The effect of presence of permeable media on natural convection flow of MHD along an upward wavy surface

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Research Article

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The effect of presence of permeable media on natural convection flow of MHD along an upward wavy surface

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Abstract

The natural convection flow over the vertical wavy surface in permeable media of viscous incompressible fluid with thermal conductivity has been investigated. The governing boundary layer equations are transformed into a non-dimensional form using a suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations is solved numerically employing the finite difference, known as the fully implicitmethod. The numerical results for velocity **u,** temperature **θ**, Nusselt number **Nu** for different magnitudes of magnetic field parameter **M**, Prandtl number **Pr**, inverse Darcy number Da^{-1}, surface amplitude **a**, thermal conductivity variation parameter **γ**, and heat generation parameter **Q.**

Keywords: Natural convection; Magnetohydrodynamics (MHD); Heat transfers; Wavy surface; permeable media; Thermal conductivity.

Introduction

Roughened surfaces are encountered in several heat transfer collectors, flat plate condensers in refrigerators, and heat exchangers. One normal illustration of a heat exchanger is radiator utilized in vehicles.

The natural convection in permeable media on Magnetohydrodynamic(MHD). Boundary layers on various geometrical shapes have been studied by many investigators and it has been a very popular research topic for many years. The problem of MHD free convection in a strong cross field was investigated by Kuiken [1]. The natural convection of a vertical wavy surface was first studied by Yao [2]. Moulic et al. [3] have likewise concentrated on the mixed convection along the wavy surface.Molla et al [4] explored the natural convection stream along an upward wavy surface with uniform surface temperature in presence of heat generation/absorption. Al-Nimr and Hader[7] additionally researched MHD free convection flow in open-ended vertical permeable channels. Tashtoush and Al- odat[5] investigated the magnetic field effect on heat and fluid flow over a wavy surface with variable heat flux. Pop I et al[8] have also studied the problem of Magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface. G. Tanda and G. Vittori [9] studied the fluid flow and heat transfer in a two-dimensional wavy channel. F. M . Hady et al [10] likewise explored “MHD boundary layer flow and heat transfer over anuninterrupted moving wavy surface in permeable media. C. –Y. Cheng [11] studied also “the natural convection heat and mass transfer near aupward wavy surface with fixed wall temperature and concentration in a permeable medium. The natural convection along a vertical complex wavy surface was studied by Yao [6]. Heat generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity investigated by Md. Abdul Alim et al [13]

The governing partial differential equations were transformed into a non-dimensional form by using dimensionless variables. The transformed boundary layer equations are solved numerically by finite difference (fully implicit method) [12]. The variation of velocity **u,** temperature θ and Nusselt number **Nu** as a function of Prandtl number **Pr**, inverse Darcy numberDa^{-1}, magnetic field parameter **M**, wavy surface amplitude **a**, thermal conductivity variation parameter**γ** and heat generation parameter **Q.**

Nomenclature

a the dimensionless amplitude of the wavy surface B_o magnetic induction M magnetic field parameter L characteristic reference wave-length Nu local Nusselt number P pressure Pr Prandtl number Gr Grashof number σ_o electrical conductivity M magnetic field parameter g gravitational acceleration T temperature u,v axial and normal dimensionless velocity Components, respectively $\bar{u},\bar{v}$ the velocity components along $(\bar{x},\bar{y})$ x,y dimensionless coordinates $\bar{x},\bar{y}$ dimensional coordinates Da Darcy number Q heat generation parameter Q_o heat generation constant	C_p specific heat at constant pressure k permeability of the saturated permeable Medium K thermal conductivity **Greek symbol** υ kinematics viscosity ρ density $\sigma(x)$ surface geometry function θ dimensionless temperature β coefficient of thermal expansion α thermal diffusivity γ thermal conductivity variation parameter **superscripts** $-$ dimensional quantity **Subscripts** w wall surface ∞ free stream x derivative with respect to x

Mathematical formulation and analysis

In the study of two-dimensional steady laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical wavy surface in permeable media in presence of the magnetic field of strength $B_\circ$, it is assumed that the surface temperature of the vertical wavy surface T_w is uniform, where $T_w \succ T_\infty$.

The boundary layer analysis outlined below $\sigma\left(\bar{x}\right)$ is arbitrary, but our detailed numerical work. Assumes that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$\bar{y} = \bar{\sigma}\left(\bar{x}\right) = a \sin\left(\frac{n\pi\bar{x}}{l}\right) \qquad (1)$$

Where l is the characteristic length associated with the wavy surface and **a** is the amplitude of the wavy surface. The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in figure(1) in addition, it is assumed that there is no heat generation, nobody forces acting on the system, no Joule heat effect, and no viscous dissipation.

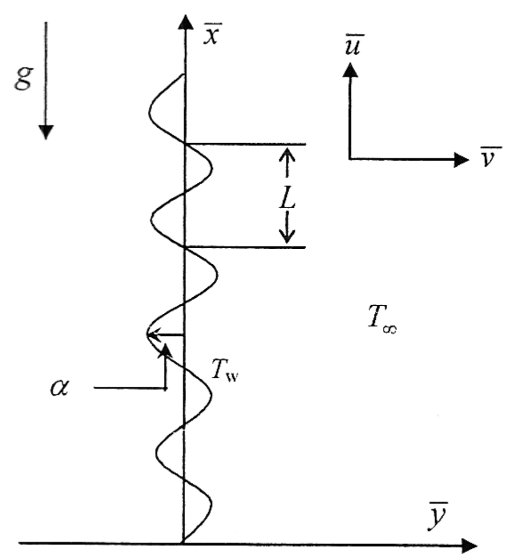

Figure (1) physical model and coordinate system.

The conservation equations for the flow with steady, laminar, and two-dimensional boundary layer under the Boussinesq approximation, the continuity, momentum, and energy equations can be written as:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \qquad (2)$$

$$\overline{u}\,\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\,\frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \upsilon \nabla^2 \overline{u} + g\beta\left(T - T_\infty\right) - \frac{\sigma_\circ \beta_\circ^2}{\rho}\overline{u} - \frac{\upsilon}{k}\overline{u} \qquad (3)$$

$$\overline{u}\,\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\,\frac{\partial \overline{v}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{y}} + \upsilon \nabla^2 \overline{v} - \frac{\upsilon}{k}\overline{v} \qquad (4)$$

$$\overline{u}\,\frac{\partial T}{\partial \overline{x}} + \overline{v}\,\frac{\partial T}{\partial \overline{y}} = -\frac{K}{\rho c_p}\nabla^2 T + \frac{Q_\circ\left(T - T_\infty\right)}{\rho c_p} \qquad (5)$$

The boundary conditions

$$at\ \ \overline{y} = \overline{y}_w = \overline{\sigma}\left(\overline{x}\right) \quad : \overline{u} = 0 \ , \ \ \overline{v} = 0 \ , \ T = T_w$$

$$at\ \ \overline{y} \to \infty \quad : \overline{u} = 0 \ , \ T = T_\infty \ , \ \overline{p} = p_\infty \qquad (6)$$

Where $\left(\overline{x}, \overline{y}\right)$ are the dimensional coordinates tangent and perpendicular to the surface and $\left(\overline{u}, \overline{v}\right)$ are the velocity components in direction of $\left(\overline{x}, \overline{y}\right)$, ∇^2 is the Laplacian operator, g is the acceleration caused by gravity, ρ is the density, $\overline{p}$ is the dimensional pressure of the fluid, $\beta_\circ$ is the strength of magnetic fluid, $\sigma_\circ$ is the electrical conduction, β is the coefficient of thermal expansion, υ is the kinematics viscosity, μ is the dynamic viscosity, K is the thermal conductivity of the fluid c_p , is the specific heat caused by constant pressure, k is the permeability of the saturated porous medium, T_w is the surface temperature, T_∞ is the temperature surrounding the fluid and p_∞ is the pressure of fluid beyond the boundary layer.

Applying Prandl's transposition theorem to convert the asymmetrical wavy surface into a flat surface and boundary-layer approximation, the upcoming dimensionless variables were introduced for non-dimensionalizing the fundamental equations shown:

$$x = \frac{\bar{x}}{L} \quad , \quad y = \frac{\bar{y} - \sigma}{L} Gr^{\frac{1}{4}} \quad , \quad p = \frac{L^2}{\rho \upsilon^2} Gr^{-1} \bar{p}$$

$$u = \frac{\rho L}{\mu} Gr^{-\frac{1}{2}} \bar{u} \quad , \quad v = \frac{\rho L}{\mu} Gr^{-\frac{1}{4}} \left(\bar{v} - \sigma_x \bar{u} \right) \quad , \quad \sigma_x = \frac{d\sigma}{dx}$$

$$Gr = \frac{g\beta \left(T_w - T_\infty \right)}{\upsilon^2} L^3 \quad , \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} . \tag{7}$$

Where (u, v) are the dimensionless velocity components in (x, y) direction, θ is the dimensionless temperature. Registeringthe dimensionless dependent and independent variables into equations (2)-(5), we are going to get the following dimensionless form of the governing equations, after neglecting terms of smaller orders of magnitude inGr , the Grashof number, the fundamental equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{\frac{1}{4}} \sigma_x \frac{\partial p}{\partial y} + \left(\sigma_x^{\ 2} + 1 \right) \frac{\partial^2 u}{\partial y^2} \tag{9}$$

$$+\theta - Mu - Da^{-1} u$$

$$\sigma_x \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -Gr^{1/4} \frac{\partial p}{\partial y} + \sigma_x \left(\sigma_x^{\ 2} + 1 \right) \frac{\partial^2 u}{\partial y^2}$$

$$-\sigma_{xx} u^2 - Da^{-1} \sigma_x u \tag{10}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(\sigma_x{}^2+1\right)\left(\gamma\theta+1\right)\frac{\partial^2\theta}{\partial y^2}+Q\theta+\frac{1}{\Pr}\left(\sigma_x{}^2+1\right)\gamma\left(\frac{\partial\theta}{\partial y}\right)^2 \qquad (11)$$

The pressure gradient is along the y-direction which can be demonstrated by reviewing Equation (10), as the result of the previous interpretations it implies that the lowest order pressure gradient along x-direction can be determined from the inviscid flow solution.

$$\sigma_x{}^2\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\sigma_x Gr^{1/4}\frac{\partial p}{\partial y}+\sigma_x{}^2\left(\sigma_x{}^2+1\right)\frac{\partial^2 u}{\partial y^2}$$

$$-\sigma_x\sigma_{xx}u^2-Da^{-1}\sigma_x{}^2u \qquad (12)$$

For our problem, the inviscid flow field is at rest and hence $\left(\frac{\partial p}{\partial x}=0\right)$ and elimination of $\left(\frac{\partial p}{\partial y}\right)$ between equations (9), (10) by multiplying equation (10) by σ_x and adding the two equations (9), (12) we have

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\left(\sigma_x{}^2+1\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{\sigma_x{}^2+1}u^2-Da^{-1}u$$

$$-\frac{M}{\sigma_x{}^2+1}u+\frac{1}{\sigma_x{}^2+1}\theta \qquad (13)$$

The governing equations will be

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (14)$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} \right)=\left({\sigma_x}^2+1 \right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x \sigma_{xx}}{{\sigma_x}^2+1}u^2-Da^{-1}u-\frac{M}{{\sigma_x}^2+1}u+\frac{1}{{\sigma_x}^2+1}\theta \quad (15)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left({\sigma_x}^2+1 \right)\left(\gamma\theta+1 \right)\frac{\partial^2 \theta}{\partial y^2}+Q\theta+\frac{1}{\Pr}\left({\sigma_x}^2+1 \right)\gamma\left(\frac{\partial \theta}{\partial y} \right)^2 \quad (16)$$

By altering the boundary conditions (6) utilizing the dimensionless form (7) we get :

$$at \quad y=0 \ : \quad u=0 \ , \quad v=0 \ , \ \theta=1$$

$$at \quad y\to\infty \ : \quad u=0 \ , \quad \theta=0 \qquad (17)$$

In practical applications, the physical quantities of principle interest are the shearing stress τ_w in terms of the skin friction coefficients C_f and the rate of heat transfer in terms of Nusselt number Nu which can be written as

$$C_f=\frac{2\tau_w}{\rho U^2} \quad and \quad Nu=\frac{q_w x}{k_\infty\left(T_w-T_\infty \right)} \qquad (18)$$

The local skin friction coefficient C_f and the rate of heat transfer in terms of the local Nusselt number Nu takes the following form:

$$C_f\left(Gr/x \right)^{1/4} \Big/ 2=\sqrt{1+\ {}_x^2}\ \frac{^2 u}{y^2} \qquad (19)$$

$$Nu\left(Gr/x \right)^{1/4}=\ \left(1+\ \ \right)\sqrt{1+\ {}_x^2}\ \frac{}{y} \qquad (20)$$

Results and Discussion

The partial differential equations (14)-(16) and the associated boundary conditions (17) were solved numerically by finite difference fully (implicit method) the derivatives in regard to x and y are approximated by central difference. Many numerical results were obtained all over the progression of this task and representative set of graphical results for the velocity u and temperature field θbesides the Nusselt number Nu are presented. Results are given for the magnetic field parameter M = 0 , 1 , 5 and 7 ; Prandtl number Pr = 0.5 , 1.5 , 3 and 4.5 ; the amplitude parameter a = 0.1 , 0.2 , 0.4 and 0.5 ; the inverse Darcy number Da^{-1} = 0.1 , 0.5 , 1 , 2 and 5 ; heat generation parameter Q = 0.2 , 0.5 , 1 , 1.5 , 2 and thermal conductivity variation parameter γ = 0 , 1 , 2 , 6. Fig. (2) introduce the impact of magnetic field parameter M on velocity description u , we can observe from this figure that the increasingof magnetic field parameter M inclines towards the increase of the velocity description u when M = 0 , 1 , 5, 7 , pr =0.7, Q = 0.1 , a = 0.2 , Da^{-1} =0.1 and γ = 2. fig. (3) presents the impact of inverse Darcy number Da^{-1} on velocity description u , we can observe from this figure that the increase of inverse Darcy number Da^{-1}inclines towards increase the velocity descriptionu whenDa^{-1} = 0.1 , 0.5 , 1 , 2 , 5 , pr =0.7, Q = 0.1 , a = 0.2 , M =0.2 and γ = 2. fig. (4) illustrated that the increase of heat generation parameter Q tends to increase the velocity description u when Q = 0.2 , 0.5 , 1 , 1.5 , 2 , pr = 0.7 , Da^{-1} =0.1 , a = 0.2 , M = 0.2 and γ = 2. Fig.(5) show that the increase of thermal conductivity variation parameter γ tends to increase of the velocity description u when γ = 0 , 1 , 2 , 6 ,pr =0.7, Da^{-1} =0.1, a = 0.2 , M =0.2 and Q = 0.1. fig.(6) presents the impact of the magnetic field parameter M on the temperature descriptionθ and it is clearly that the increase of M tends to decrease the temperature θ when M = 0 , 1 , 5, 7 , pr =0.7, Q = 0.1 , a = 0.2 , Da^{-1} =0.1 and γ = 2. Fig.(7) and fig. (8) illustrates that the temperature descriptionθ decrease with increment both of inverse Darcy number Da^{-1} and prandtle number Pr when Da^{-1} = 0.1 , 0.5 , 1 , 2 , 5 , Pr = 0.5 , 1.5 , 3 , 4.5 , Q = 0.1 , a = 0.2 , M =0.2 and γ = 2. Fig. (9) and fig. (10) present the influence of prandtle number Pr and magnetic field parameter M on Nusselt number Nu and it is clearly that the increment of prandtle number Pr and magnetic field parameter M incline towards the increment of the Nusselt number Nu when Pr = 0.5 , 1.5 , 3 , M = 0 , 1 , 5, 7 , Q = 0.1 , a = 0.2 , Da^{-1} =0.1 and γ = 2. Fig.(11) and fig. (12) illustrate that the Nusselt number Nu increase with increase both of inverse Darcy number $Da^{-1}$and heat generation parameter Q when Da^{-1} = 0.1 , 0.5 , 1 , 2 , 5 , Q = 0.2 , 0.5 , 1 , 1.5 , 2 , pr = 0.7 , a = 0.2 , M = 0.2 and γ = 2. Fig .(13) illustrates that the increment in thermal conductivity variation parameterγincline towards to the increment the Nusselt number Nu when γ = 0 , 1 , 2 , 6 , pr =0.7, Da^{-1} =0.1, a = 0.2 , M =0.2 and Q = 0.1.

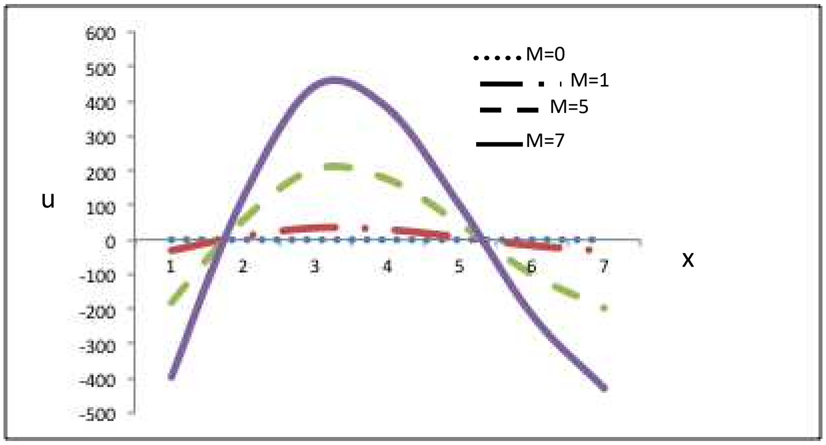

Fig. (2)Illustratesvelocity distribution u for different magnetic field parameter M, pr =0.7, Q = 0.1, a = 0.2, Da^{-1} =0.1 and γ = 2.

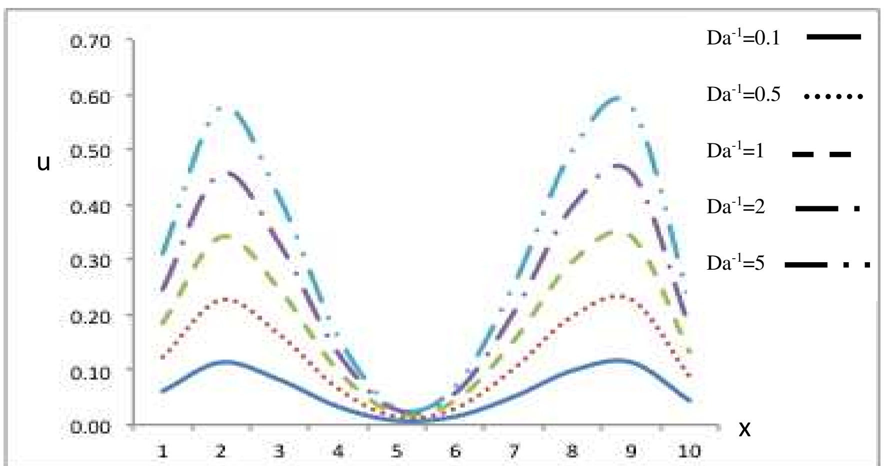

Fig. (3)Illustratesvelocity distribution u for different inverse Darcy number Da^{-1}, pr =0.7, Q = 0.1, a = 0.2, M =0.2 and γ = 2.

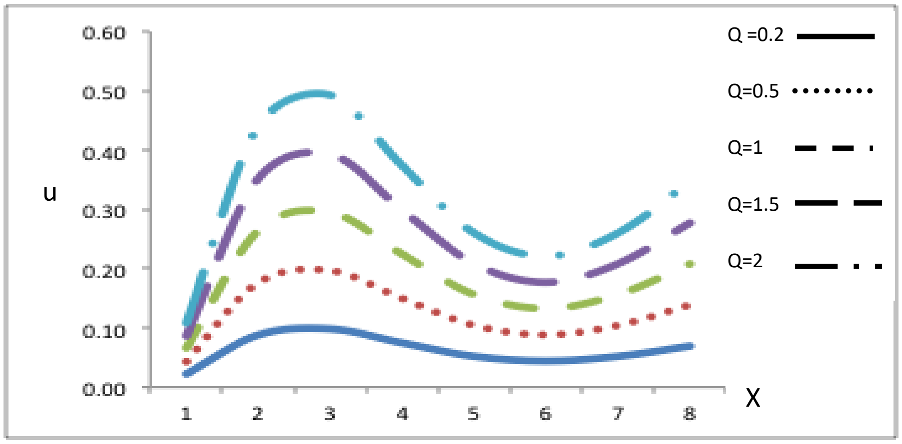

Fig. (4)Illustratesvelocity distribution u for different heat generation parameter Q, pr =0.7, Da^{-1} =0.1, a = 0.2, M =0.2 and $\gamma = 2$.

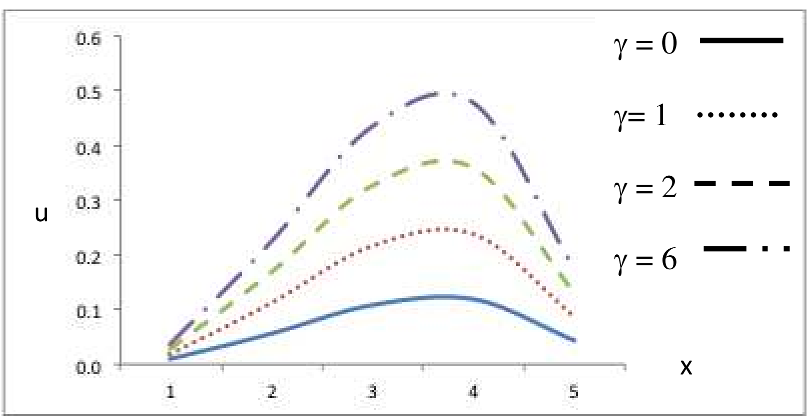

Fig. (5)Illustratesvelocity distribution u for different thermal conductivity variation parameterγ, pr =0.7, Da^{-1} =0.1, a = 0.2, M =0.2 and Q = 0.1.

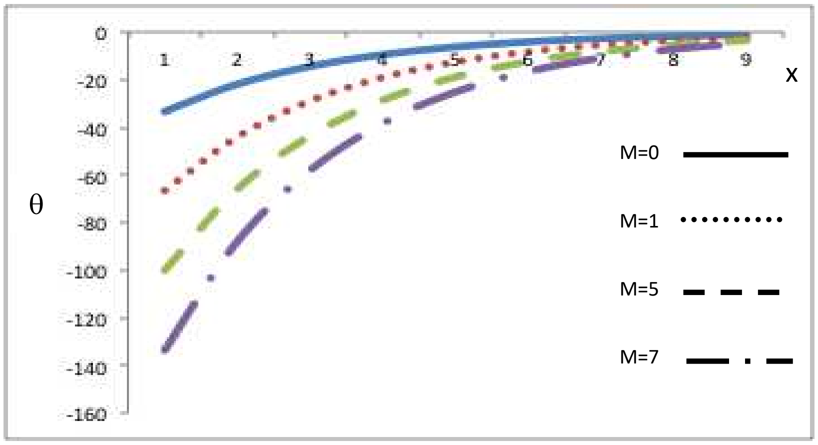

Fig. (6)Illustratesthe temperature profile θ for different magnetic field parameter M, pr =0.7, Q = 0.1, a = 0.2, Da^{-1} =0.1 and γ = 2.

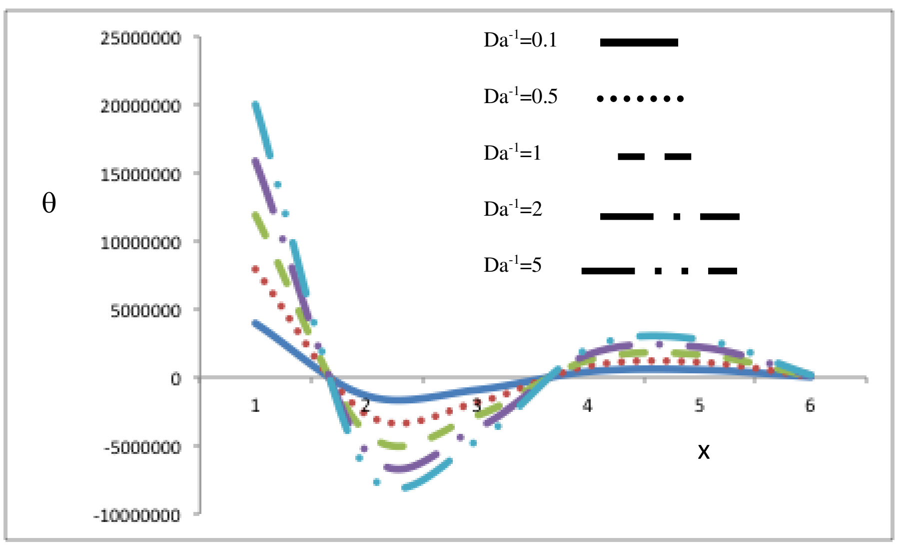

Fig. (7)Illustratesthe temperature profile θ for different inverse Darcy number Da^{-1}, pr =0.7, Q = 0.1, a = 0.2, M =0.2 and γ = 2.

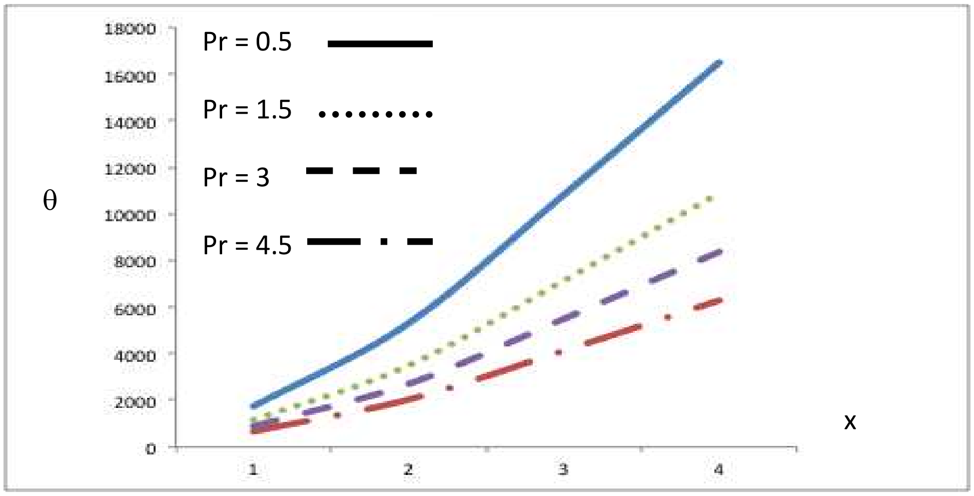

Fig. (8)Illustratesthe temperature profile θ for different prandtle number Pr, Da^{-1} =0.1, Q = 0.1, a = 0.2, M =0.2 and γ = 2.

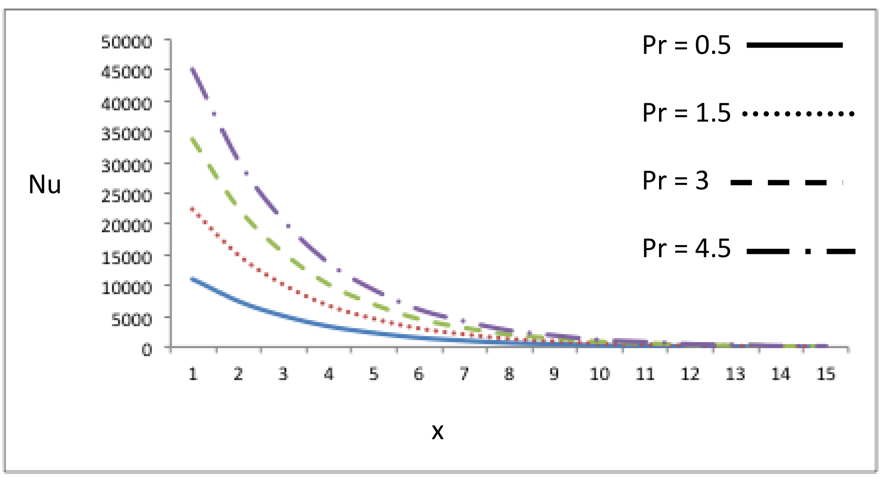

Fig. (9)IllustratesNusselt number Nu for different prandtle number Pr, Da^{-1} =0.1, Q = 0.1, a = 0.2, M =0.2 and γ = 2.

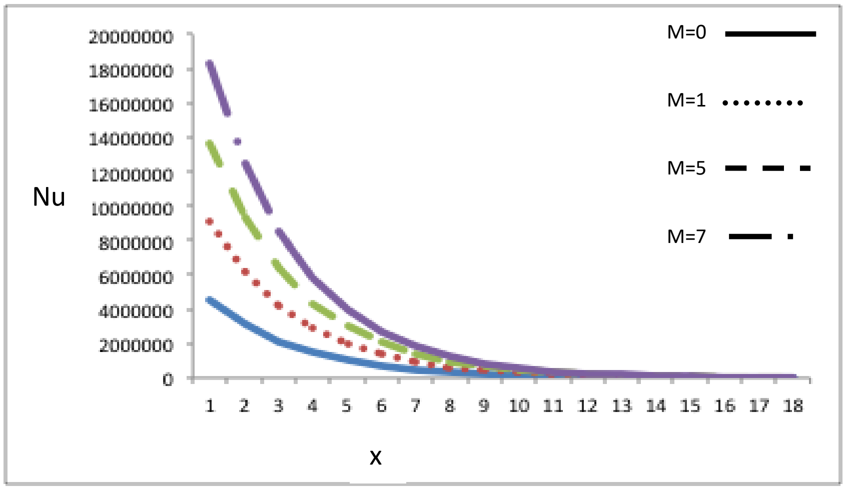

Fig. (10)IllustratesNusselt number Nu for different magnetic field parameter M, pr =0.7, Q = 0.1, a = 0.2, Da^{-1} =0.1 and γ = 2.

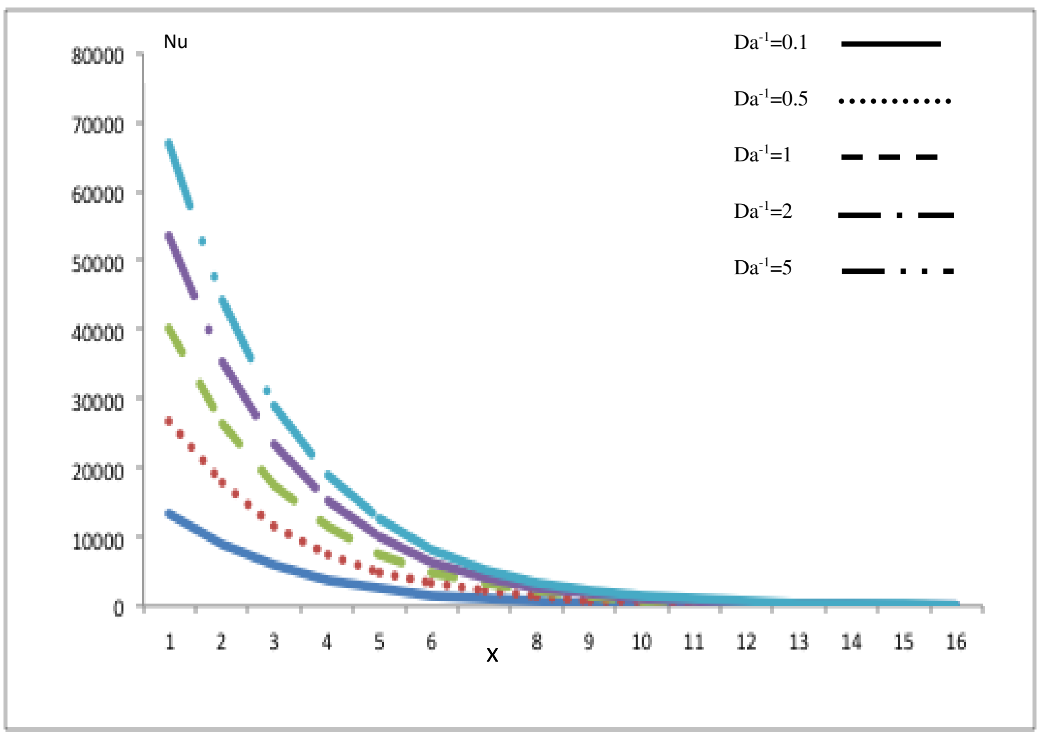

Fig. (11)IllustratesNusselt number Nu for different inverse Darcy number Da^{-1}, pr =0.7, Q = 0.1, a = 0.2, M =0.2 and γ = 2.

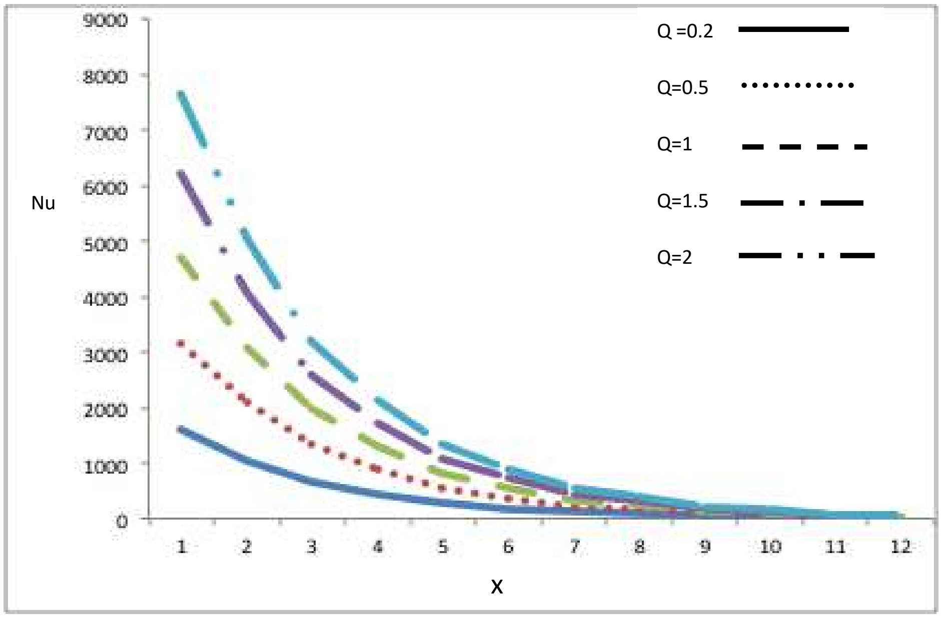

Fig. (12)IllustratesNusselt number Nu for different heat generation parameter Q, pr =0.7, Da^{-1} =0.1, a = 0.2, M =0.2 and γ = 2.

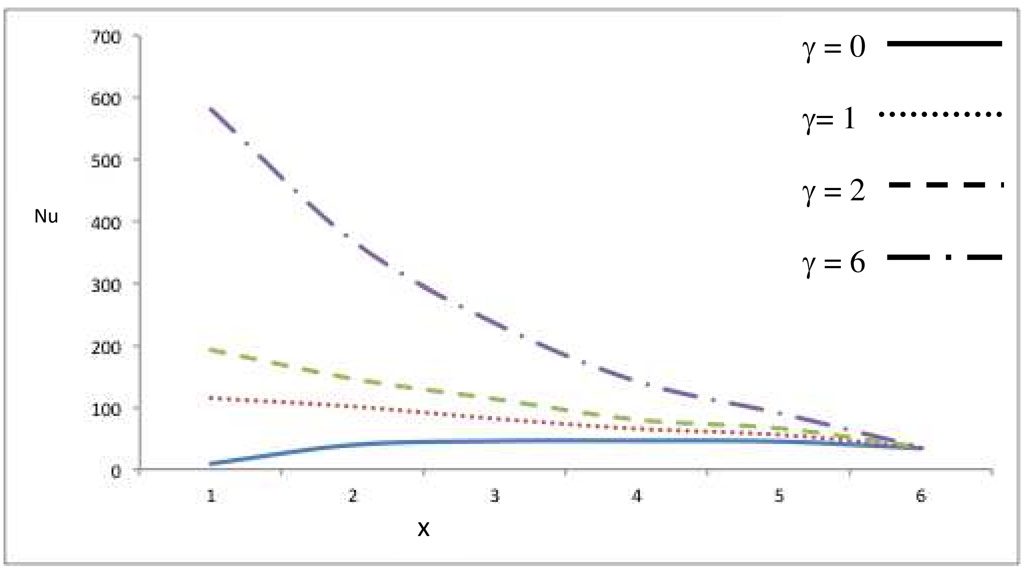

Fig. (13)Illustrates Nusselt number Nu for different thermal conductivity variation parameterγ, pr =0.7, Da^{-1} =0.1, a = 0.2, M =0.2 and Q = 0.1.

Conclusion

In this paper, we study the effect of natural convection flow along an upward wavy surface in permeable media with variable thermal conductivity. We have found the governing boundary layer equations and altered them into a non-dimensional form using a suitable set of dimensionless variables. Afterwards, we have acquired the nonlinear system of partial differential equations which can be solved numerically by finite difference, known as the fully implicit method. We achieve some results of velocity u, temperature Q, Nusselt number Nu for different values of magnetic field parameter M, prandtle number Pr, inverse Darcy number Da^{-1}, amplitude of wavelength a, heat generation parameter Q and in thermal conductivity variation parameter g.

Declarations

Ethics approval: Not applicable.

Consent for publication: Not applicable.

Availability of Data and Materials: Not applicable

Competing interests: The author declares that they have no competing interests

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Author’s contribution: RA wrote, analyzed and solved the equations numerically and analytically of this manuscript.

Acknowledgements: Not applicable

Author’s information: **Rasha Adel** graduated from Helwan University, Cairo, Egypt, in 1999. She received the M.Sc. degree in mathematics from Helwan University, Cairo, Egypt, in 2007, and the Ph.D. degree in applied mathematics from Helwan University, Cairo, Egypt, in 2012. Her research interests are focused on fluid mathematics.

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