Combined Polar Codes and 2-12 QAM over AWGN and Rayleigh Fading Channels

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RESEARCH

Combined polar codes and 2-12 QAM over AWGN and Rayleigh fading channels

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Abstract

This article investigates a communication system using polar codes combined with a 2-12 QAM modulation scheme over channels disturbed by additive white Gaussian noise (AWGN) as well as over channels disturbed by Rayleigh fading in addition to AWGN. The 2-12 QAM modulation is compatible with legacy 16 QAM still widely used, and when combined with appropriate error correcting codes produces results that approach the Shannon limit.

Keywords: Polar Codes; Quadrature Amplitude Modulation (QAM); Rayleigh Fading Channel

1 Introduction

Quadrature amplitude modulation (QAM) has been included in current wireless communication standards due to its high bandwidth efficiency [1]. The 16 QAM constellation with Gray code mapping, performs well and in combination with LDPC codes or turbo codes it can approach the Shannon limit [2], [3]. High power linear amplifiers are required at the transmitter side for some practical applications employing QAM modulation [4]. However, due to non-linear distortions caused by the output of power amplifiers, the Shannon limit is far from being reached [5]. Various linearization techniques [6], [7] have been introduced to reduce the non-linear effect, but changing hardware makes implementation difficult due to non-compatibility with systems already in use [8].

The recently proposed multidimensional 2-12 QAM modulation [8] presents relevant energy gains using just software mapping changes in the existing 16 QAM modulation. The 2-12 QAM modulation scheme uses two 16 QAM constellations, and eliminates the four highest energy symbols in each 16 QAM constellation to reduce the peak-to-average power ratio (PAPR). The 2-12 QAM maps 7 bits into a 4-dimensional symbol.

In a wireless mobile communication system, the transmitted signals suffer alterations due to different deleterious effects caused by the channel. As a consequence such signals might reach the receiver through multiple paths with different phases and amplitudes [9] thus causing signal fading. Signal fading can be characterized by a random process governing the magnitude of the transmitted signal [10] reaching the receiver. If there is no line of sight signal reception then the envelope of the received signal can be modeled by a Rayleigh probability distribution function [11].

To minimize the effect of noise in transmissions, error correcting codes are used to add redundant bits to the sequence to be sent in order to improve reliability [12].

The error correction capability of polar codes allowed them to be selected by the 3rd Generation Partnership Project (3GPP) as the code for the control channel in the 5th generation (5G) of mobile communication systems [13], [14]. Polar codes present low complexity encoding and decoding algorithms [15].

In usual practice, the block length of polar code is a power of 2, therefore not being immediately compatible for combining with some modulation schemes, such as 2-12 QAM, for example. In this case, a kernel matrix of adequate size combined with a standard kernel matrix is recommended [16]. Another alternative is the Arikan punctured kernel matrix [17], however both solutions require a high computational complexity. Another solution, which consists of appending a suffix of zeros to each codeword, can be employed in order to minimize computational complexity [18].

A polar code is designed to operate efficiently in a specific channel, and thus it is a challenge to use polar codes in a practical situation where the communication channel changes with time, as occurs in the presence of fading during transmission in free space. The problem presented by the use of polar codes over fading channels is addressed in [19], considering an additional constraint on the permissible average and peak power. Their model reduces the fading channel to a model consisting of a cascade of an AWGN channel followed by an erasure channel. In this article we apply polar codes to a fading channel without providing any further mitigation in order to cope with fading conditions. Our goal is to analyze the performance of polar codes combined with 2-12 QAM modulation in AWGN channels and Rayleigh fading channels disturbed by AWGN, when compared to legacy 16 QAM modulation under similar noise conditions, and discuss the results obtained by computer simulation.

The remaining sections of this paper are organized as follows. In Section 3, the 2-12 QAM modulation is described. A review of polar codes is presented in Section 4. The system model is described in Section 5. The simulated results are presented in Section 6. Finally, the conclusions are presented in Section 7.

2 Methods/Experimental

The studies that take place on this manuscript require data obtained from computational simulations with the goal of properly describe the BER performance of polar codes combined with 2-12 QAM modulation in AWGN channels and Rayleigh fading channels disturbed by AWGN, when compared to legacy 16 QAM modulation under similar noise conditions. The data acquired through simulations are then compared to other set of data that can be found on [8] for example. The authors would like to take the opportunity to state that there was no need to collect data from human beings given the purely computational nature of our study.

3 2-12 QAM

Fig. 1 illustrates a scheme used for data transmission in several communication systems employing a rate $3/4$ error correcting code, a 16 QAM modulator and a power amplifier (PA). The error correcting code is often referred as a forward error correcting (FEC) code [12]. The system shown in Fig. 1 has a significant drawback as the power amplifier operates with an average power well below its maximum power [20], in order to avoid non-linear distortions. Aiming to operate effectively a power back-up is required from the power amplifier, with sufficient power to feed the 16

QAM modulation when highest energy symbols occur. This leads to a significant reduction in the overall system efficiency [21]. Several techniques have been introduced in order to reduce this undesirable effect, such as non-linearity correction and reduction of PAPR, among others [6], [7], [22], [23], however significant changes are required to overcome this negative effect and, because of that, questions about compatibility with current systems are raised.

In a 16 QAM system [24], a signal $s_i(t)$, $0 < t \leq T$, received in a time slot T is represented as

$$s_i(t) = A_x \cos(2\pi f_c t) + B_y \sin(2\pi f_c t), \quad (1)$$

where $1 \leq i \leq 16$, A_x and B_y are the amplitudes corresponding to axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, and f_c denotes the carrier frequency. The corresponding 16 QAM constellation is illustrated in Fig. 2, and this format consists of 16 2-dimensional vectors, denoted by

$$s_i \in \{[A_x, B_y]^T\}, \quad (2)$$

where $i \in \{1, \dots, 16\}$, $A_x \in \{(2x - \sqrt{M} - 1)d\}_{x=1}^{\sqrt{M}}$, $B_y \in \{(2y - \sqrt{M} - 1)d\}_{y=1}^{\sqrt{M}}$ and the distance between adjacent symbols is equal to $2d$. The parameter d is related to the signal energy [24].

In order to use the 2-12 QAM modulation the transmission system of Fig. 1 should be replaced by the transmission system illustrated in Fig. 3, consisting of a rate 6/7 FEC code, a 2-12 QAM modulator and a PA. The 2-12 QAM modulation is based on the legacy 16 QAM modulation with mapping changes and offers energy gain and compatibility with current systems [8].

The 2-12 QAM is a multidimensional modulation obtained by eliminating the four highest energy symbols from two 2-dimensional 16 QAM constellations similar to the one depicted in Fig. 2. The eliminated symbols are those indicated by s_1 , s_4 , s_{13} and s_{16} , situated in the regions marked with $\times$ in Fig. 4. As a result we have two constellations with twelve symbols each (2-12 QAM) to be mapped into a 4-dimensional signal.

The 16 QAM modulation combined with a rate 3/4 FEC has spectral efficiency of 3 bits/symbol [25]. In order to maintain the same spectral efficiency in a 2-12 QAM

Figure 2 Diagram of 2-dimensional 16 QAM constellation

Figure 3 Block diagram of a transmitter for a 2-12 QAM system with a rate 6/7 FEC code

system, a rate 6/7 FEC and a new mapping is employed, by which a 7 bit block is associated to 2 twelve-ary symbols as illustrated in Fig. 3, by the block labeled as 7B - 2TW mapping. One advantage of this format is the possibility of using a 16 QAM modulator/demodulator hardware and obtain 2-12 QAM modulation through a software update [8].

Figure 4 Diagram of 4-dimensional 2-12 QAM constellation

As explained in [8], a 2-12 QAM signal $s_{ij}(t)$, $0 < t \leq 2T$, received in time slot $2T$ is represented as

$$s_{ij}(t) = \begin{cases} A_x \cos(2\pi f_c t) + B_y \sin(2\pi f_c t), & 0 < t \leq T \\ C_s \cos(2\pi f_c t) + D_z \sin(2\pi f_c t), & T < t \leq 2T, \end{cases} \quad (3)$$

where the indices i and j are each associated with a distinct constellation in Fig. 4, $i \in \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$, $j \in \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$. A_x and B_y are amplitudes allowed in the axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, for one constellation, and C_s and D_z are amplitudes allowed in the axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, in the other constellation in Fig. 4, and f_c denotes the carrier frequency.

In the 2-12 QAM modulation format there are 144 (12^2) possibilities of 2-12-ary symbols, however we further eliminate the 16 highest energy symbols in order to end up with 128 symbols, which is a power of 2, as required for the mapping of bit blocks to 2-12-ary symbols. Thus, the signal constellation of 2-1 2QAM after pruning consists of 128 (2^7) 4-dimensional vectors selected, where each symbol s_{ij} is one-to-one associated to 7 information bits and is denoted by

$$s_{ij} \in \{[A_x, B_y]^T, [C_s, D_z]^T\}, \quad (4)$$

where $i \in \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$ and $j \in \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$, $A_x \in \{(2x - \sqrt{M} - 1)d\}_{x=1}^{\sqrt{M}}$, $B_y \in \{(2y - \sqrt{M} - 1)d\}_{y=1}^{\sqrt{M}}$, $C_s \in \{(2s - \sqrt{M} - 1)d\}_{s=1}^{\sqrt{M}}$ and $D_z \in \{(2z - \sqrt{M} - 1)d\}_{z=1}^{\sqrt{M}}$ and $2d$ is the distance between two adjacent symbols.

The corresponding probability density function is given by

$$\rho_{r/s_{i,j}}(\mathbf{r}) = \left(\frac{1}{\pi N_0}\right)^2 \exp \left[\frac{-(r_1 - A_x)^2 + (r_2 - B_y)^2}{N_0} \right] \times \exp \left[\frac{-(r_3 - C_s)^2 + (r_4 - D_z)^2}{N_0} \right], \quad (5)$$

where r_1, r_2, r_3 and r_4 are the components of the received vector $\mathbf{r}$ after transmission over an AWGN channel, given by

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} A_x + n_1 \\ B_y + n_2 \\ C_s + n_3 \\ D_z + n_4 \end{bmatrix}, \quad (6)$$

where n_1, n_2, n_3 and n_4 are the additive Gaussian noise components with variance $\sigma^2 = N_0/2$ and mean $\mu = 0$.

4 Polar Codes

Polar codes, proposed by Arikan [15], can achieve the symmetric capacity of binary discrete memoryless channels (B-DMC) through channel polarization. Channel polarization refers to the operation that allows synthesizing N independent copies of a B-DMC W , from a set of N binary inputs $\{W_N^{(i)}, 1 \leq i \leq N\}$ [15]. Following [15] a generic B-DMC, denoted as $W : \mathcal{X} \rightarrow \mathcal{Y}$, is considered with a binary input alphabet $\mathcal{X}$, i.e., $\mathcal{X} = \{0, 1\}$, output alphabet $\mathcal{Y}$, and transition probabilities $W(y|x), x \in \mathcal{X}, y \in \mathcal{Y}$. The goal of this technique is to select the most suitable channels for transmission of K information bits, with N encoded bits, and the code rate R is defined as $R = K/N$. The remaining $N - K$ positions in a codeword are filled with a fixed value each, known by the decoder in advance, which are called frozen bits [15]. The choice of the most suitable channels is made by the use of the Bhattacharyya parameter, defined as

$$Z(W) \triangleq \sum \sqrt{W(y|0)W(y|1)}. \quad (7)$$

There are two phases to this operation, namely a channel combining phase and a channel splitting phase. The first phase consists of combining N copies of W to produce the channel $W_N : X^N \rightarrow Y^N$ in a recursive manner, where $N = 2^n, n \geq 1$. In its first level, two copies of W_1 are combined to form the channel $W_2 : X^2 \rightarrow Y^2$ with the transition probabilities

$$W_2(y_1, y_2|u_1, u_2) = W^N(y_1|u_1 \oplus u_2)W(y_2|u_2), \quad (8)$$

where u_1, u_2 are the inputs to channel W_2 and y_1, y_2 are its outputs.

The next phase, after having synthesized the channel vector W_N out of W^N , consists of splitting W_N back into a set of N channels $W_N^{(i)} : X \rightarrow Y^N \times X^{(i-1)}, 1 \leq i \leq N$, with (y_1^N, u_1^{i-1}) as the output of $W_N^{(i)}$ and u_i as its input. The corresponding transition probabilities are defined by

$$W_N^{(i)}(y_1^N, u_1^{i-1}|u_i) \triangleq \sum_{u_{i+1}^N \in X^{N-i}} \frac{1}{2^{N-1}} W_N(y_1^N|u_1^N). \quad (9)$$

Given the bit sequence $\mathbf{u}$, the codeword $\mathbf{x}$ is generated by the following operation

$$\mathbf{x} = \mathbf{u}\mathbf{G}_N, \quad (10)$$

where $\mathbf{G}_N$ is the generator matrix of order N for the encoder [14], given by

$$\mathbf{G}_N = \mathbf{B}_N \mathbf{F}^{\otimes n}, \quad (11)$$

where $\mathbf{B}_N$ is a bit-reversal permutation, $\mathbf{F}^{\otimes n}$ is the n -th Kronecker power of $\mathbf{F}$ [15], and

$$\mathbf{F} \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}. \quad (12)$$

The decoding process takes place with the use of the Successive Cancellation (SC) algorithm, also proposed in [15]. In order to estimate the decoded message, an SC decoder calculates the probability of the value of a certain bit by making use of values found on the encoder structure. In case of decoding a frozen bit, the value 0 is automatically assigned by the decoder to that frozen bit.

5 System Model

The block diagram of the proposed system is shown in Fig. 5.

The idea is to investigate the performance of the proposed system to transmit information, when polar codes are combined with 2-12 QAM modulation over AWGN channels and Rayleigh fading channels affected by AWGN. Initially, an information source generates a sequence $\mathbf{u}$ of equiprobable bits to be encoded as indicated in (10). Then the generated codeword is split into 7 bit blocks to be mapped into 2-12-ary symbols according to the 2-12 QAM scheme described earlier. Because the block length of the polar code used in this article is a power of 2, which is not an integer multiple of 7, at least in principle, it is not compatible with the 2-12 QAM modulation. In order to circumvent this difficulty a possible approach is to develop a scheme that combines the standard polarization kernel with an elaborate 7×7 design kernel [15], [16]. In order to reduce the complexity of code construction and decoding algorithm in this article an uncoded suffix of zeros corresponding to $s = 7 - (N \bmod 7)$ is added to each codeword [18]. Then this modified codeword

$\mathbf{x}'$, with suffix s , has length N_s given by

$$N_s = N + s. \quad (13)$$

After being mapped according to the 2-12 QAM scheme, the modulated sequence is transmitted through AWGN channels and Rayleigh fading channels disturbed by AWGN, with transmission rate $R = K/N_s$. The channel output is denoted by the vector $\mathbf{r}_k$ in (14), $1 \leq k \leq N_s/7$.

$$\mathbf{r}_k = \begin{bmatrix} r_{1k} \\ r_{2k} \\ r_{3k} \\ r_{4k} \end{bmatrix} = \begin{bmatrix} \alpha A_{xk} + n_{1k} \\ \alpha B_{yk} + n_{2k} \\ \alpha C_{sk} + n_{3k} \\ \alpha D_{zk} + n_{4k} \end{bmatrix}, \quad (14)$$

where n_{1k} , n_{2k} , n_{3k} and n_{4k} are the additive Gaussian noise components with variance $\sigma^2 = N_0/2$ and mean $\mu = 0$, A_{xk} , B_{yk} , C_{sk} and D_{zk} are the set of true hypothesis, α is the fading amplitude of a channel obeying the Rayleigh distribution with probability density function given by [26]

$$p(\alpha) = 2\alpha e^{-\alpha^2}, \quad (15)$$

considering a normalized fading, that is, $E[\alpha^2] = 1$, where $E[\cdot]$ denotes the expected value operator [27].

After transmission over AWGN channels or Rayleigh fading channels disturbed by AWGN, and assuming equiprobable bits, $P(b = 0) = P(b = 1) = 1/2$, the Log Likelihood Ratio (λ_t) for bit x_t is calculated according to [28] as follows

$$\lambda(x_t) = \ln \left(\frac{\sum_k p(r_k | \alpha, x_t = 0)}{\sum_k p(r_k | \alpha, x_t = 1)} \right), \quad (16)$$

where $1 \leq t \leq N_s$. The decision rule employed is $b_t = 0$ if $\lambda \geq 0$ and $b_t = 1$ otherwise, and the probability $p(r_k | \alpha_k, x_k = b)$ can be written as (17). The last s bits provided by the 2-12 QAM demodulator are punctured and a sequence of length N is decoded through the conventional SC algorithm.

$$p(r_k | \alpha_k, x_k = b) = \sum_{ik} \sum_{jk} \frac{1}{2\pi\sigma^2} \exp \left[-\frac{(r_{1k} - \alpha_k A_{xk})^2 + (r_{2k} - \alpha_k B_{yk})^2}{2\sigma^2} \right] \frac{1}{2\pi\sigma^2} \exp \left[-\frac{(r_{3k} - \alpha_k C_{sk})^2 + (r_{4k} - \alpha_k D_{zk})^2}{2\sigma^2} \right]. \quad (17)$$

6 Results and Discussion

Fig. 6 shows performance curves expressed by bit error rate (BER) versus E_b/N_0 for a system using uncoded 2-12 QAM with average energy $E_s \approx 7.33$ dB and variance $\sigma^2 \approx 0.52/10^{(E_b/N_0)}$. For comparison purposes, a system using uncoded 16 QAM modulation format with Gray code mapping, with average energy $E_s = 10$

dB and variance $\sigma^2 \approx 1.25/10^{(E_b/N_0)}$ is also shown in Fig. 6. Observing the curves in Fig. 6, we notice that the system employing uncoded 2-12 QAM modulation presents an energy gain of approximately 3.1 dB for $\text{BER} = 10^{-4}$ when compared to the system that uses uncoded 16 QAM modulation, both systems operating over AWGN channels.

We consider next a coded communication system as indicated in Fig. 3 fed with an information sequence $\mathbf{u}$ of length $K = 880$ as the input to the encoder, using a polar code of rate $R = 6/7$ and block length 1024. A codeword $\mathbf{x}$ of length $N = 1024$ has a suffix of 5 bits added in order to be compatible with 2-12 QAM, thus resulting in a lengthened codeword $\mathbf{x}'$ of length $N_s = 1029$. The modulated sequence passes through an AWGN channel with variance $\sigma^2 \approx 0.61/10^{(E_b/N_0)}$. At the receiver, after calculation of the LLRs and puncturing the suffix, the SC algorithm is employed. The corresponding performance curves, expressed by BER versus E_b/N_0 , are also shown in Fig. 6. The system using 2-12 QAM with a polar code, when compared to uncoded 2-12 QAM presents energy gains of about 1.5 dB and 2.3 dB for $\text{BER} = 10^{-3}$ and 10^{-4} , respectively, with a block length $N = 1024$.

For comparison purposes, a coded system as illustrated in Fig. 1, with 16 QAM modulation is considered which employs Gray code mapping and a polar code with block length $N = 1024$ and rate $R = 3/4$ in AWGN channel with variance $\sigma^2 \approx 1.67/10^{(E_b/N_0)}$. The corresponding BER versus E_b/N_0 curve is shown in Fig. 6. The system employing 2-12 QAM modulation and a polar code with block length $N = 1024$ presents a gain of approximately 1.3 dB for $\text{BER} = 10^{-3}$.

Fig. 6 also shows performance curves when a polar code of block length $N = 512$ is employed. When considering 2-12 QAM modulation, the suffix to be added has length $s = 6$ bits and the length of codeword $\mathbf{x}'$ is $N_s = 518$. Considering the same spectral efficiency, performance curves for the system using 2-12 QAM with code rate $R = 6/7$ presents an energy gain of approximately 0.8 dB for $\text{BER} = 10^{-3}$

when compared to a system using 16 QAM with code rate $R = 3/4$, and block length $N = 512$. The 2-12 QAM combined with a polar code with block length $N = 1024$ presents an energy gain of about 0.6 dB for $\text{BER} = 10^{-4}$ when compared to the system that uses 2-12 QAM and a polar code with block length $N = 512$. The system using uncoded 2-12 QAM has an energy gain of about 1.2 dB for $\text{BER} = 10^{-2}$ when compared with the 16 QAM combined and a polar code with block length $N = 1024$.

Figure 7 BER versus E_b/N_0 performance over Rayleigh fading channels disturbed by AWGN of uncoded 2-12 QAM and uncoded 16 QAM, and 2-12 QAM and 16 QAM combined with polar codes

Fig. 7 shows performance curves for combined polar codes and 2-12 QAM system, indicating an energy gain of approximately 2 dB for $\text{BER} = 10^{-2}$ when compared to the system using 16 QAM modulation format with Gray code mapping, both over uncoded AWGN channels and Rayleigh fading channels disturbed by AWGN.

Fig. 7 illustrates the performance of a system using polar codes of rate $R = 6/7$, block length $N = 1024$ with an appended suffix of $s = 5$ bits to achieve compatibility with the 2-12 QAM scheme, and considering AWGN channels and Rayleigh fading channels disturbed by AWGN, when the SC decoding algorithm is used. This system presents an energy gain of about 2 dB for $\text{BER} = 10^{-3}$ over the 16 QAM modulation with Gray code mapping with polar code of rate $R = 3/4$ and block length $N = 1024$ also over AWGN channels and Rayleigh fading channels disturbed by AWGN.

The system performance when 2-12 QAM and a polar code with block length $N = 1024$ are employed, shows an energy gain of about 5.5 dB for $\text{BER} = 10^{-2}$ when compared to uncoded 2-12 QAM, both over AWGN channels and Rayleigh fading channels disturbed by AWGN.

The performance of 16 QAM combined with a polar code of block length $N = 1024$ over AWGN channels offers an energy gain of about 6 dB for $\text{BER} = 10^{-2}$ in comparison to uncoded 16 QAM over the same channels conditions. For Rayleigh fading channels disturbed by AWGN this system offers an energy gain of about 6 dB for $\text{BER} = 10^{-2}$ in comparison to uncoded 16 QAM over the same channels

conditions. Fig. 7 also shows the performance of systems using 2-12 QAM and 16 QAM both combined with a polar code of block length $N = 512$ and over AWGN channels and Rayleigh fading channels disturbed by AWGN.

7 Conclusions

This article presents performance results for a new scheme for data transmission over channels disturbed by AWGN or by Rayleigh fading plus AWGN, using 2-12 QAM modulation and polar codes. The main results are summarized as follows.

- AWGN channels, at $\text{BER} = 10^{-4}$
 - a) Uncoded 2-12 QAM has a 3 dB gain in E_b/N_0 over uncoded 16 QAM.
 - b) Coded 16 QAM has a 4 dB gain in E_b/N_0 over uncoded 16 QAM, for the same spectral efficiency, employing a polar code of block length 1024.
 - c) Coded 2-12 QAM has a 1.5 dB gain in E_b/N_0 over coded 16 QAM, for the same spectral efficiency, employing polar codes of block length 1024 in both cases.
- Rayleigh fading with AWGN channels, at $\text{BER} = 10^{-3}$
 - a) Uncoded 2-12 QAM has an estimated gain of 2.5 dB in E_b/N_0 over uncoded 16 QAM.
 - b) Coded 16 QAM has an estimated gain of at least 10 dB in E_b/N_0 over uncoded 16 QAM, for the same spectral efficiency, employing a polar code of block length 1024.
 - c) Coded 2-12 QAM has a 2 dB gain in E_b/N_0 over coded 16 QAM, for the same spectral efficiency, employing either polar codes of block length 1024 or 512.

For future work other decoding schemes such as Successive Cancellation List decoding can be used [29] as well as generalized fading channel distributions [30].

Abbreviations

3GPP: 3rd Generation Partnership Project; 5G: 5th Generation; AWGN: Additive White Gaussian Noise; B-DMC: Binary Discrete Memoryless Channels; BER: Bit Error Rate; FEC: Forward Error Correcting; PA: Power Amplifier; PAPR: Peak-to-Average Power Ratio; QAM: Quadrature Amplitude Modulation; SC: Successive Cancellation

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Authors' contributions

All authors proposed the main idea. FCMO is the main writer and the corresponding author of this paper. All authors read and approved the final manuscript.

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Availability of data and materials

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests

The authors declare that they have no competing interests.

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8 Figure Title and Legend

8.1 Figure 1: Block diagram of a transmitter for a 16 QAM system with a rate 3/4 FEC code

A scheme used for data transmission in several communication systems employing a rate 3/4 FEC, a 16 QAM modulator and a PA.

8.2 Figure 2: Diagram of 2-dimensional 16 QAM constellation

The corresponding 16 QAM constellation, and this format consists of 16 2-dimensional vectors denoted by $s_i \in \{[A_x, B_y]^T\}$ where $i \in \{1, \dots, 16\}$, $A_x \in \{(2x - \sqrt{M} - 1)d\}_{x=1}^{\sqrt{M}}$, $B_y \in \{(2y - \sqrt{M} - 1)d\}_{y=1}^{\sqrt{M}}$ and the distance between adjacent symbols is equal to $2d$.

8.3 Figure 3: Block diagram of a transmitter for a 2-12 QAM system with a rate 6/7 FEC code

A transmission system consisting of a rate 6/7 FEC code, a 2-12 QAM modulator and a PA.

8.4 Figure 4: Diagram of 4-dimensional 2-12 QAM constellation

The corresponding 2-12 QAM constellation, obtained by eliminating the four highest energy symbols from two 2-dimensional 16 QAM constellations. The eliminated symbols are those indicated by s_1 , s_4 , s_{13} and s_{16} , situated in the regions marked with X. As a result we have two constellations with twelve symbols each (2-12 QAM) to be mapped into a 4-dimensional signal.

8.5 Figure 5: Block diagram of the proposed transmission system employing coded 2-12 QAM modulation

The block diagram of the proposed system, consisting of a sequence $\mathbf{u}$ generated by an information source to be encoded using polar code. Then the generated codeword is split into 7 bit blocks to be mapped into 2-12-ary symbols according to the 2-12 QAM modulation, but to be compatible with this scheme, an uncoded suffix of zeros s is added to each codeword. Then this modified codeword $\mathbf{x}'$, is mapped according to the 2-12 QAM modulation and transmitted through AWGN channels and Rayleigh fading channels disturbed by AWGN. Thus, the Log Likelihood Ratio (λ_t) for bit x_t is calculated, the last s bits provided by the 2-12 QAM demodulator are punctured and a sequence of length N is decoded through the conventional SC algorithm.

8.6 Figure 6: BER versus E_b/N_0 over AWGN channels of 2-12 QAM and 16 QAM

The performance curves expressed by bit error rate (BER) versus E_b/N_0 for a system using uncoded 2-12 QAM and for comparison purposes, a system using uncoded 16 QAM modulation format with Gray code mapping, both systems operating over AWGN channels. The figure also shows the performance curves of a coded system with 2-12 QAM and a polar code rate $R = 6/7$ and codeword of length $N = 1024$ with a suffix of 5 bits added in order to be compatible with 2-12 QAM modulation. For comparison, a coded system with 16 QAM modulation and a polar code with block length $N = 1024$ and rate $R = 3/4$ is also illustrated in the figure. Figure also shows performance curves when a polar code of block length $N = 512$ is employed.

8.7 Figure 7: BER versus E_b/N_0 over AWGN and Rayleigh fading channels of 2-12 QAM and 16 QAM

The performance curves for 2-12 QAM system, when compared to the system using 16 QAM modulation format with Gray code mapping, both over uncoded AWGN channels and Rayleigh fading channels disturbed by AWGN. Figure also illustrates the performance of a system using 2-12 QAM, polar codes of rate $R = 6/7$, block length $N = 1024$ with an appended suffix of $s = 5$ bits to achieve compatibility with the 2-12 QAM scheme, and considering AWGN channels and Rayleigh fading channels disturbed by AWGN, when the SC decoding algorithm is used. For comparison, is considered the 16 QAM modulation with Gray code mapping with polar code of rate $R = 3/4$ and block length $N = 1024$ also over AWGN channels and Rayleigh fading channels disturbed by AWGN. Figure also shows the performance of systems using 2-12 QAM and 16 QAM both combined with a polar code of block length $N = 512$ and over AWGN channels and Rayleigh fading channels disturbed by AWGN.

Figures

Figure 1

Block diagram of a transmitter for a 16 QAM system with a rate 3/4 FEC code

Figure 2

Diagram of 2-dimensional 16 QAM constellation

Figure 3

Block diagram of a transmitter for a 2-12 QAM system with a rate 6/7 FEC code

Figure 4

Diagram of 4-dimensional 2-12 QAM constellation

Figure 5

Block diagram of the proposed transmission system employing coded 2-12 QAM modulation

Figure 6

BER versus E_b/N_0 performance over AWGN channels of uncoded 2-12 QAM and uncoded 16 QAM, and 2-12 QAM and 16 QAM combined with polar codes

Figure 7

BER versus E_b/N_0 performance over Rayleigh fading channels disturbed by AWGN of uncoded 2-12 QAM and uncoded 16 QAM, and 2-12 QAM and 16 QAM combined with polar codes