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Solving Fundamental Life Testing Issues: An Application in Medical Science

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Abstract

In biomedical sciences, the application of probability models for survival experience has become increasingly significant. Survival models of classes of life distributions summarize the pattern of survival, recommend additional research, and offer theories and hypotheses. Nowadays, reliability is extremely crucial because complicated scientific gadgets, such as precise medical equipment, airplane equipment, and air defense equipment, are employed for exact scientific reasons. Furthermore, low or no reliability might result in an increase in the cost of device parts (spare parts), the loss of a person, or a threat to national security.

For decades, researchers have examined the use of the Laplace transform approach to test exponentiality. We proposed new tests for new better than utilized in expectation class in this post (NBUE). Critical values that have been censored and those that have not been censored are calculated. To evaluate the performance of this test, the powers for numerous regularly used alternatives are determined. The proposed test appears to have high efficacy and good powers for many regularly used alternatives. Finally, sets of medical real data are utilized to demonstrate how the proposed tests can be applied to solve real-world problems.

Keywords

NBUE; Laplace transform; Exponential distribution; Hypothesis testing; Pittman asymptotic efficiency; Linear failure rate (LFR), Gamma and Weibull distributions.

1. Introduction and background

In reliability theory, there are two types of tests for ageing criterion. Modeling lifetime data with quantile functions is one of them (see Sankaran and Midhu [1] and Sreelakshmi et al. [2]). The other test is to see if $\bar{F}(x)$ follows an exponential distribution or if it belongs to a certain ageing class.

In reliability theory and biostatistics, modeling of lifespan data is applied in a variety of ways. The result of significance in these applications is the time T until an event occurs. This action includes everything from deaths to the appearance of a cancer to the onset of a sickness to the recurrence of a disease to conception to quitting smoking. The survival function is a fundamental number used to describe time-to-event events.

This function, often known as the survivor function or survivorship function, is defined as:

$$\bar{F}_t(x) = \frac{\bar{F}(x+t)}{\bar{F}(t)}; \bar{F}(t) \neq 0.$$

The ability to age is a crucial feature of surviving distribution. In order to categorize distributions, the manuscript introduces a number of classes.

Definition 1.1

A survival function $\bar{F}(x)$ for a non-negative random variable X is considered new better than used, ($X \in NBU$) iff

$$\bar{F}(x)\bar{F}(t) \geq \bar{F}(x+t); t > 0. \quad (1.1)$$

Definition 1.2

A survival function $\bar{F}(x)$ for a non-negative random variable X is considered new better than used in expectation, ($X \in NBUE$) iff

$$\mu \bar{F}(x) \geq \bar{v}(x); \quad x, \mu > 0 \quad (1.2)$$

where $\bar{v}(x) = \int_x^\infty \bar{F}(u) du$

In the literature, many articles proposed the probabilistic features of the NBUE class of life distributions, see, for example, Hollander and Proschan [3], Fernández-Ponce et al. [4] and M. Hassan [5], among others.

This research study focuses on we've got some genuine data, and we want to see if H_0 : data is exponential or H_1 : data is NBUE. We need to establish test statistics to choose between H_0 and H_1 or to reach a choice. The test statistic is a random variable that determines how closely a given sample result matches one of the hypotheses being tested.

Recently Atallah et al. [6] devised a new way to test exponentiality that is more generic and flexible than the goodness approach and Al-Gashgari et al. [7] and Gadallah [8] followed them.

This article is set up as follows: in section 2, we provide a test statistic for verifying H_0 : F is exponential versus H_1 : F has NBUE property for complete data using the Laplace Transform approach. For sample sizes 5(5)100, Monte Carlo null distribution critical points are provided using Mathematica 8, as well as the power of the test and the Pitman asymptotic efficiency for common alternatives. In section 3, a proposed test for right-censored data is offered. Finally, in section 4, we discuss sets of medical real data to demonstrate the significance of our test.

2. A Measure of Deviation from Exponentiality

First, we construct a measure of deviation from exponentiality towards NBUE class. We use Monte Carlo null distribution critical points, Pitman asymptotic efficiency, and powers for common alternatives to achieve this goal.

2.1 Testing Complete Data

Let $F(x) = 1 - e^{-\beta x}$ for $\beta, x > 0$, be the distribution function for the class of exponential distributions ζ . Our formal purpose is to test $H_0 : F \in \zeta$ versus $H_1 : F \in NBUE \setminus \zeta$

A measure of deviation is provided by the following lemma. Hence it could be used to development a testing procedure.

Lemma 2.1.

$$\delta(s) = \frac{1}{s^2} E[(s+1) - (s+1)e^{-sx} - sx] \quad (2.1)$$

Proof:

Define a measure of deviation from H_0 as

$$\delta(s) = \int_0^\infty [\mu \bar{F}(x) - \bar{v}(x)] e^{-sx} dx,$$

we can take $\mu = 1$ without losing generality because the measure is scale invariant,

$$\begin{aligned} \delta(s) &= \int_0^\infty e^{-sx} \bar{F}(x) dx - \int_0^\infty \int_x^\infty e^{-sx} \bar{F}(u) du dx, \\ &= I_1 - I_2, \end{aligned}$$

where,

$$\begin{aligned} I_1 &= \int_0^{\infty} \bar{F}(x) e^{-sx} dx \\ &= \frac{1}{s} E[1 - e^{-sx}], \end{aligned}$$

and

$$\begin{aligned} I_2 &= \int_0^{\infty} \int_x^{\infty} \bar{F}(u) e^{-sx} du dx, \\ &= \frac{1}{s^2} E[e^{-sx} + sx - 1] \end{aligned}$$

Hence, the result follows.

If $X_1, X_2, \dots, X_n$ are random samples from F, then the empirical estimator form $\delta(s)$ in (2.1) can be obtained as:

$$\hat{\delta}_n(s) = \frac{1}{s^2 n} \sum_i \{(s+1)(1 - e^{-sX_i}) - sX_i\}.$$

So, $\hat{\delta}_n(s)$ can be written as

$$\hat{\delta}_n(s) = \frac{1}{n s^2} \sum_i \phi(X_i),$$

where

$$\phi(X_i) = \{(s+1)(1 - e^{-sX_i}) - sX_i\}.$$

To get the limiting distribution of $\hat{\delta}_n(s)$, one can be recourse to the U-statistic theory.

Set

$$\phi(X) = (s+1)(1 - e^{-sX}) - sX.$$

The asymptotic normality of the statistic given in (2.1) can be demonstrated in the next theorem.

Theorem 2.1.

The statistic has the following qualities according to U-statistics theory.

i- $(\hat{\delta}_n(s) - \delta(s))$ is asymptotically normal, as $n \rightarrow \infty$, with mean 0 and σ^2 given by,

$$\sigma^2 = \text{Var}[\phi(X)] = E\left(\frac{s+1}{s^2} - \frac{s+1}{s^2} e^{-sx} - \frac{x}{s}\right)^2$$

ii- Under H_0 , the variance reduces to (2.2)

$$\sigma_0^2 = \frac{2}{2s^2 + 3s + 1}.$$

Proof:

i- Using U-statistic theory (Lee [9]), we get

$$E[\phi(X)] = E\left(\frac{s+1}{s^2} - \frac{s+1}{s^2} e^{-sx} - \frac{x}{s}\right),$$

$$\sigma^2 = \text{Var}[\phi(X)] = E\left(\frac{s+1}{s^2} - \frac{s+1}{s^2} e^{-sx} - \frac{x}{s}\right)^2.$$

ii- Under H_0 it is clear that $\mu_0 = E[\phi(X)] = 0$, and the variance is

$$\sigma_0^2(s) = \frac{2}{1 + 3s + 2s^2}. \quad (2.2)$$

2.2 Critical Points

The upper percentile points of our test $\hat{\delta}_n(s)$, $n = 5(5)100$ of NBUE class of life distributions are determined using a 10000 simulated sample from the standard exponential distribution.

Tables 1 gives these percentile points of statistics $\hat{\delta}_n(s)$ at $s = 0.5$ and 0.9

Table 1. The Upper Percentile Points of $\hat{\delta}_n(s)$

$\hat{\delta}_n(0.5)$				$\hat{\delta}_n(0.9)$		
n	90%	95%	99%	90%	95%	99%
5	0.279825	0.304076	0.338167	0.237167	0.256942	0.28723
10	0.241742	0.26591	0.299279	0.197007	0.219304	0.248741
15	0.21439	0.240285	0.277052	0.173834	0.199192	0.230774
20	0.197137	0.222957	0.261329	0.154963	0.179302	0.212976
25	0.181544	0.20597	0.244408	0.138918	0.162472	0.199224
30	0.167484	0.194504	0.232533	0.130133	0.154276	0.192599
35	0.155425	0.183761	0.225934	0.121819	0.142971	0.177445
40	0.147586	0.172854	0.215481	0.113036	0.134869	0.168717
45	0.140122	0.16712	0.210354	0.110527	0.132517	0.167341
50	0.134274	0.15932	0.199281	0.101871	0.123498	0.157767
55	0.127303	0.152902	0.192771	0.0990066	0.119881	0.153957
60	0.122194	0.146696	0.184618	0.0974162	0.118562	0.149805
65	0.11823	0.14292	0.181738	0.0918818	0.112444	0.143462
70	0.114374	0.139343	0.175979	0.0885922	0.107244	0.140813
75	0.112557	0.136265	0.174548	0.0852451	0.104024	0.13518
80	0.10844	0.132925	0.172311	0.0836206	0.102547	0.13514
85	0.105188	0.127049	0.166552	0.0813093	0.098198	0.130949
90	0.103513	0.125009	0.163713	0.0787947	0.0965464	0.124997
95	0.10165	0.122707	0.158526	0.0773597	0.0949787	0.122551
100	0.0975	0.117743	0.157147	0.074867	0.0929875	0.125029

Percentile Points decrease as the sample size grows and increase as the confidence level grows, as seen in Table 1. Furthermore, as s grows, the asymptotic normality of our test improves.

2.3 Power Estimates

Table 2 will be used to assess the powers of the proposed test $\hat{\delta}$ of NBUE test at the significant level of 0.05. Based on 10000 simulated samples of sizes $n=10,20$, and 30, these powers were computed for the LFR, Makeham, Gamma, and Weibull distributions.

Table 2. Powers estimates at $\alpha = 0.05$				
Distribution	n	$\theta = 2$	$\theta = 3$	$\theta = 4$
LFR	10	0.592	0.675	0.637
	20	0.946	0.987	0.987
	30	0.998	0.999	1
Makeham	10	1	1	1
	20	1	1	1
	30	1	1	1
Weibull	10	0.228	0.839	0.855
	20	0.930	1	1
	30	0.994	1	1
Gamma	10	0.832	0.900	0.925
	20	0.775	0.99	99
	30	995	1	1

Our test provides ideal capabilities for the Makeham family and good powers for the LFR, Weibull, and Gamma families, as shown in Table 2. As the sample size grows, so does the power estimate. As the families are carried away by the exponential distribution, the powers become bigger.

2.4 Pittman Efficiency

The PAE of our test $\delta(s)$ is investigated in this subsection, where

$$\begin{aligned}
 \text{PAE}(\delta) &= \frac{\left| \frac{\partial}{\partial \theta} \delta \right|_{\theta \rightarrow \theta_0}}{\sigma_0} \\
 &= \frac{1}{\sigma_0} \left| \frac{1}{s^2} \int_0^\infty e^{-sx} \bar{F}'_{\theta_0}(x) dx - \frac{1}{s(1-s)} \int_0^\infty x \bar{F}'_{\theta_0}(x) dx \right|
 \end{aligned}$$

Where $\bar{F}'_{\theta_0}(x) = \frac{d}{d\theta} \bar{F}_\theta(u) \Big|_{\theta \rightarrow \theta_0}$.

The LFR, Makeham, Weibull and Gamm distributions are generally used as alternatives to the exponential model. The LFR and Makeham distributions are clearly in the NBUE class for $\theta > 0$, whereas the Weibull and gamma distributions are in the NBUE class for $\theta > 1$.

The null distribution is obtained at $\theta = 0$ for the first two; whereas at $\theta = 1$, the Weibull and gamma distributions reduce to the exponential distribution.

- (i) $\text{PAE}(\delta, \text{LFR}) = \frac{1}{\sigma_0} \left| \frac{2+s}{(1+s)^2} \right|$
- (ii) $\text{PAE}(\delta, \text{Makeham}) = \frac{1}{\sigma_0} \left| \frac{3+s}{2(1+s)(2+s)} \right|$
- (iii) $\text{PAE}(\delta, \text{weibull}) = \frac{1}{\sigma_0} \left| \frac{\ln(1+s) - s(\gamma - 1)}{s(1+s)} \right|, \quad \gamma(\text{Euler constant})$

$$(iv) \quad PAE(\delta, \text{Gamm}) = \frac{1}{\sigma_0} \left| \frac{\ln(1+s)-s}{s^2} \right|.$$

Table 3 compares the efficiency of our suggested test $\hat{\delta}_n(s)$ with those Ahmed et al. [10] ($\delta_n^{(1)}$) and Mugdadi and Ahmad [11] (δ_3), for various selected values s .

Table 3. PAE of $\delta_n^{(1)}$, $\delta_{F_n}^{(2)}$ and $\hat{\delta}(s)$							
Distribution	$\delta_n^{(1)}$	δ_3	$\hat{\delta}_n(s)$				
			$s = 0.1$	$s = 0.5$	$s = 0.9$	$s = 1.5$	$s = 1.9$
LFR	1.299	0.408	1.409	1.361	1.310	1.252	1.223
Weibull	0.9665	0.17	1.016	1.007	0.975	0.923	0.894
Makeham	0.577	0.039	0.545	0.572	0.577	0.574	0.571
Gamm	----	-----	0.381	0.463	0.519	0.582	0.610

As demonstrated in Table 3, our test has high performance for the four alternatives families and it is more efficient than both $\delta_n^{(1)}$ and δ_3 .

3. Testing for Censored Data

One of the most major developments is owing to a distinctive feature of survival data in the life sciences: when some study participants have not experienced the event of interest at the end of the research or at the time of analysis, the data becomes invalid. At the end of the trial, some patients may still be alive or disease-free. It is uncertain how long the subjects survived. When participants are lost to follow-up after a research period, this is referred to as censored observations or censored times.

3.1 Test Statistic

A test statistic is proposed to test: H_0 versus H_1 with randomly right censored data, in this section. Suppose n components are put on test, and $X_1, X_2, \dots, X_n$ denote their complete-life time. Let that $X_1, X_2, \dots, X_n$ be independent and identically distributed (i.i.d.) according to a continuous life distribution F .

Let $Y_1, Y_2, \dots, Y_n$ be (i.i.d.) according to a continuous life distribution G . Also, we assume that X_i s and Y_i s are independent. In the randomly right censored model, we observe the pairs $(Z_i, I_i), i = 1, \dots, n$ where $Z_i = \min(X_i, Y_i)$. And the indicator function

$$I_i = \begin{cases} 1 & \text{if } Z_i = X_i \text{ (i th observation is uncensored)} \\ 0 & \text{if } Z_i = Y_i \text{ (i th observation is censored).} \end{cases}$$

Let $Z_{(0)} < Z_{(1)} < \dots < Z_{(n)}$ denoted the ordered of Z 's and I_i is the indicator function corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier estimator in the case of censored data $(Z_i, I_i), i = 1, \dots, n$ as follows:

$$\delta_c(s) = \frac{1}{ns^2} ((s+1)(1-v) - s\xi) \quad (3.1)$$

Where

$$v(s) = \int_0^\infty e^{-sx} dF(x),$$

$$\hat{v}(s) = \sum_{m=1}^l e^{-sZ_{(m)}} \left(\prod_{p=1}^{m-2} C_p^{\delta_p} - \prod_{p=1}^{m-1} C_p^{\delta_p} \right)$$

$$\xi = \sum_{j=1}^l \prod_{k=1}^{j-1} C_m^{\delta_m}(Z_{(j)} - Z_{(j-1)})$$

Again, the 95%, 98% and 99% percentage points of the test statistic in (3.1) are simulated for some selected values s based on 10000 simulated and sample sizes $n = 5(5)100$ from the standard exponential distribution in Table (4) below.

Table 4. The Upper Percentile Points of $\hat{\delta}_c$

$\hat{\delta}_c; s = 0.5$				$\hat{\delta}_c; s = 0.9$		
n	95%	98%	99%	95%	98%	99%
5	0.556089	0.723507	0.749522	0.141975	0.200356	0.223169
10	0.222706	0.253582	0.277925	0.0632499	0.0714814	0.0778456
15	0.112603	0.141131	0.159027	0.0348527	0.041507	0.0448878
20	0.0747683	0.0890169	0.0987488	0.0215112	0.0260348	0.0283035
25	0.0555926	0.0636641	0.0679127	0.0142808	0.0167054	0.0188996
30	0.039203	0.0472691	0.0549178	0.011458	0.0146262	0.0161358
35	0.0306762	0.0370581	0.0446611	0.00988711	0.0116444	0.0134167
40	0.0263251	0.0310865	0.0335195	0.0075689	0.00907673	0.00965258
45	0.0207818	0.0245241	0.0270926	0.00617013	0.00732922	0.00816913
50	0.0176545	0.0202812	0.0231021	0.0053267	0.00622565	0.00683493
55	0.0152143	0.0189062	0.0215259	0.00438757	0.00572819	0.0067370
60	0.012891	0.0163699	0.018469	0.00391244	0.00476194	0.0053709
65	0.012267	0.0151002	0.0165272	0.00344586	0.00436945	0.00467291
70	0.0100757	0.0127268	0.0141637	0.00316022	0.00394992	0.00438839
75	0.0093662	0.0108985	0.0112213	0.00285087	0.00325591	0.0035898
80	0.00826502	0.0104013	0.0112688	0.00252824	0.00307005	0.00344041
85	0.00749924	0.00900839	0.0100838	0.00230725	0.00285733	0.0031758
90	0.00644113	0.00781295	0.0085409	0.00206423	0.00251488	0.00271548
95	0.0061869	0.00759966	0.00814865	0.00195439	0.00231045	0.00263562
100	0.00552738	0.00694048	0.00812523	0.00175933	0.00209644	0.00228567

Table 4. shows that the Percentile Points decrease as the sample size increases and they increase as the confidence level increases. Also, our test of $\hat{\delta}_c(s)$ behaves better in terms of asymptotic normality whenever s increases.

3.2 Power Estimates

The powers estimate of the proposed test $\hat{\delta}$ will be carried out in Table 5 at the significant level $\alpha = 0.05$. These powers estimated for Weibull, LFR and Gamma distributions based on 10000 simulated samples for sizes $n = 10, 20$ and 30 .

Table 5. Powers estimates at $\alpha = 0.05$				
n	θ	Distribution		
		Weibull	LFR	Gamma
10	1	0.9354	0.9983	0.9317
	2	0.9998	0.9999	0.8932
	3	1	1	0.8774
20	1	0.8945	0.9964	0.9971
	2	0.9996	0.9998	0.9958
	3	1	1	0.9951
30	1	0.8783	0.9944	1
	2	0.9994	0.9999	1
	3	1	0.999	1

Our test has good powers for the Weibull, LFR, and Gamma families, as shown in Table 5.

4. Applications

In order to show the usefulness of the results in this article, we apply the findings to some of real data sets.

4.1 Complete Data

Example 1: Consider the data in Abouammoh et al. [12] these data represent the ages (in years) of 40 patients suffering from blood cancer (leukemia) from one of ministry of health hospitals in Saudi Arabia and the ordered observations are:

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036
2.162	2.211	2.370	2.532	2.693	2.805	2.910	2.912	3.192
3.263	3.348	3.348	3.427	3.499	3.534	3.767	3.751	3.858
3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753
4.929	4.973	5.074	4.381					

We calculate the statistic in (2.1) $\hat{\delta}(0.5) = 1.8$ and $\hat{\delta}(0.9) = 1.6$ which both are greater than the critical value of Table 1 in the two cases of $\hat{\delta}(0.5)$ and $\hat{\delta}(0.9)$ as $n = 40$. Hence, we conclude that this data set have UBAE property.

4.2 Censored Data

Example 2: In this application, we consider the data in Hassan [13] and Mahmoud et al. [14] which represents the ages (in days) of 51 liver cancers patients taken from Elminia cancer center Ministry of Health Egypt, whom entered the medical investigation in (1999). Only 39 patients are observed (right-censored) in the investigation, while the remaining 12 patients are dropped out (missing from the investigation). The order data are given below.

The ordered non-censored data are:

10	14	14	14	14	14	15	17	18	20
20	20	20	20	23	23	24	26	30	30
31	40	49	51	52	60	61	67	71	74
75	87	96	105	107	107	107	116	150	

The ordered censored data (missing from the investigation) are:

30	30	30	30	30	60	150	150	150	150	150	185
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Applying statistic in (3.1), we calculate $\hat{\delta}_c(0.5) = 0.00012$ and $\hat{\delta}_c(0.9) = 3.65264 \times 10^{-7}$ which both are smaller than the critical value of Table 4 in the two cases of $\hat{\delta}_c(0.5)$ and $\hat{\delta}_c(0.9)$. Then we conclude that this data set have exponential property.

Example 3: consider the data in Kamran Abbas et al. [15] and in Lee and Wolfe [16], the survival times, in weeks, of 61 patients with inoperable lung cancer treated with cyclophosphamide. There are 33 uncensored observations and 28 censored observations, representing the patients whose treatment was terminated because of a devolving condition.

The ordered non-censored data are:

0.43	2.86	3.14	3.14	3.43	3.43	3.71	3.86	6.14	6.86	9.00
9.43	10.71	10.86	11.14	13.00	14.43	15.71	18.43	18.57	20.71	29.14
29.71	40.57	48.57	49.43	53.86	61.86	66.57	68.71	68.96	72.86	72.86

The ordered censored data (missing from the investigation) are:

0.14	0.14	0.29	0.43	0.57	0.57	1.86	3.00	3.00	3.29	3.29
6.00	6.00	6.14	8.71	10.57	11.86	15.57	16.57	17.29	18.71	21.29
23.86	26.00	27.57	32.14	33.14	47.29					

Applying statistic in (3.1), we calculate $\hat{\delta}_c(0.5) = 1.62637 \times 10^{17}$ and $\hat{\delta}_c(0.9) = 9.03541 \times 10^{16}$ which both are greater than the critical value of Table 4 in the two cases of $\hat{\delta}_c(0.5)$ and $\hat{\delta}_c(0.9)$. Then we conclude that this data set have UBAE property.

5. Conclusion

In this paper, a non-parametric testing for the NBUE based on Laplace transform is developed in both complete and censored cases. The percentage points of the proposed statistics are simulated. The efficacies of our developed tests are compared to some other classes based on Pitman asymptotic relative efficiency using some well-known life distributions; namely, Linear failure rate family (LFR) and Weibull family. Finally, the findings of the paper are applied to some medical real data sets.

6. Methods Section

In two independent cases, a group of cancer patients was observed and their survival times were recorded. They were calculated a test value $\hat{\delta}(s)$ and $\hat{\delta}_c(s)$ and the Monte Carlo

critical values were compared. It was also determined what the test's Pittman asymptotic relative effectiveness and power were.

Declaration

Ethical Approval and Consent to participate

Not applicable.

Availability of data and materials

Data are available along the paper.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

The main manuscript content was written by M. E. Bakr and A. M. Gadallah, while simulations were made by Abdulhakim A. Al-Babtain. The manuscript was reviewed by all of the authors.

References:

- [1] Sankaran, P. G., and Midhu, N. N. (2016). Testing exponentiality using mean residual quantile function. *Statistical Papers*, 57, 235–247.
- [2] Sreelakshmi, N., Sudheesh K. Kattumannil and Asha G. (2018). Quantile based tests for exponentiality against DMRQ and NBUE alternatives. *Journal of the Korean Statistical Society*, 47(2), 180-200.
- [3] Hollander, M. and Prochan, F. (1975). Test for mean residual life. *Biometrika*, 62, 585-593.
- [4] Fernández-Ponce JM, Pellerey F, Rodríguez-Grinolo MR (2011) On a new NBUE property in multivariate sense: an application. *Comput Stat Data Anal* 55(12):3283–3294.
- [5] Hassan, M.KH. (2015). A generalized Hollander-Proschan test for NBUE alternative based on U-statistics approach. *International Journal of Reliability and Applications* 16(2), 113-122.
- [6] Atallah M. A., Mahmoud M. A. W., Alzahrani B. M. (2014). A new test for exponentiality versus NBU_{mgf} life distribution based on Laplace transform. *Quality and Reliability Engineering International*. 30(8), 1353-1359.
- [7] Al-Gashgari F. H., Shawky A. I. (2016). Mahmoud M. A. W.: A non-parametric test for exponentiality against NBUE class of life distribution based on Laplace transform. *Quality and Reliability Engineering International*. 32(1), 29-36.
- [8] Gadallah A. M. (2017). Testing EBU_{mgf} Class of Life Distributions based on Laplace Transform Technique. *Journal of Statistics Applications & Probability*. 6(3), 471-477.
- [9] Lee A. J. (1990). *U-Statistics Theory and Practice*, Marcel Dekker, Inc. New York, NY.
- [10] Ahmad, I.A., Alwasel, I.A. and Mugdadi, A.R. (2001). A goodness of fit approach to major life testing problems. *International Journal of Reliability and Applications*. 2, 81-97.
- [11] Mugdadi, A.R. and Ahmad, I.A. (2005). Moment inequalities derived from comparing life with its equilibrium form, *J. Stat. Plann. Inf.*, **134**, 303-317.
- [12] Abouammoh A.M., Abdulghani SA., Qamber I.S.: On partial orderings and testing of new better than renewal used classes. *Reliability Engineering. System Safety*, 43(1), 37–41 (1994).
- [13] Hassan, M.KH.: On testing exponentiality ageing UBA class of life distribution based on Laplace transform, *Sankhya B*, 79, 142-155 (2017).

- [14] Mahmoud, M.A.W., Hassan, E.M.A., Gadallah A.M.: On NRBUL class of life distributions. *Journal of the Egyptian Mathematical Society*, 26(3), 483-490 (2018).
- [15] Kamran Abbas, Zamir Hussain, Noreen Rashid, Amjad Ali, Muhammad Taj, Sajjad Ahmad Khan, Sadaf Manzoor, Umair Khalil, and Dost Muhammad Khan.: Bayesian Estimation of Gumbel Type-II Distribution under Type-II Censoring with Medical Applications. *Journal of Computational and Mathematical Methods in Medicine*, 7, 1-11(2020).
- [16] Lee E. T., Wolfe R. A.: A simple test for independent censoring under the proportional hazards model, *Biometrics*, 54, 1176-1182 (1998).