Pairwise $Q_{\infty}O$ - Compact Spaces in Bitopological Spaces

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PAIRWISE Q^*O - COMPACT SPACES IN BITOPOLOGICAL SPACES

P. PADMA AND S. UDAYAKUMAR

ABSTRACT. Murugalingam and Lalitha [18, 19] introduced the concepts of Q^* - closed sets in a topological space. The same concept was extended to bitopological spaces by K. Kannan and K. Chandrasekhara Rao [16]. In this paper we introduced a new type of compact spaces called pairwise Q^*O - compact spaces and study its properties .

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1. INTRODUCTION

The notions of compactness is useful and fundamental notions of not only general topology but also of other advanced branches of mathematics . Many researchers have investigated the basic properties of compactness . The productivity and fruitfulness of these notions of compactness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness have been introduced and investigated. Balachandran , Sundaram and Maki [2] introduced a class of compact space called GO - compact space and GO - connected space using g - open cover. The main focus of this paper is to introduce a new type of compact spaces called pairwise Q^*O - compact spaces and study its properties .

2. PRELIMINARIES

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, $\tau_i - int(A)$ and $\tau_i - cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i , respectively. A^C denotes the complement of A in X unless explicitly stated.

Definition 1. [16] A subset A of a bitopological spaces (X, τ_1, τ_2) is called i) $\tau_1\tau_2-Q^*$ - closed if $\tau_1 - \text{int} (A) = \phi$ and A is τ_2 - closed . ii) $\tau_1\tau_2 - Q^*$ - open if $X - A$ is $\tau_1\tau_2 - Q^*$ - closed in X .

Definition 2. [18] The set of all Q^* - closed sets with X is a topology . It is denoted by τ_{Q^*} . For simplicity , write $\tau_{Q^*} = \sigma^*$.

Definition 3. [15] A bitopological space X is pairwise S^*GO - compact if every pairwise s^*g - open cover of X has a finite sub cover .

Definition 4. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise pre semi - closed if $f : (X, \tau_1) \rightarrow (Y, \sigma_1)$ and $f : (X, \tau_2) \rightarrow (Y, \sigma_2)$ are pre semi closed .

Definition 5. [22] Let (X, τ_1, τ_2) be a bitopological space . Then (X, τ_1, τ_2) is said to be pairwise Q^* - connected if X cannot be expressed as the union of 2 non empty disjoint sets A and B such that $(A \cap \tau_1 - Q^*cl(B)) \cup (\tau_2 - Q^*cl(A) \cap B) = \phi \dots \dots (1)$ If (1) is satisfied , we call A and B as pairwise Q^* separated sets . If $X = A \cup B$, where A, B satisfy (1) , then X is called a pairwise Q^* disconnected space . In this case , we write $X = A \setminus B$ a pairwise Q^* separation of X .

Definition 6. (X, τ) is called an s - normal space if given two disjoint closed sets A and B in X , there exist disjoint semi open neighbourhoods U and V of A and B respectively .

Definition 7. [23] A space (X, τ_1, τ_2) is said to be pairwise $Q^* - R_1$ if for each $x, y \in X$, $\tau_i - Q^*cl(x) \neq \tau_j - Q^*cl(y)$, there exist disjoint sets $U \in Q^*O(X, \tau_j)$ and $V \in Q^*O(X, \tau_i)$ such that $\tau_j - Q^*cl(x) \subset \tau_U$ and $\tau_j - Q^*cl(y) \subset V$ where $i, j = 1, 2$ and $i \neq j$.

Definition 8. [25] A bitopological space X is said to be pairwise countably compact if every proper τ_i - closed set is τ_j - countably compact. $i \neq j$, $i, j = 1, 2$. If $i = 1, j = 2$, then X is said to be $(2, 1)$ - countably compact .

Definition 9. [13] A bitopological space X is said to be pairwise compact in the sense of Fletcher, Hoyle and Patty [FHP] (to be abbreviated as FHP compact) if every pairwise open cover of X has a finite subcover.

3. PAIRWISE Q^*O - COMPACT SPACES

In 1995 , sg compact spaces were introduced independently by Caldas[4] . According to him , a topological space (X, τ) is called sg - compact if every cover of X by sg open sets has a finite subcover. Devi , Balachandran and Maki [5] defined the same concept and they used the term SGO - compactness . In the sequel pairwise Q^*O - compact spaces is introduced in this section .

Definition 10. A nonempty collection $\zeta = \{A_i, i \in I; \text{an index set}\}$ is called a pairwise Q^* - open cover of a bi space (X, τ_1, τ_2) . If $X = \cup A_i$ and $\zeta \subseteq \tau_1 - Q^*O(X, \tau_1, \tau_2) \cup \tau_2 - Q^*O(X, \tau_1, \tau_2)$ and ζ contains atleast one member of $\tau_1 - Q^*O(X, \tau_1, \tau_2)$ and one member of $\tau_2 - Q^*O(X, \tau_1, \tau_2)$.

Definition 11. A bitopological space (X, τ_1, τ_2) is pairwise Q^*O - compact if every pairwise Q^* open cover of X has a finite subcover .

Definition 12. A set A of a bitopological space (X, τ_1, τ_2) is pairwise Q^*O - compact relative to X if every pairwise Q^* open cover of A has a finite subcover as a subspace.

Example 1. Let $\zeta = \{G_\alpha : \alpha \in \Lambda\}$ be a pairwise Q^* - open covering for X so that each G is a pairwise Q^* - open set and $X = \cup \{G_\alpha : \alpha \in \Lambda\}$. G_α^c is the complement of G is a finite set by definition of cofinite topology . Therefore , $G_\alpha^c = \{x_1, x_2, \dots, x_n\}$ ie) a finite set . Now each element of G_α^c is also an element of X whose cover is ζ and hence each member of G_α^c is contained in one or other member of G_α . At the most for each $x_i \in G_\alpha^c \exists$ a set G_{α_i} in ζ such that $x_i \in G_{\alpha_i}$. Hence $G_{\alpha_0}^c \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \dots \cup G_{\alpha_n}$. Above relation shows that the finite collection $\zeta^* = \{G_{\alpha_0}, G_{\alpha_1}, \dots, G_{\alpha_n}\}$ is a finite pairwise Q^* - open covering for X and hence (X, τ_1, τ_2) is pairwise Q^*O - compact .

Theorem 1. If Y is $\tau_i - Q^*$ closed subset of a pairwise Q^*O - compact space (X, τ_1, τ_2) then Y is $\tau_j - Q^*O$ compact .

Proof. Let X be a pairwise Q^*O - compact space . Let $\zeta = \{A_i, i \in I; \text{an index set}\}$ be the $\tau_j - Q^*$ open cover of Y . Since Y is $\tau_1 - Q^*$ closed subset , Y^c is $\tau_i - Q^*$ open . Also $\zeta \cup Y^c = Y^c \cup \{A_i, i \in I; \text{an index set}\}$ be a pairwise Q^* - open cover of X . Since X is pairwise Q^*O - compact , $X = Y^c \cup A_1 \cup \dots \cup A_n$. Hence $Y = A_1 \cap \dots \cap A_n$. Therefore , Y is $\tau_j - Q^*O$ compact .

Since every $\tau_i - Q^*$ closed set is τ_i - closed . We have the following.

Theorem 2. If Y is $\tau_i - Q^*$ closed subset of a pairwise Q^*O - compact space (X, τ_1, τ_2) then Y is $\tau_j - Q^*O$ compact .

Theorem 3. Pairwise Q^* - continuous image of a pairwise Q^*O - compact space is pairwise Q^*O - compact .

Proof. Let (X, τ_1, τ_2) be a pairwise Q^*O - compact . Let $f : (X, \tau_1, \tau_2) \xrightarrow{\text{onto}} (X^*, \tau_1^*, \tau_2^*)$ be a pairwise Q^* - continuous . Let $\{G_i\}$ be a pairwise Q^* - open cover of X^* . $\implies \{f^{-1}(G_i)\}$ is pairwise Q^* - open cover of X . $\implies$ a finite sub cover of X [because X is pairwise Q^*O compact] $\{f^{-1}(G_1), f^{-1}(G_2), \dots, f^{-1}(G_n)\}$. $\implies G_1, \dots, G_n$ is a subcover of G_i . $\implies X^*$ is pairwise Q^*O - compact .

Remark 1. Pairwise Q^*O - compact space and pairwise Q^*T_2 - space which is not pairwise Q^* - connected . The following example supports our claim .

Example 2. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a, c\}\}$ and $\tau_2 = \{\phi, X, \{b\}\}$. Since X is pairwise Q^*O - compact , we have X is finite . Since X is pairwise Q^*T_2 , $\exists$ a τ_1 - Q^* open set $U = \{a, c\}$ and τ_2 - Q^* open set $V = \{b\}$ such that $a \in U$, $b \in V$ and $U \cap V = \phi$. $\implies X = U \cup V$ with $U \cap V = \phi$. Hence X is pairwise Q^* - disconnected .

Remark 2. A pairwise Q^*O - compact subset of a bitopological space X is need not be $\tau_2 - Q^*$ closed . The following example supports or claim .

Example 3. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a, b, c\}, \{a, b, d\}, \{c, d\}\}$. Let $A = \{a, c, d\}$. Now $A \subset \{a, b\} \cup \{c, d\}$. Hence by definition A is pairwise Q^*O - compact set. But A is not $\tau_2 - Q^*$ closed its complement b is not $\tau_2 - Q^*$ open .

Remark 3. A pairwise Q^*O - compact space which is not pairwise Q^* - Hausdorff . The following example supports our claim .

Example 4. In example 1 , X is finite therefore if it is pairwise Q^*O - compact , But X is not pairwise Q^* - Hausdorff because for distinct points a and b or disjoint $\tau_1 - Q^*$ open and $\tau_2 - Q^*$ open sets containing a and b or disjoint neighborhoods of a and b . Hence it is not pairwise Q^* Hausdorff .

Definition 13. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is pre Q^* - closed if $f(U)$ is Q^* - closed in X for every Q^* - closed set in Y .

Definition 14. A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is pairwise pre Q^* - closed if $f : (X, \tau_1) \longrightarrow (Y, \sigma_1)$ and $f : (X, \tau_2) \longrightarrow (Y, \sigma_2)$ are pre Q^* - closed .

Theorem 4. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be pairwise Q^* - continuous , bijective and pairwise pre Q^* - closed . Then the image of a pairwise Q^*O - compact space under f is pairwise Q^*O - compact .

Proof. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be pairwise Q^* - continuous , bijective and pairwise pre Q^* - closed . Let X be a pairwise Q^*O - compact . Let $\zeta = \{A_i, i \in I; \text{an index set}\}$ be the pairwise Q^* - open cover of Y . Then $Y = \cup A_i$ and $\zeta \subseteq \sigma_1 - Q^*O(X, \tau_1, \tau_2) \cup \sigma_2 - Q^*O(X, \tau_1, \tau_2)$ and $f^{-1}(\zeta)$ contains atleast one member of $\tau_1 - Q^*O(X, \tau_1, \tau_2)$ and one member of $\tau_2 - Q^*O(X, \tau_1, \tau_2)$. Therefore , $X = f^{-1}(\cup A_i) = \cup f^{-1}(A_i)$ and $f^{-1}(\zeta) \subseteq \tau_1 - Q^*O(X, \tau_1, \tau_2) \cup \tau_2 - Q^*O(X, \tau_1, \tau_2)$ and $f^{-1}(\zeta)$ contains atleast one member of $\tau_1 - Q^*O(X, \tau_1, \tau_2)$ and one member of $\tau_2 - Q^*O(X, \tau_1, \tau_2)$. Therefore , $f^{-1}(\zeta)$ is the pairwise Q^* - open cover of X . Since

X is pairwise Q^*O - compact , we have $X = f^{-1}(\cup A_i)$, $i = 1$ to n . $\Rightarrow Y = f(x) = \cup A_i$, $i = 1$ to n . Hence ζ has the finite subcover . Therefore , Y is the pairwise Q^*O - compact .

Theorem 5. *If (X, τ_1, τ_2) is pairwise Q^*O - compact space then both (X, τ_1) and (X, τ_2) are Q^*O - compact . .*

Proof. To prove (X, τ_1) is Q^*O - compact space we must prove for any Q^* - open cover of X has a finite sub cover . Let $\{U_i/i \in \Lambda\}$ is a Q^* - open cover of X , implies $\{f^{-1}(U_i)/i \in \Lambda\}$ is a Q^* - open cover of X , so (X, τ_1) is Q^*O - compact . And by the same way we prove (X, τ_2) is Q^*O - compact .

Theorem 6. *Pairwise Q^* - continuous image of a pairwise S^*GO - compact space is a pairwise S^*GO - compact space .*

Proof. Let (X, τ_1, τ_2) is pairwise S^*GO - compact and let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be pairwise Q^* - continuous , surjection . We shall show that (Y, σ_1, σ_2) is a pairwise S^*GO - compact space . Let $\{U_i/i \in \Lambda\}$ be any pairwise Q^* - open cover of Y then $\{f^{-1}(U_i)/i \in \Lambda\}$ is a pairwise Q^* - open cover of X . Since every Q^* open set is S^*g - open , we have $f^{-1}(U_i)$ is a pairwise S^*g - open cover of Y . So there exists $i_1, i_2, \dots, i_n \in \Lambda$ such that $\{f^{-1}(U_{i_j}), j = 1, 2, \dots, n\}$ covers X and since f is onto then $\{(U_{i_j}), j = 1, 2, \dots, n\}$ is a finite subcover of Y . Hence Y is a pairwise S^*GO - compact .

Theorem 7. *If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise Q^* - continuous and X is pairwise Q^* - connected , then Y is pairwise Q^* - connected .*

Proof. Suppose that Y is not pairwise Q^* - connected. Let $Y = A \cup B$ where A and B are disjoint non-empty $\tau_i - Q^*$ - open and $\tau_j - Q^*$ - open sets in Y . Since f is pairwise Q^* - continuous and onto , $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non - empty τ_i - open and τ_j - open sets in X . This contradicts the fact that X is Q^* - connected . Hence Y is Connected .

Theorem 8. *If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise Q^* - irresolute surjection and X is pairwise Q^* - connected , then Y is pairwise Q^* - connected .*

Proof. If possible, assume that Y is not pairwise Q^* - connected. Then $Y = A \cup B$ where A and B are nonempty , disjoint and $\tau_i - Q^*$ - open and $\tau_j - Q^*$ - open sets in Y . f is pairwise Q^* - irresolute , where $f^{-1}(A)$ and $f^{-1}(B)$ are $\tau_i - Q^*$ open and $\tau_j - Q^*$ open sets in X . Since f is onto , $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty. Now $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$. Thus X is a union of disjoint, nonempty and $\tau_i - Q^*$ - open and $\tau_j - Q^*$ - open sets in X . This contradicts the fact that X is pairwise Q^* - connected . Therefore, Y is pairwise Q^* - connected.

Theorem 9. *If A and B are two pairwise Q^*O - compact subsets of a bitopological space (X, τ_1, τ_2) then $A \cup B$ is pairwise Q^*O - compact subset of X .*

Proof. Given A and B are two pairwise Q^*O - compact subsets of a bitopological space (X, τ_1, τ_2) . We shall prove that $A \cup B$ is pairwise Q^*O - compact subset of X , We have to prove that for any pairwise Q^* - open cover of $A \cup B$ has a finite sub cover . Let $\{U_i/i \in \Lambda\}$ be any pairwise Q^* - open cover of $A \cup B$. Then $A \cup B \subseteq \{U_i/i \in \Lambda\}$ and therefore $A \subseteq \cup U_i$ and $B \subseteq \cup U_i$, which implies that $\{\cup U_i/i \in \Lambda\}$ is an pairwise Q^* - open cover of A and B , where $i, j = 1, 2$ and $i \neq j$. But A and B are pairwise Q^*O - compact subsets . Therefore there exist $i_1, i_2, \dots, i_n \in \Lambda$ and $t_1, t_2, \dots, t_n \in \Lambda$ such that $\{U_{i_1}, U_{i_2}, \dots, U_{i_n}\}$ and $\{U_{t_1}, U_{t_2}, \dots, U_{t_n}\}$ is a finite sub cover of A and B respectively , then $\{U_{i_1}, U_{i_2}, \dots, U_{i_n}\} \cup \{U_{t_1}, U_{t_2}, \dots, U_{t_n}\}$ is a finite sub cover of $A \cup B$. Therefore $A \cup B$ is an pairwise Q^*O - compact subset of X .

Theorem 10. *Every pairwise Q^*O - compact subset of a pairwise Q^* - Hausdorff space is pairwise Q^* - closed .*

Proof. Suppose that A be a pairwise Q^*O - compact subset of a pairwise Q^* - Hausdorff space X . Since X is pairwise Q^* - Hausdorff , the subspace A is pairwise Q^* - Hausdorff . By hypothesis , A is a pairwise Q^*O - compact . Hence A is pairwise compact . Let $x \in X - A$. For every $a \in A$ we have $a \neq x$. But X is pairwise Q^* - Hausdorff . Hence there exists a $\tau_i - Q^*$ open nbds U_α of a and a $\tau_j - Q^*$ - open nbds V_α of x such that $U_\alpha \cap V_\alpha = \emptyset \dots (1)$, where $i, j = 1, 2$ and $i \neq j$. But then the collection $\zeta = \{U_a : a \in A\}$ is an pairwise Q^* - open cover of A . But A is pairwise compact . Hence ζ has a finite sub collection $\{U_{a_1}, U_{a_2}, \dots, U_{a_n}\}$ covering A . Put $U = U_{a_1} \cup U_{a_2} \cup \dots \cup U_{a_n}$. Then U is an $\tau_i - Q^*$ open set with $A \subset U$. Consider the corresponding $\tau_j - Q^*$ open sets $V_{a_1}, V_{a_2}, \dots, V_{a_n}$. Write $V = V_{a_1} \cap V_{a_2} \cap V_{a_n}$. Then V is an $\tau_j - Q^*$ open set with $x \in V$. By virtue of (1) , $U \cap V = \emptyset \implies x \in U \subset X - V \subset X - A \implies x \in U \subset X - A \implies X - A$ is $\tau_i - Q^*$ open. A is $\tau_i - Q^*$ closed . Similarly , A is $\tau_j - Q^*$ closed . Hence A is pairwise Q^* - closed .

Theorem 11. *Let X be a pairwise Q^*O - compact space and Y be a pairwise Q^* - Hausdorff space . If $h : X \rightarrow Y$ is a pairwise Q^* - continuous bijection , then h is a pairwise Q^* - homeomorphism .*

Proof. It is given h is a pairwise Q^* - continuous and bijection . To P.T h is a pairwise Q^* - homeomorphism . It is enough to show that h is an pairwise Q^* - open map . Let G be any $\tau_i - Q^*$ open set in X . Let $A = X - G$. Then $X - G$ is a $\tau_i - Q^*$ closed set in X . Since X is pairwise Q^*O - compact , by theorem 2 , A is also pairwise Q^*O - compact . Using theorem 4 , since h is pairwise Q^* - continuous , we find that $h(A)$ is pairwise Q^*O - compact in Y . But Y is pairwise

Q^* - Hausdorff and by theorem 6 , it follows that $h(A)$ is a τ_i - Q^* closed in Y . But then $Y - h(A) = h(X) - h(A) = h(X - A) = h(G)$. Therefore , $h(G)$ is τ_i - Q^* open in Y . Consequently , h is an τ_i - Q^* open map . Similarly , h is an τ_j - Q^* open map . Therefore , h is pairwise Q^* - open map . Already h is pairwise Q^* - continuous and bijection . Hence h is a pairwise Q^* - homeomorphism .

Theorem 12. *A pairwise Q^* - continuous , pairwise Q^* - irresolute image of an pairwise Q^*O - compact space is pairwise Q^*O - compact .*

Theorem 13. *A pairwise Q^* - closed subset of pairwise Q^*O - compact space is pairwise Q^*O - compact relative to X .*

Proof. Let A be a pairwise Q^* - closed subset of a pairwise Q^*O - compact space X . Then A^c is pairwise Q^* - open in X . Let S be a cover of A by pairwise Q^* - open sets in X . Then , $\{S, A^c\}$ is a pairwise Q^* - open cover of X . Since X is pairwise Q^*O - compact , it has a finite subcover, say $\{G_1, G_2, \dots, G_n\}$. If this subcover contains A^c , we discard it. Otherwise leave the subcover as it is. Thus , we have obtained a finite pairwise Q^* - open subcover of A and so A is pairwise Q^*O - compact relative to X .

Theorem 14. *If a map $f : X \rightarrow Y$ is pairwise Q^* - irresolute and a subset B is pairwise Q^*O - compact relative to X , then the image $f(B)$ is pairwise Q^*O - compact relative to Y .*

Proof. Let $\{A_i, i \in I; \text{an index set}\}$ be any collection of pairwise Q^* - open subsets of Y such that $f(B) \subseteq \cup\{A_i, i \in I; \text{an index set}\}$. Then , $B \subseteq \cup\{f^{-1}(A_i) : i \in I\}$. By using assumptions there exists a finite subset I_o of I such that $B \subseteq \{f^{-1}(A_i) : i \in I_o\}$. Therefore, we have $f(B) \subseteq \{A_i : i \in I_o\}$ which shows that $f(B)$ is pairwise Q^*O - compact relative to Y .

Definition 15. *(X, τ_1, τ_2) is called an pairwise s - normal space if given two disjoint τ_1 - closed set A and τ_2 - closed set B in X , there exist disjoint τ_1 - semi open neighbourhood U of A and τ_2 - semi open neighbourhood V of B respectively .*

Theorem 15. *Suppose that A is a pairwise Q^*O - compact subset of a pairwise Q^* - Hausdorff space X . Let $x \in X - A$. Then there exist disjoint τ_1 - Q^* open neighborhood U of A and τ_2 - Q^* open neighborhood V of X respectively .*

Proof. By hypothesis , X is pairwise Q^* - Hausdorff . Let $a \in A$ arbitrarily . Then there exist disjoint τ_1 - Q^* open neighborhoods U_a of A and τ_2 - Q^* open neighborhoods V_x of X respectively . The collection $\zeta = \{U_a : a \in A\}$ is a pairwise Q^* - open cover of A . But A is pairwise Q^*O - compact . Accordingly , this collection ζ has a finite sub cover $\{U_{a_1}, U_{a_2}, \dots, U_{a_n}\}$. Let $U = U_{a_1} \cup \dots \cup U_{a_n}$. Put

$V = V_{a_1} \cap \dots \cap V_{a_n}$. Then $A \subset U$ and $x \in V$. Also U is $\tau_1 - Q^*$ open and V is $\tau_2 - Q^*$ open. Since $U_{a_i} \cap V_{a_i} = \phi$ for $1 \leq i \leq n$. We obtain that $U \cap V = \phi$. We have proved the result.

Lemma 16. *If, in a pairwise $Q^* - R_1$ space (X, τ_1, τ_2) , A is a $\tau_i - Q^*O$ - compact subset of X and $x \in X$ such that $\sigma_i^* - cl(x) \cap A = \phi$, then there exists a $\tau_i - Q^*$ - open set U and a $\tau_j - Q^*$ - open set V such that $x \in V$, $A \subset U$, $U \cap V = \phi$; $i \neq j$, $i, j = 1, 2$.*

Proof. For each $y \in A$, $\sigma_i^* - cl(x) \neq \sigma_j^* - cl(y)$ so that there exists a $\tau_i - Q^*$ - open set U_y and a $\tau_j - Q^*$ - open set V_y such that $x \in V_y$, $y \in U_y$, $U_y \cap V_y = \phi$; $i \neq j$, $i, j = 1, 2$. Thus $\{U_y : y \in A\}$ is a $\tau_i - Q^*$ - open cover of the $\tau_i - Q^*O$ - compact set A , so that there exists a finite number points of A such that $A \subseteq \cup_{i=1}^n U_{y_i} = U$ (say). Let $V = \cap_{k=1}^n V_{y_k}$. Then U is a $\tau_i - Q^*$ - open set and V is a $\tau_j - Q^*$ - open set such that $x \in V$, $A \subseteq U$, $U \cap V = \phi$; $i \neq j$, $i, j = 1, 2$.

Theorem 17. *Every pairwise Q^*O - compact, Q^* - Hausdorff space is pairwise s - normal.*

Proof. Suppose that X is a pairwise Q^*O - compact, pairwise Q^* - Hausdorff space. Let A be $\tau_1 - Q^*$ closed and B be $\tau_2 - Q^*$ closed subsets of X . Since every Q^*O - compact space is semi compact we have A is semi compact. It follows, by theorem 3.18, that for each $x \in B$ there exist disjoint τ_1 - semi open set U_x and τ_2 - semi open set V_x such that $x \in U_x$ and $B \subset V_x$. The collection $\zeta = \{U_x : x \in B\}$ is a pairwise semi open cover of B . But B is pairwise - semi compact. Hence the collection ζ has a sub cover $\{U_{x_1}, U_{x_2}, \dots, U_{x_n}\}$. Let $U = U_{x_1} \cup \dots \cup U_{x_n}$. Put $V = V_{x_1} \cup \dots \cup V_{x_n}$. Then U is τ_1 - semi open and V is τ_2 - semi open with $A \subset U$ and $B \subset V$. Since $U_{x_i} \cap V_{x_i} = \phi$ for $1 \leq i \leq n$. It follows that $U \cap V = \phi$. Hence theorem holds.

Theorem 18. *If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \tau_1^*, \tau_2^*)$ is pairwise Q^* - closed and pairwise Q^* - continuous and (Y, τ_1^*, τ_2^*) is Pairwise - Q^*O - compact then (X, τ_1, τ_2) is also so.*

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \tau_1^*, \tau_2^*)$ is pairwise Q^* - closed and pairwise Q^* - continuous mapping. Let $\{H_\alpha : \alpha \in I\}$ be any pairwise Q^* - open covering of $f(X)$ in (Y, τ_1^*, τ_2^*) . Then $\{f^{-1}(H_\alpha) : \alpha \in I\}$ is a pairwise Q^* - open covering of (X, τ_1, τ_2) . Since X is pairwise Q^*O - compact, therefore there is a finite sub covering $\{f^{-1}(H_{\alpha_k}) : k = 1, 2, \dots, n\}$ of X . Hence $\{H_{\alpha_k} : k = 1, 2, \dots, n\}$ forms a required sub covering of $\{H_\alpha : \alpha \in I\}$. Hence the theorem.

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