\ 1 \end{bmatrix} \quad (29)$$

Where Δ_i is the comprehensive error; $d_{ix}, d_{iy}, d_{iz}, \Delta\phi, \Delta\theta$ and $\Delta\psi$ are the

translation and rotation values between the two coordinates.

5.2 Posture adjustment based on comprehensive error model

Fig.12 Relationship between actual and theoretical position of posture adjustment

As shown in Fig.12, there is a posture error between the theoretical coordinate and the actual coordinate in the workpiece coordinate system (WCS). Posture error includes coordinate value and direction. According to the above comprehensive error model, the angle error is $\Delta R = \{\Delta\alpha, \Delta\beta, \Delta\gamma\}$, the displacement error is $\Delta P = \{\Delta x, \Delta y, \Delta z\}$. That is, there are rotation error ΔR and displacement ΔP error between the actual coordinate system and the theoretical coordinate system of key machining feature.

It is assumed that the theoretical coordinate system of machining feature i is recorded as $PCS_i^0(x_i^0, y_i^0, z_i^0)$ and the actual coordinate system is $PCS_i(x_i, y_i, z_i)$. Then the relationship between $PCS_i^0(x_i^0, y_i^0, z_i^0)$ and $PCS_i(x_i, y_i, z_i)$ is

$$PCS_i = PCS_i^0 + \Delta P_i \quad (30)$$

At the same time, $PCS_i^0(x_i^0, y_i^0, z_i^0)$ and $PCS_i(x_i, y_i, z_i)$ satisfies:

$$PCS_i^0 = \Delta P + \Delta R \bullet PCS_i \quad (31)$$

Where

$$\Delta R = \Delta R_{\Delta\alpha} \bullet \Delta R_{\Delta\beta} \bullet \Delta R_{\Delta\gamma}$$

$$= \begin{bmatrix} \cos(\Delta\alpha) & -\sin(\Delta\alpha) & 0 \\ \sin(\Delta\alpha) & \cos(\Delta\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos(\Delta\beta) & 0 & \sin(\Delta\beta) \\ 0 & 1 & 0 \\ -\sin(\Delta\beta) & 0 & \cos(\Delta\beta) \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta\gamma) & -\sin(\Delta\gamma) \\ 0 & \sin(\Delta\gamma) & \cos(\Delta\gamma) \end{bmatrix} \quad (32)$$

$\cos(\Delta\alpha)$, $\sin(\Delta\alpha)$, $\cos(\Delta\beta)$, $\sin(\Delta\beta)$, $\cos(\Delta\gamma)$, $\sin(\Delta\gamma)$ are high-order small quantities and can be ignored. Therefore, the above equation can be simplified to

$$\Delta R = \begin{bmatrix} 1 & \Delta\gamma & \Delta\beta \\ \Delta\gamma & 1 & \Delta\alpha \\ -\Delta\beta & \Delta\alpha & 1 \end{bmatrix} \quad (33)$$

It can be deduced from the equation that the posture to be adjusted for machining key feature i is:

$$PCS_i^o = \begin{bmatrix} 1 & \Delta\gamma & \Delta\beta \\ \Delta\gamma & 1 & \Delta\alpha \\ -\Delta\beta & \Delta\alpha & 1 \end{bmatrix} PCS_i + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (34)$$

Considering that the absolute position accuracy and direction accuracy of milling features meet the design requirements, an error compensation strategy is proposed. According to the posture error, the NC code of machining feature is modified based on the WCS to realize the posture error compensation of the milling system under the target pose. The specific methods are as follows: (1) the milling target pose is imported into the post processor in the form of NC code as the system input; (2) according to the above error prediction value, the error amount to be compensated for the target pose is determined; (3) after the compensation is completed, the motion command is input to the motion control system in the form of NC code to control the motion of the machine tool to obtain the right pose.

6. Case study

To validate the feasibility of the proposed comprehensive compensation method, an example is given to show the application. The object of which is an aerospace thin-walled shell part with multiple typical machining features shown in Fig.1.

6.1 The interaction of geometric errors in multi feature machining

The relationship between machining features and locating / datum in clamping planning is shown in Table 1. The first clamping and the second clamping share the positioning / reference ②, and the features ④ and ⑨ processed in the first clamping are the positioning / reference plane of the second clamping. After the first clamping process, there will be machining error. When the second clamping takes the feature with the error as the reference / locating plane, the machining error caused by the first clamping will be transferred and superimposed into the second clamping machining.

According to the datum / locating error analysis in section 3, the positioning / datum plane affects the parallelism error of features ④, ⑤ and ⑩, and also affects the verticality of features ⑥, ⑧ and ⑨ in the first clamping of the sample part. Due to the difference in the distance between the feature and the datum, there is a proportional difference in the error caused by the positioning / datum error. In addition, different layouts of positioning elements will be different. Table 2 and Table 3 respectively show the error analysis results caused by different features of complex thin-walled parts in datum conversion. Here, all positioning / datum error analysis is carried out in the workpiece coordinate system (WCS).

Table 1 Locating / datum planes and machining features in different clamps

Surface property	First clamping						Second clamping	
Machined feature	④	⑤	⑥	⑧	⑨	⑩	①	⑦
First locating surface	①						④	
ξ_{ij1}	0	0	90	90	0	90	0	90
Second locating surface	②						⑨	
ξ_{ij2}	90	90	0	0	90	0	90	0
Third locating surface	③						②	
ξ_{ij3}	30	90	90	0	0	90	90	0

6.2 The interaction of machining errors in multi feature machining

The three holes (H1~3) of the sample part are clamped and machined at one time through five-axis end milling, as shown in Fig.13. The diameter d_1 , d_2 , and d_3 of the three holes are $\Phi 40\text{mm}$, $\Phi 40\text{mm}$ and $\Phi 25\text{mm}$. Using a cemented carbide tool with diameter $\Phi 12\text{mm}$, the three holes are machined with three axial milling forces of 50N, 100N and 150N respectively. The influence of three holes on the coaxiality of existing features (H4 and H5) under different milling forces and machining sequences is studied. The deformation is investigated according to the 16 observation points shown in Fig.13. Two circular sections are examined for each hole, and each circular section is fitted by four observation points. The connecting line of the two center of H₄ is regarded as the reference axis. According to the definition of coaxiality, 2 times ($2 \times \max(t_1, t_2)$) of the maximum distance from the center of two cross-section circles of H₅ to the reference axis is the coaxiality. The least square method is used to fit the circle.

Assumptions are: (a) no other deformation on the machined feature except clamping and milling; (b) fixtures, machine tools and cutting tools have good rigidity, that is, the deformation of fixtures, machine tools and cutting tools is not considered; and (c) the errors are generated by the current processing, and the previous processing quality is regarded as the ideal value. The coaxiality error caused by clamping and cutting deformation is predicted by FEA and verified by experiment using CMMs. The analysis results are shown in Fig.14.

Table 2 The geometric error caused by positioning /datum plane in the first clamping

Name	Positioning / datum				First clamping					
	Ideal coordinate	Vector direction	Coordinate error	Vector error	④	⑤	⑥	⑧	⑨	⑩
First datum	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	0.233	0.145	0.2875	0.14	0.182	0.294
Second datum	$\begin{pmatrix} 67.5 \\ 0 \\ 65 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	0.203	0.105	0.307	0.11	0.202	0.219
Third datum	$\begin{pmatrix} 0 \\ 32 \\ 65 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	0	0	0.040	0	0.141	0.045
Locating surface	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$						

Table 3 The geometric error caused by positioning /datum plane in the second clamping

Name	Positioning /datum				Second clamping	
	Ideal coordinate	Vector direction	Coordinate error	Vector error	①	⑦
First datum	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0.233 \\ 0.203 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	0.456	0.451
Second datum	$\begin{pmatrix} 67.5 \\ 0 \\ 65 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	-0.030	-0.025
Third datum	$\begin{pmatrix} 0 \\ 32 \\ 65 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$	-0.010	-0.010
Locating surface	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$

Fig.13 Numbering and observation points of feature holes

Fig.14 Effect of different machining sequence and milling force on coaxiality of H4 and H5

According to the simulation results shown in Fig.14, the hole machining sequence of "213" has the least impact on the coaxiality of holes 4 and 5 under the action of milling forces 50N and 100N, both of which are reduced by 0.2%. When the milling force is 150N, the coaxiality of holes 4 and 5 can be reduced by 0.63% in the machining sequence of "312" and "321". It can be seen that the milling force directly affects the machining sequence of features. Different milling forces and reasonable feature machining sequence can reduce the mutual machining influence between features.

6.3 Error reduction based on posture adjustment

In this section, the coaxiality of H₄ and H₅ is mainly investigated, the tolerance is 0.2 mm. To. Since the adjacent feature H₁, H₂ and H₃ affect the coaxiality of features H₄ and H₅, the machining analysis is carried out with the machining sequence of 321 and the milling force of 150N. In the coordinate system shown in Fig.15, the posture matrix of H₁ ∩ H₅ is listed in Table 4. According to the calculation and analysis in sections 5.1, 5.2 and Eq.(29), the final error value after machining the characteristic holes is shown in Table 5. Therefore, the comprehensive coaxiality error of H₄ and H₅ is

$$\overset{1}{\Delta}_{tol} = \overset{1}{\Delta}_g + \overset{1}{\Delta}_m = 0.290\text{mm} + 0.008\text{mm} = 0.298\text{mm} \quad (35)$$

Table 4 Posture matrixes of machining feature holes

Feature	Position
H1	$\begin{bmatrix} -0.717 & 0 & 0 & 0 & 164 & -37 \\ 0 & -0.5 & 0 & -164 & 0 & 21 \\ 0 & 0 & 0.5 & 37 & -21 & 0 \\ 0 & 0 & 0 & -0.717 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}$
H2	$\begin{bmatrix} 0.717 & 0 & 0 & 0 & 173 & 27 \\ 0 & 0.5 & 0 & -173 & 0 & -46 \\ 0 & 0 & -0.5 & -27 & 46 & 0 \\ 0 & 0 & 0 & 0.717 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix}$
H3	$\begin{bmatrix} 0.717 & 0 & 0 & 0 & 140 & 15 \\ 0 & 0.5 & 0 & -140 & 0 & -25 \\ 0 & 0 & -0.5 & -15 & 25 & 0 \\ 0 & 0 & 0 & 0.717 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix}$
H4	$\begin{bmatrix} 1 & 0 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

H5

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -130 & 0 \\ 0 & 1 & 0 & 130 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig.15 Experimental clamping in milling process of thin-walled parts

Thus, it can be seen, the accuracy requirement of the coaxiality cannot meet the tolerance requirements. According to the posture error, the NC code of H5 is modified based on the WCS to realize the posture error compensation of the milling system under the target pose. The coaxiality of the thin-walled part is tested by the equipment three-coordinate measuring instrument. The test results are shown in Table 6. Through the coaxiality test value, it can be seen that the coaxiality prediction of H4 and H5 is relatively close to the experimental value, which proves the effectiveness of the prediction algorithm. The coaxiality error after compensation is significantly reduced, and the accuracy is improved by 53.69%, which can meet the machining requirements, indicating that the compensation algorithm is also effective.

Table 5 Error value of feature $H_1 \sim H_5$ after machining

Error vector	H_1	H_2	H_3	H_4	H_5
x	$\begin{pmatrix} -0.1227 \end{pmatrix}$	$\begin{pmatrix} 0.1547 \end{pmatrix}$	$\begin{pmatrix} 0.2387 \end{pmatrix}$	$\begin{pmatrix} -0.0348 \end{pmatrix}$	$\begin{pmatrix} 0.1048 \end{pmatrix}$
y	$\begin{pmatrix} -0.0980 \end{pmatrix}$	$\begin{pmatrix} -0.1220 \end{pmatrix}$	$\begin{pmatrix} -0.1700 \end{pmatrix}$	$\begin{pmatrix} 0.06068 \end{pmatrix}$	$\begin{pmatrix} -0.0828 \end{pmatrix}$
z	$\begin{pmatrix} 0.2620 \end{pmatrix}$	$\begin{pmatrix} 0.0660 \end{pmatrix}$	$\begin{pmatrix} 0.0300 \end{pmatrix}$	$\begin{pmatrix} -0.0680 \end{pmatrix}$	$\begin{pmatrix} 0.1680 \end{pmatrix}$
α	$\begin{pmatrix} 0.0001 \end{pmatrix}$	$\begin{pmatrix} 0.0059 \end{pmatrix}$	$\begin{pmatrix} 0.0059 \end{pmatrix}$	$\begin{pmatrix} 0.0010 \end{pmatrix}$	$\begin{pmatrix} 0.0070 \end{pmatrix}$
β	$\begin{pmatrix} 0.0010 \end{pmatrix}$	$\begin{pmatrix} 0.0050 \end{pmatrix}$	$\begin{pmatrix} 0.0050 \end{pmatrix}$	$\begin{pmatrix} 0.0030 \end{pmatrix}$	$\begin{pmatrix} 0.0070 \end{pmatrix}$
γ	$\begin{pmatrix} 0.0030 \end{pmatrix}$	$\begin{pmatrix} -0.0010 \end{pmatrix}$	$\begin{pmatrix} -0.0010 \end{pmatrix}$	$\begin{pmatrix} 0.0050 \end{pmatrix}$	$\begin{pmatrix} 0.005 \end{pmatrix}$

Table 6 Comparison of coaxiality pre-and post- compensation

Comparison	Forward	Measurement	Backward
Coaxiality	0.298	0.305	0.161

7. Conclusions

In this study, a comprehensive error model is established to predicting coupling

effects between features caused by machining, mainly on the final machining error of features. By analyzing the influencing factors between features, two coupling modes are determined: geometric interaction and machining interaction. On this basis, a comprehensive error model is developed by combining the theory of SDT and homogeneous coordinate conversion. The control actions were determined using the method of adjusting posture according to the comprehensive error model.

In order to verify the accuracy and effectiveness of the error model and control method, a case of a thin-walled part was presented. In the case study report, the geometric interaction and machining interaction in the multi feature machining process of thin-walled parts are analyzed respectively, and the coupling error prediction value is given. According to the error prediction value, the corresponding error control method is implemented. The results show that the method in this paper can effectively improve the machining accuracy of features. The experiment results show that the machining accuracy of the feature match well with the really measured ones, and the error is reduced by 47.21% with the help of the proposed control method.

Author contribution

Chengjun Zhang contributes to investigation, data curation, formal analysis and writing original draft. Xiaoyan Zuo contributes to conceptualization, methodology, writing-review and editing. Chi Zhang contributes to investigation. Xiangyang Zhou contributes to supervision. Yin Zhou and Tao Xiong particularly contributes to reviewing the simulation works in Section 4.

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