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Research Article

Keywords:

Posted Date: February 25th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1382481/v1>

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Noise self-cancelling metrology using twisted light

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Noise fundamentally limits the ultimate optical precision. Here, we unveil a real-time noise self-cancelling metrology by exploiting the unique spiral phase of light possessing orbital angular momentum (OAM). With a compact twisted interferometer and a novel measurement scheme exploiting an ensemble of N phase-orthogonal single-pixel intensity pairs on petals of the daisy flower-like interference pattern, we cancel various noises (intensity noise, interferometer noise, and detector noise) by three orders of magnitude, enabling detection of a weak input signal, in excellent agreement with our theoretical analysis. Remarkably, our approach allows detection of an aperiodic, non-repetitive dynamic event masked by $100\times$ larger noise background faithfully with picoscale resolution, which is otherwise undetectable. Because no upper limit on the quantum numbers of OAM state exists, in principle, the noise cancellation scales up indefinitely. These results open promising ways to enhance precision metrology, secure communication, and develop analogous ideas for twisted acoustic waves, electron beams, and matter waves.

Introduction. Noise is a key bottleneck that limits precision of optical systems^{1,2}. One of the most challenging task in precision metrology has always been to isolate a non-repetitive single aperiodic input signal of arbitrary shape deeply submerged in noise in real-time^{3,4}. Conventional approaches minimize the detrimental effect of noise by passive or active isolation of system, by averaging repeated measurements, using lock-in detection for periodic signals, or approaches designed to exploit prior information about the nature of signal or noise in a setup⁵⁻⁸. Besides, in Gaussian beam laser interferometers (Mach-Zehnder/Michelson type), intensity noise has been reduced by subtracting two quadrature maintained outputs⁹, exploiting squeezed states of light¹⁰, and by operating optical micro-cavities near exceptional points^{11,12}. However, a Gaussian beam interferometer does not allow a large number of orthogonal and independent outputs thereby inherently lacking noise cancelling capability.

Although, OAM of light has played a vital role over the past three decades leading to a wide variety of applications in communication, micro-manipulation and imaging¹³⁻¹⁷. Spiral interferometry has been advantageous to distinguish elevations from depression in an image by means of spiraling interference pattern^{18,19}. The Michelson-type interferometers with OAM light has been used to measure direction and magnitude of tiny displacements via rotating daisy-flower like interference pattern and to quantify the OAM states of light itself^{20,21}. However, the potential of twisted light for noise self-cancelling metrology remains largely untapped.

Here, we demonstrate a comprehensive approach to real-time noise self-cancelling metrology by exploiting the unique spiral phase structure of twisted light beam. We devise a single cylindrical lens based compact interferometer and demonstrate a novel detection scheme by constructing N single-pixel intensity differential pairs having orthogonal azimuthal phases on the petals of the daisy-flower like interference pattern. We demonstrate a real-time self-cancellation of intensity noise, interferometer noise and detector noise by about three or-

ders of magnitude while selectively resolving the weak arbitrary-shaped input signal down to picometer scale. As a promising demonstration, we faithfully isolate a single non-repetitive nano-mechanical event inside a twisted cavity, two-orders of magnitude below the noise-floor of the setup, which would be otherwise undetectable. Because there is no known upper limit on the quantum numbers of twisted light, in principle, the noise-cancellation capability of the present approach scales indefinitely.

Results

Compact twisted interferometer. Fig. 1(a) shows our compact twisted interferometer. It is based on a novel multifunctional Cylindrical-interference-lens (CiLens) whose flat surface was partially silver coated (see Methods). The CiLens is a four-in-one lens that integrates four independent functions of a beam-splitter, reference mirror, light collector, and helicity inverter²². Compared to the twisted Michelson-type interferometers where these functions are generally obtained with multiple optics²³, CiLens renders our interferometer the most compact possible. We obtained a stable daisy-flower like interference pattern by collinearly overlapping a reference beam of topological charge $+\ell$ (obtained from the flat surface of the CiLens) on to a retro-reflected (measurement) beam with the opposite charge $-\ell$ collected through the CiLens. For a given OAM light, the daisy flower-like interference pattern consist of $2|\ell|$ petals²⁴ with $p + 1$ radial lobes on each petal.

The entire interference pattern systematically rotated clockwise or counterclockwise in a screw-like fashion when the mirror M was displaced (Δh) along positive or negative z -direction²⁵, respectively (Figs. S1-2). The overall noise budget of the twisted interferometer (Fig. 1(a)) consists of three main sources: intensity noise in the OAM beam $\xi(t)$, interferometer noise (thermal, acoustic, air-current fluctuations) $\eta(t)$, and detector noise $\gamma(t)$. The arbitrary signal $S(t)$ to be detected is fed to the piezo-controller holding the mirror M or (optionally) via photo-thermal excitation of interferometer induced by another Gaussian (pump) laser beam.

Working principle. We first elaborate the working

FIG. 1: **Schematics of compact twisted interferometer and detection scheme.** (a) CiLens based twisted interferometer producing daisy-flower like interference pattern with noise budget. Input signal $S(t)$ is given to the piezopositioner holding M or optionally as laser-induced intracavity photo-deformation signal. Input OAM beam was generated by the spatial light modulator. (b) Detection scheme leading to noise-cancellation. Creation of N orthogonal differential intensity pairs with detectors $D_1, D_2, \dots$ on petals of interference pattern followed by ensemble averaging producing real-time noise cancelled output.

principle of the noise self-cancellation scheme (Fig. 1(b), 2(b)). The electric field of the reference OAM beam of topological charge ℓ (obtained from the flat surface of the CiLens) is given as,

$$E_{p,+ \ell}(r, \phi, z, t) = E_0 (1 + \xi(t)) R_{p,\ell}(r, z) e^{i\ell\phi} \quad (1)$$

where $R_{p,\ell}(r, z)$ includes the z and radial dependence including the Gouy phase term for given ℓ and p state as in^{26,27}, E_0 is the amplitude of the field and $\xi(t)$ is a white Gaussian electric field noise. The noise has zero-mean $\langle \xi(t) \rangle = 0$ and delta-correlated in time $\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t')$ with noise intensity D .

The measurement beam collected through the the CiLens, after reflecting from mirror M, has $-\ell$ topological charge. Besides, it travels extra distance of $2h(t)$ in the cavity. It can be written as:

$$E_{p,- \ell}(r, \phi, z, t) = E_0 (1 + \xi(t)) R_{p,\ell}(r, z) e^{i(-\ell\phi - 2kh(t))} \quad (2)$$

where $k = 2\pi/\lambda$ is the wave vector. On the fixed detector plane ($z = z_0$), these two conjugate beams interfere to produce a noisy daisy flower-like intensity distribution, $I(r, \phi, t) = |E_{p,\ell}(r, \phi, t) + E_{p,-\ell}(r, \phi, t)|^2$, which to the first order (in noise) is given by,

FIG. 2: **Intensity noise-cancellation using orthogonal differential pixel pair.** (a) Simulated (noise-free) and experimental daisy-flower pattern for $\ell = 5$, $p = 0$. (b) Simulated and experimental single-pixel intensities $I_{D1}(t)$ and $I_{D2}(t)$ with intensity noise. Lower plots show corresponding differential intensity showing linear noise-cancelling regions (green vertical bars). Inset: zoom of experimental data showing residual noise.

$$I(r, \phi, t) \approx I_{p,\ell}^0(r) (1 + 2|\xi(t)|) \times \cos^2(\ell\phi + kh(t)) + \gamma(t) \quad (3)$$

where $I_{p,\ell}^0(r) = 4E_0^2 |R_{p,\ell}(r)|^2$ is the radial intensity distribution which depends on ℓ and p . Besides, the multiplicative intensity noise $\xi(t)$, the argument of the $\cos^2$ term also contains a second independent white Gaussian noise (interferometer noise) in $h(t) = h_0 + \Delta h(t)$, with $\Delta h(t) = S(t) + \eta(t)$, with fixed $h_0 \sim f^{28,29}$. $S(t)$ is a weak input signal of arbitrary shape and $\eta(t)$ denotes a second independent white Gaussian noise with zero mean and strength D_2 . When the interference pattern is detected, for example with a CCD camera, a third independent and small noise $\gamma(t)$ is present in each pixel of the image which can be further reduced using cooled detectors. In the following, we describe how the spiral wave front allows real-time noise-cancelling metrology leading to enhanced signal detection.

To eliminate the intensity noise we exploit azimuthal phase orthogonality in the twisted wave front to create anti-correlated intensity pairs. For this, we choose a pair of single-pixels located at suitable angular positions θ_1 and θ_2 and radially centred ($r = r_0$) on each

petal (at $\phi_i = \pm m\pi/(4|\ell|)$, $m = 1, 3, \dots$). The corresponding intensities $I_{D1}(\theta_1, t)$ and $I_{D2}(\theta_2, t)$ are linearly anti-correlated when Δh is scanned, because of their orthogonal azimuthal phase difference ($|\theta_1 - \theta_2| = \pi/2$). Incidentally, each anti-correlated pixel pairs is symmetrically located on either side of the maximum petal intensity. These intensities, $I_{D1}(\theta_1, t)$ and $I_{D2}(\theta_2, t)$, exhibit a near perfect anti-correlation and these can be easily obtained from Eq. 3 as,

$$I_{D1,D2}(\theta_{1,2}, t) = I_{p,\ell}^0(r_0)(1 + 2|\xi(t)|) \times \cos^2(\theta_{1,2} + k\Delta h(t)) + \gamma(t) \quad (4)$$

Expanding the above equation around its linear regimes ($\theta_{1,2} \simeq \pm\pi/4$), and considering only the first order terms in noise (small noise), we obtain differential intensity, $\Delta I_1 = I_{D2}(t) - I_{D1}(t) \simeq 2kI^0\Delta h(t) + 2\gamma(t)$. Note that the differential intensity is free from laser intensity noise while doubling the input signal.

To eliminate remaining noise, we exploit the topological charge, i.e., how many twists the phase of light undergoes in one wavelength. Due to the higher charge one can create a large number of N orthogonal pairs, $\delta I_n(t)$ on the interfere pattern. For example, taking a modest one pair per lobe (Fig. 4(b)), an ensemble average over $N = 2(p+1)\ell$ differential pairs progressively cancel other noises in real time, $I(t) = \sum_{n=1}^N \Delta I_n(t)$ where the summation runs over $n = 1, 2, 3, \dots, N$. This ensemble averaging over pixel-pairs leads to $I(t) = 2kI^0S(t) + 2kI^0\langle\eta(t)\rangle_N + 2\langle\gamma(t)\rangle_N$, where the weak input signal $S(t)$ is strengthened but the last two noisy terms vanish as $1/\sqrt{N}$. In effect, a single twisted interferometer behaves as if there were N parallel interferometers functioning cooperatively to cancel noises while enhancing the weak input signal. The above analysis is also valid when noises additionally contain specific undesired narrow-band frequencies.

Experimental validation. To experimentally validate the noise cancellation metrology, we begin with a stable (noise-free) interference pattern, say for $\ell = 5$ and $p = 0$ generated with our setup (see Fig. 2(a)). The interference pattern has 10 petals and the experimental intensity perfectly matched the simulated one (see Fig. S3 for other ℓ values). The intensity noise in the input beam is induced by the SLM which in addition to broadband noise, also contains unwanted narrow-band intensity fluctuations due to its intrinsic refresh rate (see lower inset in Fig. 3(b)). Additionally, a white Gaussian laser intensity noise of was also introduced by modulating laser intensity using noise generator (100 kHz bandwidth, ~ 80 nm noise-floor).

The multiplicative character of intensity noise is clearly evident in the anti-correlated pair, $I_{D1}(t)$ and $I_{D2}(t)$, measured using single-pixel photodiodes (active area 0.8 mm^2 much smaller than the petal width pixel width $\sim 20 \text{ mm}$ on the detection plane) when $\Delta h(t)$ was linearly scanned, as shown in Fig. 2(b). Both the experimental data and simulations show excellent agreement

FIG. 3: Picometer scale calibration of noise-cancelling twisted interferometer. (a) Real-time detection of noise cancelled 2 nm displacement step with $sub-50 \text{ pm}$ resolution (inset). Single-pixel noise level of about 70 nm is reduced to about 1 nm. (b) Calibration curve showing the measured mirror displacement (red circle) with the known input mirror displacement (blue square), the solid blue line is the linear fit. Error bars indicate experimental noise floor. The lower inset: typical power spectral density of interferometer output without (blue) and with noise-cancellation (orange).

over $\cos^2$ angular intensity distribution of the multiplicative noise, with intensity nulls coinciding with the dark fringe of the observed interference pattern. Remarkably, their differential intensity, $\Delta I_1(t)$ showed noise-free regime (highlighted regions), where the intensity response of the twisted interferometer to mirror displacement was not only linear but also the maximum. Notably, at the middle of the noise-free region, the intensity noise almost vanishes. Furthermore, ensemble averaging over N differential pairs on the interference pattern (Fig. 4(b)) leads to noise cancelling output of the interferometer.

Comparison of our parallel detection scheme with the usual one-pixel detection of intensity under identical conditions is shown in Fig. 3 where we aim to detect a displacement step of $\Delta h = 2 \text{ nm}$ amplitude. While the one-point intensity (I_{D1}) shows a large laser-noise background of $\sim 70 \text{ nm}$ that totally masks the 2 nm signal. Remarkably, our scheme with single differential pair reduces noise-floor to about 5 nm. The noise is further reduced by $1/\sqrt{N}$ below 1 nm for $N \sim 30$ ensemble (Fig. 3(a)). The histograms of the noise and the zoom

FIG. 4: **Detection of a non-repetitive aperiodic input event masked by noise.** (a) Schematic of generation of weak input aperiodic photo-event in an intracavity sample (SS). (b) N pixel pair ensemble on the interference pattern ($\ell = 3, p = 2$). (c) Comparison of single-pixel detection (blue curve), single differential detection (orange curve) and ensemble averaged detection (red curve). Inset: zoom showing sub-50 pm resolution.

near the nanoscale step shows a few tens of pm resolution which is about $1000\times$ below the original noise floor (~ 70 nm).

We performed a systematic calibration of our noise-cancelling twisted interferometer by giving an input signal of known amplitude and recovering it with our detection scheme. Clearly, while the single-pixel detection was rather limited, the differential detection ΔI_1 could measure down to ~ 9 nm, which was further extended to picoscale ~ 150 pm resolution (Fig. 3(b)). It is worth mentioning that pico-scale displacements are much below the repeatable displacements provided by the used piezo-positioner (1 nm), nevertheless the presented OAM metrology allows isolating such feeble input signals in real-time.

Isolating aperiodic event 100x below noise.

Fig. 4 shows one of the most powerful capability of our approach that enables reliable real-time detection of single, non-repetitive, and arbitrary-shaped input signal, deeply submerged into the noise-floor of the interferometer. To demonstrate this, we photo-excite an intracavity solid-sample in order to create a nanometric aperiodic three-impulse (100 ms each Fig. 4(a)) event using another (pump) laser. This effectively perturbs the length ($h(t)$) of the twisted cavity with amplitude ~ 2 nm, roughly two orders of magnitude below the over-

all noise-floor of the setup, and was hardly visible in I_{D_1} (or I_{D_2}) as shown by blue curve of Fig. 4(c). As shown in Fig. 4(c) lower curve, our twisted interferometer resolves the reversible photo-thermal dynamics with a precision of 40 pm, which would be otherwise undetectable. The high sensitivity, simplicity and scalability of the twisted scheme makes it attractive for quasi-noise-free measurements on a liquid, solid, gas, or plasma samples³⁰.

Discussion. Because no upper limit on the maximum topological charge (ℓ) and the radial quantum number (p) of the OAM state of light is known, our capability of noise-cancellation, in principle, scales up unlimited. It is worth mentioning that the twisted light with $\ell = 288$ has been directly produced from the laser³¹. Recently, using a phase-step mirror OAM states up to $\ell = 10,000$ has been generated³². As shown in Fig. 4(b), twisted interference pattern consist of $2|\ell|$ petals and $(p+1)$ lobes on each petal. A discrete sampling approach using only one phase-orthogonal pair per lobe allows at least $N = 2(p+1)\ell$ differential pairs, leading to scalable reduction of noises as $1/\sqrt{N}$. However, in the quasi-continuous pair formation approach, N can be much larger, when every pixels located on the phase-orthogonal radial-lines is used, practically limited by pixel size. For example, taking $\ell = 10^4$ and $p = 100$ and the conservative discrete approach, a massive improvement in the signal-to-noise ratio of ~ 1000 fold is possible. Furthermore, exploiting vector vortex beam could open intriguing ways of noise-cancellation scheme.

Additionally, it is worth mentioning that by further improving hardware, such as using frequency-stable twisted beam, operating the compact interferometer at low-temperatures, and using cryogenic detectors the detection capability can be further boosted. Moreover, our OAM based approach is inclusive in the sense that our conventional ideas of active and/or passive noise isolation can be easily incorporated.

In conclusion, we have successfully introduced a new capability of twisted interferometer exploiting alternative degrees of freedom intrinsic to OAM to enables a comprehensive real-time auto-cancellation of noise allowing isolation of weak signal deeply submerged in noise. Our single cylindrical lens based setup is compact and produces a stable daisy flower-like interference pattern. We introduce a multi-pixel parallel detection scheme exploiting the spiral wave front to cancel-noise while self-calibrating the twisted interferometer with picometer resolution. We show that our scheme cancels intensity noise, interferometer noise and detector noise and allows detection of isolated aperiodic signal many orders of magnitude below the noise floor.

We envision that this work will open routes to novel applications of twisted interferometry in scalable noise-cancelling metrology, OAM based noise-free signal processing schemes and novel communication devices. This work provides a generic platform to exploit spiral wave front in acoustic waves, twisted electrons beams and matter waves interferometry for intriguing applications.

Methods. *Experimental setup:* The cylindrical interference lens was fabricated using a plano-convex cylindrical lens of focal length $f = 50$ mm, dimension 30×40 mm $\times$ mm, whose plane surface was Ag-coated with a reflection to transmission intensity ratio of 40 : 60. The twisted light of a given state was produced by SLM (Pluto-2, Holoeye) which converted an incoming Gaussian beam from a low-power He-Ne laser, linearly polarised (10 mW, $\lambda = 632$ nm, and $1/e^2$ full waist $\simeq 1.0$ mm). The single-pixel intensities were detected with a pair of photodiode-iris system of active area (0.8 mm²) while the width of each petal was around 20 mm. Additionally, the interference pattern was video recorded and analyzed frame-by-frame for large N measurements.

The mirror M was mounted on a 3-axis nano-positioner piezo stage (Thorlabs) with 20 μ m travel range. An arbitrary function generator (Keysight 33500B) was used to drive the piezo stage with a minimum possible signal amplitude of 1 mV. The function generator was also used to add random noise in the laser intensity as well. The random aperiodic events in the cavity were created by exciting a semi-transparent poly-methyl-methacrylate slide of thickness $t = 3$ mm and 80% intensity transmission. The pump laser excited the sample with 100 ms, 5 mW. The pump beam was a CW diode-pumped solid-state laser

$\lambda = 532$ nm and linearly polarized. The exposure time was controlled using an electro-mechanical shutter (Thorlabs). The experiment was performed on a floated optical table (Thorlabs) and adequate arrangements were made to protect the setup from environment noise. Numerical simulations were performed in Python.

Competing Interest statement The authors declare no competing interests.

Data Availability The data generated and/or analysed during the current study are available from the authors upon reasonable request.

Author contribution: KPS designed and guided the research. KC performed initial calibration experiments. PM advanced all the experiments including the single-event detection. PM developed theoretical model and performed all the simulations. All authors discussed results and wrote the manuscript.

Acknowledgments

KPS acknowledges Max-Planck Society for financial support through Partner's group. We acknowledge discussions with M. S. Sidhu, A. Mandal, S. Goyal and A. Singh.

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